Conditional prob is extremely useful.

For example: the auto-fill function (AFF) of MS word:

Example: By Wiki, "e" 12%

"t" 9%

"a" 8%

So if I have not typed any letter, the best guess of AFF is "e".

Nonethelss, once I typed the first letter being "e", is the second letter also going to be "e"? (double e is unlikely in English) What is the next letter should be? ∴ Need to use "conditional prob"

Ex:

	t	s	d
i	3/20	3/20	5/20
a	4/20	2/20	3/20

$S = \{at, as, ad, it, is, id\}$

By counting the historical data

Q: If the AFF likes to pick one word, which word should we choose?

A: "id" ∵ $P(id) = 5/20$ is the larges.

035)

Q2: After typing the first letter being "a", which letter should AFF choose?

A: Conditioned on the first letter being a.

the conditional prob becomes (after zooming & renormalization

P(· | the first letter being a)

	t	s	d
a	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

So we should choose "t" conditioned on the first letter is "a"

Q3: A detective found that the 2nd letter is "d" but the first letter is missing. What is the most likely first letter.

A: Conditioning on the 2nd letter being "d"

The conditional prob becomes

	d
i	$5/8$
a	$3/8$

We should choose

i (with the largest conditional prob)

* Another reason why conditional prob is important is that it can be used to construct W.A.

Continue the two-team, Sunny/Rainy example

	Sunny	Rainy	
A wins	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
B wins	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
	$\frac{5}{12}$	$\frac{7}{12}$	

Q: How likely it is a sunny day

Q rainy day?

Conditioning on Sunny

A wins	$3/5$
B wins	$2/5$

Rainy

A wins	$3/7$
B wins	$4/7$

Q: What if we move to Florida, which has $9/10$ Sunny prob.

$1/10$ Rainy prob

What is a reasonable W.A?

Ans:

	S	R
A wins	$3/5 \times \frac{9}{10}$	$\frac{3}{7} \times \frac{1}{10}$
B wins	$2/5 \times \frac{9}{10}$	$\frac{4}{7} \times \frac{1}{10}$
	$9/10$	$1/10$

Q: $P(A \text{ wins}) = \frac{3}{5} \times \frac{9}{10} + \frac{3}{7} \times \frac{1}{10} = \frac{204}{350}$

In many cases, the statistics of the "conditional prob" is easier to find & can be used to construct new W.A.

Example: Nationally, students who attend lectures have 20% of getting A. Students who do not attend letures have 10% of getting A. In ECE 302, 80% of students attend lectures. What is the prob that an ECE 302 student gets an A.

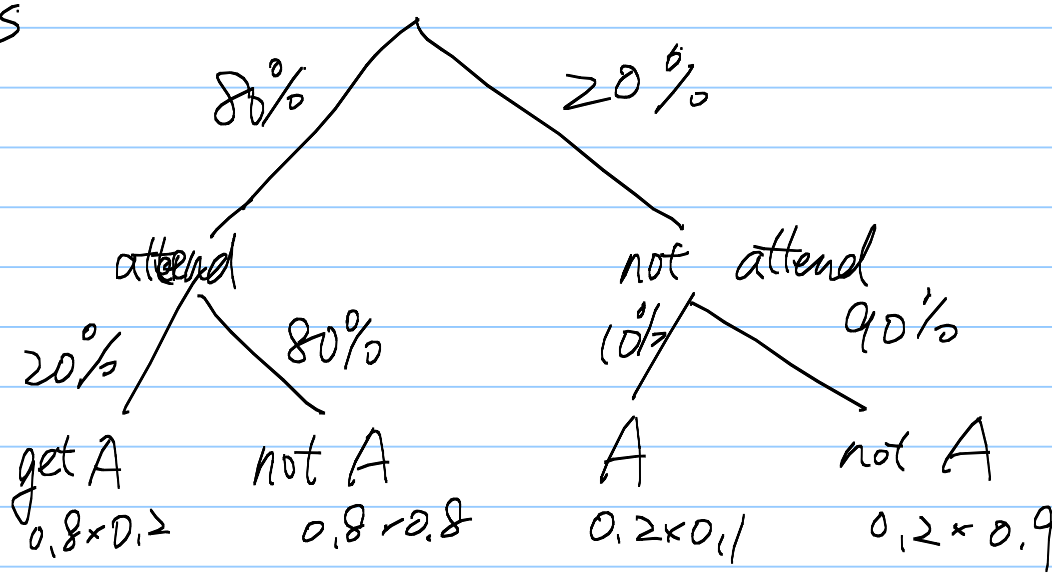
Ans: Sample space

	Lecture	Not Lecture
A	0.2×0.8	0.1×0.2
not A	0.8×0.8	0.9×0.2
	80%	20%

$$P(\text{a student gets } A) = 0.2\times0.8 + 0.1\times0.2$$
$$= 18\% \#$$

Many students prefer a tree method rather than the table method to solve the same problem. (see textbook Example 2.25)

Ans



$P(\text{get } A) = 0.8\times 0.2 + 0.2\times 0.1 = 18\%$

* Another application of conditional prob.

