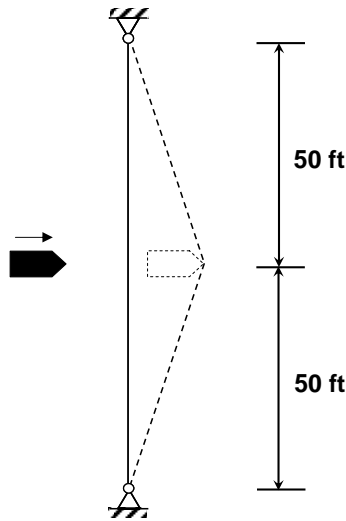


## CE573 – Structural Dynamics [Fall 2008]

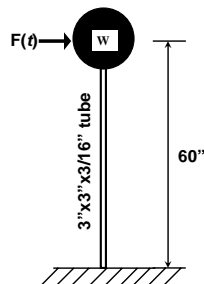
- 1) A rigid vehicle weighing 2000 lb, moving horizontally at a velocity of 12 ft/sec, is stopped by a barrier consisting of wire ropes stretched between two rigid anchors 100 ft apart. The wire ropes have a total cross-sectional area of  $1.25 \text{ in}^2$  and a modulus of elasticity of 26,000 ksi, and are stretched to an initial tension of 1000 lb. The vehicle moves normal to the barrier and strikes it at mid-height. Find the maximum deflection of the barrier. Assume ideal conditions, that is, a rigid vehicle, weightless barrier, no friction, no damping, and perfectly elastic ropes.



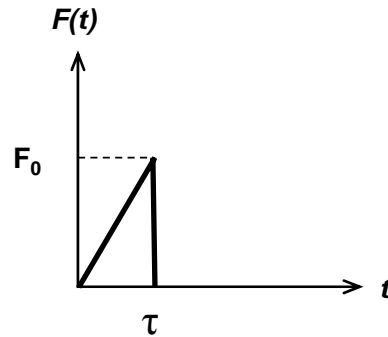
- 2) A 60-in long 3 in x 3 in x 3/16 in tube cantilever structure supports a 2000-lb weight attached at the tip. The properties of the tube are as follows: cross-sectional area,  $A = 2.02 \text{ in}^2$ ; moment of inertia,  $I = 2.60 \text{ in}^4$ ; section modulus,  $S = 1.73 \text{ in}^3$ ; and, modulus of elasticity,  $E = 29,500 \text{ ksi}$ .

The system is subjected to a sinusoidal force at the tip, acting horizontally in one of the planes of symmetry. The force has an amplitude of 250 lb and oscillates at 3 cycles per second. Assuming that the system is damped to 2 percent of critical damping, find the maximum steady-state tip displacement and the maximum steady-state bending stress in the cantilever.

Treat the attached weight as concentrated at the tip of the support structures and neglect the weight of the tube. Neglect  $P-\Delta$  effects, that is, neglect the bending moment due to the eccentricity of the gravity force on the tip load with respect to the base of the cantilever.



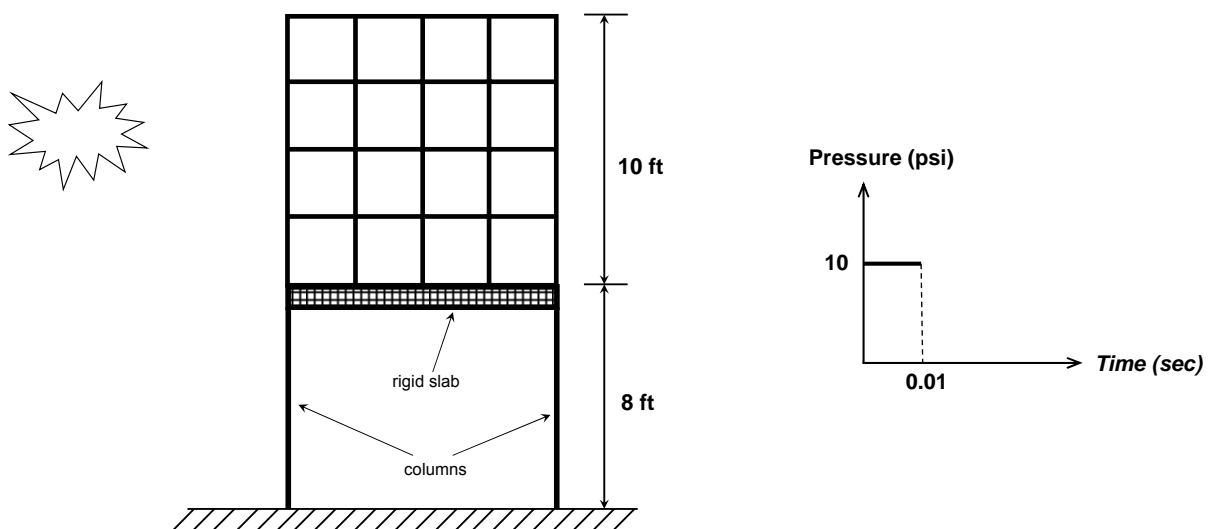
- 3) Find the general expression for displacement for  $t > \tau$ , i.e. for the free-vibration stage, of an undamped single-degree-of-freedom structure forced with the finite duration load shown below. Do not assume that the loading is impulsive.



