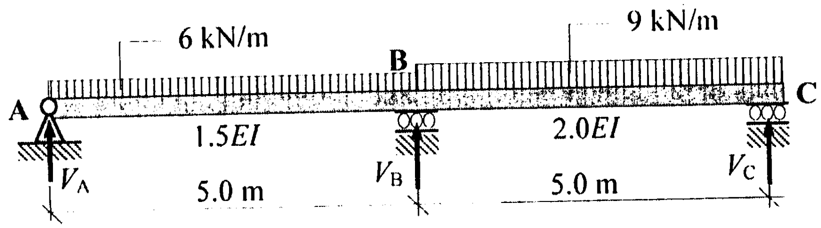
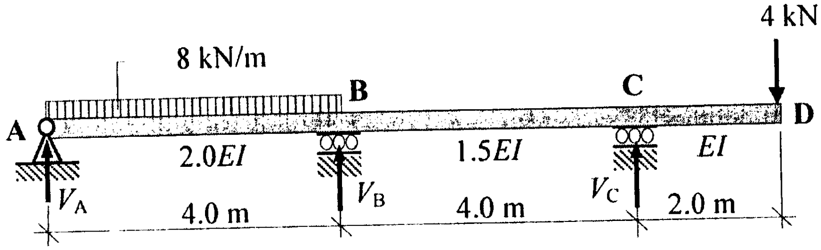
4.6.4 Problems: Unit Load Method for Singly-Redundant Beams

A series of singly-redundant beams are indicated in Problems 4.24 to 4.27. Using the applied loading given in each case:

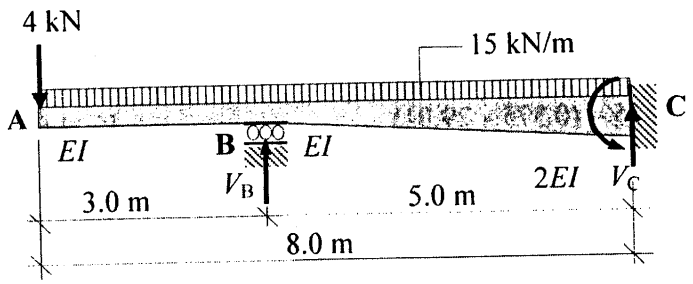
i) determine the support reactions,
ii) sketch the shear force diagram and
iii) sketch the bending moment diagram.



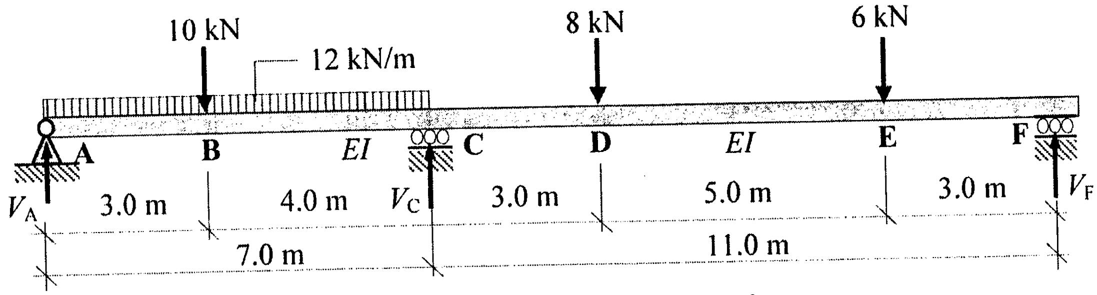
Problem 4.24



Problem 4.25



Problem 4.26



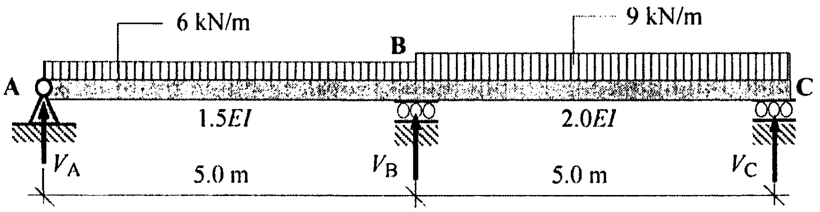
Support C settles by 4.0 mm and $EI = 100.0 \times 10^3$ kNm^2

