Finally we like to find the error magnitude $e_{l_i} \neq 0$. (If non-binary)

Recall $Z_0(x) = S(x) \cdot \Lambda(x) \mod x^{2t}$

We can thus compute $Z_0(x)$.

Also we compute

$$\Lambda'(x) \triangleq \lambda_1 \cdot x^0 + \lambda_2 \cdot 2 \cdot x^1 + \cdots \; t_0 \cdot \lambda_{t_0} x^{t_0 - 1}$$

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But we also know

$$Z_0(x) = \sum_{j=1}^{t_0} \bar{e}_j \cdot \bar{\beta}_j \cdot \prod_{j' \neq j}^{t_0} (1 - \bar{\beta}_{j'} x) \quad \text{①}$$

$$\Lambda(x) = \prod_{j=1}^{t_0} (1 - \bar{\beta}_j x)$$

$$\Lambda'(x) = \sum_{j=1}^{t_0} (-\bar{\beta}_j) \cdot \prod_{j' \neq j}^{t_0} (1 - \bar{\beta}_{j'} x) \quad \text{②}$$

Comparing ①, ②

$$\bar{e}_j = e_{l_j} = \frac{Z_0(\bar{\beta}_j^{-1})}{-\Lambda'(\bar{\beta}_j^{-1})}$$

$\{ S_i : i = 1, \cdots, 2t \}$

$\rightarrow \Lambda(x)$ with $\deg(\Lambda(x)) = t_0$

$\rightarrow$ roots are $\beta^{k_i}$ and location indices $l_j = -k_j$

$\rightarrow Z_0(x)$ and $\Lambda'(x)$

$\rightarrow$

$$e_{l_j} = \frac{Z_0\left(\left(\beta^{l_j}\right)^{-1}\right)}{-\beta^{l_j} \cdot \Lambda'\left(\left(\beta^{l_j}\right)^{-1}\right)}$$

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