

HW9Q6 Prob 4,100

Note Title

3/4/2011

X is a binomial R.V with para n, p (Think $X = X_1 + \dots + X_n$)

$Y = \frac{X}{n}$ Q: Find $E(Y)$, $\text{Var}(Y)$

Ans: $E(Y) = \frac{E(X)}{n} = \frac{1}{n} \times (np) = p.$

$$\text{Var}(Y) = \left(\frac{1}{n}\right)^2 (\text{Var}(X))$$

$$= \frac{1}{n^2} \times (np(1-p)) = \frac{p(1-p)}{n}$$

Q: $P(|Y - p| > a)$?

Ans: $P(|Y - p| > a)$

$$\leq P(|Y - p| \geq a)$$

$$= \frac{\text{Var}(Y)}{a^2}$$

$$= \frac{p(1-p)}{n \times a^2}$$

by Chebyshev inequality

When $n \rightarrow \infty$

$$P(|Y - p| > a) = 0$$

You should think "a" being a small number
 \Rightarrow When n is large, the average is close to its individual expected value