ECE 302-003, Homework #12
It is a self-exercise. No need to turn in the homework.

https://engineering.purdue.edu/~chihw/23ECE302F/23ECE302F.html

Question 118: [Intermediate/Exam Level] Suppose Θ is uniformly distributed in the interval $(0, 2\pi)$. Let $X = \cos(\Theta)$ and $Y = \sin(\Theta)$.

1. Find $E(Y)$.
2. Find $E(XY)$.
3. Let $h(x) = E(Y|X = x)$, where x is the input parameter that is between $(-1, 1)$. Find out the expression of $h(x)$.
4. Does $E(h(X)) = E(Y)$?

Question 119: [Intermediate/Exam Level] Continue from the previous question.

1. Are X and Y orthogonal?
2. Are X and Y correlated?
3. Find out the covariance and the correlation coefficient of X and Y.

Question 120: [Basic] Problem 5.47.

5.47. Let X and Y be independent random variables. Find an expression for the probability of the following events in terms of $F_X(x)$ and $F_Y(y)$.

(a) $\{a < X \leq b\} \cap \{Y > d\}$.

(b) $\{a < X \leq b\} \cap \{c \leq Y < d\}$.

(c) $\{|X| < a\} \cap \{c \leq Y \leq d\}$.

Question 121: [Intermediate/Exam Level] Problem 5.58.

5.58. Find $E[X^2 e^Y]$ where X and Y are independent random variables, X is a zero-mean, unit-variance Gaussian random variable, and Y is a uniform random variable in the interval $[0, 3]$.

Question 122: [Basic] Problem 5.40.

5.40. Determine which of the joint pmfs in Problem 5.11 correspond to independent pairs of random variables.

5.11. **(a)** Find the marginal pmf's for the pairs of random variables with the indicated joint pmf.

(i)

X/Y	-1	0	1
-1	1/6	1/6	0
0	0	0	1/3
1	1/6	1/6	0

(ii)

X/Y	-1	0	1
-1	1/9	1/9	1/9
0	1/9	1/9	1/9
1	1/9	1/9	1/9

(iii)

X/Y	-1	0	1
-1	1/3	0	0
0	0	1/3	0
1	0	0	1/3

(b) Find the probability of the events $A = \{X > 0\}$, $B = \{X \geq Y\}$, and $C = \{X = -Y\}$ for the above joint pmf's.

Question 123: [Basic] Problem 5.61.

5.61. For the three pairs of discrete random variables in Problem 5.11, find the correlation and covariance of X and Y, and indicate whether the random variables are independent, orthogonal, or uncorrelated.

Question 124: [Basic] Problem 5.65.

5.65. Find the correlation and covariance of X and Y in Problem 5.26. Determine whether X and Y are independent, orthogonal, or uncorrelated.

5.26. Let X and Y have joint pdf:

$$f_{X,Y}(x, y) = k(x + y) \qquad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1.$$

(a) Find k.

(b) Find the joint cdf of (X, Y).

(c) Find the marginal pdf of X and of Y.

(d) Find $P[X < Y]$, $P[Y < X^2]$, $P[X + Y > 0.5]$.

Question 125: [Intermediate/Exam Level] Problem 5.71.

5.71. The output of a channel $Y = X + N$, where the input X and the noise N are independent, zero-mean random variables.

(a) Find the correlation coefficient between the input X and the output Y.

(b) Suppose we estimate the input X by a linear function $g(Y) = aY$. Find the value of a that minimizes the mean squared error $E[(X - aY)^2]$.

(c) Express the resulting mean-square error in terms of σ_X/σ_N.

Question 126: [Basic] Problem 5.76(a).

5.76. **(a)** Find $p_X(x \mid y)$ for the communication channel in Problem 5.3.

(b) For each value of y, find the value of x that maximizes $p_X(x \mid y)$. State any assumptions about p and p_e.

(c) Find the probability of error if a receiver uses the decision rule from part b.

5.3. The input X to a communication channel is "-1"or "1", with respective probabilities 1/4 and 3/4. The output of the channel Y is equal to: the corresponding input X with probability $1 - p - p_e$; $-X$ with probability p; 0 with probability p_e.

(a) Describe the underlying space S of this random experiment and show the mapping from S to S_{XY}, the range of the pair (X, Y).

(b) Find the probabilities for all values of (X, Y).

(c) Find $P[X \neq Y]$, $P[Y = 0]$.

Question 127: [Intermediate/Exam Level] Problem 5.80(a,b,d). Also find $E(Y)$ and $E(h(X))$ where $h(x) = E(Y|X = x)$.

5.80. **(a)** Find $f_Y(y \mid x)$ in Problem 5.26.

(b) Find $P[Y > X \mid x]$.

(c) Find $P[Y > X]$ using part b.

(d) Find $E[Y \mid X = x]$.

5.26. Let X and Y have joint pdf:

$$f_{X,Y}(x, y) = k(x + y) \qquad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1.$$

(a) Find k.

(b) Find the joint cdf of (X, Y).

(c) Find the marginal pdf of X and of Y.

(d) Find $P[X < Y]$, $P[Y < X^2]$, $P[X + Y > 0.5]$.