Problem 4.27

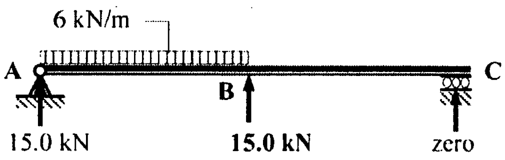
4.6.5 Solutions: Unit Load Method for Singly-Redundant Beams

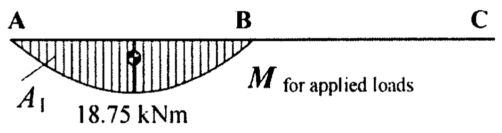
Solution

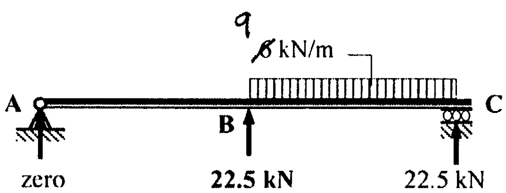
Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.24 **Page No. 1**

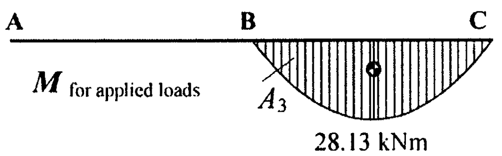


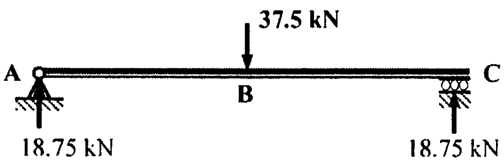
Determine the value of the support reactions and sketch the shear force and bending moment diagrams.
Assume that the reaction at B is the redundant reaction.

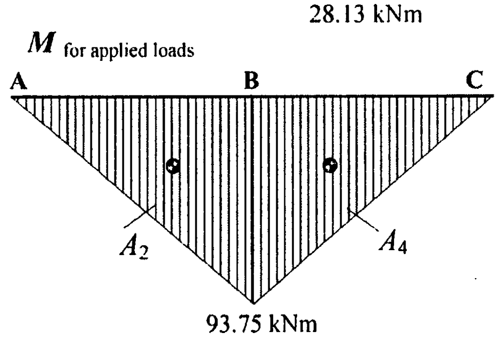


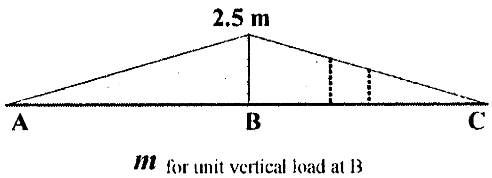


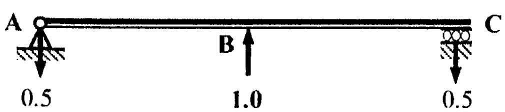












$$\int_0^L \frac{Mm}{EI}dx = \int_A^B \frac{Mm}{1.5EI}dx + \int_C^B \frac{Mm}{2.0EI}dx$$

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.24
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Using the coefficients given in Table 4.1:

$$\int_0^L \frac{Mm}{EI}dx = \sum_0^L (Coefficient \times a \times b \times L)/EI$$

$$\int_0^L \frac{Mm}{EI}dx = [-(0.333 \times 18.75 \times 2.5 \times 5.0) - (0.333 \times 93.75 \times 2.5 \times 5.0)]/1.5EI$$

