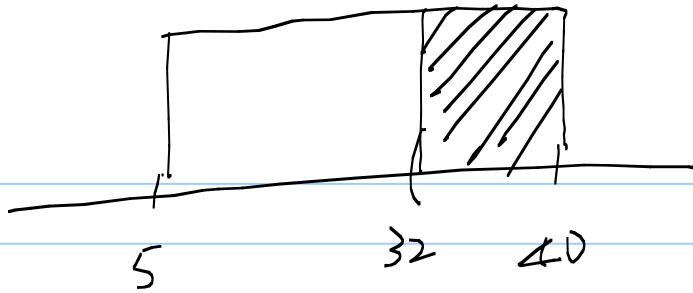


$$P(T>32)=\int_{32}^{40}\frac{1}{35}\,dx = \frac{8}{35}$$

$$P(T>32)=\int_{32}^{\infty} f_X(x)\,dx = \int_{32}^{40}\frac{1}{35}\,dx = \frac{8}{35} \#$$

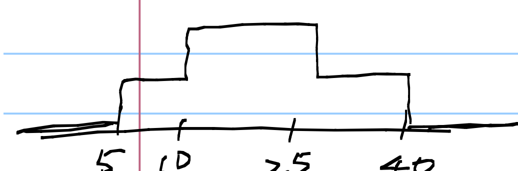
Example: Temperature is between 5 & 40. Suppose the prob that it is between (10, 25) is twice as likely is it is between (5, 10) or (25, 40)

Find $P(T > 32)$

Ans: Step 1: the sample space is the same. $S = \mathbb{R}$

Step 2: The curve must be

$$f_X(x) = \begin{cases} 2c & \text{if } 10 \le x < 25 \\ c & \text{if } 5 \le x < 10 \text{ or } 25 \le x < 40 \\ 0 & \text{otherwise} \end{cases}$$



$\because \int_{-\infty}^{\infty} f_X(x) = 1$ we must have

$$\int_{5}^{10} c\,dx + \int_{10}^{25} 2c\,dx + \int_{25}^{40} c\,dx = 1$$

$$\Rightarrow 50c = 1 \quad c = \frac{1}{50}$$

$$f_X(x) = \begin{cases} \frac{1}{25} & \text{if } 10 \le x < 25 \\ \frac{1}{50} & \text{if } 5 \le x < 10,\ 25 \le x < 40 \\ 0 & \text{otherwise} \end{cases}$$

Step 3: Count the weight

$$P(T > 32) = \int_{32}^{\infty} f_X(x)\,dx$$

$$= \int_{32}^{40} \frac{1}{50}\,dx = \frac{8}{50} \times$$

$$= 16\%$$

Q25

Q For any valid W.A. show that

$$P(-\infty < X \leq r) \leq P(-\infty < X \leq s)$$

if $r \leq s$

Ans: Since $r \leq s$

$$P(-\infty < X \leq s) = P(-\infty < X \leq r) + \underbrace{P(r < X \leq s)}_{\geq 0}$$

Since

$$\Rightarrow P(-\infty < X \leq s) \geq P(-\infty < X \leq r)$$

Q: Suppose

$$P(-\infty < X \leq r) = p_r$$

$$P(-\infty < X \leq s) = p_s$$

what is $P(r < X \leq s)$?

Ans: $\because$ $p_s = p_r + P(r < X \leq s)$

$\therefore$ $P(r < X \leq s) = p_s - p_r$

Example: A continuous R.V.

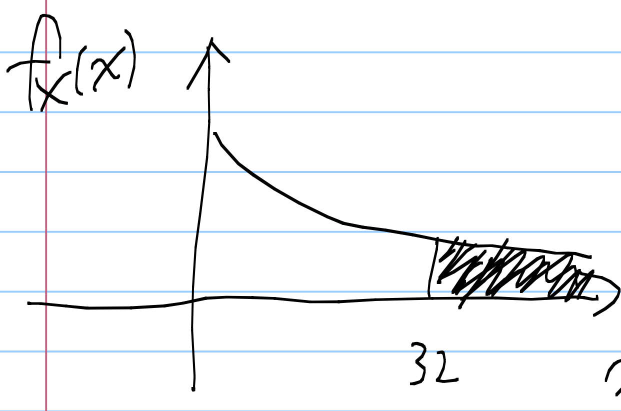
has sample space $S = [0, \infty)$ (all positive real numbers)

and the prob density function is

$$f_X(x) = \begin{cases} 3e^{-3x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Q: What is the prob $X > 32$

Ans:



$$\int_{32}^{\infty} f_X(x)\,dx$$

$$= \int_{32}^{\infty} 3e^{-3x}\,dx$$

$$= e^{-96} \quad \#$$

* Conditional prob
(Or the "Relative Frequency")

Example: Two teams play a game.
& the weather can be sunny/rainy.

Q: Sample space?

A: $S = \{(A\text{win}, \text{sunny}), (A\text{ win}, \text{rainy}), (B\text{win}, \text{sunny}), (B\text{ win}, \text{rainy})\}$

Q: Construct a valid W.A?

