1) A two-story building frame is shown below. The mass of the frame is assumed to be lumped at the floor levels and the floor slabs are considered rigid. The floor masses and the story stiffnesses are indicated on the figure. Someone has suggested that, in terms of the degrees of freedom x_1 and x_2 as shown on the sketch, the "modeshapes" are in the form of $\underset{\sim}{\phi_1} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$ and $\underset{\sim}{\phi_2} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$.

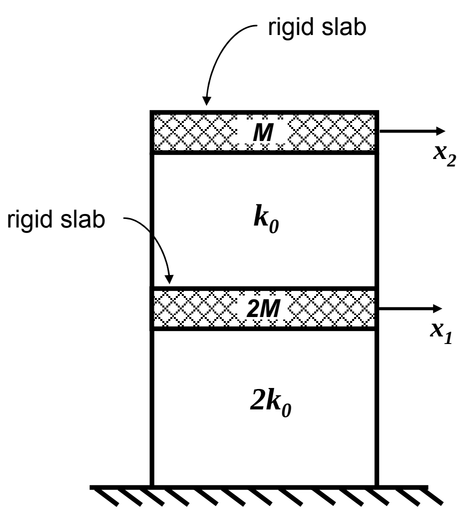
a. Are the suggested "modeshapes" correct? In other words, are they valid modeshapes for this structure?

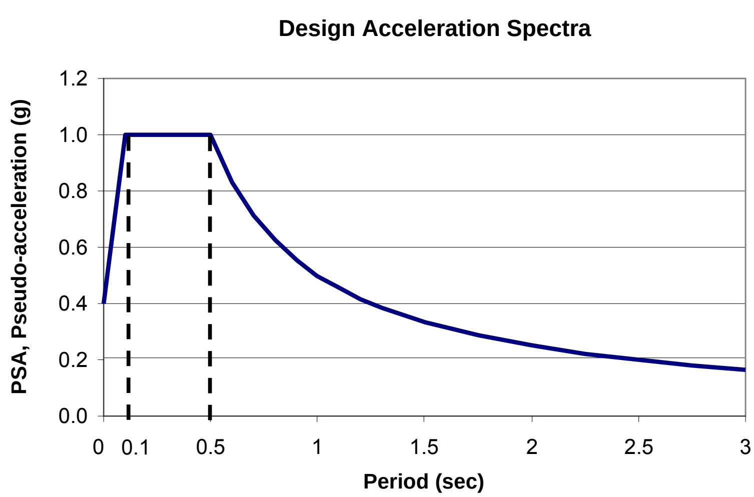
b. Find the natural frequencies in terms of k_0 and *M*. Note that if you think the given "modeshapes" are correct, you may use them in conjunction with Rayleigh's quotient approach, modal mass-modal stiffness approach, or any approach you like in finding the corresponding frequencies. If you think the suggested "modeshapes" are not valid, you need to find the natural frequencies and modeshapes using eigenvalue approach or any other approach you like.

c. Assume that the modal damping is 2% of the critical in both modes. If the fundamental period of the structure is given as 0.4 sec, <u>considering contribution from both modes</u> and using the design spectrum for 2% of the critical damping given below to estimate:

   i. the maximum displacement at roof level expected during a design-level earthquake;

   ii. the maximum base shear force as a fraction of total mass of the structure, i.e. 3*M*, during a design-level earthquake.

