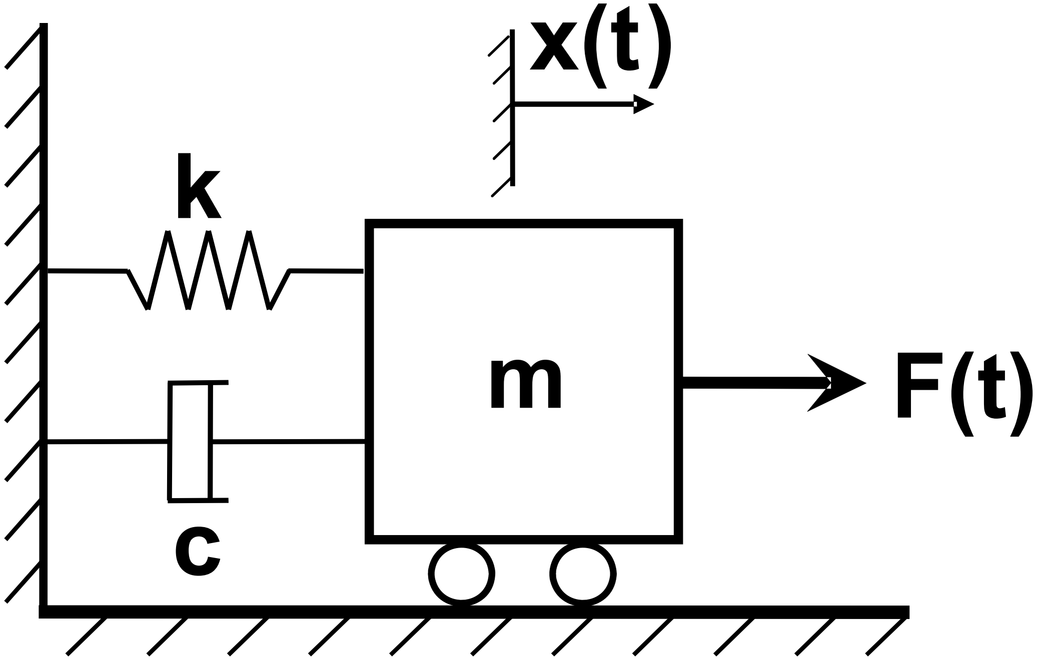
NUMERICAL SOLUTION BASED ON APPROXIMATING DERIVATIVES

constant average-acceleration method

~nonlinear analysis~



k^* = 2000 lb/ft

m = 2 lb/(ft/sec/sec)

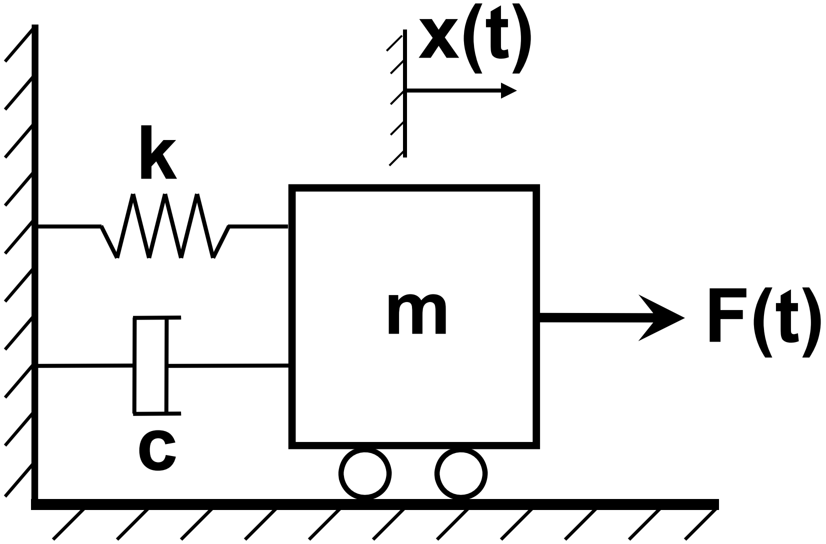
c = 0 lb/(ft/sec)

