L14

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* Error Decoding Via Table-based search.

Method # 1: Standard Array.

$C = \{\vec{0}, \vec{c}_1, \cdots, \vec{c}_{2^k-1}\}$

Step 1: List all codewords as columns

$$\vec{0}, \vec{c}_1, \vec{c}_2, \cdots, \vec{c}_{2^k-1}$$
$$\vec{e}, \vec{e}+\vec{c}_1, \cdots, \vec{e}+\vec{c}_{2^k-1}$$

Step 2: Set $V_{error} = \{0, 1\}^n$

Step 3: Choose $\vec{e} \in V_{error}$ with the smallest weight

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Step 4: Insert $\vec{e}+0, \vec{e}+\vec{c}_1, \cdots, \vec{e}+\vec{c}_{2^k-1}$ as a new row.

Step 5: Remove 2^k vectors from V_{error}.

$V_{error} = V_{error} \setminus \{\vec{e}+\vec{0}, \vec{e}+\vec{c}_1, \cdots, \vec{e}+\vec{c}_{2^k-1}\}$.

& Go back to Step 3 if the new V_{error} is non-empty.

Properties: ① Each entry of the table is distinct. (A nontrivial property follows from the linearity of the code.)

② We have exactly $\frac{2^n}{2^k} = 2^{n-k}$ rows

When decoding, the decoder observes $\vec{y}_{obs}$. It searches the entire table to

$\vec{y}_{obs}$.' It searches the entire table to find the entry that contains $\vec{y}_{obs}$.

② Output the column index m. i.e. $\vec{c}_m$ is the decoded codeword.

Method 2: A simplified (more efficient) version of Method 1.

$\boxed{\text{Syndrome Decoding}}$

$$\vec{y}_{obs} = \vec{c}_m + \vec{e}$$

$$H \cdot \vec{y}_{obs} = H \cdot \vec{c}_m + H \cdot \vec{e}$$

$$= 0 + H \cdot \vec{e} \triangleq \vec{s} \quad \text{syndrome}$$

Properties: ① Every row corresponds to the same $\vec{e}$, thus the same $\vec{s}$

$\vec{s}$

② Totally 2^{n-k} rows

thus at most 2^{n-k} $\vec{s}$

③ Each row has a unique $\vec{s}$, and thus we have exactly 2^{n-k} distinct $\vec{s}$

(A non-trivial property due to the linearity of the code)

Improved Version of Method #1:

① Pre-compute the mapping $f(\vec{e}) = H \cdot \vec{e} = \vec{s}$ for all e.

② For the received $\vec{y}_{obs}$, we compute the $\vec{e}$ of the corresponding row by

$$f^{-1}(H \cdot \vec{y}_{obs}) = \vec{e}$$

③ Finally $\vec{c}_m = \vec{y}_{obs} - \vec{e}$

$$= \vec{y}_{obs} - f^{-1}(H \cdot \vec{y}_{obs})$$

Remark: We still have to have to do ~~table~~ table look-up. for f^{-1} where

$$f^{-1}: \{0,1\}^{n-k} \longmapsto \{0,1\}^{n}.$$

Comparison, the table look-up for the previous method is

$$T: \{0,1\}^n \longmapsto \{0,1\}^n$$

$$\vec{y}_{obs} \longmapsto \vec{c}_m$$

Exercise: Apply the syndrome decoder to binary Hamming Codes and q-ary Hamming Codes

Hamming Codes.