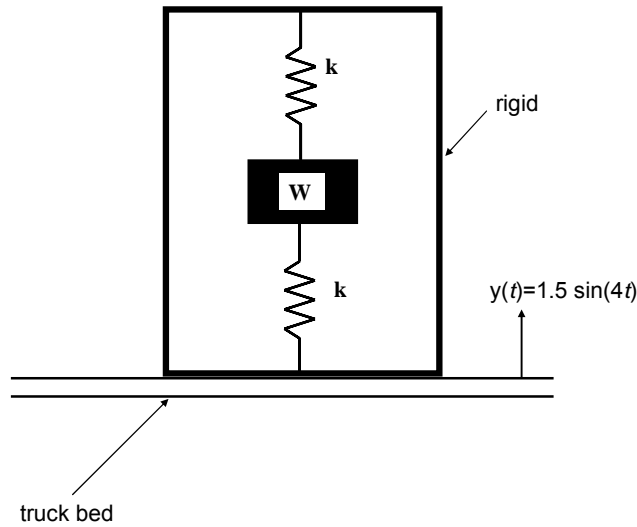
- 4) The 8-ft high two column + slab storage system shown below supports a collection of rigid crates bundled as a 10 ft x 10 ft x 10 ft cube and weighing 50 kips total. Each column has a lateral stiffness of 5 kips/in. An explosion near the structure loads one side of the structure area over 0.01 sec with an average pressure of 10 psi. Initially the crate arrangement is intact. However once the structure reaches its first maximum displacement (i.e., the first maximum displacement away from the blast side), the straps holding the crate assembly breaks and 1/4 of the crates fall from the supporting slab.

Find the amplitude of the next maximum displacement (i.e., the first maximum displacement in the direction of the blast location). Also state how long it takes for the structure to arrive at that displacement state after the blast?

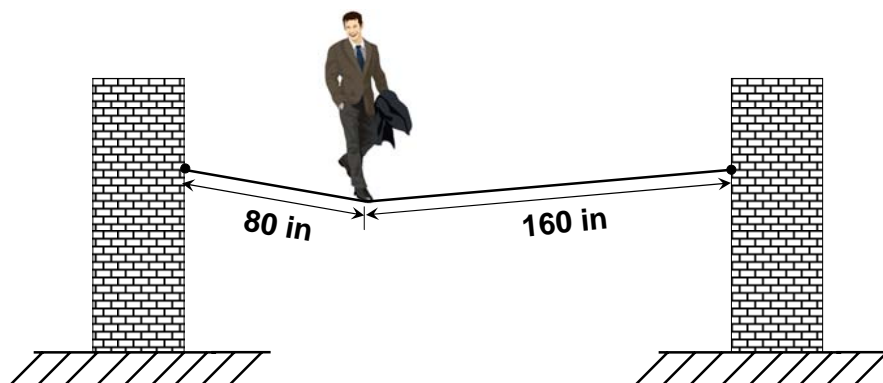
Compared to the crates, columns and the slab have negligible weight. Ignore the blast load on the frame of the structure and damping in the structure. Assume the slab to be rigid and neglect the axial and shear deformation in the columns, and assume the structure behaves linear elastically.



- 5) A package weighing 50 lb is suspended in a box, as shown below, by two springs with a stiffness of 250 lb/in each. The box is placed inside a truck that produces vertical harmonic vibrations during transport of amplitude  $y(t) = 1.5 \sin(4t)$  in. Determine the maximum steady-state displacement, velocity and acceleration experienced by the package. Ignore the transient response. Ignore damping.



