ECE 639, Homework #6 (CRN: 25576)
Due date: Wednesday 11/16/2022 during the lecture

`https://engineering.purdue.edu/~chihw/22ECE639F/22F_ECE639.html`

Question 27: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 5.12] For a cyclic code, if an error pattern $e(x)$ is detectable, show that its i-th cyclic shift $e^{(i)}(x)$ is also detectable for all i.

Question 28: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 5.14] For any cyclic code, let $\vec{v}$ be an n-dimensional codeword of the given cyclic code, and $\vec{v}^{(l)}$ is the l-cyclically-shifted version of $\vec{v}$. Prove that if we choose $l_0 > 0$ as the smallest, non-zero l such that $\vec{v} = \vec{v}^{(l)}$, then we must have $l_0 | n$, i.e., l_0 is a factor of n.

Question 29: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 5.20] Let C_1 and C_2 be two cyclic codes of length n. Define $C_3 = C_1 \cap C_2$ as the codewords that belong to both C_1 and C_2. Prove that (1) C_3 is cyclic; and (2) Find the generator polynomial of C_3.

Question 30: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 4.7] Form the generator matrix of the RM(1,4) code of length 16. Answer the following questions:

1. What is the minimum distance of the code?
2. Decode the received vector $r = (0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1)$.

Question 31: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 4.9] Prove that the dual code of $\mathrm{RM}(m - r - 1, m)$ is $\mathrm{RM}(r, m)$.