ECE 302-003, Homework #11
Due date: Saturday 12/02/2023, 11:59pm;
Submission via Gradescope

https://engineering.purdue.edu/~chihw/23ECE302F/23ECE302F.html

Question 110: [Intermediate/Exam Level]

Suppose X is a uniform random variable with parameters $a = 1, b = 2$. Given $X = x_0$, the conditional probability density function of Y, is an exponential random variable with $\lambda = x_0$.

1. Find the sample space of (X, Y).
2. What is the joint probability density function of X and Y?
3. What is the probability that $P(X < 1.5 \text{ and } Y \leq 2)$?

Question 111: [Basic] Problem 5.31.

5.31. Let X and Y be the pair of random variables in Problem 5.17.
- **(a)** Find the joint pdf of X and Y.
- **(b)** Find the marginal pdf of X and of Y.
- **(c)** Find $P[Y < X^2]$.

5.17. A point (X, Y) is selected at random inside a triangle defined by $\{(x, y) : 0 \leq y \leq x \leq 1\}$. Assume the point is equally likely to fall anywhere in the triangle.
- **(a)** Find the joint cdf of X and Y.
- **(b)** Find the marginal cdf of X and of Y.
- **(c)** Find the probabilities of the following events in terms of the joint cdf: $A = \{X \leq 1/2, Y \leq 3/4\}$; $B = \{1/4 < X \leq 3/4, 1/4 < Y \leq 3/4\}$.

Question 112: [Basic] Problem 5.41.

5.41. Michael takes the 7:30 bus every morning. The arrival time of the bus at the stop is uniformly distributed in the interval [7:27, 7:37]. Michael's arrival time at the stop is also uniformly distributed in the interval [7:25, 7:40]. Assume that Michael's and the bus's arrival times are independent random variables.
- **(a)** What is the probability that Michael arrives more than 5 minutes before the bus?
- **(b)** What is the probability that Michael misses the bus?

Question 113: [Basic] Problem 5.48(a,b,d).

5.48. Let X and Y be independent random variables that are uniformly distributed in $[-1, 1]$. Find the probability of the following events:

(a) $P[X^2 < 1/2, |Y| < 1/2]$.

(b) $P[4X < 1, Y < 0]$.

(c) $P[XY < 1/2]$.

(d) $P[\max(X, Y) < 1/3]$.

Question 114: [Intermediate/Exam Level] Problem 5.18.

5.18. A dart is equally likely to land at any point (X_1, X_2) inside a circular target of unit radius. Let R and Θ be the radius and angle of the point (X_1, X_2).

(a) Find the joint cdf of R and Θ.

(b) Find the marginal cdf of R and Θ.

(c) Use the joint cdf to find the probability that the point is in the first-quadrant of the real plane and that the radius is greater than 0.5.

Question 115: [Basic] Problem 5.20(b,c).

5.20. The pair (X, Y) has joint cdf given by:

$$F_{X,Y}(x, y) = \begin{cases} (1 - 1/x^2)(1 - 1/y^2) & \text{for } x > 1, y > 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Sketch the joint cdf.

(b) Find the marginal cdf of X and of Y.

(c) Find the probability of the following events: $\{X < 3, Y \le 5\}$, $\{X > 4, Y > 3\}$.

Question 116: [Basic] Problem 5.17.

5.17. A point (X, Y) is selected at random inside a triangle defined by $\{(x, y) : 0 \le y \le x \le 1\}$. Assume the point is equally likely to fall anywhere in the triangle.

(a) Find the joint cdf of X and Y.

(b) Find the marginal cdf of X and of Y.

(c) Find the probabilities of the following events in terms of the joint cdf: $A = \{X \le 1/2, Y \le 3/4\}$; $B = \{1/4 < X \le 3/4, 1/4 < Y \le 3/4\}$.

Question 117: [Intermediate/Exam Level] Problem 5.35.

5.35. The input X to a communication channel is +1 or −1 with probability p and $1 - p$, respectively. The received signal Y is the sum of X and noise N which has a Gaussian distribution with zero mean and variance $\sigma^2 = 0.25$.

(a) Find the joint probability $P[X = j, Y \leq y]$.

(b) Find the marginal pmf of X and the marginal pdf of Y.

(c) Suppose we are given that $Y > 0$. Which is more likely, $X = 1$ or $X = -1$?