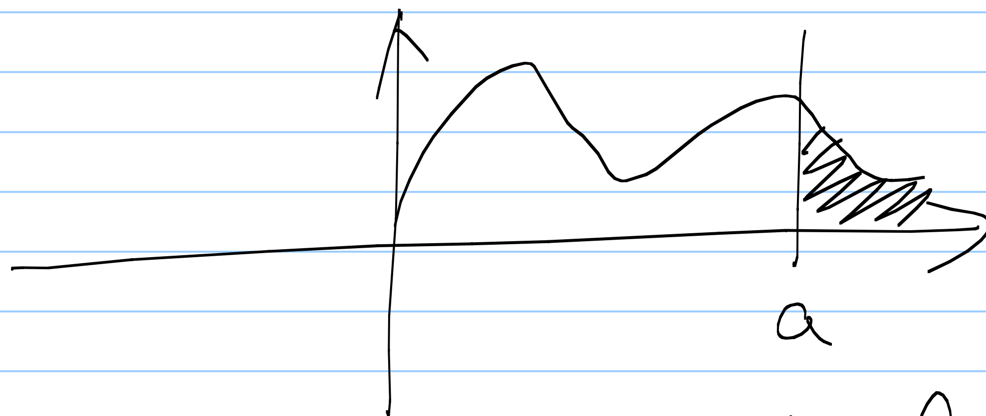
Specifically, what is the ~~the~~ largest possible value of $P(X \geq a)$ while keeping the same mean value $E(X) = m$

say $a = 3$ say $m = 5$

Ans: $\boxed{P(X \geq a) \leq \frac{E(X)}{a}}$



pf: suppose $P(X \geq a)$ is fixed to K

how to design a W.A with the smallest E(X)

Ans: Choose $P(X=0) = 1-K$

$P(X=a) = K$.

the smallest possible $E(X)$ is $0 \times (1-K) + aK$.

$\Rightarrow$ For any other W.A.

$$E(X) \geq a \cdot P(X \geq a)$$

③ Chebyshev Inequality: A refinement of the Markov Inequality, which gives you more accurate estimate at the price of requiring more info. ② m ③ σ^2 (but does not need X to be positive)

$$P(|X-m| \geq a) \leq \frac{\sigma^2}{a^2}$$

pf: let $Y=(X-m)^2$

$$\Rightarrow P(|X-m| \geq a) = P(Y \geq a^2)$$

$$\leq \frac{E(Y)}{a^2} \quad \text{by Markov inequality}$$

$$\because E(Y) = E((X-m)^2) = Var(X)$$

$$= \frac{Var(X)}{a^2}$$

Ex: I am recording the values of my stock portfolio, which has 100 stocks. One day my computer crashes, I only remember the average price is 50.

Q: At most how many stocks can have the price ≥ 90?

Ans: By Markov inequality

$$P(X \geq 90) \leq \frac{50}{90}$$

$\Rightarrow$ At most $100 \times \frac{50}{90}$ stocks can have value larger than 90.

Q: Suppose I also remember the standard deviation is 20.
What is the maximum number of stocks having value ≥ 90.

Ans: $P(X \geq 90) = P(X-50 \geq 40)$

$$\leq P(|X-50| \geq 40)$$

$$\leq \frac{20^2}{40^2} = \frac{1}{4} = 0.25$$

Q: Suppose I also know it is a bell-shaped distribution
What is $P(X \geq 90)$?

Ans: X is Gssn with $\mu = 50$, $\sigma = 20$

$\Rightarrow X = 50 + 20Z$ where Z is standard Gssn

$$P(X > 90) = P(50 + 20Z > 90)$$

$$= P\left(Z > \frac{90-50}{20}\right)$$

$$= P(Z > 2) = Q(2) = 0.0228$$