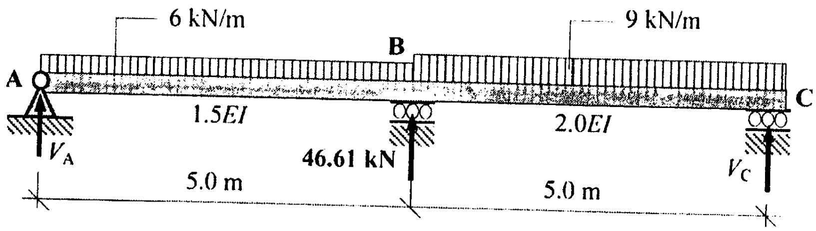
$$+ [-(0.333 \times 28.13 \times 2.5 \times 5.0) - (0.333 \times 93.75 \times 2.5 \times 5.0)]/2.0EI$$

$$= -\mathbf{565.85/EI}$$

$$\int_0^L \frac{m^2}{EI}dx = +(0.333 \times 2.5 \times 2.5 \times 5.0)/1.5EI + (0.333 \times 2.5 \times 2.5 \times 5.0)/2.0EI$$

$$= +\mathbf{12.14/EI}$$

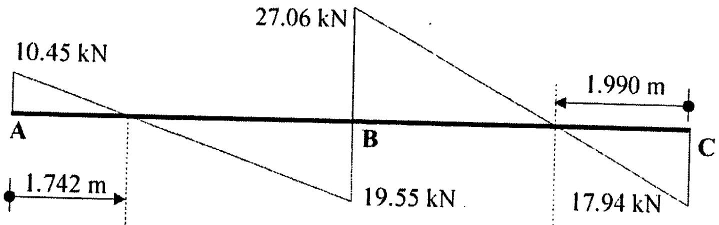
$$V_B = -\int_0^L \frac{Mm}{EI}dx \bigg/ \int_0^L \frac{m^2}{EI}dx = -(-565.85/EI)/(12.14/EI) = +\mathbf{46.61\ kN} \uparrow$$



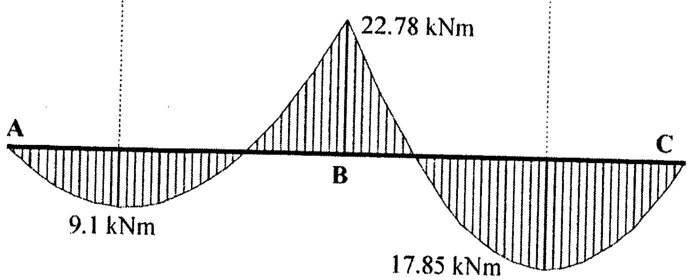
$V_A = +15.0 + 18.75 - (0.5 \times 46.61)$ $\quad\therefore$ $\mathbf{V_A = +10.45\ kN}$ ↑

$V_C = +22.5 + 18.75 - (0.5 \times 46.61)$ $\quad\therefore$ $\mathbf{V_C = +17.94\ kN}$ ↑

$M_B = +93.75 - (2.5 \times 46.61)$ $\quad\therefore$ $\mathbf{M_B = -22.78\ kNm}$



