$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$l = 4$ Hamming Code $(15, 11)$

$$H = 4\left\{ \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & \\ 0 & 1 & 1 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \end{bmatrix}}_{2^4 - 1 \text{ (non-zero columns only)}} \right.$$

$\Rightarrow$ d_{min} of Hamming codes is always 3.

proof: 1 column cannot be L.I.

2 distinct columns in $\{0, 1\}^{\ell}$ cannot be L.I.

We can find 3 columns that are L.I.

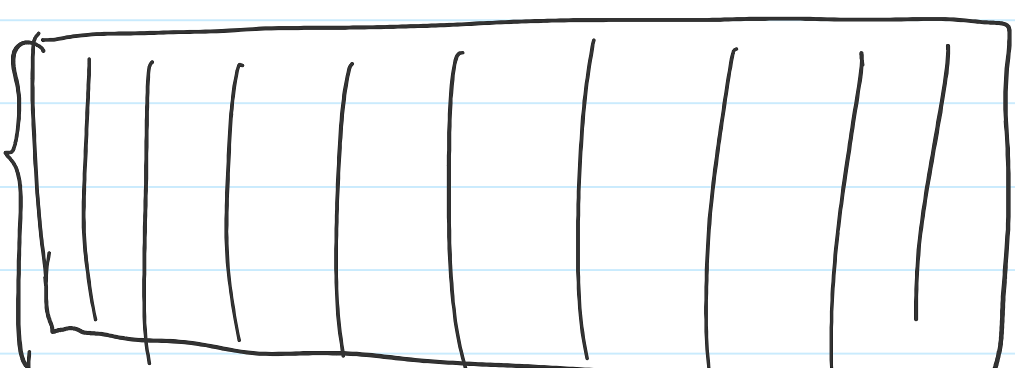
Summary of Hamming codes

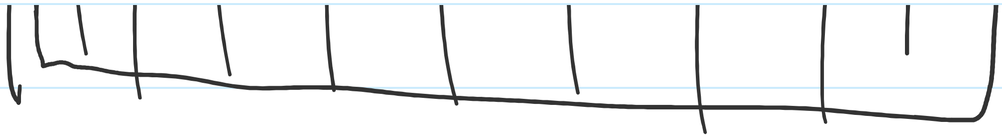
1. Decided by a single para l
2. $n-k=l$, $n=2^l-1$, $k=2^l-1-l$
3. $d_{min}=3$
4. Can correct $t=1$ error.

Extension to q-ary Hamming Codes.

* Construction by specifying the H matrix
* Decided by a single parameter l
*

$H = l$





Contains all l-dimensional non-zero vectors with the <u>leading coeff</u> $= 1$.

Totally we have $n = \frac{q^l - 1}{q - 1}$ columns.

$$k = \frac{q^l - 1}{q - 1} - l$$

* $d_{min} = 3$, can correct one position error.