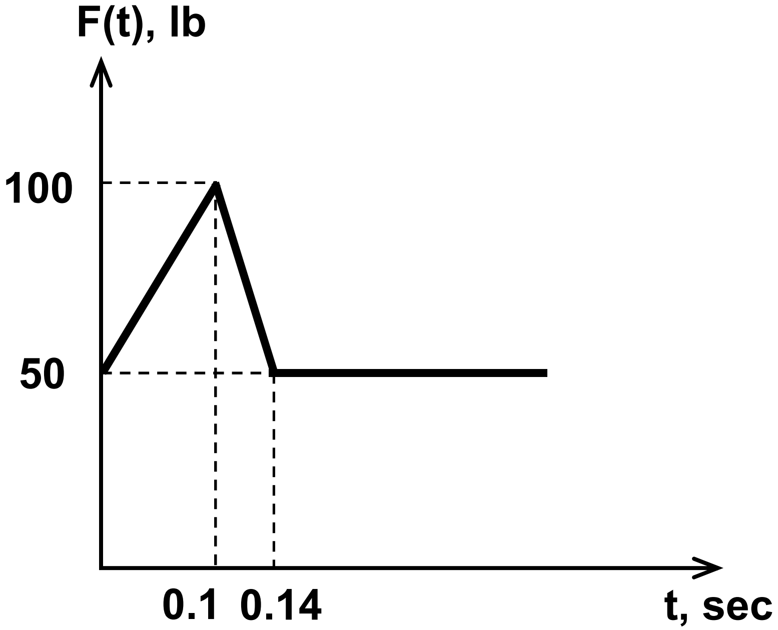
x(0) = 0 ft

v(0) = 0 ft/sec

Δt = 0.01 sec

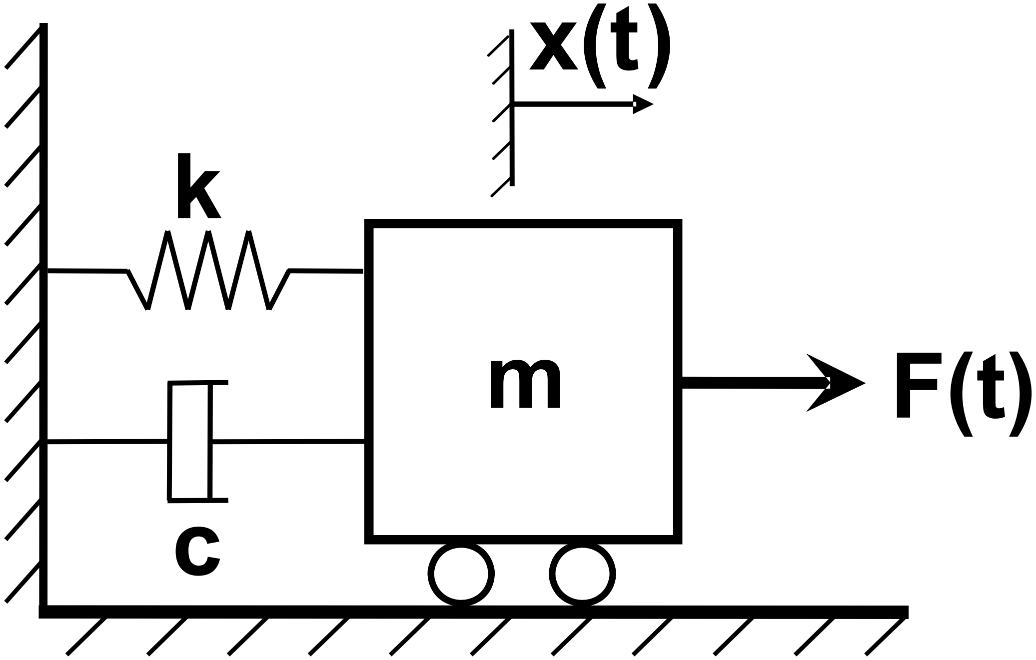
(*) Spring is elasto-plastic w/ maximum resistance 110 lb


x(t)
k
m
F(t)
c


F(t), lb
100
50
0.1
0.14
t, sec

Linear elastic spring case

k=2000 lb/ft



k = 2000 lb/ft

m = 2 lb/(ft/sec/sec)

c = 0 lb/(ft/sec)

