ECE 695C, Midterm #2

6:30-7:30pm Wednesday, November 18, 2009, ME 312,

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. Some time-consuming and bonus questions are assigned a small number of points. Make sure you manage your time well during the exam.
4. There are 2 questions and 8 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

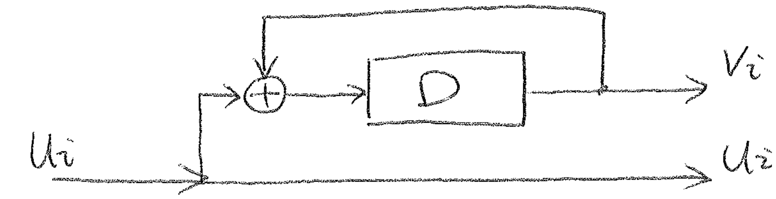
Student ID:

E-mail:

Signature:

Question 1: [70%]

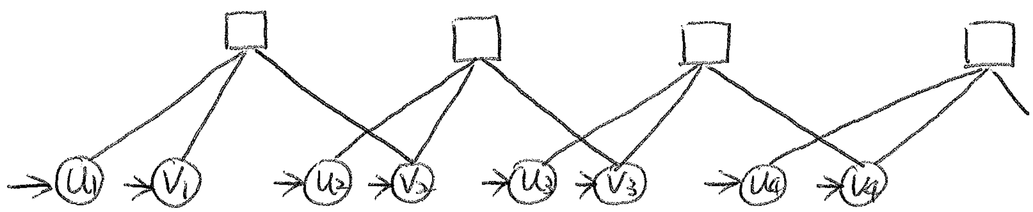
Consider the following turbo code that consists of two Recursive Systematic Codes (RSCs) and each RSC is described by the following figure with a single shift register.



1. [10%] What is the rate of the given RSC? What is the rate of the turbo code?

This question will use density evolution (DE) to analyze the performance of the given turbo code performance on binary erasure channels (BECs) with erasure probability ϵ.

2. [10%] Prove/show that the given RSC is equivalent to an LDPC code with the following factor graph.



3. [10%] Draw the factor graph of an LDPC code that is equivalent to the turbo code by combining the two RSCs. Your factor graph should have two halves. The upper half captures the first RSC/LDPC code and the lower half captures the second RSC/LDPC code.

4. [Advanced, 20%] [This sub-question is long. After finishing reading this sub-question and the included hints, you may want to proceed to Q1.5 first.]

   Turbo decoding is slightly different from LDPC decoding in the following sense. Instead of performing message-passing (MP) decoding in parallel, the MP decoding is performed for the upper half first and then performed for the lower half. Namely, the turbo decoding schedule is [Var.+Chk. in the upper half] $\longrightarrow$ [Var.+Chk. in the lower] $\longrightarrow$ [Var.+Chk. in the upper] $\longrightarrow \cdots$.

   For comparison, an LDPC decoding schedule is [All Var.] $\longrightarrow$ [All Chk.] $\longrightarrow$ [All Var.] $\longrightarrow \cdots$.

   In this question, we consider only "turbo decoding." Based on the observation, write down the justification for the following two DE formulas, where each p represents an erasure probability.

   - Between RSCs:

$$p_{u\to c}^{[t+1]} = \epsilon \cdot \left(1 - \left(1 - p_{v\to c}^{[t]}\right)^2\right) \tag{1}$$

- Within a RSC:

$$p_{v\to c}^{[t]} = \epsilon \cdot \left(1 - \left(1 - p_{v\to c}^{[t]}\right)\left(1 - p_{u\to c}^{[t]}\right)\right) \tag{2}$$

$$\text{or equivalently} \quad p_{v\to c}^{[t]} = \frac{\epsilon \cdot p_{u\to c}^{[t]}}{1 - \epsilon \cdot \left(1 - p_{u\to c}^{[t]}\right)}. \tag{3}$$

Hint 1: When t is odd, $p_{u\to c}^{[t]}$ and $p_{v\to c}^{[t]}$ correspond to the erasure probability of the messages in the upper half LDPC code. When t is even, $p_{u\to c}^{[t]}$ and $p_{v\to c}^{[t]}$ correspond to the erasure probability of the messages in the lower half LDPC code.

Hint 2: $p_{u\to c}^{[t]}$ denotes the erasure probability of the message from u nodes to check nodes. $p_{v\to c}^{[t]}$ denotes the erasure probability of the message from v nodes to check nodes.

5. [10%] Sketch a generalized EXIT chart such that the x-axis is $p_{v\to c}$ and the y-axis is $p_{u\to c}$ when $\epsilon = 0.1$. Your EXIT chart should contain two curves. For each curve, arbitrarily choose 3 points and mark their coordinates.

6. [5%] In the traditional EXIT chart, we would like to "escape" to the upper right corner. In this generalized EXIT chart, to which corner we would like to escape?

7. [Advanced, 5% + 10% bonus] Show that the asymptotic threshold ϵ^* of the turbo code (predicted by the DE formulas) is no larger than 0.5.

   Namely, show that when $\epsilon > 0.5$, the decoder is not "stable," in the sense that when $p_{u\to c}^{[t]}$ and $p_{v\to c}^{[t]}$ are small, there is no open tunnel in the generalized EXIT chart if $\epsilon > 0.5$.

Question 2: [30%] Consider an irregular LDPC code with codeword length 10^5 bits, which has 50000 variables of degree 3 and 50000 variable nodes of degree 4, and 35000 check nodes of degree 10.

1. [10%] Write down the variable and check node degree polynomials $\lambda(x)$ and $\rho(x)$. What is the rate of this LDPC code?

2. [10%] For a BEC with erasure probability ϵ, write down the density evolution formula in terms of $\lambda(x)$ and $\rho(x)$, and $p_{v\to c}^{[t]}$, the erasure probability of the v-to-c messages in the t-th round.

3. [10%] Consider a regular $(3, 6)$ code. When the erasure probability $p_{v\to c}^{[t]}$ is very small, by Taylor's expansion, the DE formula can be approximated by

$$p_{v\to c}^{[t+1]} = \epsilon \left(5p_{v\to c}^{[t]}\right)^2. \tag{4}$$

   Use the above approximation to show that if for some t_0, we have

$$p_{v\to c}^{[t_0]} \leq \frac{1}{25\epsilon}, \tag{5}$$

   then we must also have

$$p_{v\to c}^{[t]} \leq \frac{1}{25\epsilon} e^{-2^{t-t_0}}, \forall t \geq t_0. \tag{6}$$

   Namely, the convergence rate is doubly exponential.

4. [Bonus 10%] Show that although the DE predicts only the bit error rate, for regular (3,6) codes, the convergence of the bit-error rate implies the convergence of the frame error rate.

   Hint: A few hand-waving sentences should be sufficient for this subquestion.