ECE 695C, Homework #6, due date: 3/27/2012

https://engineering.purdue.edu/∼chihw/12ECE695C/12ECE695C.html

Question 1: Go to the course website and download the HMAT.mat and the YOBS.mat files, the former of which contains the parity-check matrix of a regular (3,6) LDPC code of codeword length $n = 512$ while the latter of which is the observation vector $\overrightarrow{y}_{obs}$ of size 512.

Suppose the BPSK modulation is used with i.i.d. Gaussian noise and 3dB signal-to-noise ratio. Namely, $Y_i = (-1)^{X_i} + \frac{1}{\sqrt{2}} N_i$ where N_i is i.i.d. standard Gaussian. Use the sum-product decoder with 80 iterations to recover the uncorrupted codeword. Print the first 20 bits of the decoded codeword.

Question 2: Consider an irregular bipartite-graph LDPC code ensemble with the edge-based degree distributions:

$$\lambda(x) = 0.41667x + 0.16667x^2 + 0.41667x^5$$
$$\text{and} \quad \rho(x) = x^5.$$

1. Find the code rate of this irregular code ensemble.
2. Implement the BEC-based density evolution analysis in MATLAB and use binary search to identify the largest erasure probability ϵ^* that this code can correct.
3. How far is the performance this code away from the Shannon capacity?

   Hint: For any BEC with erasure probability ϵ, find out the Shannon capacity by computing the mutual information $I(X;Y)$. The Shannon capacity should be $C(\epsilon)$, a function of ϵ. For a code of rate R, the corresponding Shannon (erasure) threshold is $\epsilon_{\text{Shannon}}$ that satisfies $C(\epsilon_{\text{Shannon}}) = R$. That is, $\epsilon_{\text{Shannon}}$ is the largest possible erasure probability value for which a rate-R code can still decode successfully. We are interested in the gap to the capacity, which is usually defined as $\frac{\epsilon_{\text{Shannon}} - \epsilon^*}{\epsilon_{\text{Shannon}}}$.
4. How to design a code of length 1000 bits that has (approximately) the above degree distributions? Namely, how many variable nodes will be of degree 2? How many variable nodes will be of degree 3? So on and so forth.