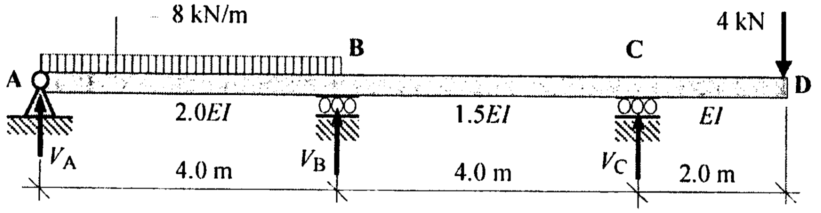
Shear Force Diagram



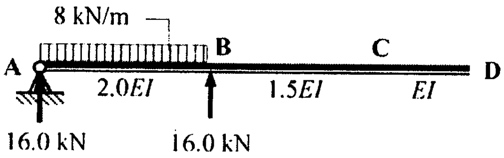
Bending Moment Diagram

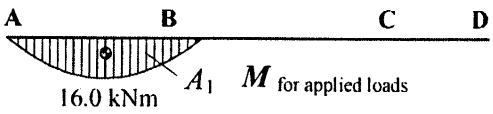
Solution

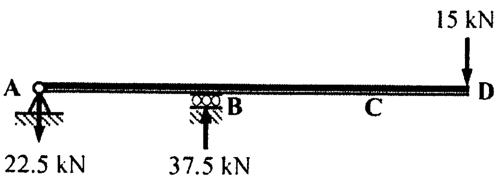
Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.25 **Page No. 1**

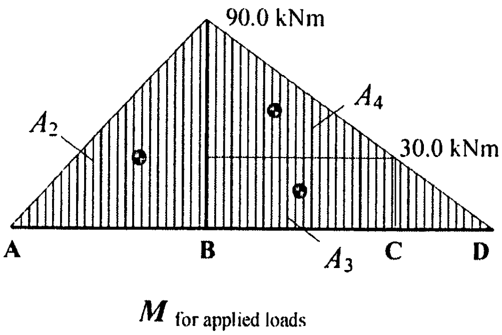


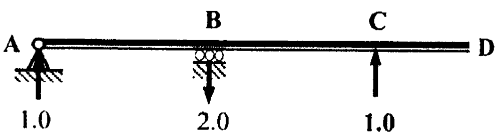
Determine the value of the support reactions and sketch the shear force and bending moment diagrams.
Assume that the reaction at C is the redundant reaction.

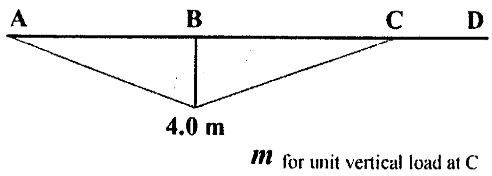












$$\int_0^L \frac{Mm}{EI}dx = \int_A^B \frac{Mm}{2.0EI}dx + \int_C^B \frac{Mm}{2.0EI}dx + \int_D^C \frac{Mm}{EI}dx$$

zero since m is equal to zero

Using the coefficients given in Table 4.1:

$$\int_0^L \frac{Mm}{EI}dx = \sum_0^L (Coefficient \times a \times b \times L)/EI$$

$$\int_0^L \frac{Mm}{EI}dx = [+(0.333 \times 16.0 \times 4.0 \times 4.0) - (0.333 \times 90.0 \times 4.0 \times 4.0)]/2.0EI$$

$$+ [-(0.5 \times 30.0 \times 4.0 \times 4.0) - (0.333 \times 60.0 \times 4.0 \times 4.0)]/1.5EI$$

$$= -\mathbf{570.26/}\boldsymbol{EI}$$

Solution

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Problem Number: 4.25

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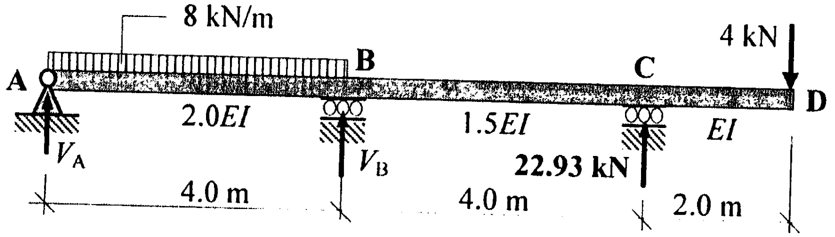
$$\int_0^L \frac{m^2}{EI}dx = +(0.333 \times 4.0 \times 4.0 \times 4.0)/2.0EI + (0.333 \times 4.0 \times 4.0 \times 4.0)/1.5EI$$

$$= +\mathbf{24.87/EI}$$

$$V_C = -\int_0^L \frac{Mm}{EI}dx \Big/ \int_0^L \frac{m^2}{EI}dx = -(-570.26/EI)/(24.87/EI) = +\mathbf{22.93\ kN} \uparrow$$

