Interconnect Modeling

Modeling of Interconnects

- Interconnect R, C and L computation
- Interconnect models
  - Lumped RC model
  - Distributed circuit models
  - Higher-order waveform in distributed RCL trees
- Accuracy and fidelity
Interconnect Resistance Computation

Resistance \( R_\square \cdot \frac{L}{W} \)

Maxwell’s Equation of Differential Form

- Maxwell’s equation for static electric field:
  \[ \nabla \cdot (\varepsilon \cdot E) = \rho \]
  - In Cartesian coordinate system, gradient operator is
    \[ \nabla = \frac{\partial}{\partial x} \hat{n}_x + \frac{\partial}{\partial y} \hat{n}_y + \frac{\partial}{\partial z} \hat{n}_z \]
  - \( \varepsilon \): permittivity of the field region
  - \( E \): strength of electric field
  - \( \rho \): charge density of the field region
Poisson’s Equation

\[ \nabla \cdot (\varepsilon \cdot E) = \rho \]

• Base on \( E = -\nabla \psi \), the Poisson’s equation is
\[ \nabla \cdot (\varepsilon \cdot \nabla \psi) = -\rho \]
- \( \varepsilon \): permittivity of the field region
- \( E \): strength of electric field
- \( \rho \): charge density of the field region

Laplace’s Equation

\[ \nabla \cdot (\varepsilon \cdot \nabla \psi) = -\rho \]

• If assume \( \rho = 0 \) and homogenous dielectric, potential \( \psi \) satisfies Laplace’s equation
\[ \nabla \cdot (\varepsilon \cdot \nabla \psi) = \varepsilon \cdot \nabla^2 \psi = 0 \]
\[ \Delta \psi = 0 \]
- \( \Delta = \nabla^2 \) is Laplacian operator
- In Cartesian coordinate system, Laplace’s equation is
\[ \frac{\partial^2 \psi}{\partial^2 x} + \frac{\partial^2 \psi}{\partial^2 y} + \frac{\partial^2 \psi}{\partial^2 z} = 0 \]
Exact Extraction of Resistance

- Solving Laplace’s equation, $\nabla^2 \Psi = 0$
- Relaxation method:
  - Potential at every point is the average potential of its neighbors
  - Consider square grids
    $$\Psi(x, y) = \left[\Psi(x+1, y) + \Psi(x-1, y) + \Psi(x, y+1) + \Psi(x, y-1)\right]/4$$
  - Point on an edge, mirror the potential about the edge, e.g., right vertical edge
    $$\Psi(x, y) = \left[2\Psi(x-1, y) + \Psi(x, y+1) + \Psi(x, y-1)\right]/4$$
  - Repeatedly replace the potential of all points with the average of their neighbors

Equipotentials and Current Flow

Current flow  potential
Break Lines for Various Shapes

Potential Distribution for Parallel and Diagonal Contacts
### Equivalent Perpendicular Contacts

$$I_p = \min(W_r, L)$$  
$$\Delta = I_p / 2$$

### Resistance Computation for Non-rectangular Shapes

[Horowitz-Dutton, T-CAD July 83]

- **Rectangle**: 
  $$R = \frac{L}{W}$$

- **Square**: 
  $$R = \frac{4L}{L + 4W_i}$$

- **Isosceles Triangle**: 
  $$R = \frac{2L}{L + 2W_i}$$

*All lines are rectilinear or diagonal (45°)*
Calculating the Resistance of a Convex Region

Resistances for Non-rectangular Shapes (cont’d)

<table>
<thead>
<tr>
<th>Shape</th>
<th>Ratio</th>
<th>Resistance</th>
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</thead>
<tbody>
<tr>
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<td>1</td>
</tr>
<tr>
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<td>1.5</td>
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<tr>
<td>B</td>
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<tr>
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<tr>
<td>C</td>
<td>1.5</td>
<td>2.11</td>
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<tr>
<td>C</td>
<td>2</td>
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<tr>
<td>C</td>
<td>3</td>
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<tr>
<td>D</td>
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<tr>
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### Accuracy of Resistance Extraction

<table>
<thead>
<tr>
<th>Shape</th>
<th>Ratio</th>
<th>Extracted Resistance</th>
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<th>%Error</th>
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<td>2.71</td>
<td>2.65</td>
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</table>

Exact resistances are obtained by solving Laplace’s equation.