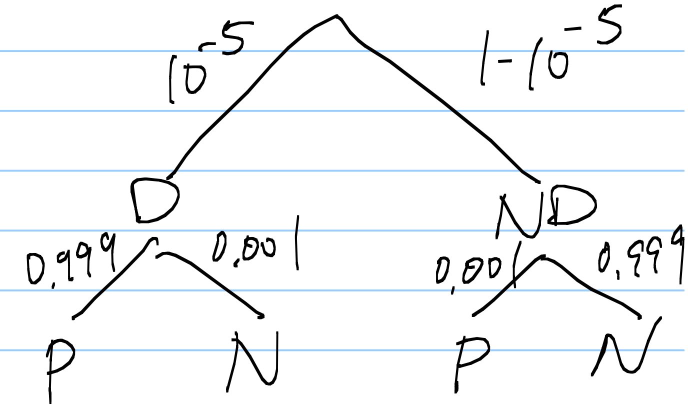
* Diagnosing a rare disease

The sample space and the W.A for this question

Table method

	D	ND
P	0.999×10^{-5}	$0.001\times(1-10^{-5})$
N	0.001×10^{-5}	$0.999\times(1-10^{-5})$
	10^{-5}	$1-10^{-5}$

Tree



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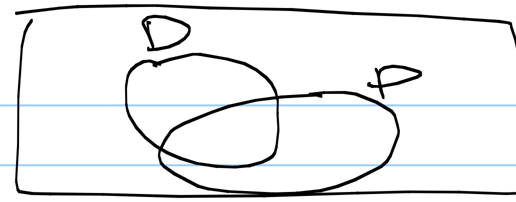
Q: $P(D \mid P)$

$$= \frac{P(D \text{ and } P)}{P(P)}$$



$$\simeq \frac{0.999\times10^{-5}}{0.999\times10^{-5}+0.001\times0.99999} \approx 0.989\%$$

Q36

Problem 2.80

Computer chips: 50% from Factory A
10% from Factory B
40% from Factory C

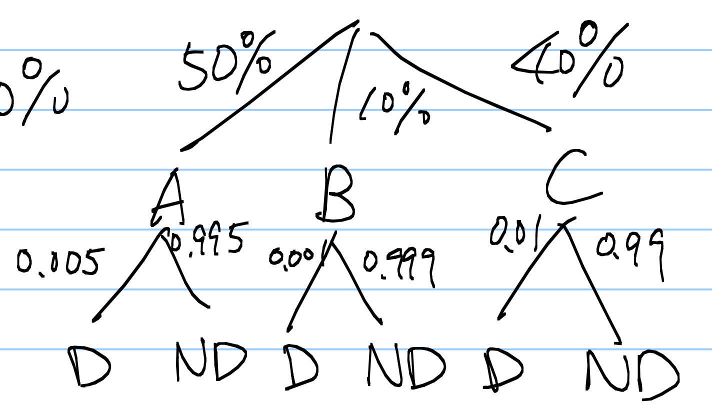
We know that chips from A has defective rate 0.005
- - - - - B - - - - 0.001
C - - - - 0.010

Q: P(It is from A | a chip is defective)

Ans: Solution 1. Table method

	A	B	C
Defective	50% × 0.005	10% × 0.001	40% × 0.010
ND	50% × 0.995	10% × 0.999	40% × 0.99
	50%	10%	40%

Solution 2: Tree method



$$P(A \mid \text{detected}) = \frac{P(A \,\&\, \text{detected})}{P(\text{detected})}$$

$$= \frac{50\% \times 0.005}{50\% \times 0.005 + 10\% \times 0.001 + 40\% \times 0.010}$$

$$= \frac{25}{66}$$

Q: Can we derive a formula to speed-up the counting process?

* $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ ← Renormalization

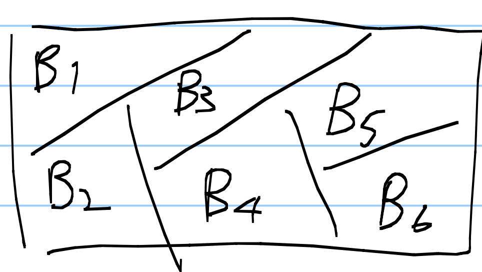
$\Leftrightarrow P(A \cap B) = P(A|B)\,P(B)$ ← Tree-method W.A construction

* Bayes Rule

Def: $B_1, \cdots, B_n$ form a $\boxed{\text{partition}}$

if ① they are mutually exclusive/disjoint

② $\bigcup_{i=1}^{n} B_i = S$



V4!

Theorem 1:

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)$$

Theorem 2:

$$P(B_j / A) = \frac{P(A \cap B_j)}{P(A \cap B_1) + \cdots + P(A \cap B_n)}$$

$$\frac{P(A | B_j) P(B_j)}{P(A|B_1)P(B_1) + \cdots + P(A|B_n)P(B_n)}$$

For our example HW3Q11,

"A": defected

"B_1, B_2, B_3": are the partition that the chip is made by factory A to C respectively. (Mutually exclusive & covers the entire sample space)

$$P(B_1 | A) = \frac{P(A|B_1) \times P(B_1)}{P(A|B_1) \times P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)}$$

$$= \frac{50\% \times 0.005}{50\% \times 0.005 + 10\% \times 0.001 + 40\% \times 0.010}$$