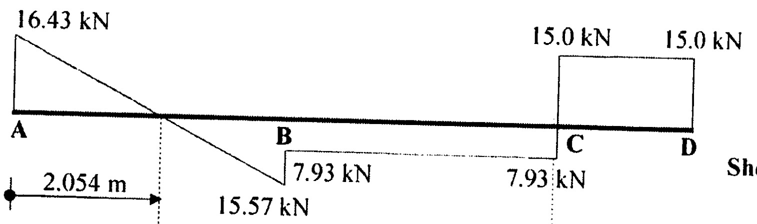


$V_A = +16.0 - 22.5 + (1.0 \times 22.93)$ $\quad\therefore$ $V_A = +\mathbf{16.43\ kN} \uparrow$

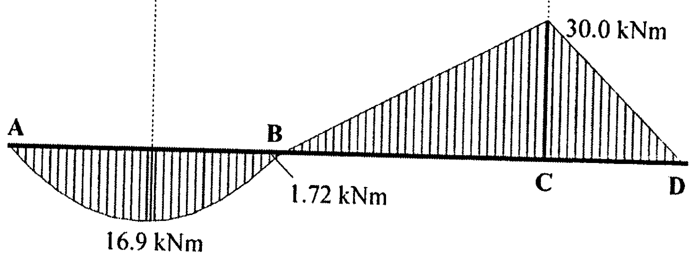
$V_B = +16.0 + 37.5 - (2.0 \times 22.93)$ $\quad\therefore$ $V_C = +\mathbf{7.64\ kN} \uparrow$

$M_B = -90.0 + (4.0 \times 22.93)$ $\quad\therefore$ $M_B = +\mathbf{1.72\ kNm}$

$M_C = -30.0$ $\quad\therefore$ $M_C = -\mathbf{30.0\ kNm}$



Shear Force Diagram



Bending Moment Diagram

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.26

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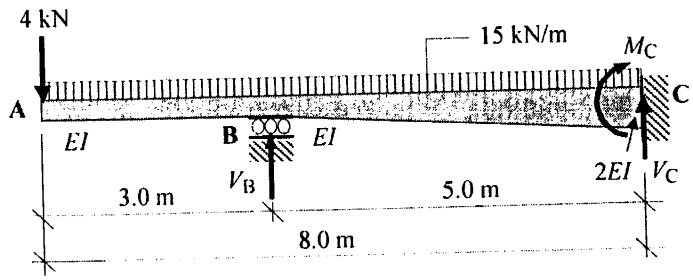
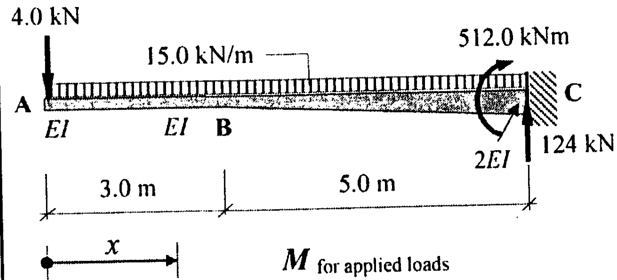


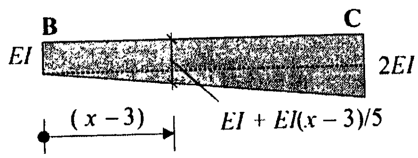
The *EI* value of the beam BC varies linearly from *EI* at support B to 2.0*EI* at C.

Determine the value of the support reactions and sketch the shear force and bending moment diagrams.

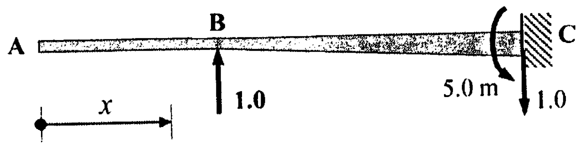
Assume that the reaction at B is the redundant reaction.



