Note Title 1/26/2011

* Basic prob concepts: W.A., Counting, Conditional prob, independence.

* We are now ready for some intermediate-level discussion, for which, we focus only on "Random Variables": Random experiments that have output being a <u>number</u>

* R.V is very useful as ① many experiments indeed output numbers. Ex: temperature, voltage,

② Moreover, in a digital world. more and more things are converted to numbers.

Ex: Black $\longrightarrow$ 0
White $\longrightarrow$ 255
light gray $\longrightarrow$ 200
dark gray $\longrightarrow$ 50

③ Easy manipulation. Suppose X, Y are Random variables, we can define a new R.V. $Z = X^2 + Y^2$ and ask question like $P(Z<1)$

④ We can take weighted average

Ex: Flip a fair coin.

If the outcome is $\{H, T\}$. then the weighted average is meaningless.

(average of head & tail)

However, if we convert it to a R.V. $S=\{0, 1\}$ with weight assignment $\frac{1}{2}$ for $X=0$, $\frac{1}{2}$ for $X=1$. The weighted average becomes

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For the following, we first consider "discrete R.V" (such that $S=$ integers)

* Discrete R.V:

① Sample space is

② The weight assignment is

Q: how to plot the pmf?

③ The " " of a discrete R.V. is the _____

Ex: If X is a fair dice,

④ Expectation is simply weighted average.

Expectation of X^2 is

Expectation of e^X is

Ex: X is a fair die

$E(e^{-X}) =$

$E(X^3) =$

Ex: Throw an unfair die with weight assignment

$$P_1 = \frac{2}{7}, \quad P_2 = \frac{1}{7} = P_3 = P_4 = P_5 = P_6$$

The casino gives you $f(X)$ dollars depending on the outcome of the die.

$$f(X) = \begin{cases} 1 & \text{if } 1 \le X \le 3 \\ X^2 & \text{if } 4 \le X \le 6 \end{cases}$$

Q: What is the <u>expected</u> return?

Ans:

Important properties of expectation:

①

Namely: The (weighted) avg of a fair dice is $\Rightarrow$ The weighted average of $E(2X)$

Note Title 2/2/2011

(2)

(3)

(4)

Example $p_k = \begin{cases} 0.5^k & \text{for all } k \geq 1 \\ 0 & \text{otherwise.} \end{cases}$

$E(3^X) =$

(5)

Ex: X is a fair dice.

What is Var(X)

Ans: Step 1:

Step 2:

An alternative formula of