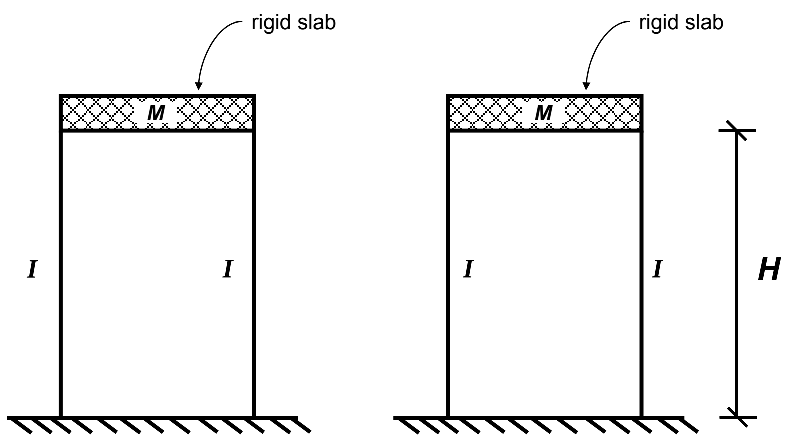




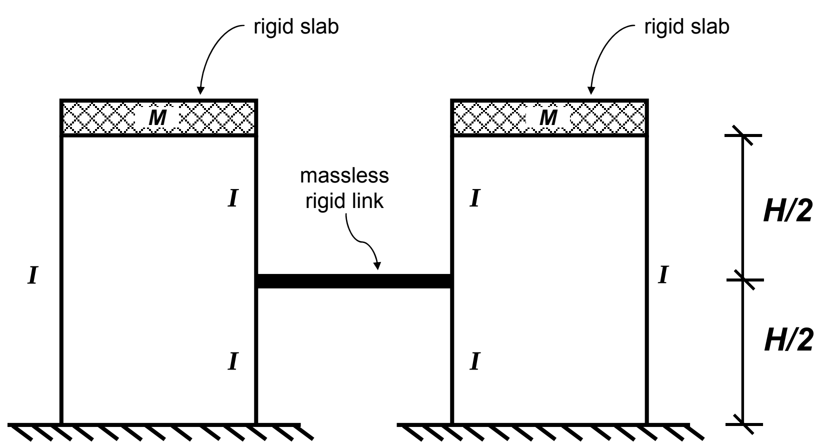
2) Two identical canopies were built initially separate from each other as shown below (see *separate canopies* below). Each canopy has a rigid roof slab with mass M supported by a pair of massless columns of length H and responding in flexure only. Axial deformations are negligible. Columns have moment of inertia I and modulus of elasticity E. Later on, a massless rigid link was constructed to connect the canopies at mid-height of their neighboring columns (see *linked canopies* below).

For the "linked canopies" structure,

a. What is the *minimum* number of degrees of freedom you need to express the dynamic response accurately?
b. Write the set of equations that, collectively, describe the free vibration response. Provide the stiffness and inertia matrices. Do not solve the equations.
c. Find the modal frequencies.
d. Find the modeshapes and sketch them.



[*separate canopies*]

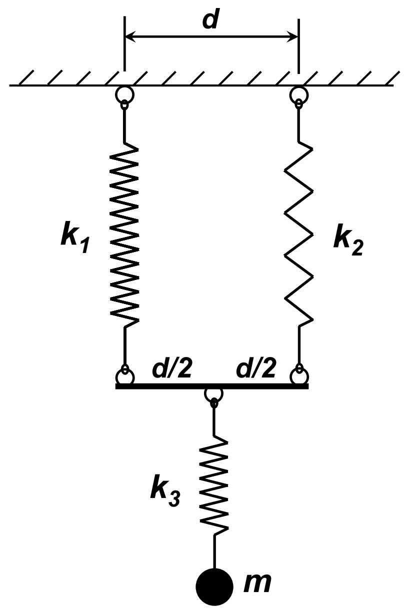


[*linked canopies*]

3) Two massless linear springs with spring constants k_1 and k_2 are used to hang a massless rigid bar of length d from a slab. A third massless linear spring with spring constant k_3 is connected to the rigid bar in the middle and is used to support mass m, as shown below.

Note that the springs are connected to the bar and the slab with frictionless hooks. Therefore, the massless rigid bar is free to rotate.

Considering only the vertical motion of the mass, find the expression for the natural frequency of the system for small motion (i.e. springs remain vertical).

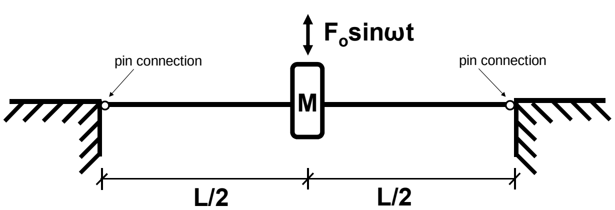


4) A single-degree-of-freedom structure with mass **m**, stiffness **k**, and damping coefficient **c** undergoes free vibrations. It is observed that the peak displacement decreases by 50% over five cycles of motion. If the mass of the structure is quadrupled, i.e. its mass is increased to **4m**, but its stiffness and damping coefficient are kept same as before, what will be the reduction (in %) of the peak displacement of the new structure over ten cycles of free vibration?