M for applied loads



The value of *EI* at a distance of *x* m from A is given by: *EI* (0.4 + 0.2*x*)



m for unit vertical load at B

(*Mm*/*EI*) is not a continuous function the product integral must be evaluated between each of the discontinuities, i.e. A to B and B to C.

The value of *I* at position '*x*' along the beam between B and C is given by:
EI (0.4 + 0.2*x*)

$$\int_0^L \frac{Mm}{EI}dx = \int_A^B \frac{Mm}{EI}dx + \int_B^C \frac{Mm}{EI(0.4+0.2x)}dx$$

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.26

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Consider the section from A to B: $\mathbf{0 \le x \le 3.0\ m}$

m = zero $\qquad\qquad \therefore\ Mm$ = zero

$$\int_A^B \frac{Mm}{EI}dx = \text{zero}$$

Consider the section from B to A: $\mathbf{3.0 \le x \le 8.0\ m}$

$M = -4.0x - 15.0x^2/2 = -4.0x - 7.5x^2$

$m = +1.0(x-3)$

$Mm = (x-3)(-4.0x - 7.5x^2) = 12.0x + 18.5x^2 - 7.5x^3$

$m^2 = +(x-3)^2$

$$\int_B^C \frac{Mm}{EI(0.4+0.2x)}dx = \int_{3.0}^{8.0} \frac{12.0x + 18.5x^2 - 7.5x^3}{EI(0.4+0.2x)}dx$$

Let $v = (0.4 + 0.2x)$ $\qquad \therefore\ x = (5v - 2)$ $\qquad$ and $\qquad dx = 5dv$

$x^2 = (25v^2 - 20v + 4.0)$

$x^3 = (125v^3 - 150v^2 + 60v - 8.0)$

when $\quad x = 3.0 \quad v = 1.0 \qquad$ and when $\quad x = 8.0 \quad v = 2.0$

$Mm = 12.0x + 18.5x^2 - 7.5x^3$

$\quad = 12.0(5v - 2) + 18.5(25v^2 - 20v + 4.0) - 7.5(125v^3 - 150v^2 + 60v - 8.0)$

$\quad = (-760v + 110 + 1587.5v^2 - 937.5v^3)$

$$\int_B^C \frac{Mm}{EI(0.4+0.2x)}dx = \int_{3.0}^{8.0} \frac{12.0x + 18.5x^2 - 7.5x^3}{EI(0.4+0.2x)}dx$$

$$= \int_{1.0}^{2.0} \frac{-760v + 110 + 1587.5v^2 - 937.5v^3}{EIv} 5.0dv$$

$$= \frac{5.0}{EI}\int_{1.0}^{2.0}\left(-760 + \frac{110}{v} + 1587.5v - 937.5v^2\right)dv$$

$$= \frac{5.0}{EI}\left[-760v + 110\,ln\,v + \frac{1587.5v^2}{2.0} - \frac{937.5v^3}{3.0}\right]_{1.0}^{2.0}$$

$$= \frac{5.0}{EI}\left[(-768.8) - (-278.8)\right] = +\frac{2450.0}{EI}\ \text{m}$$

Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.26 **Page No. 2**

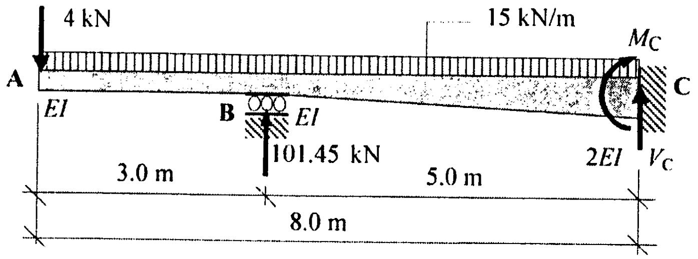
$m^2 = +(x-3)^2 \quad = x^2 - 6x + 9.0 \ = 25v^2 - 50v + 25.0$