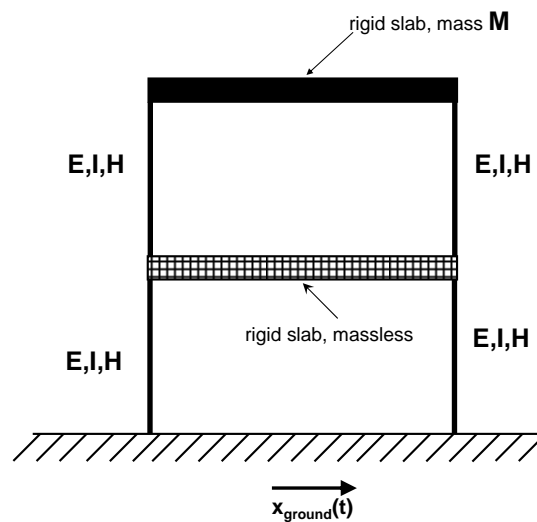
- 6) [BONUS] A panicked businessman weighing 200 lb walks on a tightrope between two structures to avoid the people on main street. If the natural frequency of his vertical vibrations at the particular position shown below is 2 Hz, find the tension in the rope.



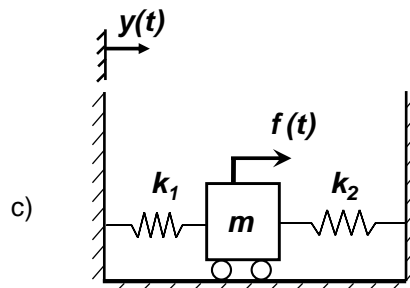
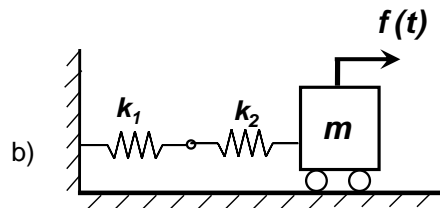
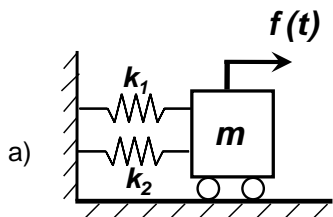
**CE573 – Structural Dynamics [Fall 2007]**

1) The planar two-story frame structure sketched below has rigid slabs. The slab at mid-height is massless; the roof slab has mass  $M$ . All four columns in the frame are identical: they have the same modulus of elasticity  $E$ , moment of inertia  $I$ , and height  $H$ , and no mass. The column-slab and column-ground connections are rigid (no relative rotation between column ends and slab/ground is possible). Assume that there is no damping in the structure and ignore the axial deformations in the columns.

- What is the least number of coordinates, i.e. degrees of freedom, do you need to express the dynamic behavior of this structure properly?
- Find the natural period of the structure.
- During an earthquake, the ground moves. You are given the ground displacement  $x_{\text{ground}}(t)$ . Write the differential equation of motion for the structure. (Do not solve the equation.)



2) For each system shown below, write the differential equation of motion and provide the expression for the natural period. Note that  $m$  is mass,  $k_1$  and  $k_2$  are spring constants,  $f(t)$  is external force, and  $y(t)$  is support displacement.



Note the displacement  $y(t)$  of the left-hand support.

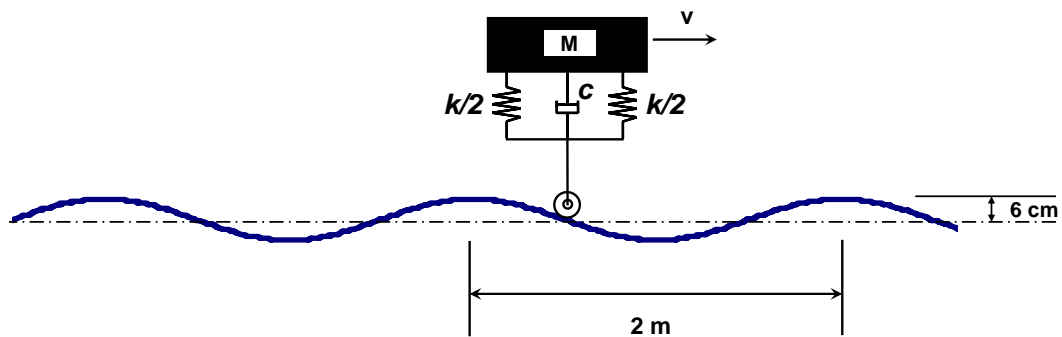
- 3) On a nice weekend day, while you are riding your bicycle around in West Lafayette casually, you make a wrong turn and end up riding on the infamously rough Lindberg Road by the Celery Bog.

The Lindberg Road surface level varies harmonically with  $\pm 6$  cm undulations about the level surface elsewhere. The distance between consecutive peaks of these undulations is measured to be 2 m.