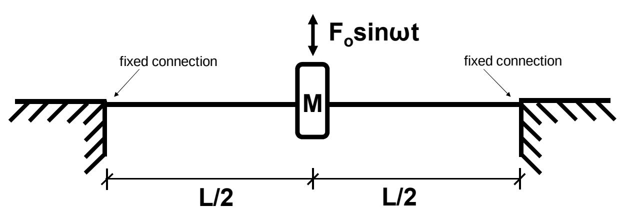
5) The space station has a prismatic beam attached to its satellite inspection platform with pinned connections at both ends. The beam has length **L**, modulus of elasticity **E**, moment of inertia **I**, negligible mass and negligible damping.

To inspect a satellite, astronauts anchor the satellite to the beam at its mid-span point. When a particular satellite with mass **M** is anchored to the beam, the spinning machinery in the satellite exerts a harmonic force of amplitude $\mathbf{F_o}$ at a frequency of $\boldsymbol{\omega}$ on the beam (see the sketch on the left, below). The astronauts observe that the resulting displacements in the beam are too large. To reduce the displacements they decide to convert the supports of the beam to fixed-fixed type (see the sketch on the right, below). They calculate that if the supports are converted to fixed-fixed type, the mid-span displacement will reduce to 35% of the mid-span displacement observed in the original pinned-pinned beam.

As a mission support engineer you are asked to calculate if the maximum force transmitted to the platform through beam supports will change when the supports are converted to fixed-fixed type. If you think the maximum transmitted force will change, what is the amount of change?

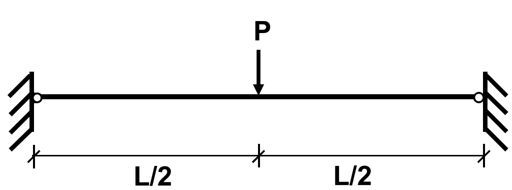


[*pinned-pinned beam*]



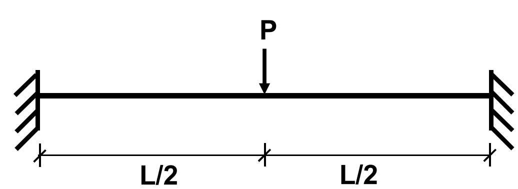
[*fixed-fixed beam*]

Hint: Under static loading



$$\Delta_{\text{mid-span}} = \frac{PL^3}{48EI}$$

[*pinned-pinned beam*]



$$\Delta_{\text{mid-span}} = \frac{PL^3}{192EI}$$

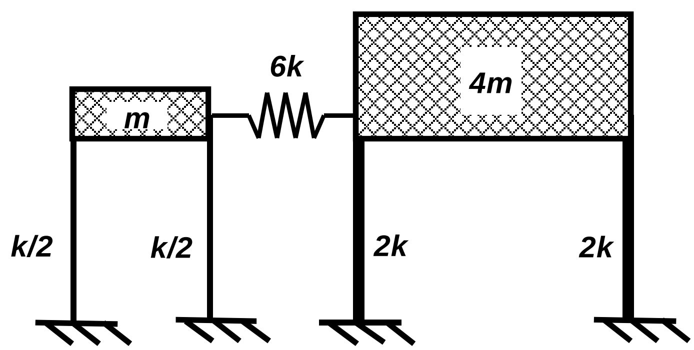
[*fixed-fixed beam*]

6) A small single-story building, with stiffness **k**, mass **m**, and natural period of 0.3 sec, is found to be seismically vulnerable because it is estimated that the structure would displace too much during a medium-intensity earthquake and experience damage. The owner of the building hires a contractor to stiffen the building. As his fix, the contractor installs an elastic link with stiffness **6k** between the small building and an adjacent stiffer and bigger building, which has stiffness **4k** and mass **4m** (see sketch below). If you need to, assume identical damping ratio throughout, i.e. for both individual buildings and when they are linked together.

