Q1.

$P(\hat{M}=1 \text{ or } 0 \mid M=2)$

$\leq P(\#\text{ of } 1\text{'s during } 1 \text{ to } 500$

$\text{is} \geq 250)$

$+ P(\#\text{ of } 1\text{'s during } 501 \text{ to } 1000$

$\text{is} \leq 250)$

$= 2 \cdot \left(2 \times \sqrt{p(1-p)}\right)^{500}$ #

Q2:

$$P(\overline{X}_n \geq 1) \approx (e^{-c_X})^n \approx \left(\min_{s_1 \geq 0} \frac{E(e^{s_1 X})}{e^{s_1}}\right)^n$$

and $$(e^{-c_X}) = \left(\min_{s_1 \geq 0} \frac{E(e^{s_1 X})}{e^{s_1}}\right) \quad \text{— ①}$$

$$P(\overline{Y}_n \geq 1) \approx (e^{-c_Y})^n = \left(\min_{s_2 \geq 0} \frac{E(e^{s_2 X})}{e^{s_2}}\right)^n$$

and $$(e^{-c_Y}) = \left(\min_{s_2 \geq 0} \frac{E(e^{s_2 X})}{e^{s_2}}\right)^n \quad \text{— ②}$$

$$P(\overline{Z}_n \geq 1) \approx (e^{-c_Z})^n = P\left(\frac{\sum_{i=1}^{0.5n}(X_i + Y_i)}{0.5n} \geq 2\right)$$

$$= \left(\min_{s_3 \geq 0} \frac{E(e^{s_3 X})E(e^{s_3 Y})}{e^{s_3} \cdot e^{s_3}}\right)^{0.5n}$$

$$\Rightarrow e^{-c_Z} = \left(\min_{s_3 \geq 0} \frac{E(e^{s_3 X})E(e^{s_3 Y})}{e^{s_3} \quad e^{s_3}}\right)^{0.5}$$

$$\geq (e^{-c_X} \cdot e^{-c_Y})^{0.5} \quad \text{by ① and ②}$$

take $-\log(\cdot)$

$$\Rightarrow C_Z \leq 0.5(C_X + C_Y) \quad \#$$

Q3

Statement 1: Define

$$h_3 = h_1 \circ h_2 \in H.$$

$$\Rightarrow \quad x \circ h_2 = (y \circ h_1) \circ h_2 = y\, h_3 \qquad \#$$

Statemen 2: Define

$$h_5 = h_1^{-1} \circ h_4.$$

$$\Rightarrow \quad y \circ h_4 = (x \circ h_1^{-1}) h_4 = x \cdot h_5 \qquad \#$$

Statemen3: A restatement of Statements 1 and 2.

Q4:

$$
\begin{array}{r}
110 \\
202 \\
\hline
220 \\
220 \\
\hline
22220
\end{array}
$$

$$
\begin{array}{r|l}
 & 22 \\
\hline
1022 & 22220 \\
 & 2011 \\
\hline
 & 2110 \\
 & 2011 \\
\hline
 & 102
\end{array}
$$

~~12×20=10~~

$12 \cdot 20 = 11$