

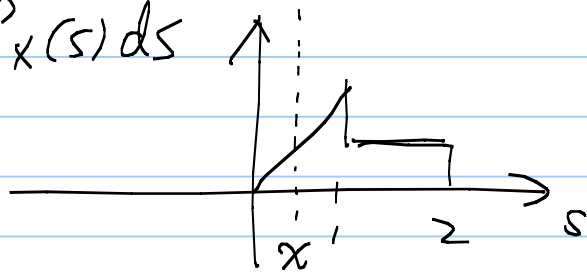
Q41:

Note Title

$$f_x(x) = \begin{cases} x & \text{if } x \in [0, 1] \\ \frac{1}{2} & \text{if } x \in (1, 2] \\ 0 & \text{otherwise.} \end{cases}$$

1/31/2011

$$F_x(x) = \int_{-\infty}^x f_x(s) ds$$



Q1: Find $F_x(x)$ if $x < 0$

$$\begin{aligned} \text{Ans: } F_x(x) &= \int_{-\infty}^x 0 ds \quad \text{if } x < 0 \\ &= 0. \end{aligned}$$

Q2: Find $F_x(x)$ if $x \in [0, 1]$

$$\text{Ans: } F_x(x) = \int_0^x s ds = \frac{1}{2} x^2$$

Q3: Find $F_x(x)$ if $x \in (1, 2]$

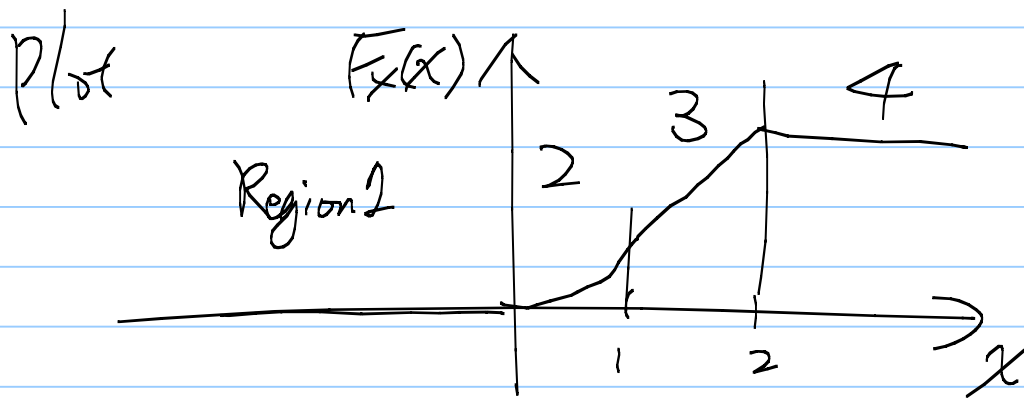
$$\begin{aligned} \text{Ans: } F_x(x) &= \int_0^1 s ds + \int_1^x 0.5 ds \\ &= \frac{1}{2} + 0.5(x-1) \end{aligned}$$

Q4: Find $F_x(x)$ if $x > 2$

$$\begin{aligned} \text{Ans: } F_x(x) &= \int_0^1 s ds + \int_1^2 0.5 ds \\ &= 0.5 + 0.5 = 1 \end{aligned}$$

Q5: Complete expression of $F_x(x)$?

Ans:
$$F_x(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2}x^2 & \text{if } 0 \leq x < 1 \\ \frac{1}{2}x & \text{if } 1 \leq x < 2 \\ 1 & \text{if } 2 \leq x \end{cases}$$



Q47:

Prob 2,85

two events

2/7/2011

We know A, B are independent, prove that A, B^c are also independent

Ans: A, B are indep

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

We need to prove

$$P(A \cap B^c) = P(A) \cdot P(B^c)$$

To that end, we notice

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$= P(A) - P(A)P(B)$$

$$= P(A) \cdot (1 - P(B))$$

$$= P(A) \cdot (P(B^c)) \quad \#$$

Q49: Prob 3, 13

X is a discrete R.V w. pmf $p_k = \frac{C}{k^2}$
for $k=1, 2, 3, \dots$

Q1: Find the value of C numerically.

Ans: $\sum_{k=1}^{\infty} p_k = 1 \Rightarrow C \cdot \sum_{k=1}^{\infty} \frac{1}{k^2} = 1.$

Since we do not know how to compute $\sum_{k=1}^{\infty} \frac{1}{k^2}$, we assume $\sum_{k=1}^{\infty} \frac{1}{k^2} = \sum_{k=1}^{100000} \frac{1}{k^2}$

$$C \times \sum_{k=1}^{100000} \frac{1}{k^2} = C \times 1.6449 = 1$$

$$C = \frac{1}{1.6449} = 0.6079$$

$$\Rightarrow p_k = \frac{0.6079}{k^2} \quad \text{for } k=1, 2, \dots$$

Q2: $P(X > 4) = ?$

$$\text{Ans} = \sum_{k=5}^{\infty} \frac{0.6079}{k^2} = 1 - \sum_{k=1}^4 \frac{0.6079}{k^2}$$

$$= 0.1346$$

Q3: $P(6 \leq X \leq 8) = ?$

$$\text{Ans: } \sum_{k=6}^8 \frac{0.6079}{k^2} = 3.879\%$$

Q51: Prob. 3.20

Two dice are tossed & let X be the difference of the dots facing up

Find the pmf of X .

Ans:

	1	2	3	4	5	6
1	$\frac{1}{36}$					
2		$\frac{1}{36}$				
3						
4						
5						
6						

$$P(X=0) = \frac{1}{36} \times 6 = \frac{1}{6} = p_0$$

$$P(X=1) = \frac{1}{36} \times 5 = p_1$$

$$P(X=-1) = \frac{1}{36} \times 5 = p_{-1}$$

$$P(X=3) = \frac{3}{36} =$$

$$P(X=4) = \frac{2}{36}$$

$$P(X=5) = \frac{1}{36}$$