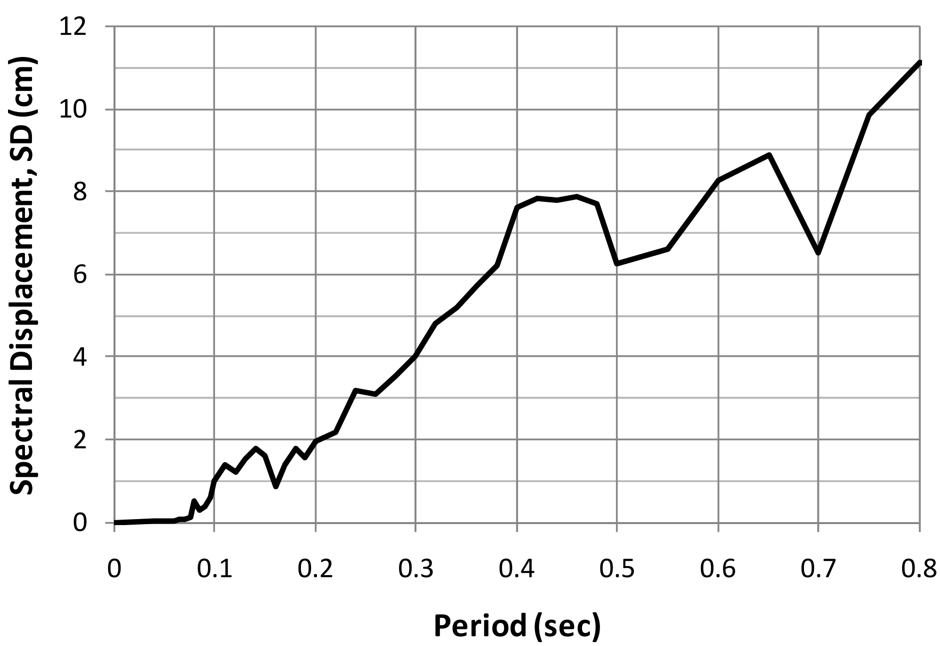
About a year later an earthquake of medium-intensity occurs nearby. The displacement response spectrum for the ground motion recorded during the earthquake near the buildings is given below. Assume that the buildings are very close to each other and therefore experience identical ground motion.

Upon inspection, the columns in the small single-story building are found damaged. The owner sues the contractor claiming that the contractor's fix was useless. The contractor defends that his fix helped the small building. He claims that his fix reduced the peak displacement of the building compared to what it would have experienced without his fix. The judge on the case has invited you as an expert witness to tell the court whether the elastic link installed by the contractor was useful or not.

What will you say to the court? Was the contractor's fix useless or was it helpful? Do not forget to provide evidence proving your conclusion.



[*small building linked to the bigger building by contractor's fix*]



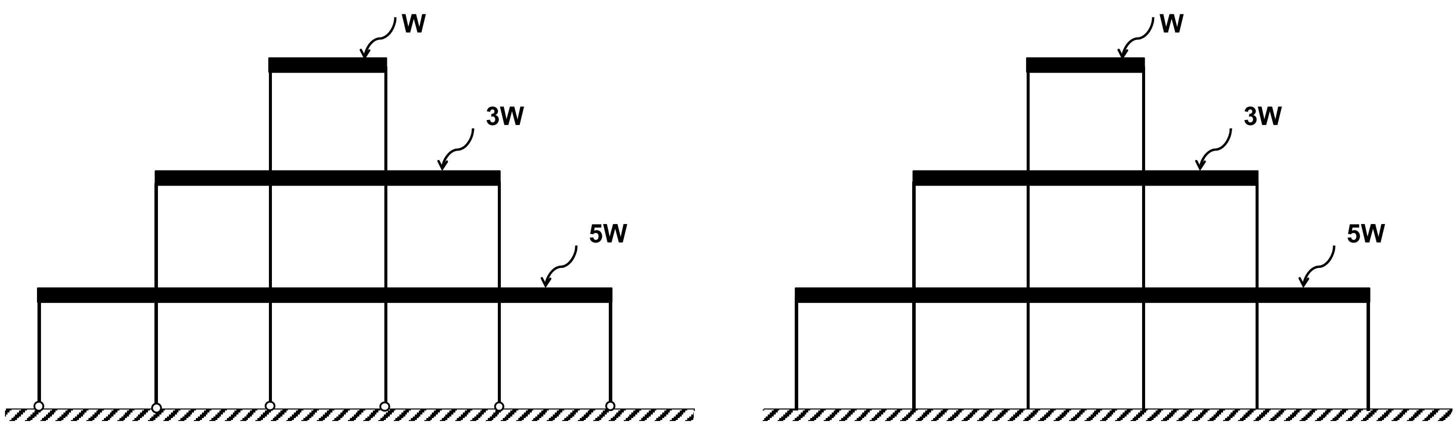
[*displacement response spectrum*]