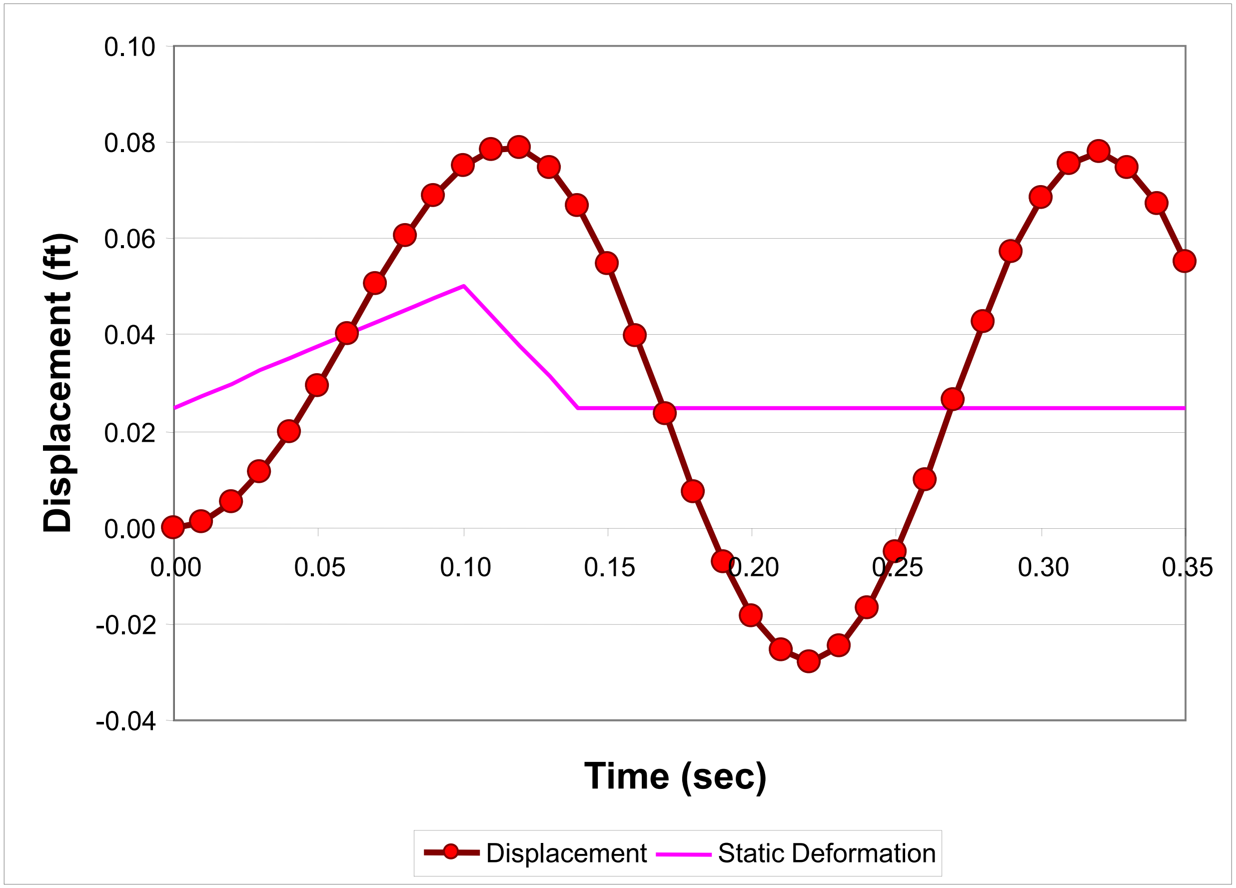
x(0) = 0 ft

v(0) = 0 ft/sec

Δt = 0.01 sec

t_i (sec)	**F_i (lb)**	**a_i (ft/sec/sec)**	**v_i (ft/sec)**	**x_i (ft)**	**ΔF*_i**	**k***	**Δx_i**	**Δv_i**	**Δa_i**	**xstatic_i**	**fspring**
0.00	50	25.00	0.00	**0.00**	105.0	82000.0	0.00	0.26	1.22	0.025	0.0
0.01	55	26.22	0.26	**0.00**	314.8	82000.0	0.00	0.26	-1.34	0.0275	2.6
0.02	60	24.88	0.51	**0.01**	513.8	82000.0	0.01	0.23	-3.77	0.03	10.2
0.03	65	21.12	0.74	**0.01**	682.7	82000.0	0.01	0.18	-5.83	0.0325	22.8
0.04	70	15.29	0.92	**0.02**	805.0	82000.0	0.01	0.12	-7.32	0.035	39.4
0.05	75	7.97	1.04	**0.03**	868.8	82000.0	0.01	0.04	-8.10	0.0375	59.1
0.06	80	-0.12	1.08	**0.04**	867.8	82000.0	0.01	-0.04	-8.08	0.04	80.2
0.07	85	-8.21	1.04	**0.05**	802.2	82000.0	0.01	-0.12	-7.28	0.0425	101.4
0.08	90	-15.49	0.92	**0.06**	678.3	82000.0	0.01	-0.18	-5.77	0.045	121.0
0.09	95	-21.26	0.74	**0.07**	508.2	82000.0	0.01	-0.23	-3.70	0.0475	137.5
0.10	100	-24.96	0.50	**0.07**	291.0	82000.0	0.00	-0.30	-9.80	0.05	149.9
0.11	87.5	-34.76	0.21	**0.08**	13.0	82000.0	0.00	-0.38	-6.41	0.04375	157.0
0.12	75	-41.16	-0.17	**0.08**	-316.4	82000.0	0.00	-0.42	-2.39	0.0375	157.3
0.13	62.5	-43.56	-0.60	**0.07**	-664.8	82000.0	-0.01	-0.43	1.86	0.03125	149.6
0.14	50	-41.70	-1.02	**0.07**	-985.9	82000.0	-0.01	-0.36	12.02	0.025	133.4
0.15	50	-29.68	-1.38	**0.05**	-1223.3	82000.0	-0.01	-0.22	14.92	0.025	109.4
0.16	50	-14.76	-1.60	**0.04**	-1341.4	82000.0	-0.02	-0.07	16.36	0.025	79.5
0.17	50	1.60	-1.67	**0.02**	-1328.6	82000.0	-0.02	0.10	16.20	0.025	46.8
0.18	50	17.80	-1.57	**0.01**	-1186.1	82000.0	-0.01	0.25	14.47	0.025	14.4
0.19	50	32.27	-1.32	**-0.01**	-928.0	82000.0	-0.01	0.38	11.32	0.025	-14.5
0.20	50	43.59	-0.94	**-0.02**	-579.3	82000.0	-0.01	0.47	7.06	0.025	-37.2
0.21	50	50.65	-0.47	**-0.03**	-174.1	82000.0	0.00	0.52	2.12	0.025	-51.3
0.22	50	52.77	0.05	**-0.03**	248.1	82000.0	0.00	0.51	-3.03	0.025	-55.5
0.23	50	49.75	0.56	**-0.02**	646.1	82000.0	0.01	0.46	-7.88	0.025	-49.5
0.24	50	41.87	1.02	**-0.02**	981.0	82000.0	0.01	0.36	-11.96	0.025	-33.7
0.25	50	29.91	1.38	**0.00**	1220.3	82000.0	0.01	0.22	-14.88	0.025	-9.8
0.26	50	15.02	1.60	**0.01**	1340.5	82000.0	0.02	0.07	-16.35	0.025	20.0
0.27	50	-1.32	1.67	**0.03**	1329.9	82000.0	0.02	-0.09	-16.22	0.025	52.6
0.28	50	-17.54	1.57	**0.04**	1189.6	82000.0	0.01	-0.25	-14.51	0.025	85.1
0.29	50	-32.05	1.33	**0.06**	933.2	82000.0	0.01	-0.38	-11.38	0.025	114.1
0.30	50	-43.43	0.95	**0.07**	585.8	82000.0	0.01	-0.47	-7.14	0.025	136.9
0.31	50	-50.57	0.48	**0.08**	181.2	82000.0	0.00	-0.52	-2.21	0.025	151.1
0.32	50	-52.78	-0.04	**0.08**	-241.1	82000.0	0.00	-0.51	2.94	0.025	155.6
0.33	50	-49.84	-0.55	**0.07**	-639.8	82000.0	-0.01	-0.46	7.80	0.025	149.7
0.34	50	-42.04	-1.01	**0.07**	-976.1	82000.0	-0.01	-0.36	11.90	0.025	134.1
0.35	50	-30.14	-1.37	**0.06**	-1217.2	82000.0	-0.01	-0.23	14.84	0.025	110.3

