HW6

Monday, November 21, 2022 10:18 AM

Q27: By indution, we only need to prove that

If $e(x) = e_0 + e_1 x + \cdots + e_{n-1} x^{n-1}$

is not a codeword of $\mathbb{C}$, then

$$e'(x) = e_{n-1} + e^0 x + \cdots + e_{n-2} x^{n-1}$$

is not a codeword of C.

Suppose $e'(x)$ is a codeword of C. Because $\mathbb{C}$ is cyclic, $e(x)$ must be a codeword of $\mathbb{C}$, which leads to a contradiction.

Q.E.D.

Q28 Suppose l_0 is the smallest such l. As $n = a \cdot l_0 + r$

for some $0 < r < l_0$.

$$\Rightarrow \vec{V} = \vec{V}^{(n)} = \vec{V}^{(a \cdot l_0 + r)} = \left(\vec{V}^{(a \cdot l_0)}\right)^{r}$$

$$= \vec{V}^{(r)}$$

which contradicts that l_0 is the smallest such l. Q.E.D

Q29: For any codeword $\vec{c_3} \in \mathcal{C}_3$

$\vec{c_3}^{(+1)} \in \mathcal{C}_2$ & $\vec{c_3}^{(+1)} \in \mathcal{C}_1$.

$\Rightarrow \vec{c_3}^{(+1)} \in \mathcal{C}_3 \Rightarrow \mathcal{C}_3$ is cyclic.

Suppose $g_1(x)$ generates $\mathcal{C}_1$.

$g_2(x)$ generates $\mathcal{C}_2$

Claim: $g_3(x) \triangleq L.C.M(g_1(x), g_2(x))$

generates $\mathcal{C}_3$.

generates C_3.

Proof:

"$\Rightarrow$" $\because$ $\begin{cases} g_1(x) \mid m_3(x) g_3(x) \\ g_2(x) \mid m_3(x) g_3(x) \end{cases}$

therefore any codeword

$m_3(x) g_3(x)$ belongs to C_3

"$\Leftarrow$" suppose $C_3(x) \in C_3$

$\Rightarrow \begin{cases} g_1(x) \mid C_3(x) \\ g_2(x) \mid C_3(x) \end{cases}$

$\Rightarrow \mathrm{LCM}(g_1(x), g_2(x)) \mid C_3(x)$

$\Rightarrow C_3(x)$ can be written as

$m_3(x) \cdot g_3(x)$ Q.E.D.

Q30: $d_{min} = 2^{m-r} = 2^3 = 8$

$\boxed{m_{\{0\}} = 0}$:

$b_0 + b_1 = 0$
$b_2 + b_3 = 0$
$b_4 + b_5 = 0$
$b_6 + b_7 = 1$
$b_8 + b_9 = 1$
$b_{10} + b_{11} = 0$
$b_{12} + b_{13} = 0$
$b_{14} + b_{15} = 0$

$\boxed{m_{\{1\}} = 1}$:

$b_0 + b_2 = 1$
$b_1 + b_3 = 1$
$b_4 + b_6 = 1$
$b_5 + b_7 = 0$
$b_8 + b_{10} = 1$
$b_9 + b_{11} = 0$
$b_{12} + b_{14} = 1$
$b_{13} + b_{15} = 1$

$\boxed{m_{\{2\}} = 0}$:

$b_0 + b_4 = 0$
$b_1 + b_5 = 0$
$b_2 + b_6 = 0$
$b_3 + b_7 = 1$
$b_8 + b_{12} = 0$
$b_9 + b_{13} = 1$
$b_{10} + b_{14} = 0$
$b_{11} + b_{15} = 0$

$\boxed{m_{\{3\}} = 0}$:

$b_0 + b_8 = 0$
$b_1 + b_9 = 1$
$b_2 + b_{10} = 0$
$b_3 + b_{11} = 0$
$b_4 + b_{12} = 0$
$b_5 + b_{13} = 0$
$b_6 + b_{14} = 0$
$b_7 + b_{15} = 1$

$m_{\{1\}} \cdot G_{\{1\}} =$

0011	0011	0011	0011

$\Rightarrow y' =$

0000	0001	0100	0000

$\Rightarrow m_\phi = 0.$

Q31 $G_1 = RM(m-r-1, m)$

$G_2 = RM(r, m).$

Prove that

$$g_1(S_1) \cdot g_2(S_2) = 0,$$

for all $|S_1| \le m-r-1$ and $|S_2| \le r.$

Since $g_1(S_1) \cdot g_2(S_2)$

$$= \sum_{\substack{\vec{b} \in \{0,1\}^m \\ \text{mod } 2}} \mathbb{1}\{ b_i = 1 \text{ for all } i \in S_1 \cup S_2 \}$$

$$= 2^{m - |S_1 \cup S_2|} \mod 2$$

since $|S_1 \cup S_2| \leq m-1$

$$\Rightarrow g_1(S_1) \cdot g_2(S_2) = 0.$$