ECE 639, Homework #5 (CRN: 25576)
Due date: Friday 11/04/2022 during the lecture

https://engineering.purdue.edu/~chihw/22ECE639F/22F_ECE639.html

Question 19: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 3.3] Consider a systematic (8,4) code with the whose parity-check equations are

$$p_0 = x_1 + x_2 + x_3; \tag{1}$$
$$p_1 = x_0 + x_1 + x_2; \tag{2}$$
$$p_2 = x_0 + x_1 + x_3; \tag{3}$$
$$p_3 = x_0 + x_2 + x_3; \tag{4}$$

Construct the functions of how to compute the syndromes s_0, s_1, s_2, and s_3.

Question 20: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 3.10] Continue from the above question. Construct and describe the syndrome decoder of the above code.

Question 21: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 3.14] Continue from the above question. Show that this code is *self-dual*. That is, $C = C^{\perp}$.

Question 22: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 3.5] Let C be a linear code with both even and odd weight codewords. Show that the number of even-weight codewords is equal to the number of odd-weight codewords.

Question 23: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 3.7] Prove that the Hamming distance satisfies the triangle inequality; that is, let $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ be three n-tuples over GF(2), and show that

$$d(\mathbf{x}, \mathbf{y}) + d(\mathbf{y}, \mathbf{z}) \geq d(\mathbf{x}, \mathbf{z}). \tag{5}$$

Question 24: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 4.4] Compute the error probability of (15,11) Hamming code over a Binary Symmetrc Channel with crossover probability $p = 10^{-2}$.

Question 25: Consider a binary CRC code generated by a generator polynomial $g(x) = x^{16} + x^{15} + x^2 + 1$. We know that for some codeword length n (unit: bits), this CRC code will be cyclic. Find the smallest of such n by writing a short MATLAB program.

Question 26: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 5.7] Consider a binary (n, k) cyclic code C generated by $g(x)$. Let

$$g^*(x) = x^{n-k} g(x^{-1}). \tag{6}$$

Show that $g^*(x)$ also generates an (n, k) cyclic code C'.