

HW5

Tuesday, November 8, 2022 12:55 PM

Q19

$$S_0 = x_1 + x_2 + x_3 + P_0$$

$$S_1 = x_0 + x_1 + x_2 + P_1$$

$$S_2 = x_0 + x_1 + x_3 + P_2$$

$$S_3 = x_0 + x_2 + x_3 + P_3$$

Q20

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1 % Byunghyun Lee's code in fall 2022
2 clear all
3 m = [[ 0 0 0 0];[0 0 0 1];[0 0 1 0];[0 0 1 1];[0 1 0 0];[0
4 1 0 1];[0 1 1 0];[0 1 1 1];[1 0 0 0];[1 0 0 1];[1 0 1 0];[1
5 0 1 1];[1 1 0 0];[1 1 0 1];[1 1 1 0];[1 1 1 1]];
6
7 P = [0 1 1 1;1 1 1 0;1 1 0 1; 1 0 1 1];
8 G = [P,eye(4)];
9 H = [eye(4),P];
10
11 Std array = zeros(8,16,16);
12 Std array dec = zeros(16,16);
13 error basis=[[0 0 0 0 0 0 0 1];[0 0 0 0 0 0 1 0];[0 0 0 0 0
14 1 0 0];[0 0 0 0 1 0 0 0];[0 0 0 1 0 0 0 0];[0 0 1 0 0 0 0
15 0];[0 1 0 0 0 0 0 0];[1 0 0 0 0 0 0 0]];
16
17 for n=1:length(m)
18     c1(n,:)=mod(m(n,:) *G, 2);
19 end
20
21
22 Error_H2 = repmat([0 0 0 0 0 0 0 0],28,1);
23 changes = nchoosek(1:8,2);
24 ind = sub2ind([28,8],repmat((1:28)',1,2),changes);

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23 changes = nchoosek(1:8,2);
24 ind = sub2ind([28,8], repmat((1:28)',1,2), changes);
25 Error_H2(ind) = ~Error_H2(ind);
26
27
28 for n=1:length(c1(:,1))
29     for k=1:length(error basis(:,1))
30         Std array(:,k,n) =
31 mod(error basis(k,:)+c1(n,:),2)';
32         tmp = num2str(mod(error basis(k,:)+c1(n,:),2));
33         tmp(isspace(tmp)) = '';
34         Std array_dec(k,n) = bin2dec(tmp);
35     end
36 end
37
38

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39 for k = 1:8
40     for p=1:length(Error_H2(:,1))
41         error dec = num2str(Error_H2(p,:));
42         error dec(isspace(error dec)) = '';
43         error dec = bin2dec(error dec);
44         if sum(error dec == Std_array_dec,'all')==0
45             error basis(k+8,:)=Error_H2(p,:);
46             for n=1:length(c1(:,1))
47                 Std array(:,k+8,n) =
48 mod(Error_H2(p,:)+c1(n,:),2)';
49                 tmp =
50 num2str(mod(Error_H2(p,:)+c1(n,:),2));
51                 tmp(isspace(tmp)) = '';
52                 Std array_dec(k+8,n) = bin2dec(tmp);
53             end
54         end
55     end
56 end
57
58 for k = 1:15
59     synd(:,k) = mod(H * error basis(k,:),2);
60 end
61
62
63

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Q21

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H H^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow C = C^\perp$$

Q22: Define

$$E = \{x \in \mathbb{C} : w(x) \text{ is even}\}.$$

$$O = \{x \in \mathbb{C} : w(x) \text{ is odd}\}.$$

choose $\vec{o}_1 \in O$ arbitrarily.

$$\forall \vec{e} \in E, \quad \vec{e} + \vec{o}_1 \in O \text{ and}$$

it is distinct.

$$\left(\text{if not } \vec{e}_1 + \vec{o}_1 = \vec{e}_2 + \vec{o}_2 \right. \\ \left. \Leftrightarrow \vec{e}_1 = \vec{e}_2 \right)$$

$$\Rightarrow |E| \leq |O|.$$

For any $\vec{o} \in O$, $\vec{o} + \vec{o}_1 \in E$

and the output is distinct.

$$\left(\text{if not } \vec{o}_2 + \vec{o}_1 = \vec{o}_3 + \vec{o}_1 \right. \\ \left. \Leftrightarrow \vec{o}_2 = \vec{o}_3 \right)$$

$$\Rightarrow |O| \leq |E| \quad \#$$

Q23 For all $\vec{x}, \vec{y}, \vec{z}$

$$d(\vec{x}, \vec{z})$$

$$= \sum_{i=1}^n \mathbb{1}\{x_i \neq z_i\}$$

$$\leq \sum_{i=1}^n \left(\mathbb{1}_{\{x_i \neq y_i\}} + \mathbb{1}_{\{y_i \neq z_i\}} \right)$$
$$= d(\vec{x}, \vec{y}) + d(\vec{y}, \vec{z})$$

Q24. Because $(15, 11)$ code can correct any 1-bit error and is "perfect"

$$\Rightarrow 1 - P_{\text{error}} = (1-p)^{15} + \binom{15}{1} p (1-p)^{14}$$

Q25

```

1  % Byunghyun Lee's code in fall 2022
2
3  clear all
4
5  a = zeros(1,17);
6  a([17 16 3 1])= 1;
7  p = 2;
8  m = 16;
9  N = [];
10
11  for n=2:p^m-1
12
13      b = gfrepconv(n);
14      b(1) = p-1;
15
16      [q,r] = gfdeconv(b,a,p);
17
18      if sum(r)==0
19          n
20          N(length(N)+1) = n;
21          break
22      end
23  end
24

```

Q26

$$x^n - 1 = g(x) \cdot h(x)$$

$$= x^{n-k} g^*(x^{-1}) x^k h^*(x^{-1})$$

$$\Rightarrow g^*(x^{-1}) h^*(x^{-1}) = 1 - x^{-n}$$

$$\Rightarrow g^*(x) h^*(x) = -(x^n - 1)$$

$\Rightarrow g^*(x)$ generates a cyclic code.