Max spring force = max deflection * k = ~0.08 ft * 2000 lb/ft = ~ 160 lb



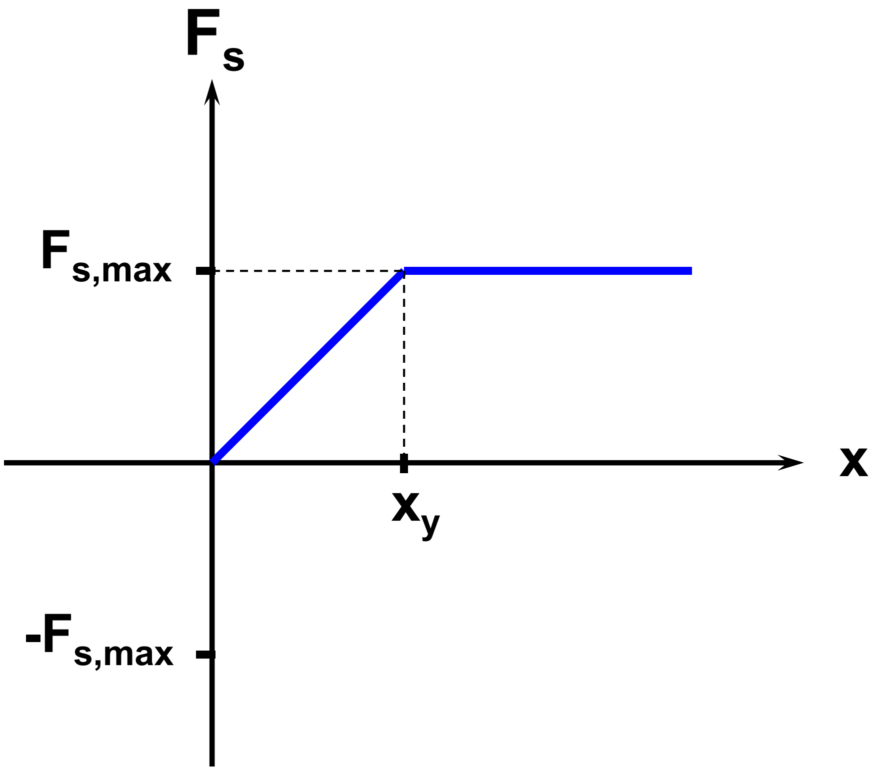
Linear elastic – plastic spring case

k^*=2000 lb/ft

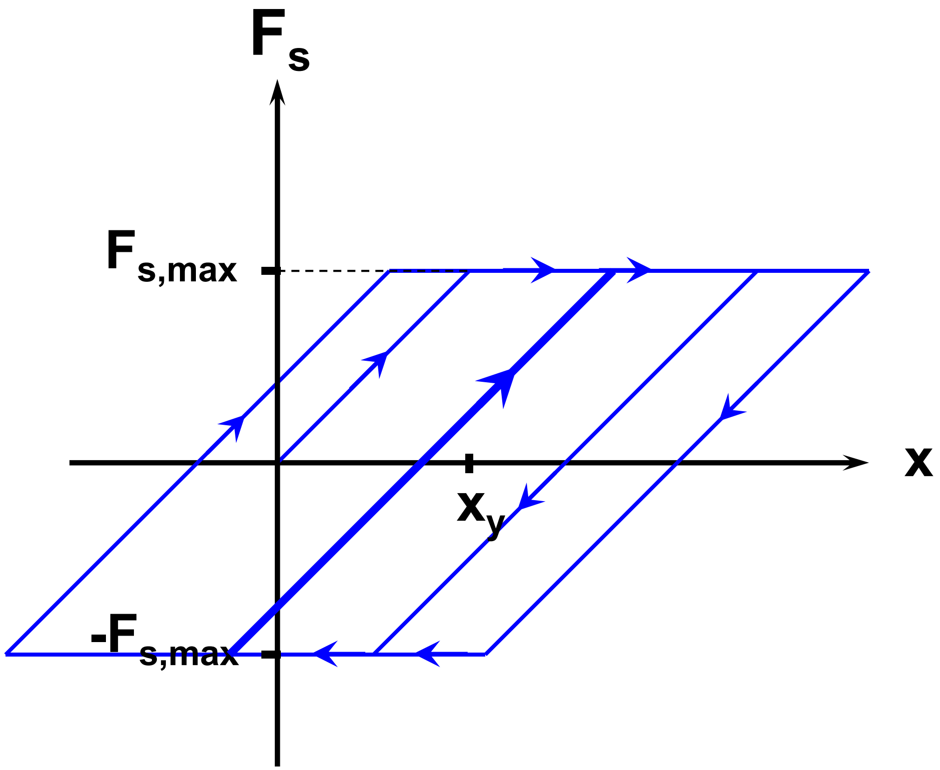
$F_{s,max}$ = 110 lb

→ spring yield def. = 0.055 ft

Elasto-Plastic Spring



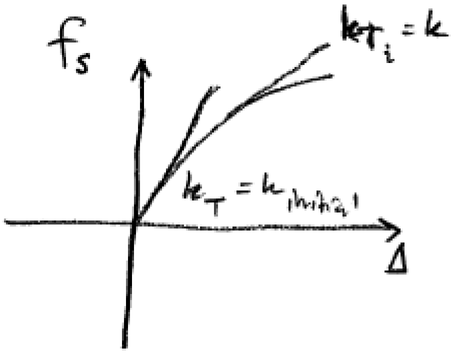
Elasto-Plastic Spring



NON-LINEAR RESPONSE

RESISTING ELEMENT IS PUSHED BEYOND ITS ELASTIC LIMIT

$$\underset{\substack{\uparrow\\ m\ddot{x}}}{f_I}(t) + \underset{\substack{\uparrow\\ c\dot{x}}}{f_D}(t) + \underset{\substack{\uparrow\\ \text{nonlinear spring}}}{f_S}(t) = F(t)$$