Assume that all of the “suspension” in your bike is thanks to two springs under the seat of your bike. When you sit on the seat of your bike for your casual ride, the springs deflect 5 cm. When you are seated, the damper under the seat provides an equivalent linear viscous damping of 10% of the critical damping.

A simple representation of your ride on the “never-ending” rough road is shown below.

- a) If you are riding your bicycle at a horizontal speed  $v$  of 2.5 m/sec, how much bumping up and down will you experience? In other words, what is the maximum vertical displacement you will experience?
- b) On another Lindberg Road bike ride, you are carrying a backpack which increases your on-seat weight by 20%. Assuming that you are still able to ride at  $v = 2.5$  m/sec, will this “loaded” ride be more or less comfortable than your previous one, i.e. the ride without the backpack?



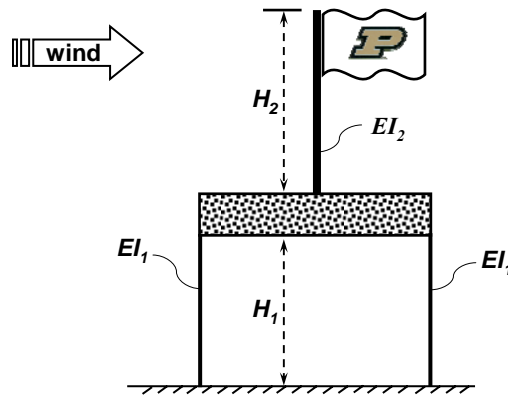
**CE573 – Structural Dynamics [Fall 2006]**

1)

- A) The total wind drag acting on a flag of mass  $m$  can be represented as  $f(t)$ . The flag is hoisted to a massless flagpole which is rigidly connected at its base to a rigid massless girder. The girder is supported by two columns. Column connections, at both ends, are rigid. The columns and the flagpole resist forces via bending. Their properties are shown on the figure.

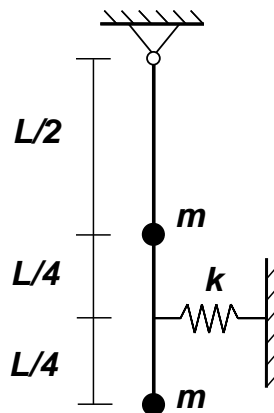
Assume that the flag can be lumped as a point-mass at the tip of the flag-pole. Ignore gravity.

- a) Write the differential equation of motion for the system.  
 b) Write the expression for the natural frequency of the system.



- B) Two point masses of  $m$  each are attached to a massless rigid rod of length  $L$ . The rod is hanging from a frictionless pin and at  $\frac{3}{4}L$  down, it is laterally supported by a spring with spring constant  $k$ . Assume that the system is in equilibrium when the rod is vertical. Do not ignore gravity.

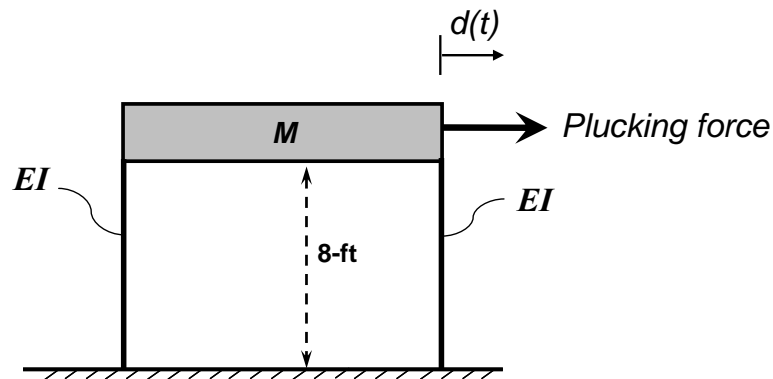
- a) Write the differential equation of motion for small angular oscillations about the equilibrium position.  
 b) Write the expression for the natural frequency of the system.



- 2) At the Bowen Laboratory, a single-story test structure with heavy roof has been subjected to dynamic tests. The structure is 8-ft tall and is supported by four concrete prismatic columns ( $E = 3,000$  ksi). The columns have a cross-section of 12-in by 12-in. An elevation view of the structure is shown below.

