

ECE 302-003, Homework #13

It is a self-exercise. No need to turn in the homework.

<https://engineering.purdue.edu/~chihw/23ECE302F/23ECE302F.html>

Question 128: [Intermediate/Exam Level] Consider two random variables X and Y with means and variances being (m_X, σ_X^2) and (m_Y, σ_Y^2) respectively. We further assume that X and Y are independent.

1. Find out the value of $E(XY)$ in terms of (m_X, σ_X^2) and (m_Y, σ_Y^2) .
2. Let $Z = X + Y$. Find out the values of $E(Z)$ and $E(Z^2)$.
3. Find out the value of $\text{Var}(Z)$.

Question 129: [Basic] Problem 5.111. There is a typo in the textbook. The coefficient of the joint pdf should be $\frac{1}{2\pi c}$ for some constant c . Unfortunately, in the textbook the constant c is missing and the question only states $\frac{1}{2\pi}$.

5.111. Let X and Y be jointly Gaussian random variables with pdf

$$f_{X,Y}(x, y) = \frac{\exp\left\{\frac{-1}{2}[x^2 + 4y^2 - 3xy + 3y - 2x + 1]\right\}}{2\pi} \quad \text{for all } x, y.$$

Find $E[X]$, $E[Y]$, $\text{VAR}[X]$, $\text{VAR}[Y]$, and $\text{COV}(X, Y)$.

Question 130: [Basic] Problem 5.112.

5.112. Let X and Y be jointly Gaussian random variables with $E[Y] = 0$, $\sigma_1 = 1$, $\sigma_2 = 2$, and $E[X|Y] = Y/4 + 1$. Find the joint pdf of X and Y .

Question 131: [Basic] Problem 5.114(a,b,d,f). You only need to print out your figures. There is no need to turn in your MATLAB codes.

5.114. Use a plotting program (as provided by Octave or MATLAB) to show the pdf for jointly Gaussian zero-mean random variables with the following parameters:

- (a) $\sigma_1 = 1, \sigma_2 = 1, \rho = 0.$
- (b) $\sigma_1 = 1, \sigma_2 = 1, \rho = 0.8.$
- (c) $\sigma_1 = 1, \sigma_2 = 1, \rho = -0.8.$
- (d) $\sigma_1 = 1, \sigma_2 = 2, \rho = 0.$
- (e) $\sigma_1 = 1, \sigma_2 = 2, \rho = 0.8.$
- (f) $\sigma_1 = 1, \sigma_2 = 10, \rho = 0.8.$

Question 132: [Intermediate/Exam Level] Suppose the means and variances of random variables X and Y is $m_X = 1, \sigma_X^2 = 2, m_Y = -2,$ and $\sigma_Y^2 = 3$ respectively. Suppose we also know that the covariance between X and Y is $\text{Cov}(X, Y) = 1.$ Let $Z = X + Y.$

1. Find the mean and the variance of $Z.$ Note: X and Y are not independent this time.
2. Find $\text{Cov}(X, Z)$ and the correlation coefficient $\rho_{X,Z}.$

Question 133: [Intermediate/Exam Level] Suppose X is equally likely to be -1 and $1.$ N is a standard Gaussian. X and N are independent. $Y = X + N.$

1. Find the mean and the variance of $Y.$
2. Find $\text{Cov}(X, Y)$ and the correlation coefficient $\rho_{X,Y}.$
3. What is the minimum mean square error estimator of X given $Y = y?$
4. The answer of the previous question can be expressed as a function $g(y).$ Use MATLAB to plot $g(y)$ for the range $y = -2$ to $2.$
5. What is the maximum a posteriori probability (MAP) detector of X given $Y = y.$

Question 134: [Intermediate/Exam Level] Problem 6.68(b,c). Please assume the joint distribution of X and Y is described by (i) in the given table. Namely, ignore the joint distributions described in (ii) and (iii) of the table.

6.68. Let X and Y be discrete random variables with three possible joint pmf's:

	(i)			(ii)			(iii)				
X/Y	-1	0	1	X/Y	-1	0	1	X/Y	-1	0	1
-1	1/6	1/6	0	-1	1/9	1/9	1/9	-1	1/3	0	0
0	0	0	1/3	0	1/9	1/9	1/9	0	0	1/3	0
1	1/6	1/6	0	1	1/9	1/9	1/9	1	0	0	1/3

- (a) Find the minimum mean square error linear estimator for Y given X .
- (b) Find the minimum mean square error estimator for Y given X .
- (c) Find the MAP and ML estimators for Y given X .
- (d) Compare the mean square error of the estimators in parts a, b, and c.

Question 135: [Intermediate/Exam Level] Problem 6.69(b,c).

6.69. Repeat Problem 6.68 for the continuous random variables X and Y in Problem 5.26.

5.26. Let X and Y have joint pdf:

$$f_{X,Y}(x, y) = k(x + y) \quad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1.$$

- (a) Find k .
- (b) Find the joint cdf of (X, Y) .
- (c) Find the marginal pdf of X and of Y .
- (d) Find $P[X < Y]$, $P[Y < X^2]$, $P[X + Y > 0.5]$.