HW7

Tuesday, December 13, 2022 11:12 AM

Q32. With the field generated by

x^6+x+1, a primitive element is

$\beta = 000010 = 2$.

The conjugacy classes are — Minimal polynomials

$\beta^0 \longrightarrow x+1$

$\beta^1, \beta^2, \beta^4, \beta^8, \beta^{16}, \beta^{32} \longrightarrow x^6+x+1$

$\beta^3, \beta^6, \beta^{12}, \beta^{24}, \beta^{48}, \beta^{33} \longrightarrow x^6+x^4+x^2+x+1$

$\beta^5, \beta^{10}, \beta^{20}, \beta^{40}, \beta^{17} \longrightarrow x^6+x^5+x^2+x+1$

$\beta^7, \beta^{14}, \beta^{28}, \beta^{56}, \beta^{49}, \beta^{35} \longrightarrow x^6+x^3+1$

$\beta^9, \beta^{18}, \beta^{36} \longrightarrow x^3+x^2+1$

$\beta^{11}, \beta^{22}, \beta^{44}, \beta^{25}, \beta^{50}, \beta^{37} \quad x^6+x^5+x^3+x^2+1$

$\beta^{11}, \beta^{22}, \beta^{44}, \beta^{25}, \beta^{50}, \beta^{37} \longrightarrow x^6 + x^5 + x^3 + x^2 + 1$

$\beta^{13}, \beta^{26}, \beta^{52}, \beta^{41}, \beta^{19}, \beta^{38} \longrightarrow x^6 + x^4 + x^3 + x + 1$

$\beta^{15}, \beta^{30}, \beta^{60}, \beta^{57}, \beta^{51}, \beta^{39} \longrightarrow x^6 + x^5 + x^4 + x^2 + 1$

$\beta^{21}, \beta^{42} \longrightarrow x^2 + x + 1$

$\beta^{23}, \beta^{46}, \beta^{29}, \beta^{58}, \beta^{53}, \beta^{43} \longrightarrow x^6 + x^5 + x^4 + x + 1$

$\beta^{27}, \beta^{54}, \beta^{45} \longrightarrow x^3 + x + 1$

$\beta^{31}, \beta^{62}, \beta^{61}, \beta^{59}, \beta^{55}, \beta^{47} \longrightarrow x^6 + x^5 + 1$

$t=2 \Rightarrow 4$ consecutive roots

$\beta^1, \beta^2, \beta^3, \beta^4 \quad g(x) = (x^6 + x + 1)(x^6 + x^4 + x^2 + x + 1)$

$= x^{12} + x^{10} + x^8 + x^5 + x^4 + x^3 + 1$

$$= x^{12} + x^{10} + x^{8} + x^{5} + x^{4} + x^{3} + 1$$

$\beta^{62}, \beta^{61}, \beta^{60}, \beta^{59}$ $\quad g(x) = x^{12} + x^{9} + x^{8} + x^{7} + x^{4} + x^{2} + 1$

Q33 $\quad S_1 = \beta^{54}$

$S_2 = \beta^{45}$

$S_3 = \beta^{38}$

$S_4 = \beta^{29}$

$\lambda_1 = \beta^{54} \quad \lambda_2 = \beta^{57}$

$$\Lambda(x) = 1 + \beta^{54} x + \beta^{57} x^{2}$$