Note Title 2/14/2011

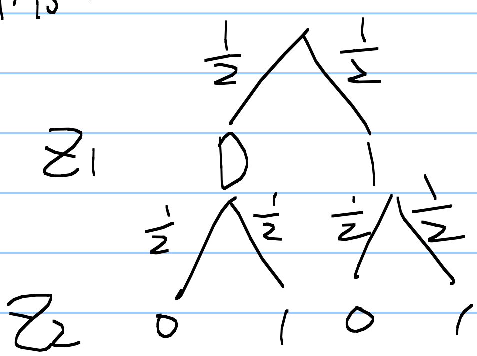
HW5 Q5: Z_1 & Z_2 are Beroulli w. para $\frac{1}{2}$. Also assume that Z_1 & Z_2 are independent.

$Y = f(Z_1, Z_2)$ where

$$f(a,b) = \begin{cases} 0 & \text{if } a = 0 = b \\ 1 & \text{if } a=1, b=0, \text{ or } a=0, b=1 \\ 2 & \text{if } a=b=1 \end{cases}$$

Q: Find the W.A of Y.

Ans: tree method of (Z_1, Z_2)



$$\Rightarrow P(Y=k) = \begin{cases} \frac{1}{4} & \text{if } k=0 \\ \frac{1}{2} & k=1 \\ \frac{1}{4} & k=2 \end{cases}$$

Sample space: $\{0, 1, 2\}$

$\Rightarrow Y$ is a binomial w. para $n=2$, $p=\frac{1}{2}$

$\boxed{* \; f(a,b) = a+b \text{ is the \# of heads in 2 trials.}}$

Note Title 2/2/2011

HW5Q7 Prob 3.52

A sequence of bits is transmitted, each bit may be corrupted with error prob. (bit error rate) $p = 0.01$

Let N denote the number of error-free bits before the 1st erroneous bit.

Q1: pmf of N?

Ans: N is best modeled by a geometric R.V. with $p = 0.01$

$$\Rightarrow p_k = 0.01(1-0.01)^k \quad k = 0, 1, 2 \cdots$$

Q2: $E(N) = ?$

Ans: For a geometric R.V. its expectatio is always $\frac{1-p}{p}$

$$\Rightarrow E(N) = \frac{0.99}{0.01} = 99$$

Q3: $P(N \geq 1000) = ?$

Ans: $= \sum_{k=1000}^{\infty} 0.01(1-0.01)^k$

$$= 0.99^{1000} = 4.317 \times 10^{-5}$$

Q4: If we want $P(N \geq 1000) > 0.99$

what much do we need to reduce the p value.

Ans: For a general p.

$$P(N \geq 1000) = \sum_{k=1000}^{\infty} p(1-p)^k$$

$$= (1-p)^{1000}$$

we like $(1-p)^{1000} > 0.99$

$$p < 1-(0.99)^{\frac{1}{1000}} \approx 10^{-5}$$

HW5Q 10 Prob 3.65

1. Each LCD panel has 1000×750 pixels
2. let X denote the # of faulty pixels.
3. A panel is accepted if $X \leq 15$
4. Each individual pixel has faulty prob 10^{-5}

Q: Prob (accepted)

Ans: Which W.A to use?

Choice 1: binomial w. $n = 1000 \times 750$

$p = 10^{-5}$

$$P(\text{accepted}) = \sum_{k=0}^{15} \binom{1000 \times 750}{k} (10^{-5})^{k} (1-10^{-5})^{1000 \times 750 - k}$$

$\Rightarrow$ Numerical precision problem

Choice 2: Poisson is the limiting behavior. For the entire panel (fixed interval) the avg # of errors (customers) is $1000 \times 750 \times 10^{-5} = 7.5 = \alpha$

$$P(\text{accepted}) \approx \sum_{k=0}^{15} \frac{7.5^{k}}{k!} e^{-7.5}$$

$$= 99.54\%$$