MT2

Monday, November 21, 2022 10:56 AM

Q1: Since $g(1)=0$

$\Rightarrow c(1) = m(1)g(1) = 0.$

$\Rightarrow$ all codewords of $\mathbb{C}$ are ~~of~~ even weight.

Q2: Since $(x+1) \mid (x^n - 1)$

and since $p(x) \mid (x^n - 1)$

$\Rightarrow L.C.M(x+1, p(x)) \mid x^n - 1$

$\because$ $g.c.d(x+1, p(x)) = 1$
($p(x)$ is irreducible)

$\Rightarrow g(x) = (x+1) \cdot p(x) \mid x^n - 1$

$\Rightarrow \mathbb{C}$ is cyclic.

Q3 $d_{min} \leq$ weight of any row vector of G matrix

$$= \sum_{\bar{b} \in \{0,1\}^m} 1_{\{b_i = 1 \text{ for all } i \in S\}}$$

$$= 2^{m-|S|}$$

by choosing $|S| = r$

$$\Rightarrow d_{min} \leq 2^{m-r}$$

Q4 $\Phi_1(x)$ has roots $\beta^1, \beta^2, \beta^4, \beta^8 \dots$

$\Phi_4(x)$ has roots $\beta^4, \beta^8 \dots$

$$\Rightarrow \Phi_1(x) = \Phi_4(x)$$

$\Phi_3(x)$ has roots $\beta^3, \beta^6, \beta^{12}, \beta^{24} \ldots$

$\Phi_6(x)$ has roots $\beta^6, \beta^{12}, \beta^{24} \ldots$

$$\Rightarrow \Phi_3(x) = \Phi_6(x)$$

For any even $j = i \cdot 2^r$

where i is odd

and $r \geq 1$.

$\Phi_i(x)$ has roots $\beta^i, \beta^{i \cdot 2}, \beta^{i \cdot 2^2}$

$\Phi_j(x)$ has roots $\beta^{i(2^r)} \ldots$

$\Phi_j(x)$ has roots $\beta^{i(2^r)}$ $\beta^{i(2^{r+1})}$ $\cdots$

$\Rightarrow \Phi_i(x) = \Phi_j(x)$.