ECE 639, Homework #3 (CRN: 25576)
Due date: Wednesday 10/05/2022 during the lecture

`https://engineering.purdue.edu/~chihw/22ECE639F/22F_ECE639.html`

Question 10: Implement MATLAB programs that performs the following tasks.

1. Take any generating matrix as input and output the set of all legitimate codewords.
2. Take the following two inputs: The conditional probabilities $p_{0\to 0}$, $p_{0\to 1}$, $p_{1\to 0}$, and $p_{1\to 1}$ of an i.i.d. binary-input/binary-output channel; The observation vector $\overrightarrow{y}$. Implement the codeword-wise ML decoder that outputs the most likely codeword.
3. Take a generating matrix as input, plot the frame error rate (FER) and the bit error rate (BER) of the corresponding code assuming that we use the codeword-wise ML decoder.

Use your program to compare the BER and FER of the following two codes described by their *parity-check* matrices. We assume the channel model being a binary symmetric channel with cross over probability p. (That is, $p_{0\to 0} = 1-p$, $p_{0\to 1} = p$, $p_{1\to 0} = p$, and $p_{1\to 1} = 1-p$.)

The Hamming code with the following parity-check matrix:

$$H_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}. \tag{1}$$

The Reed-Muller code with the following parity-check matrix:

$$H_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}. \tag{2}$$

Specifically, please plot the FER and BER curves of the above two codes for the range $p = 0.01$ to $p = 0.5$.

Question 11: Repeat the previous question by focusing on the bit-wise ML decoder instead.

Question 12: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 2.5] Let m be a positive integer. If m is not a prime, prove that the set $\{0, 1, \cdots, m-1\}$ is not a field under modulo-m addition and multiplication.

Question 13: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 2.6] Consider the integer group $G = \{0, 1, \cdots, 31\}$ under modulo-32 addition. Show that $H = \{0, 4, 8, 12, 16, 20, 24, 28\}$ forms a subgroup of G.

Question 14: [Lin, Costello Jr., Error Control Coding 2nd Ed., Problem 2.9] Solve the following simultaneous equations of X, Y, Z, and W with modulo-2 arithmetic:

$$X + Y + W = 1, \tag{3}$$
$$X + Z + W = 0, \tag{4}$$
$$X + Y + Z + W = 1, \tag{5}$$
$$Y + Z + W = 0. \tag{6}$$