$$m\ddot{x}_i + c\dot{x}_i + f_s(t_i) = F_i$$

$$m\ddot{x}_{i+1} + c\dot{x}_{i+1} +$$

$$m\Delta\ddot{x}_i + c\Delta\dot{x}_i +$$

assume that

constant and is

$$\boxed{f_S(t_i + \Delta t_i) - f_S(t_i) = k_{T_i}\,\Delta x_i}$$

use the constant average-acceleration (or trapezoidal, Newmark ½, ¼)
but replace k w/ k_{T_i} (in step 2)
also need to use proper values for ΔF_i (in step 3) which
depend on $f_s(t_i+\Delta t_i)$, $f_s(t_i)$

REVISED STEPS

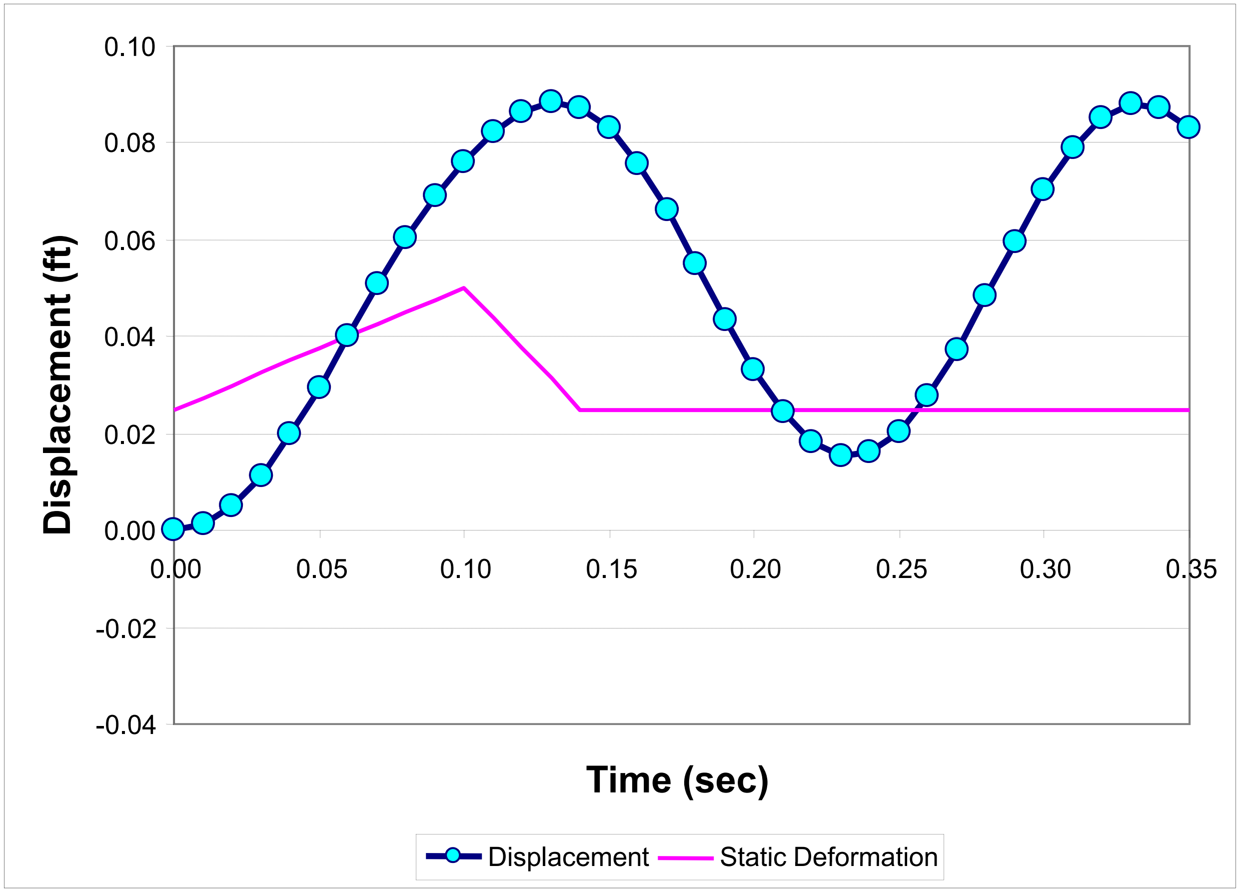
<u>step 2</u>

$$k^* = k_T + \left(\frac{2 \cdot c}{\Delta t_i}\right) + \left(\frac{4m}{\Delta t_i^2}\right)$$

