ECE 639, Homework #4 (CRN: 25576)
Due date: Wednesday 10/19/2022 during the lecture

`https://engineering.purdue.edu/~chihw/22ECE639F/22F_ECE639.html`

Question 15: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 2.4] Construct the prime field $GF(11)$ with modulo-11 addition and multiplication. Find all the primitive elements and determine the orders of other elements.

Question 16: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 2.10] Show that $X^5 + X^3 + 1$ is irreducible over $GF(2)$.

Question 17: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 2.11] let $f(X)$ be a polynomial of degree n over $GF(2)$. The reciprocal of $f(X)$ is defined as

$$f^*(X) = X^n f(X^{-1}). \tag{1}$$

1. Prove that $f^*(X)$ is irreducible over $GF(2)$ if and only if $f(X)$ is irreducible over $GF(2)$.
2. Prove that $f^*(X)$ is primitive if and only if $f(X)$ is primitive.

Question 18: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 2.18] Consider a finite field $GF(2^4)$ generated by $1+X+X^4$. Let $\alpha = 2 = 0010 \in GF(2^4)$. (One can easily verified that α is a primitive element.) Divide the polynomial $f(X) = \alpha^3 X^7 + \alpha X^6 + \alpha^7 X^4 + \alpha^2 X^2 + \alpha^{11} X + 1$ over $GF(2^4)$ by the polynomial $g(X) = X^4 + \alpha^3 X^2 + \alpha^5 X + 1$ over $GF(2^4)$. Find the quotient and the remainder of the division.