### Current Spreading from a Small Contact

[Diagram showing current spreading from a small contact]
### Typical sheet resistance for conductors

<table>
<thead>
<tr>
<th>Material</th>
<th>Sheet Resistance Ω/SQ.</th>
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<tbody>
<tr>
<td></td>
<td>Min.</td>
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<tr>
<td>Metal[Al]</td>
<td>0.03</td>
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<tr>
<td>Silicides</td>
<td>2.00</td>
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<tr>
<td>Diffusion (n+ and p+)</td>
<td>10.00</td>
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<tr>
<td>Polysilicon</td>
<td>15.00</td>
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</tbody>
</table>

### What’s Capacitance?

- Simplest model: parallel-plate capacitor
  - It has two parallel plates and homogeneous dielectric between them
  - The capacitance is \( C = \varepsilon \frac{A}{d} \)
    - \( \varepsilon \): permittivity of dielectric
    - \( A \): area of plate
    - \( d \): distance between plates
  - The capacitance is the capacity to store charge
    - charge at each plate is \( Q = CV \)
    - one is positive, the other is negative
• For multiple conductors of any shapes and materials, and in any dielectric, there is a capacitance between any two conductors

• Mutual capacitance between m1 and m2 is \( C_{12} = \frac{q_1}{v_2} \)
  – \( q_1 \) is the charge of m1
  – \( v_1 = 0 \) and \( v_3 = 0 \)

**Capacitance Matrix**

• Capacitance is often written as a symmetric matrix

\[
C = \begin{pmatrix}
c_{11} & -c_{12} & -c_{13} \\
-c_{21} & c_{22} & -c_{23} \\
-c_{31} & -c_{32} & c_{33}
\end{pmatrix}
\]

• \( c_{ii} = \sum_{j=1}^{m} c_{ij} (j \neq i) \) is the self-capacitance for a conductor,
  – e.g., \( c_{11} = c_{12} + c_{13} \)

• The charge is given by \( Q^m = C^{\text{sym}} (V^m)^T \)
  – e.g., \( q_i = c_{i1}v_1 - c_{i2}v_2 - c_{i3}v_3 = c_{i2}(v_1 - v_2) + c_{i3}(v_1 - v_3) \)
Physical Design Domain

- Conductors: metal wire, via, polysilicon, substrate
- Dielectrics: SiO₂, ...

- Total cap for a wire
  - delay, power
- Mutual cap between wires
  - signal integrity

3D Model

- 3D model
  - Wires with finite width, thickness and length
  - Compute self-cap and mutual cap

- 2D model
  - Wires with finite width and thickness, but infinite length along the 3rd dimension
  - Compute the unit-length cap (also called cap coefficient)
Approximation of 3D Structure by a 2D Model

- Pick a 3rd dimension
- Chop 3D structure into sections with distinguish profiles

Approximation of 3D structure via 2D Model

- Solve each distinguish profile via 2D model
  - two types of profiles with unit-length cap $c_1$ and $c_2$

- Total cap is $c_1(L_1+L_3)+c_2L_2$
Approximation via Quasi-3D Model

• 2D model + effects of sidewalls along the 3rd dimension
  – a correction capacitance for each sidewall

Classification (orthogonal to 3D/2D)

• Numerical method
  – accurate
  – any geometric structures
  – extremely expensive
• Formula-based method
  – efficient
  – limited accuracy and geometric structures
  – insight into dependency on design parameters
• Table lookup
  – accuracy -> numerical method
  – efficiency -> analytical method
  – More flexible handling of geometric structures than analytical method
Framework for Numerical Method

