**ECE 302-003, Homework #1**
**Due date: Wednesday 8/30/2023, 11:59pm;**
**Submission via Gradescope**

https://engineering.purdue.edu/~chihw/23ECE302F/23ECE302F.html

Review of Calculus:

*Question 1:* Compute the values of the following integrals, given the condition that $\lambda > 0$.

$$\int_1^\infty \frac{1}{x^2} dx$$
$$\int_0^\infty \lambda e^{-\lambda x} dx$$

*Question 2:* Compute the expressions of the following indefinite integrals.

$$\int y e^{-\frac{y^2}{2x^2}} dy$$
$$\int \int x e^{yz} dz dx$$

*Question 3:* Compute the values of the following integrals. Hint: It can be solved by inspection.

$$\int_{-10}^{10} y e^{-2|y|} dy$$
$$\int_{-20}^{20} x^3 e^{-\frac{x^2}{10}} dx$$

*Question 4:* Compute the values of the following integrals. Hint: Use the integration by part formula.

$$\int_0^\infty z 0.5 e^{-0.5z} dz$$
$$\int_0^\infty z^2 0.5 e^{-0.5z} dz$$

*Question 5:* Compute the bilateral Laplace transform of the following functions.

$$f(x) = \begin{cases} 0.5e^{-0.5x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$g(x) = \begin{cases} 1/3 & \text{if } x \in [0,3] \\ 0 & \text{otherwise} \end{cases}$$

Hint: The bilateral Laplace transform of any function $f(x)$ is defined as

$$L_f(s) = \int_{-\infty}^{\infty} e^{-sx} f(x) dx.$$

*Question 6:* Define a 2-D function $f(x, y)$ as follows.

$$f(x,y) = \begin{cases} 2/9 & \text{if } x \in [0,3] \text{ and } y \in [0,x] \\ 0 & \text{otherwise} \end{cases}$$

Sub-question 1: What are the values of $f(2, 2.01)$ and $f(1, 0.33)$.

Sub-question 2: Define another 2-dimensional function

$$g(x,y) = \int_{t=-\infty}^{y} \int_{s=-\infty}^{x} f(s,t) ds dt.$$

Find the values of $g(1, 2)$ and $g(2, 2.01)$.

*Question 7:* Define a 1-D function $f(x)$ as follows.

$$f(x) = \begin{cases} \frac{x}{4} & \text{if } x \in [0,1] \\ \frac{3}{8} & \text{if } x \in (1,3] \\ \frac{-x}{4} + 1 & \text{if } x \in (3,4] \\ 0 & \text{otherwise} \end{cases}$$

Sub-question 1: Plot $f(x)$ for the range of $-1 < x < 5$.

Sub-question 2: Compute the value of the following integral.

$$\int_{x=-\infty}^{\infty} x f(x) dx.$$

*Question 8:* Consider three series:

$$\sum_{k=27}^{\infty} 0.9^k$$

$$\sum_{k=0}^{\infty} k0.5^{k+1}$$

$$\sum_{k=3}^{50} y^k$$

Compute the values of the first two series, and find the expression of the third series. You may need to use the following formulas

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \quad \text{if } |r| < 1 \tag{1}$$

$$\sum_{k=1}^{\infty} akr^{k-1} = \frac{a}{(1-r)^2} \quad \text{if } |r| < 1 \tag{2}$$

$$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r} \quad \text{if } r \neq 1 \tag{3}$$

*Question 9:* Suppose $a_k$ is a series such that

$$a_k = \begin{cases} 0.8 & \text{if } k = -1 \\ 0.2 & \text{if } k = 9 \\ 0 & \text{otherwise} \end{cases}$$

Compute the values of the following expressions:

$$\sum_{k=-\infty}^{\infty} a_k \tag{4}$$

$$\sum_{k=-\infty}^{\infty} ka_k \tag{5}$$

$$\sum_{k=-\infty}^{\infty} \min(k, 3)a_k \tag{6}$$

$$\sum_{k=-\infty}^{\infty} \sin(k\pi)a_k \tag{7}$$

Note: The function "min$(\cdot,\cdot)$" returns the minimum of the two inputs. For example, $\min(1.11, 5.375) = 1.11$.

*Question 10:* [Basic] Throw a fair die and toss a fair coin together. Let $X$ and $Y$ denote the outcomes of the die and the coin respectively, where we use the convention that $Y = 1$ if the outcome of the coin is head. $Y = 0$ if the outcome of the coin is tail.

- What is the *sample space* in this experiment? (Note the definition of the sample space is the collection of all possible choices of uncertain outcomes.)
- What is the probability weight you would like to assign to each outcome of the sample space? Why do you make such a weight assignment?
- What is the probability that $X^2 + Y$ is a prime number? (Note that 1 is NOT a prime number. The smallest prime number is 2.)