Note Title 2/25/2011

HW8Q3

Let $Y = 2X+3$. Suppose we know the pdf of X. Find the pdf of Y.

Ans: Find the cdf of Y first.

$$F_Y(y) = P(Y \le y) = P(2X+3 \le y)$$

$$= P\left(X \le \frac{y-3}{2}\right)$$

$$= \int_{-\infty}^{\frac{y-3}{2}} f_X(x)\,dx$$

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

by the chain rule HW6Q1

$$= f_X\left(\frac{y-3}{2}\right) \times \frac{d}{dy}\left(\frac{y-3}{2}\right)$$

$$= \frac{1}{2} f_X\left(\frac{y-3}{2}\right)$$

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Similar to HW8Q4 Prob 4.76

Ex: X is an exponential R.V with

$$\lambda = \frac{3}{2}$$

$$Y = \max(X, 2)$$

Find the (generalized) pdf of Y,

$$E(Y) = ?$$

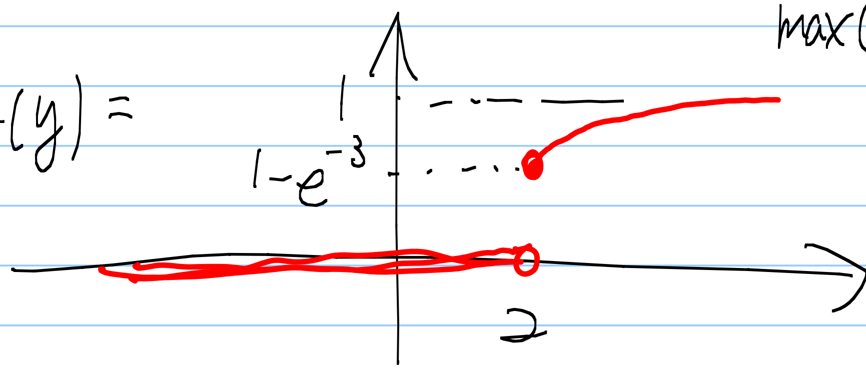
Ans: Method 1: (Introduced when discussing the cdf)

$$F_Y(y) = P(Y \le y) = P(\max(X,2) \le y)$$

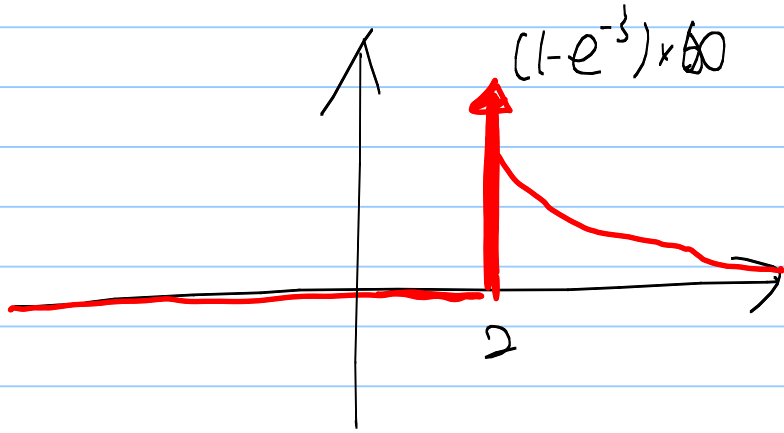
$$= \begin{cases} 0 & \text{if } y < 2 \\ \displaystyle\int_0^y \frac{3}{2} e^{-\frac{3}{2}x}\,dx & \text{if } 2 \le y \text{, then} \\ = 1 - e^{-\frac{3}{2}y} & \end{cases}$$

as long as $X \le y$, we have $\max(X,2) \le y$.

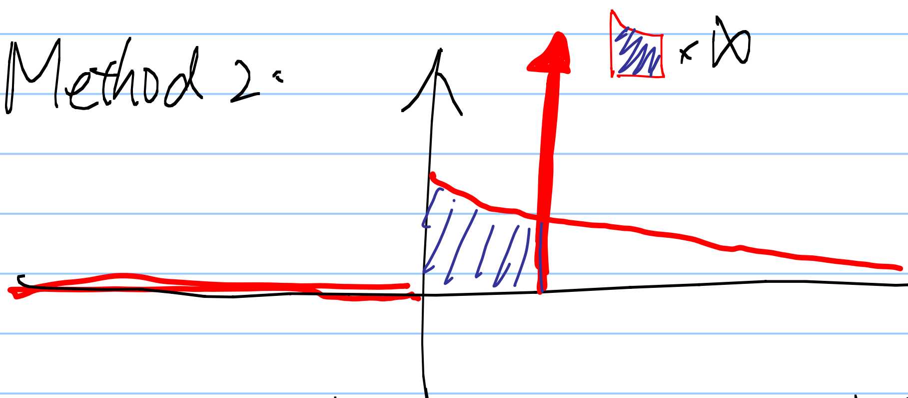
$F_Y(y) =$



$$f_Y(y) = (1-e^{-3})\,\delta(y-2) + \begin{cases} 0 & \text{if } y<2 \\ \frac{3}{2}e^{-\frac{3}{2}y} & \text{if } 2 \le y \end{cases}$$



Method 2:



any time $X<2 \Rightarrow Y=2$.

push the weight for $X<2$ to an impulse at $Y=2$.