$$\int_B^C \frac{m^2}{EI(0.4+0.2x)}dx = \int_{3.0}^{8.0} \frac{x^2 - 6.0x + 9.0}{EI(0.4+0.2x)}dx = \int_{1.0}^{2.0} \frac{25.0v - 50.0v + 25.0}{EIv} 5.0dv$$

$$= \frac{5.0}{EI}\int_{1.0}^{2.0}\left(25.0v - 50.0 + \frac{25.0}{v}\right)dv = \frac{5.0}{EI}\left[\frac{25.0v^2}{2.0} - 50.0v + 25.0 \ln v\right]_{1.0}^{2.0}$$

$$= \frac{5.0}{EI}\left[(-32.67) - (-37.5)\right] = +\frac{24.15}{EI}\text{m}$$

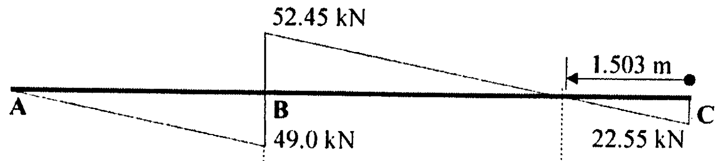
$$V_B = -\int_0^L \frac{Mm}{EI}dx \Big/ \int_0^L \frac{m^2}{EI}dx = -(-2450/EI)/(24.15/EI) = \mathbf{+101.45\ kN} \uparrow$$



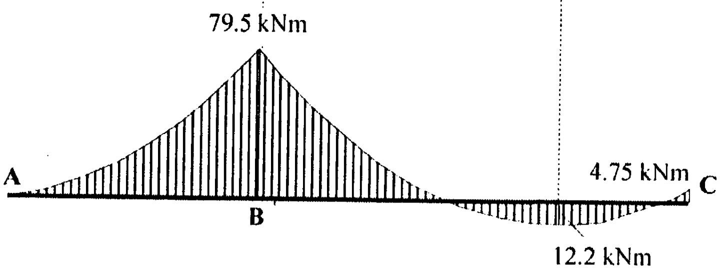
$V_C = +124.0 - (1.0 \times 101.45)$ $\therefore$ $\mathbf{V_A = +22.55\ kN}$ ↑

$M_C = -512.0 + (5.0 \times 101.45)$ $\therefore$ $\mathbf{M_C = -4.75\ kNm}$

$M_B = -(4.0 \times 3.0) - (15.0 \times 3.0)(1.5)$ $\therefore$ $\mathbf{M_B = -79.5\ kNm}$



