A question that is similar to HW5Q7.

019

Note Title 2/11/2011

Q: $X$ is an exponential R.V w. para. $\lambda$

Show that for any $a, b > 0$.

$$P(X > a+b \mid X > a) = P(X > b)$$

Ans:

* It is called the $\boxed{\phantom{memoryless}}$.

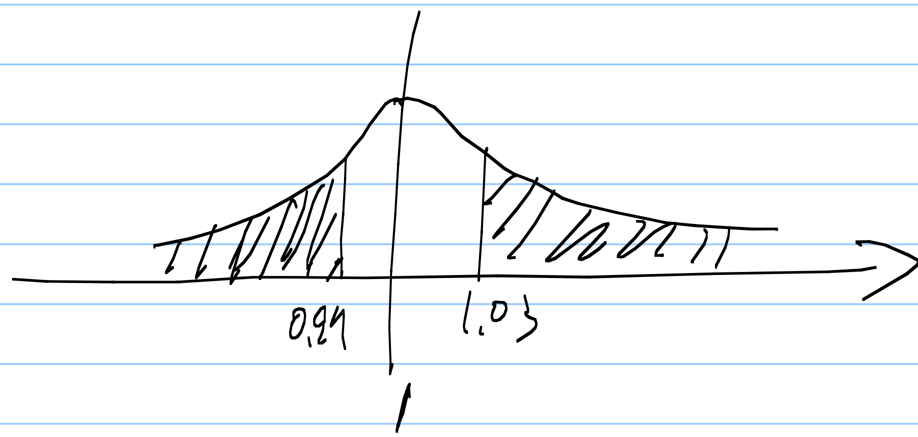
Why? Recall that $X$ generally models the time you wait for the first customer.

Given that I have waited for $a$ secs, the prob that I have to wait for additional $b$ secs does not depend on how large/small $a$ is. There is no memory to how long I have waited.

3.

Ex: The reading of the GPS device $X$ is a Gaussian R.V with $\mu = 1$ $\sigma = 0.01$

(where $\mu = 1$ is the actual location of the device




Q: What is the prob the GPS reading is 3% off the actual location.

Ans:

Other important conti R.Vs. include Laplacian, and Rayleigh R.Vs.
See p.165 for their $S$ and $f_X$ description.

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Discrete & Conti R.Vs are similar ex: $E(X+X^2)=E(X)+E(X^2)$

$$Var(X) = E((X-m)^2) = E(X^2) - m^2$$

How to use a unifying description to describe both types of R.V?

*

Discrete R.V. $\uparrow$ pmf $p_k$

Conti R.V $\uparrow$ pdf $f_X(x)$

Definition

$\boxed{I}$

* Discrete R.V. from $P_k$ to $F_X(x)$

Ex: $X$ is a bernoulli R.V. with $p=\frac{1}{3}$

Find its cdf $F_X(x)$, Plot $F_X(x)$

Ans:

Ex: $X$ is a binomial R.V with $n=2$, $p=\frac{1}{3}$

Find the cdf $F_X(x)$, Plot it.

Ans:

II From cdf $F_X(x)$ back to $p_k$.

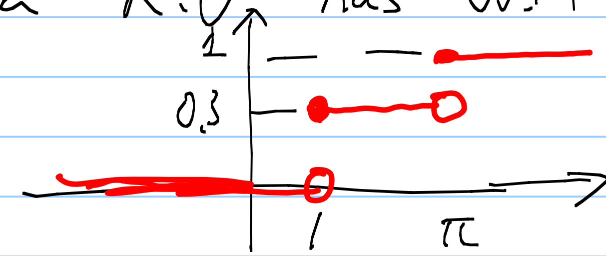
Ans: For each $k$ value, $p_k$ is the jump from to (or sometimes we write it as

Ex: I can say a R.V has W.A

① $S = \{1, \pi\}$

$P(X=1) = 0.3$

$P(X=\pi) = 0.7$




Or ② $F_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.3 & \text{if } 1 \le x < \pi \\ 0.3+0.7 & \text{if } \pi \le x \end{cases}$

Both ① and ② describe the same W.A.

As you can see, the pmf ① indeed corresponds to the jump in the cdf ②

* Summary:

① Given the pmf $P_k$, we can find the CDF $F_X(x)$ by counting.

② Given the CDF $F_X(x)$, we can find the pmf by location & the magnitude of the jumps.

Conti R.Vs:

$\boxed{\text{III}}$ From pdf $f_X(x)$ to cdf $F_X(x)$

Ex: X is a uniform R.V over (1, 4)

What is the cdf of X.

Ans: