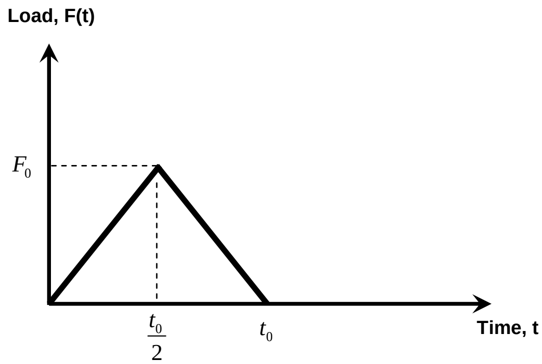
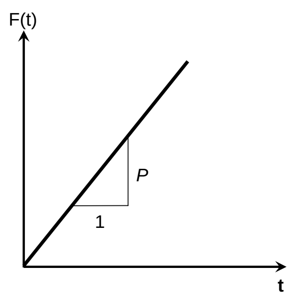
Selection of term exam questions from past years

1) An undamped single degree of freedom system that has mass m and stiffness k is subject to a triangular load shown below. Assume that the system is at rest initially.

   Find the expression for the displacement response of the SDOF system for $t \geq t_0$. Do not assume that the loading is impulsive.



Aside:



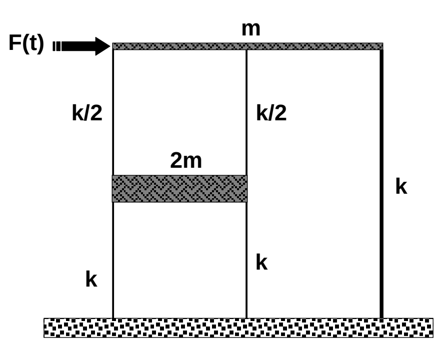
$$x(t) = \frac{P}{k}\left[t - \frac{\sin(w_n t)}{w_n}\right]$$

2)

   a) Write the Fourier series expansion for $p(t) = \frac{2}{5}\cos(2.5t) + 9 \cdot [\sin(4t) - \frac{2}{3}]$.

   b) What is the base-frequency (a.k.a. fundamental harmonic) of *p(t)*?

3) The sketch below shows a planar frame structure that has a mezzanine slab twice as heavy as its roof slab. The structure is supported by an array of columns with stiffnesses as shown on the sketch. Assume that the slabs are rigid and the axial deformations in the columns are negligible; ignore damping.



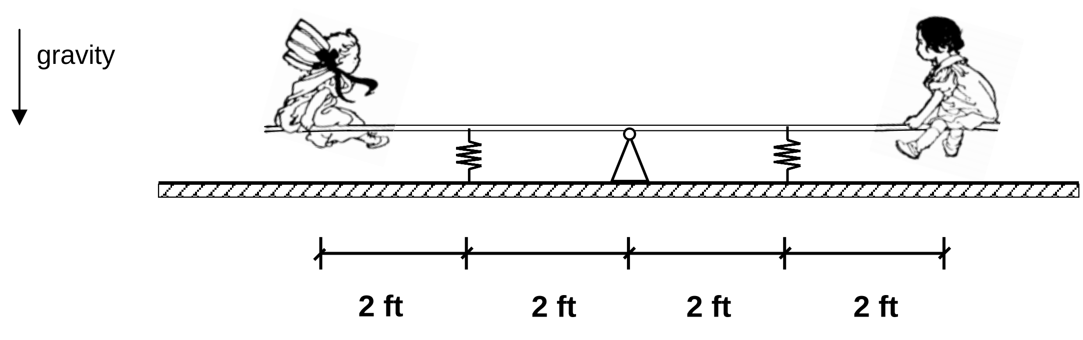
   a) What is the minimum number of coordinates, i.e. degrees of freedom, you need to express the dynamic behavior of this structure properly?

b) Write the differential equation(s) of motion when a lateral load F(t) is acting on the structure as shown above.
c) Find the natural period(s) of the system.
d) Find and sketch the modeshape(s) of the system.
e) A lateral point load is applied quasi-statically (i.e. the loading is increased from zero to its maximum value very slowly) at the roof level such that the roof reaches a displacement of unit amount relative to the ground. At that instant, what will be the displacement of the mezzanine level relative to the ground?
f) If the structure is released from the final displaced shape it has acquired in part (e), what will be the expressions for the displacements of the mezzanine and the roof levels while they are vibrating freely? Assume that at the time of the release, the structure was not moving (i.e. the slabs were not moving).

3) Two kids are playing on a seesaw as sketched below. The main element of the seesaw is a massless, rigid plank. At the mid-point of the plank is a pin support (fulcrum point). Two linear-elastic springs (each with 120 lb/in stiffness coefficient) connect the plank to the ground at half-way between the kids' seats and the pin support. Assume that each kid weighs 40 lb.

a) What is the minimum number of degrees of freedom one needs to express the dynamics of this system properly?

Assuming small displacements,
b) Write the differential equation(s) of motion.
c) Find the natural period(s).
d) Find and sketch the modeshape(s).

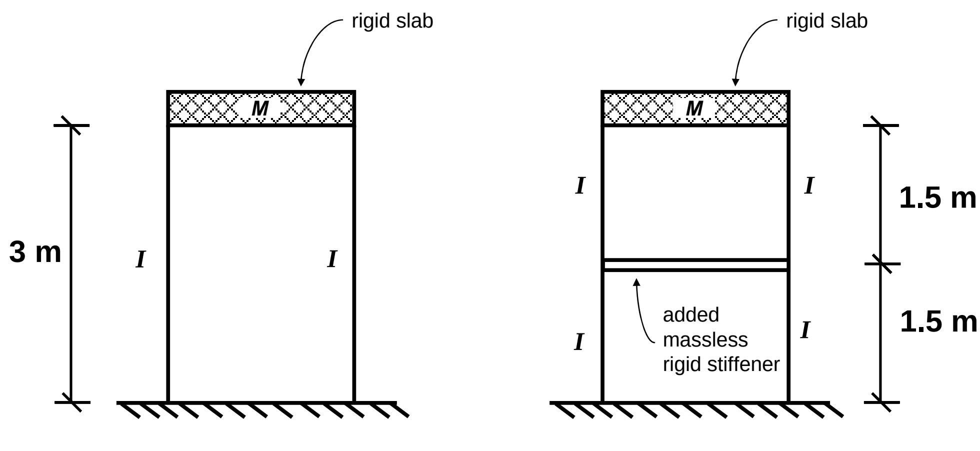


4) Soon after the construction of a single-story structure, shown below as the "as-built structure", the designer finds a major mistake in the design seismic displacement estimate. Upon reanalyzing the structure, the designer calculates that the 0.4 sec period, 5% of the critical damped "as-built" structure has a roof drift ratio greater than 1% under the design-basis earthquake. In other words, the designer calculates that at the roof level the structure sways more than 3 cm (= 300 cm x 1%), and as such, poses

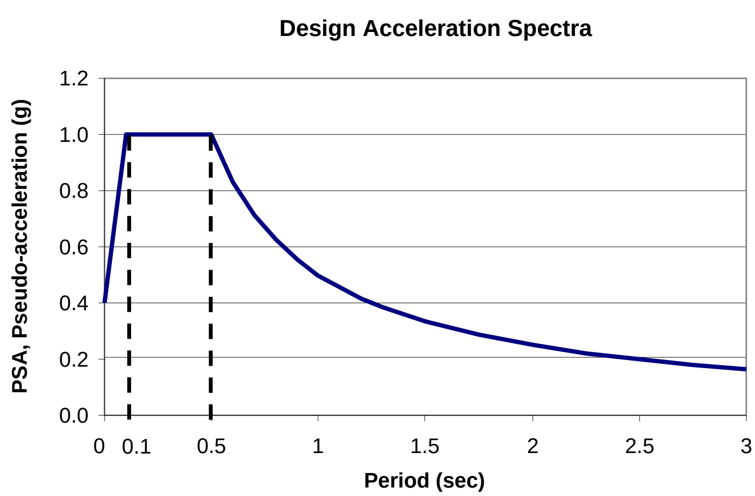
pounding hazard to neighboring buildings. To fix the problem, the designer suggests installing a massless rigid stiffener linking the columns at mid-height (see sketch shown as the "retrofitted structure"). The rigid stiffener is connected to the columns rigidly, i.e. columns do not rotate at the stiffener level.

As a reviewer, you are asked to check both claims. Using the provided design acceleration spectra, which is for 5% of the critical damped structure like the structure being reviewed,

a. check if the "as-built" structure has a roof drift exceeding 3 cm;
b. check if the suggested "retrofitted structure" has a roof drift less than 3 cm.



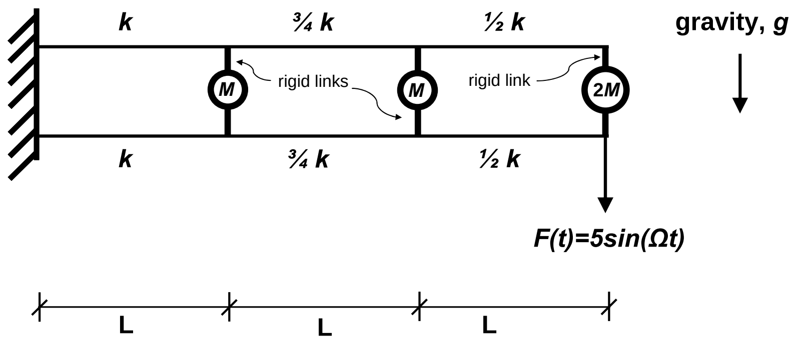
[*as-built structure*]   [*retrofitted structure*]



e) A three-bay cantilever built-up structure is shown below. Consider only flexural response in the beams, i.e. ignore axial and shear behavior. Three rigid links connecting the massless upper and lower beams are shown on the sketch. Note that the masses are concentrated on the rigid links. A source of harmonic excitation exists at the tip of the cantilever. Assume that there is no damping.

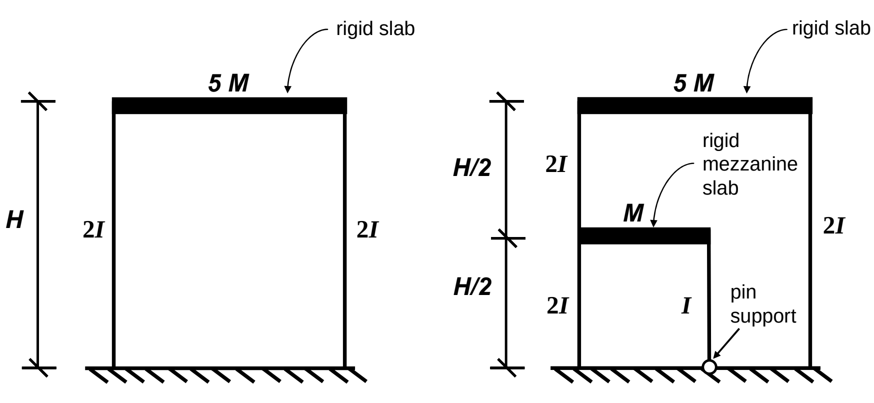
a) Write the set of differential equations of motion for the structure under the given forced setting, including the loads induced by the gravity. (Do not solve the equations.)

b) Using Rayleigh's quotient method and taking deformed shape under gravity loading applied quasi-statically (otherwise known as mass proportional loading) as your guessed modeshape, estimate the period of the fundamental mode of the structure. Do not revise your guessed modeshape, i.e. do a one-shot guess and do not iterate.



5) A single-story warehouse (labeled as "original structure" below) has been converted to an artist's loft after installing a mezzanine level (see "renovated structure" shown below). Assume that both the roof slab (mass 5M) and mezzanine level slab (mass M) are rigid and are the only elements with substantial mass in the structure. Consider response in flexure only; ignore axial and shear deformations in the columns. Note that the mezzanine slab is connected to the columns rigidly, i.e. it prevents them from rotating at the mezzanine slab level. The relative values of modulus of elasticity for the steel columns are shown on the sketch. Note that the new column that has modulus of elasticity I has a pin-support at its ground level end.

a. Find the natural periods of the renovated structure. (Do not use Rayleigh's quotient approach.) How much has the fundamental mode period changed from the original structure?
b. Find and sketch the modeshapes of the renovated structure.



[*original structure*] [*renovated structure*]