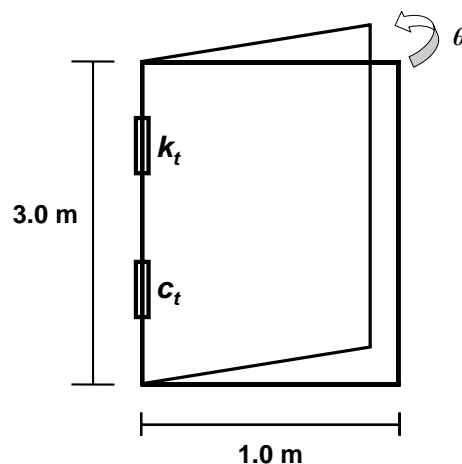
In one of the tests, the structure was pulled statically with 40 kip force causing it to deflect 0.10 inches at its roof level. The load was then removed instantaneously and the displacement response of the structure was recorded. It was found that at the end of first swing, which took 0.5 seconds to complete, the structure came within 0.02 inches of the release location (i.e. it deflected 0.08 inches from its undisturbed, vertical position). Ignore the mass of the columns and the gravity.

- Find the damping ratio of the structure.
- Find the mass of the slab.
- Find the minimum number of full-cycles the structure needs to go through such that its deflections are reduced to no more than  $\pm 0.01$  inches about its equilibrium position.



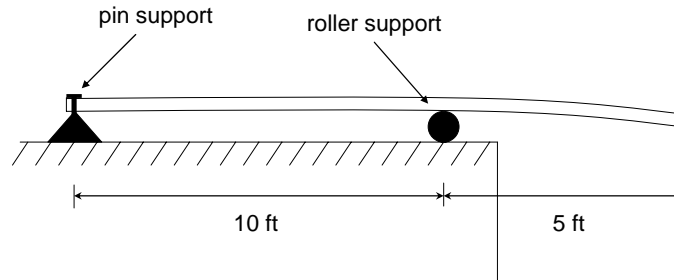
- 3) The main entrance door to the CIVL building has a torsional spring and torsional viscous damper at its hinge. The rigid door is prismatic (1 meter wide and 3 meters tall) and has a mass of 100 kg. Its centroidal mass moment of inertia about an axis parallel to its axis of rotation (and passing through its center of gravity) is  $11.0 \text{ kg}\cdot\text{m}^2$ . The torsional spring has a stiffness  $k_t$  of  $25 \text{ N}\cdot\text{m}/\text{rad}$ . The door system is critically damped.

- Write the differential equation of motion for the door's motion.
- What is the damping coefficient  $c_t$ ?
- The mailman kicks the closed door to open it and get out with his hands full. What initial angular velocity must his kick cause in the door so that the door opens to  $75^\circ$ ?



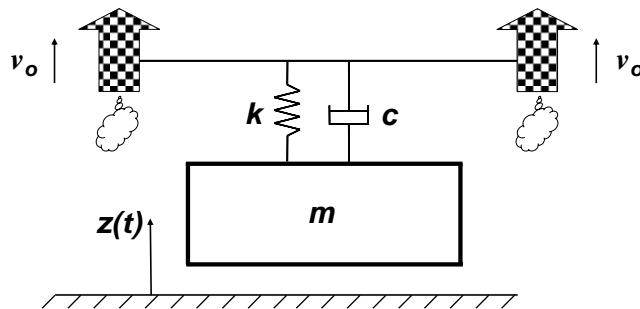
**CE573 – Structural Dynamics [Fall 2005]**

1) A child weighing 100 lb is sitting at the free end of a massless plank. Her weight causes the plank's end to deflect 1.8 inches down. Assume that the plank is made out of wood (modulus of elasticity  $E = 1.495 \times 10^6$  lb/in<sup>2</sup>) and has a cross-section 12 inches wide and 2 inches deep. What is the undamped natural period of the up-and-down movement the child would experience as a light breeze passes by?



2) You are asked to review a space elevator. This elevator can be idealized as a chamber (mass  $m$ ) supported from above by a linear spring (stiffness  $k$ ) and a linear viscous dashpot (damping constant  $c$ ) which are both attached to a set of rockets (see illustration below). The rockets fly up vertically at a constant speed  $v_o$ . Ignore the effect of gravity.

- Write the governing differential equation of motion for the elevator chamber.
- Solve the differential equation of motion to find the expression for the distance travelled by the elevator chamber, i.e.  $z(t)$ , as the rockets lift off vertically with a speed  $v_o = 50$  m/sec. Assume that the chamber has a mass 1000 kg and is initially at rest. Use  $k = 4000$  N/m and  $c = 2000$  N·sec/m.



3) An undamped single-degree-of-freedom system is excited with a triangular forcing function as shown below. The system weighs 483 lb, is initially at rest, and moves horizontally on a frictionless surface. Assuming that the linear spring stiffness constant  $k = 60$  lb/ft,  $F_o = 4500$  lb, and  $\tau = 1$  sec, find  $x(t)$ , i.e. the displacement of the mass in time. Take  $g = 32.2$  ft/sec<sup>2</sup>.