1. Assume voltage \([0, 0, \ldots, 1, \ldots, 0]\), i.e., only conductor \(i\) has unit voltage
2. Compute charge \(q_i\) for every conductor \(j\)
3. Obtain mutual cap \(c_{ij} = q_i\) and self-cap (sum of mutual cap)
4. Iterate through steps 1-3 using different voltage assignments

\[
V^m = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
C^{mm} = \begin{bmatrix}
0 & -c_{12} & 0 & 0 \\
-c_{21} & 0 & -c_{23} & -c_{24} \\
0 & -c_{32} & 0 & 0 \\
0 & -c_{42} & 0 & 0
\end{bmatrix}
\]

Solutions to Conductor Charge

- Differential-based
  - use Maxwell’s equation in differential form
- Integral-equation based
  - use Maxwell’s equation in integral form
Capacitance Extraction - Electrostatic Analysis

- For $m$ conductors solve $m$ potential problems for the conductor surface charges.
- Each problem has one conductor at 1V, the rest at zero potential.

Two Views of Capacitance

Can derive $C_{12}$ from $C_{12}$, $C_{1\infty}$, and $C_{2\infty}$.

- Set up a system of charge neutrality equations:
  $$
  \begin{bmatrix}
  C_{1\infty} + C_{12} & -C_{12} \\
  -C_{12} & C_{2\infty} + C_{12}
  \end{bmatrix}
  \begin{bmatrix}
  1 + V \\
  V
  \end{bmatrix}
  +
  \begin{bmatrix}
  q \\
  -q
  \end{bmatrix} = 0
  $$

- Solve for $V$ and $q$, then $C_{12} = q$.
Volume Methods -
Finite-Differences/Finite-Elements

- Solve Laplace’s equation in the conductor exterior
  - Approximate derivatives by finite-differences
  - Conductors Provide potential boundary conditions
  - Voltage one on conductor 1, zero on conductor 2
- 2-D example

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{m} - \psi_{i}}{x_{m} - x_{i}} - \frac{\psi_{j} - \psi_{i}}{x_{j} - x_{i}} + \frac{\psi_{i} - \psi_{j}}{0.5((x_{m} - x_{i}) + (x_{j} - x_{i}))} - \frac{\psi_{i} - \psi_{j}}{0.5((x_{i} - x_{j}) + (x_{j} - x_{i}))} = 0
\]

Volume Methods Generate Sparse Matrices
- One equation for each grid node
- In 3-D, each equation involves at least 7 variables
- Solve by matrix solution methods
  - Sparse Gaussian Elimination
  - Incomplete Cholesky-Conjugate Gradient Method (ICCG)
  - Multigrid methods
Two Different Boundary Conditions

- **Closed box**: Overestimates self C, underestimates coupling C
- **Open Box**: Underestimates self C, overestimates coupling C

Components of Interconnect Capacitance

- Classification based on profiles of interacting interconnects
  - **Area component**
    - Due to overlapping area of two interconnects on different layers
  - **Fringing component**
    - Due to side-walls of an interconnect and the surface of an interconnect on a different layer
  - **Lateral component**
    - Due to side-walls of two interconnects on the same layer
Capacitance: The Parallel Plate Model

\[ C_{\text{int}} = \frac{\varepsilon_{\text{ox}} W L}{t_{\text{ox}}} \]

\[ S_{C,\text{wire}} = \frac{S \times S_L}{S} = S_L \]

Parallel-Plate Capacitor

Laplace’s equation is

\[ \Delta \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 \]

\[ \Rightarrow \psi(x) = k_1 x + k_2 \]

\[ \psi(0) = k_1 \cdot 0 + k_2 = V \]

\[ \psi(d) = k_1 \cdot d + k_2 = 0 \]

\[ \Rightarrow \psi(x) = -\frac{V}{d} x + V \]
Parallel-Plate Capacitor

\[ \psi(x) = -\frac{V}{d} x + V \]

\[ \Rightarrow \text{unit-area charge} = -\varepsilon \cdot \nabla \psi(x) = \varepsilon \cdot \frac{V}{d} \]

\[ \Rightarrow \text{unit-area cap} = \frac{\varepsilon}{d} \]

\[ \Rightarrow \text{parallel-plate cap} = \varepsilon \frac{A}{d} \]

