

ECE 302-003, Homework #7
Due date: Wednesday 10/25/2023, 11:59pm;

<https://engineering.purdue.edu/~chihw/23ECE302F/23ECE302F.html>

Question 71: [Basic] Let X be a binomial random variable with $n = 3$, $p = 1/3$. Let Y denote an exponential random variable with $\lambda = 4/3$. Plot the cdfs of X and Y .

Question 72: [Basic] Problem 4.11.

4.11. The random variable X is uniformly distributed in the interval $[-1, 2]$.

(a) Find and plot the cdf of X .

(b) Use the cdf to find the probabilities of the following events: $\{X \leq 0\}$, $\{|X - 0.5| < 1\}$, and $C = \{X > -0.5\}$.

Question 73: [Basic] Problem 4.13.

4.13. A random variable X has cdf:

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - \frac{1}{4}e^{-2x} & \text{for } x \geq 0. \end{cases}$$

(a) Plot the cdf and identify the type of random variable.

(b) Find $P[X \leq 2]$, $P[X = 0]$, $P[X < 0]$, $P[2 < X < 6]$, $P[X > 10]$.

Question 74: [Basic] Suppose the cdf $F_X(x)$ of a random variable X is as follows.

$$F_X(x) = \begin{cases} 0 & \text{if } x < -\pi/2 \\ c(1 + \sin(x)) & \text{if } -\pi/2 \leq x < \pi/2 \\ 1 & \text{if } \pi/2 \leq x \end{cases}$$

1. Explain why c cannot be 1?
2. Explain why when $c = 1/2$, X is a continuous random variable. (Hint: you should check whether there is any jump or not.)
3. Let $c = 1/4$. Find the generalized pdf of X using the δ function.

Question 75: [Intermediate/Exam Level] Consider a random variable X with the corresponding pdf being $f_X(x) = 0.5e^{-|x|}$. $Y = \min(X, 0)$. Find out the cdf and pdf of Y . (Hint: Find the cdf first and then find the pdf.)

Question 76: [Intermediate/Exam Level] Suppose X is uniformly randomly distributed between $(0, 1)$. $Y = -\ln(x)$. Find out the cdf and pdf of Y . What type of random variables is Y ?

Question 77: [Basic] Problem 4.5.

- 4.5.** Let Y be the difference between the number of heads and the number of tails in the 3 tosses of a fair coin.
- (a) Plot the cdf of the random variable Y .
 - (b) Express $P[|Y| < y]$ in terms of the cdf of Y .

Question 78: [Intermediate/Exam Level] Problem 4.8. [Optional: You should compare your drawing for (a) and (b). Do they look similar?]

- 4.8.** Let ζ be a point selected at random from the unit interval. Consider the random variable $X = (1 - \zeta)^{-1/2}$.
- (a) Sketch X as a function of ζ .
 - (b) Find and plot the cdf of X .
 - (c) Find the probability of the events $\{X > 1\}$, $\{5 < X < 7\}$, $\{X \leq 20\}$.

Question 79: [Basic] Problem 4.9.

- 4.9.** The loose hand of a clock is spun hard and the outcome ζ is the angle in the range $[0, 2\pi)$ where the hand comes to rest. Consider the random variable $X(\zeta) = 2 \sin(\zeta/4)$.
- (a) Sketch X as a function of ζ .
 - (b) Find and plot the cdf of X .
 - (c) Find the probability of the events $\{X > 1\}$, $\{-1/2 < X < 1/2\}$, $\{X \leq 1/\sqrt{2}\}$.

Question 80: [Basic] Problem 4.17.

4.17. A random variable X has pdf:

$$f_X(x) = \begin{cases} c(1 - x^2) & -1 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Find c and plot the pdf.
- (b) Plot the cdf of X .
- (c) Find $P[X = 0]$, $P[0 < X < 0.5]$, and $P[|X - 0.5| < 0.25]$.

Question 81: [Basic] Problem 4.18.

4.18. A random variable X has pdf:

$$f_X(x) = \begin{cases} cx(1 - x^2) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) Find c and plot the pdf.
- (b) Plot the cdf of X .
- (c) Find $P[0 < X < 0.5]$, $P[X = 1]$, $P[.25 < X < 0.5]$.

Question 82: [Basic] (Similar to Problem 4.80.) Let $Y = 2X + 3$. Suppose we have already know the pdf of X being $f_X(x)$, find the pdf of Y . Hint: Find the cdf of Y first.

Question 83: [Intermediate/Exam Level] Problem 4.76. Please replace $Y = (X)^+$ by $Y = \max(X, 0)$.

4.76. Let X be a Gaussian random variable with mean 2 and variance 4. The reward in a system is given by $Y = (X)^+$. Find the pdf of Y .

Question 84: [Intermediate/Exam Level] Problem 4.86.

4.86. Let $X = U^n$ where n is a positive integer and U is a uniform random variable in the unit interval. Find the cdf and pdf of X .