

Q82

Let $Y = 2X + 3$. Suppose we know the pdf of X . Find the pdf of Y .

Ans: Find the cdf of Y first.

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(2X + 3 \leq y) \\ &= P\left(X \leq \frac{y-3}{2}\right) \\ &= \int_{-\infty}^{\frac{y-3}{2}} f_X(x) dx \end{aligned}$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) \quad \downarrow \text{ by the chain rule}$$

HW6Q1

$$\begin{aligned} &= f_X\left(\frac{y-3}{2}\right) \times \frac{d}{dy}\left(\frac{y-3}{2}\right) \\ &= \frac{1}{2} f_X\left(\frac{y-3}{2}\right) \quad \# \end{aligned}$$

Similar to Q83 Prob 4.26

Ex: X is an exponential R.V with

$$\lambda = \frac{3}{2}$$

$$Y = \max(X, 2)$$

Find the (generalized) pdf of Y ,

$$E(Y) = ?$$

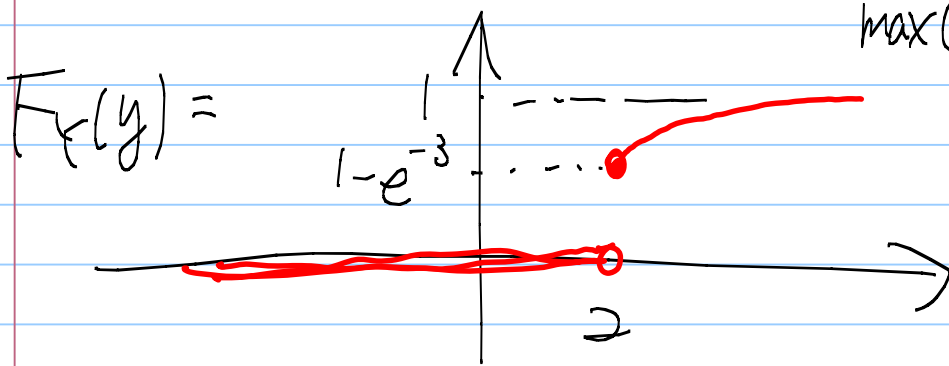
Ans: Method 1: (Introduced when discussing the cdf)

$$F_Y(y) = P(Y \leq y) = P(\max(X, 2) \leq y)$$

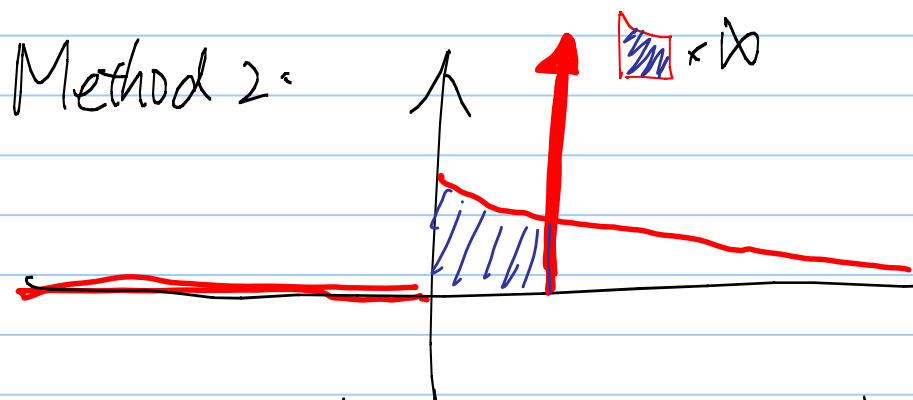
$$= \begin{cases} 0 & \text{if } y < 2 \end{cases}$$

$$\begin{cases} \int_0^y \frac{3}{2} e^{-\frac{3}{2}x} dx & \text{if } 2 \leq y, \text{ then} \\ = 1 - e^{-\frac{3}{2}y} & \text{as long as} \end{cases}$$

$X \leq y$, we have
 $\max(X, 2) \leq y$.



$$f_Y(y) = (1 - e^{-3}) \delta(y - 2) + \begin{cases} 0 & \text{if } y < 2 \\ \frac{3}{2} e^{-\frac{3}{2}y} & \text{if } 2 \leq y \end{cases}$$



any time $X < 2 \rightarrow Y = 2$.

push the weight for $X < 2$ to

an impulse at $Y = 2$.