Typical Capacitance/Unit Area for 1µm CMOS

<table>
<thead>
<tr>
<th>Interconnect Layer</th>
<th>fF/µm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polysilicon to Substrate</td>
<td>0.058 ± 0.004</td>
</tr>
<tr>
<td>Metal1 to Substrate</td>
<td>0.031 ± 0.001</td>
</tr>
<tr>
<td>Metal2 to Substrate</td>
<td>0.015 ± 0.001</td>
</tr>
<tr>
<td>Metal3 to Substrate</td>
<td>0.010 ± 0.001</td>
</tr>
<tr>
<td>N⁺ Diffusion to Substrate (@ 0 Volt)</td>
<td>0.36 ± 0.02</td>
</tr>
<tr>
<td>P⁺ Diffusion to Substrate (@ 0 Volt)</td>
<td>0.46 ± 0.06</td>
</tr>
</tbody>
</table>
Fringing Capacitance

\[
F = \varepsilon_{\text{ox}} \cdot L \left[ \ln \left( 1 + \frac{2t_{\text{ox}}}{H} + \sqrt{\frac{2t_{\text{ox}}}{H} \left( \frac{2t_{\text{ox}}}{H} + 2 \right)} \right) + \frac{W - H/2}{t_{\text{ox}}} \right]
\]

Fringing and Area Capacitances

\[
C = \varepsilon_{\text{ox}} \cdot L \left[ \left( \frac{W}{t_{\text{ox}}} \right)^{0.77} + 1.06 \left( \frac{W}{t_{\text{ox}}} \right)^{0.25} + 1.06 \left( \frac{H}{t_{\text{ox}}} \right)^{0.5} \right]
\]
Typical Fringing Capacitance/Unit Length for 1 µm CMOS

<table>
<thead>
<tr>
<th>Interconnect Layer</th>
<th>fF/µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polysilicon to Substrate</td>
<td>0.043 ± 0.004</td>
</tr>
<tr>
<td>Metal1 to Substrate</td>
<td>0.044 ± 0.001</td>
</tr>
<tr>
<td>Metal2 to Substrate</td>
<td>0.035 ± 0.001</td>
</tr>
<tr>
<td>Metal3 to Substrate</td>
<td>0.033 ± 0.001</td>
</tr>
</tbody>
</table>

Fringing-Field Effect

![Fringing-Field Effect Graph](image)
Interwire Capacitance

Coupling Capacitances

- Area component
- Fringing component
  \[ C_{X-fringing} = C_f \times L \times (e^{-x_1/x_0} - e^{-x_2/x_0}) \]
  - \( x_0 \) measures how fringe capacitance varies for incremental length of the fringing surface
- Lateral component
  \[ C_{X-lateral} \propto \frac{1}{d} \]
Interwire Capacitance for 1μm CMOS

<table>
<thead>
<tr>
<th></th>
<th>Area fF/μm$^2$</th>
<th>Fringing fF/μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal1 to Polysilicon</td>
<td>0.055</td>
<td>0.049</td>
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<tr>
<td>Metal2 to Polysilicon</td>
<td>0.022</td>
<td>0.040</td>
</tr>
<tr>
<td>Metal2 to Metal1</td>
<td>0.035</td>
<td>0.046</td>
</tr>
</tbody>
</table>

Impact of Interwire Capacitance
Significance of Coupling Capacitance

![Graph showing the significance of coupling capacitance](image)

Capacitance Crosstalk

Assume $C_X = 25\ fF$

For 5x5mm overlap of X and Y

- $C_{XY,\text{area}} = 5 \times 5 \times 0.055\ fF$
- $C_{XY,\text{fringe}} = 2 \times 5 \times 0.049\ fF$
- $C_{XY} = 1.9\ fF$

$\Delta V_X = \frac{C_X}{C_X + C_{XY}} \times 5V$

$= 0.35V$
Coupling Noise

L_{\text{max}} (\text{mm})

10\% \text{Vdd} 2x \text{min. Spacing}

10\% \text{Vdd} \text{ min. Spacing}

Technology (\mu \text{m})

0.25 0.18 0.15 0.13 0.1 0.07