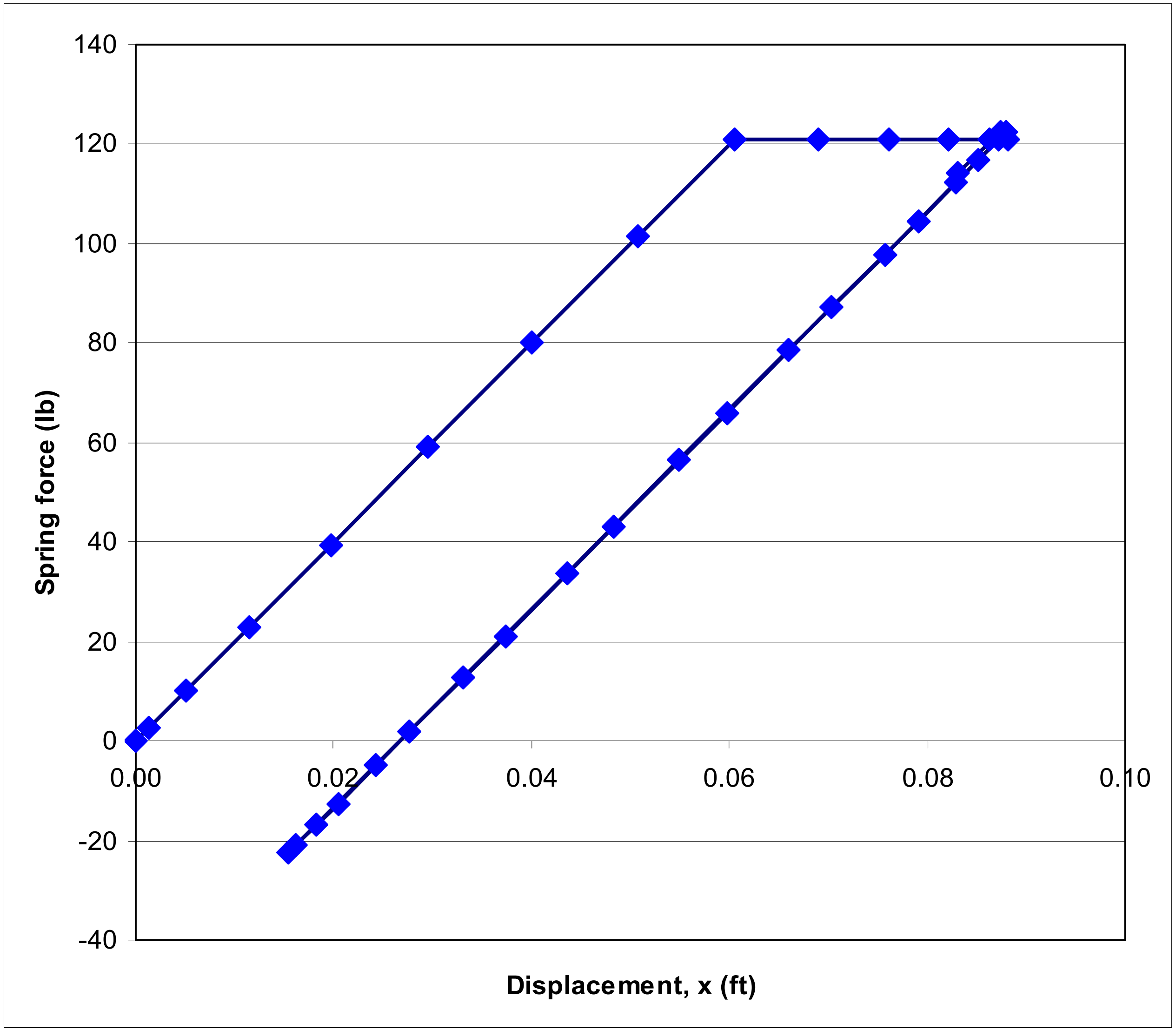
<u>step 3</u>

$$\Delta F_i^* = \Delta F_i + \left(\frac{4m}{\Delta t_i} + 2c\right)\dot{x}_i + 2m\ddot{x}_i$$