Ans: ∵ S is discrete
∴ By pmf. We use a ~~table~~ a table to represent the sample space

	S	R
A win	$\frac{1}{4}$	$\frac{1}{4}$
B win	$\frac{1}{6}$	$\frac{1}{3}$

Q: $P(A \text{ wins}) = ?$ Ans: $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Q: What is the prob ~~that~~ A wins conditioned on that it rains?

The conditional prob should be $P(A\text{ win} \mid \text{Rainy})$ as the

$$\frac{\#\text{ of times A win \& Rainy}}{\#\text{ of times Rainy}} = \frac{P(A\text{ wins \& rains})}{P(\text{rains})}$$

030)

Note Title 1/19/2011

From the W.A perspective, it is equivalent to $\boxed{\text{zooming-in}}$ & $\boxed{\text{renormalization}}$

Rainy

A wins $\frac{1}{4}$ $\longrightarrow$ $\frac{1}{4} / \left(\frac{1}{4} + \frac{1}{3}\right) = 3/7$

B wins $\frac{1}{3}$ $\longrightarrow$ $\frac{1}{3} / \left(1/4 + \frac{1}{3}\right) = 4/7$

Zoom-in

Re normalization

Mathematically

$P(A \mid B)$ The prob that event A happens conditioned on event B (happening).

$$= \frac{P(A \text{ AND } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

Example: Consider an unfair six-faced die X such that $P(X=k)$ is proportional to k. ($P(X=2)$ is twice of $P(X=1)$)

Q1: What is the conditional prob $P(X \geq 3 \mid X \text{ is a prime})$

Ans: Step 1: Sample space

$$S = \{1, \dots, 6\}$$

Step 2: The W.A.

$p_k = c \cdot k$ for some $c \geq 0$

$$\sum_{k=1}^{6} p_k = \sum_{k=1}^{6} ck = 1$$

$$\Rightarrow \quad 21c = 1 \qquad c = \frac{1}{21}$$

S:	1,	2,	3.	4.	5.	6
W.A	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$,	$\frac{6}{21}$

Ans to Q1: Zoom-In & renormalize over

2, 3, 5

$$\frac{2}{21} \quad \frac{3}{21} \quad \frac{5}{21} \xrightarrow{\text{Normalization}} \begin{matrix} 2 & 3 & 5 \\ 0.2, & 0.3, & 0.5 \end{matrix}$$

Count: $P(X \geq 3 \mid X \text{ is a prime})$

$$= 0.3 + 0.5 = 0.8$$

Q2: What is the conditional prob

$$P(X \text{ is a prime} \mid X \geq 3)$$

Ans to Q2: Zoom-In

$$3 \quad 4 \quad 5 \quad 6$$

$$\frac{3}{21} \quad \frac{4}{21} \quad \frac{5}{21} \quad \frac{6}{21} \xrightarrow{\text{normalization}} \frac{3}{18} \quad \frac{4}{18} \quad \frac{5}{18} \quad \frac{6}{18}$$

Count: $P(X \text{ prime} \mid X \geq 3)$

$$= \frac{3}{18} + \frac{5}{18} = \frac{8}{18}$$

Or by formula

$$P(X \geq 3 \mid X \text{ is prime})$$

$$= \frac{P(X \geq 3 \text{ and } X \text{ is prime})}{P(X \text{ is prime})}$$

$$= \frac{\frac{3}{21} + \frac{5}{21}}{\frac{2}{21} + \frac{3}{21} + \frac{5}{21}} = 0.8$$

$$P(X \text{ is prime} \mid X \geq 3)$$

$$= \frac{P(X \text{ is prime and } X \geq 3)}{P(X \geq 3)} = \frac{\frac{3}{21} + \frac{5}{21}}{\frac{3}{21} + \frac{4}{21} + \frac{5}{21} + \frac{6}{21}}$$

$$= 8/18$$

Example: Temperature X is a conti. R.V that has pdf

$$f_X(x) = \begin{cases} 0.5e^{-0.5x} & \text{if } x>0 \\ 0 & \text{otherwise} \end{cases}$$

Q1: What is the prob that

$$P(X>40 \mid X>32)\ ?$$

(Given that $X>32$, what's the conditional prob that $X>40$)

Q2: What is the prob that

$$P(X>32 \mid X>40)\ ?$$

Ans: By formula

$$P(A \mid B) = \frac{P(A\cap B)}{P(B)}$$

A1: $A: X>40$ $\quad B: X>32$.

$A\cap B: X>40$

$$P(X>40 \mid X>32) = \frac{P(A\cap B)}{P(B)}$$

$$= \frac{P(X>40)}{P(X>32)}$$

$$= \frac{\int_{40}^{\infty} 0.5e^{-0.5x}\,dx}{\int_{32}^{\infty} 0.5e^{-0.5x}\,dx} = \frac{e^{-20}}{e^{-16}}$$