7) Two architecturally identical looking building frames are being designed. All columns are made out of concrete. Columns have identical cross-sectional dimensions and height. The columns can be assumed axially incompressible. The floor slabs are rigid. Floor weights are shown on the sketches below. The weights of the columns are negligible compared to floor weights. The only difference between the two frames is that in one frame the ground story columns are pinned to the foundation while in the other one the ground story columns are attached to the foundation with fixed connections.

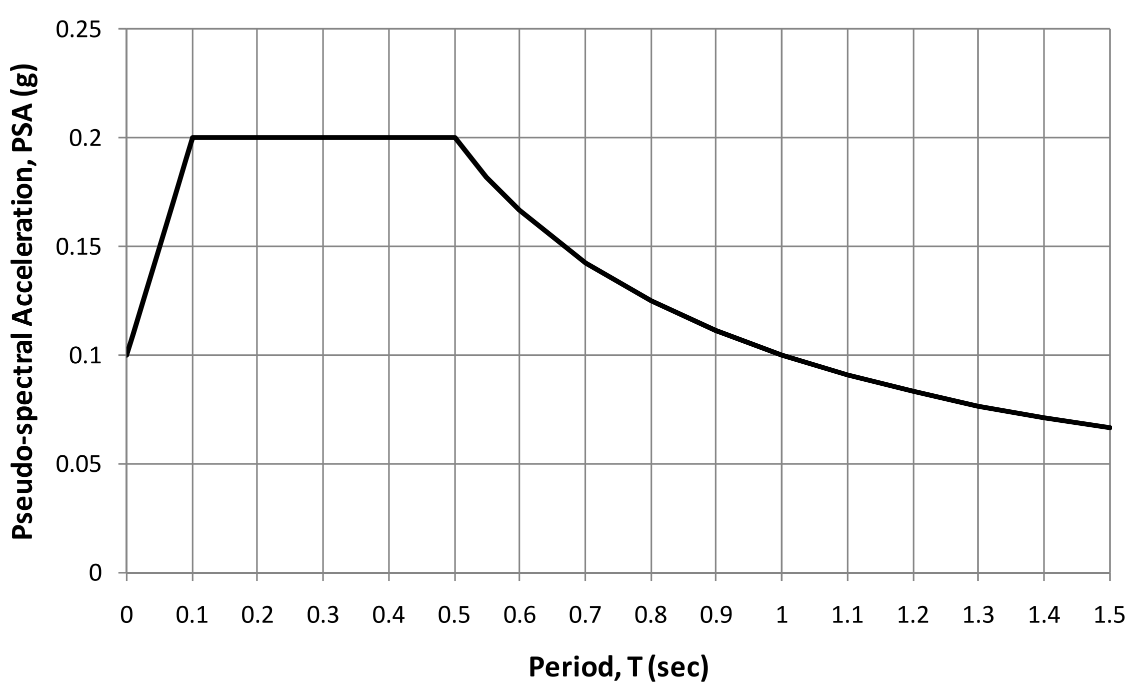
a) Find the approximate fundamental modeshape for each frame using the displacements caused by mass-proportional loading. No need to iterate to improve your approximation.

b) If the lateral stiffness of each column with both ends fixed is 1000 kip/ft and the weight of a bay-wide floor slab is 20 kip (i.e., **W**=20 kip), using your answers from above (part a), estimate the fundamental period of each frame using Rayleigh's quotient method.

c) Considering the design spectrum shown below, for each frame find the design base shear force due to fundamental mode response.



[*pinned at foundation level*]   [*fixed at foundation level*]



[*design acceleration spectrum*]