t_i (sec)	F_i (lb)	a_i (ft/sec/sec)	v_i (ft/sec)	x_i (ft)	ΔF*_i	k*	Δx_i	Δv_i	Δa_i	xstatic_i	fspring
0.00	50	25.00	0.00	**0.000**	105.0	82000	0.00	0.26	1.22	0.025	0
0.01	55	26.22	0.26	**0.001**	314.8	82000	0.00	0.26	-1.34	0.0275	3
0.02	60	24.88	0.51	**0.005**	513.8	82000	0.01	0.23	-3.77	0.03	10
0.03	65	21.12	0.74	**0.011**	682.7	82000	0.01	0.18	-5.83	0.0325	23
0.04	70	15.29	0.92	**0.020**	805.0	82000	0.01	0.12	-7.32	0.035	39
0.05	75	7.97	1.04	**0.030**	868.8	82000	0.01	0.04	-8.10	0.0375	59
0.06	80	-0.12	1.08	**0.040**	867.8	82000	0.01	-0.04	-8.08	0.04	80
0.07	85	-8.21	1.04	**0.0507**	802.2	82000	0.01	-0.12	-7.28	0.0425	101
0.08	90	-15.49	0.92	**0.060**	678.3	80000	0.01	-0.14	2.50	0.045	121
0.09	95	-12.99	0.78	**0.069**	574.3	80000	0.01	-0.12	2.50	0.0475	121
0.10	100	-10.49	0.66	**0.076**	472.9	80000	0.01	-0.14	-6.25	0.05	121
0.11	87.5	-16.74	0.52	**0.082**	339.0	80000	0.00	-0.20	-6.25	0.04375	121
0.12	75	-22.99	0.32	**0.086**	155.1	80000	0.00	-0.26	-6.25	0.0375	121
0.13	62.5	-29.24	0.06	**0.088**	-78.8	80000	0.00	-0.32	-6.25	0.03125	121
0.14	50	-35.49	-0.26	**0.087**	-350.2	82000	0.00	-0.33	4.27	0.025	121
0.15	50	-31.22	-0.59	**0.083**	-600.0	82000	-0.01	-0.28	7.32	0.025	112
0.16	50	-23.90	-0.87	**0.076**	-791.2	82000	-0.01	-0.19	9.65	0.025	98
0.17	50	-14.25	-1.06	**0.0660**	-905.2	82000	-0.01	-0.09	11.04	0.025	79
0.18	50	-3.21	-1.15	**0.0550**	-930.9	82000	-0.01	0.02	11.35	0.025	56
0.19	50	8.14	-1.12	**0.044**	-865.8	82000	-0.01	0.13	10.56	0.025	34
0.20	50	18.70	-0.99	**0.033**	-716.2	82000	-0.01	0.23	8.73	0.025	13
0.21	50	27.43	-0.76	**0.024**	-496.8	82000	-0.01	0.30	6.06	0.025	-5
0.22	50	33.49	-0.45	**0.018**	-228.8	82000	0.00	0.35	2.79	0.025	-17
0.23	50	36.28	-0.10	**0.015**	61.4	82000	0.00	0.36	-0.75	0.025	-23
0.24	50	35.53	0.25	**0.016**	345.7	82000	0.00	0.33	-4.22	0.025	-21
0.25	50	31.32	0.59	**0.020**	596.2	82000	0.01	0.28	-7.27	0.025	-13
0.26	50	24.05	0.87	**0.028**	788.6	82000	0.01	0.19	-9.62	0.025	2
0.27	50	14.43	1.06	**0.037**	904.0	82000	0.01	0.09	-11.02	0.025	21
0.28	50	3.40	1.15	**0.048**	931.2	82000	0.01	-0.02	-11.36	0.025	43
0.29	50	-7.95	1.12	**0.060**	867.6	82000	0.01	-0.13	-10.58	0.025	66
0.30	50	-18.53	0.99	**0.070**	719.4	82000	0.01	-0.23	-8.77	0.025	87
0.31	50	-27.31	0.76	**0.079**	500.9	82000	0.01	-0.30	-6.11	0.025	105
0.32	50	-33.41	0.46	**0.085**	233.6	82000	0.00	-0.35	-2.85	0.025	117
0.33	50	-36.26	0.11	**0.088**	-56.5	80000	0.00	-0.36	0.00	0.025	123
0.34	50	-36.26	-0.25	**0.087**	-346.6	82000	0.00	-0.34	4.23	0.025	123
0.35	50	-32.04	-0.59	**0.083**	-602.9	82000	-0.01	-0.28	7.35	0.025	114


0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
Displacement (ft)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Time (sec)
Displacement
Static Deformation

