Note Title 1/24/2011

Q29

$$f_X(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = 0.5 e^{-|y|}$$

Q: $\int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} e^{-s(x+y)} f_X(x) f_Y(y)\, dx\, dy$

Ans: $\int_{y=-\infty}^{\infty} \left(\int_{x=-\infty}^{\infty} e^{-sx} f_X(x)\, dx \right) e^{-sy} f_Y(y)\, dy$

$$= \underbrace{\left(\int_{x=-\infty}^{\infty} e^{-sx} f_X(x)\, dx \right)}_{\text{term 1}} \underbrace{\left(\int_{y=-\infty}^{\infty} e^{-sy} f_Y(y)\, dy \right)}_{\text{term 2}}$$

term 1: $\int_0^{\infty} e^{-(1+s)x}\, dx$

$$= \frac{1}{1+s}$$

term 2: $\int_{y=-\infty}^{0} 0.5 e^{(1-s)y}\, dy + \int_0^{\infty} 0.5 e^{(-1-s)y}\, dy$

$$= \frac{0.5}{1-s} + \frac{0.5}{1+s} = \frac{1}{1-s^2}$$

Ans: $\frac{1}{1+s} \times \frac{1}{1-s^2}$ let $s = 0$. 1 #

Q33 Problem 2.69

Note Title 1/28/2011

A number x is selected at random in $[-1, 2]$ Let the event $A = \{x < 0\}$

$B = \{|x - 0.5| < 0.5\}$ $C = \{x > 0.75\}$

Find $P(A|B)$ $P(B|C)$ $P(A|C^c)$ $P(B|C^c)$

Ans: $S = [-1, 2]$ is continuous,

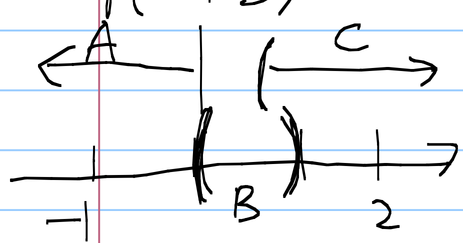
$\Rightarrow$ we use pdf.

$$f_X(x) = \begin{cases} c & \text{if } -1 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

$$\because \int_{-\infty}^{\infty} f_X(x)\,dx = 1 = \int_{-1}^{2} c\,dx \Rightarrow c = \frac{1}{3}$$

$$\therefore \quad f_X(x) = \begin{cases} \frac{1}{3} & \text{if } -1 \le x \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

$$P(A|B) = 0 = \frac{P(A \cap B)}{P(B)} = \frac{0}{\int_0^1 \frac{1}{3}\,dx}$$



B: x is from $0 < x < 1$

$A \cap B : \phi$

$$P(B|C) = \frac{0.25}{1.25} = \frac{1}{5}$$

$$= \frac{P(B \cap C)}{P(C)} = \frac{0.25 \times \frac{1}{3}}{1.25 \times \frac{1}{3}}$$

$C: \quad x > 0.75 \qquad P(C) = \int_{0.75}^{2} \frac{1}{3} dx$

$$= 1.25 \times \frac{1}{3}$$

$B \cap C: \; 0.75 < x < 1 \qquad P(B \cap C) = \int_{0.75}^{1} \frac{1}{3} dx$

$$= 0.25 \times \frac{1}{3}$$

$$P(A|C^c) = \frac{1}{1.75} = \frac{P(A \cap C^c)}{P(C^c)} = \frac{1 \times \frac{1}{3}}{1.75 \times \frac{1}{3}}$$

$C^c = \quad x \leq 0.75 \qquad P(C^c) = \int_{-1}^{0.75} \frac{1}{3} dx$

$A \cap C^c = \; x < 0 \qquad P(A \cap C^c) = \int_{-1}^{0} \frac{1}{3} dx$

$$P(B|C^c) = \frac{0.75}{1.75} = \frac{P(B \cap C^c)}{P(C^c)} = \frac{0.75 \times \frac{1}{3}}{1.75 \times \frac{1}{3}}$$

$B \cap C^c = \; 0 < x < 0.75 \qquad P(B \cap C^c)$

$$= \int_{0}^{0.75} \frac{1}{3} dx$$

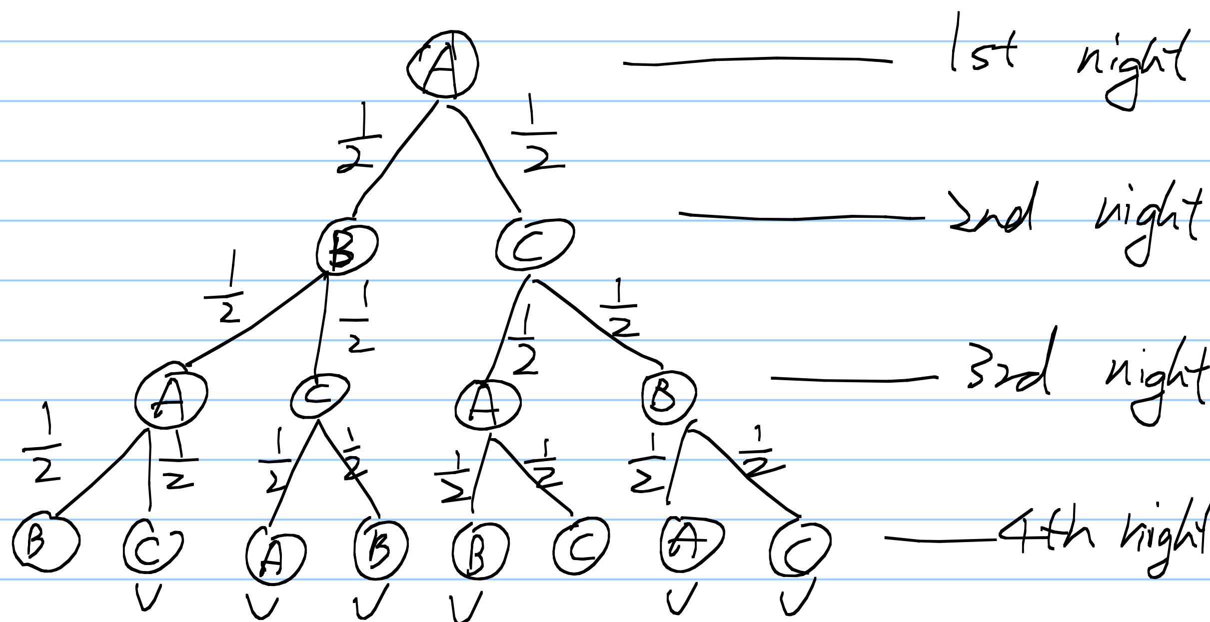
$$= \frac{1}{3} \times 0.75$$

Q37 A traveling salesperson problem

3 cities A, B, C,
4 nights, first night in A.
the next night he/she moves randomly to one of the other two cities.

Q: P(all three cities are visited)?

Ans: Tree method



Q: Prob that all cities have been visited
= P(ABAC, ABCA, ABCB, ACAB, ACBA, ACBC) = 6/8

Q: Probability that city A have been visited twice, given that the salesperson has visited all three cities.

Ans: (1/8+1/8+1/8+1/8) / (6/8) = 2/3.