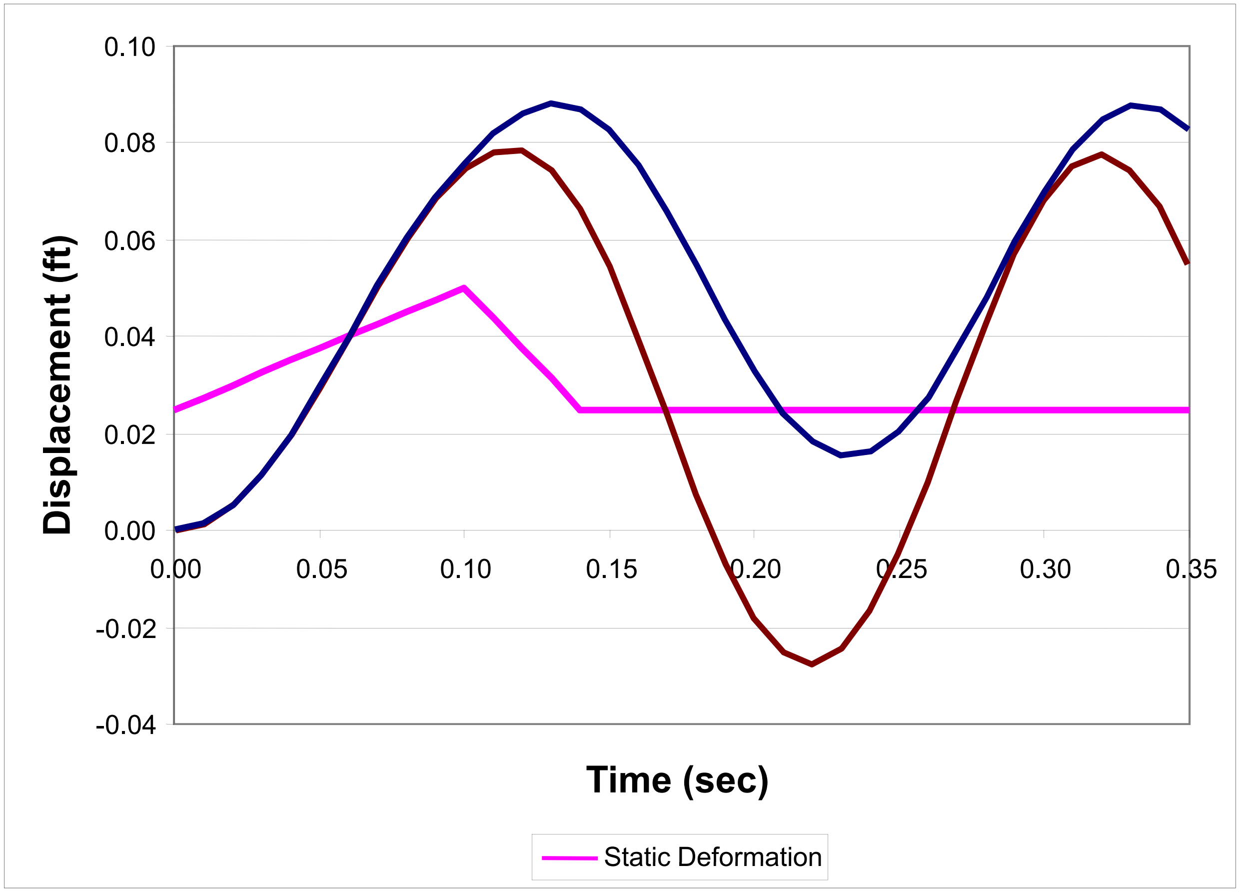
Elasto-Plastic Spring

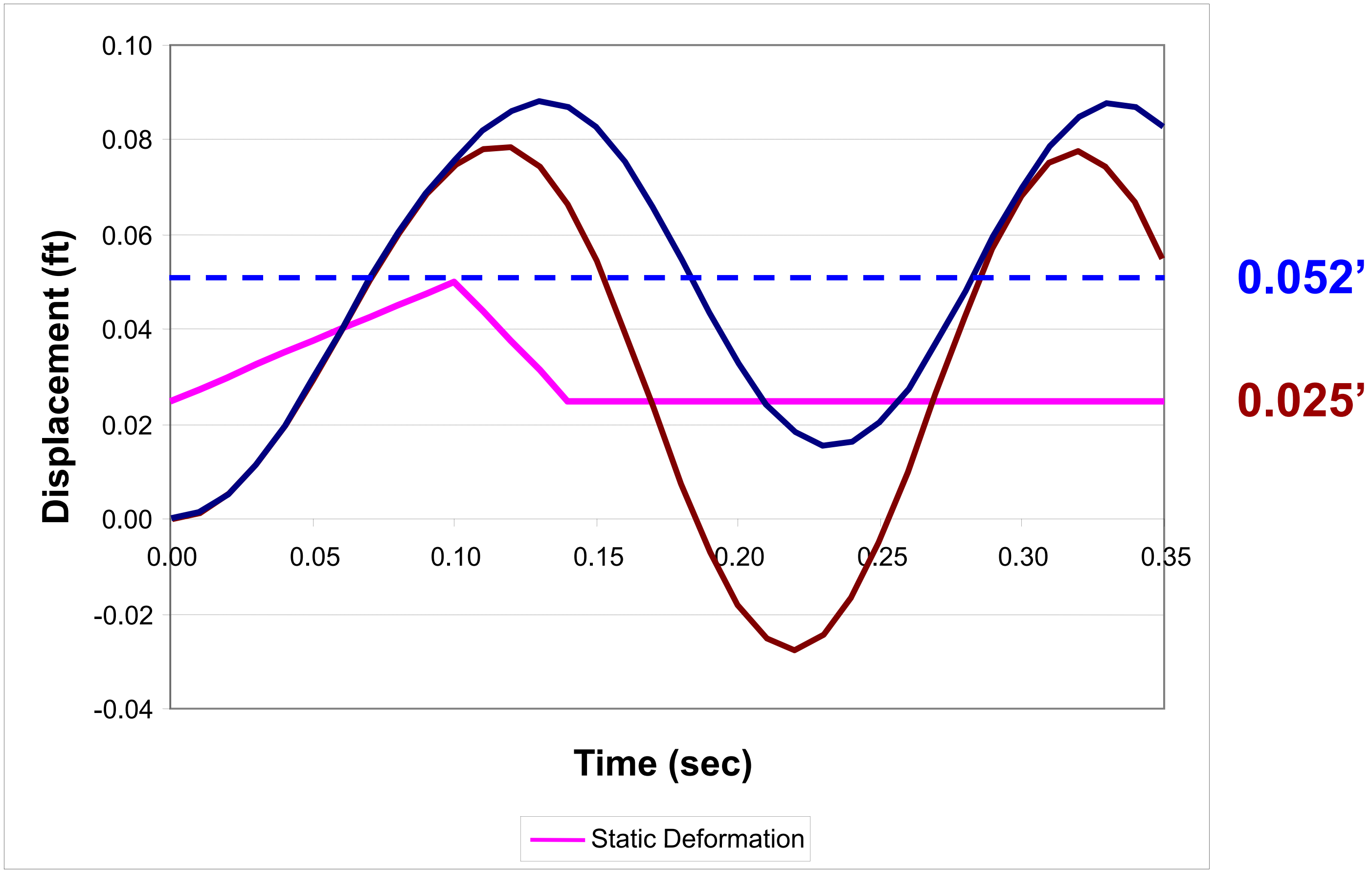


Linear elastic

vs.

Linear elastic-plastic
(elasto-plastic)


0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
Displacement (ft)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Time (sec)
Static Deformation



Max deflections:

Linear elastic: ~0.08'

Elasto-plastic: ~0.09'

Max spring forces:

Linear elastic: ~160 lb

Elasto-plastic: ~110 lb