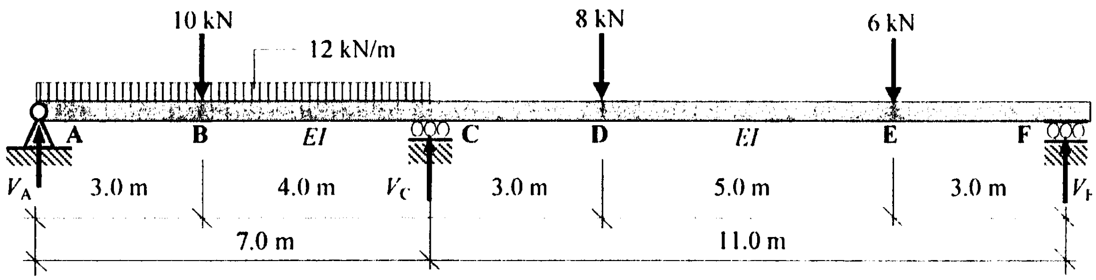
Shear Force Diagram



Bending Moment Diagram

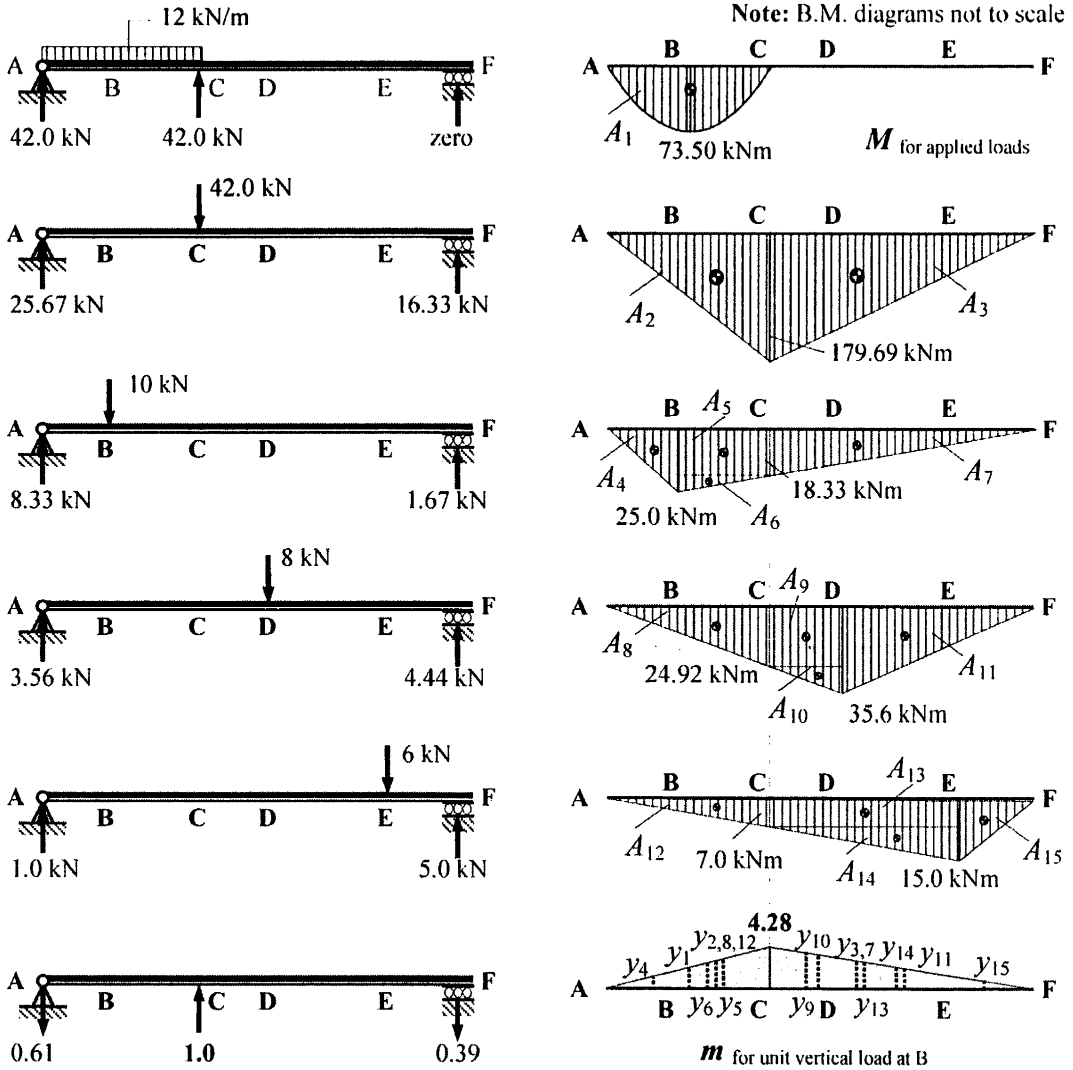
Solution

Topic: Unit Load – Singly-Redundant Beams
Problem Number: 4.27 **Page No. 1**



Support C settles by 4.0 mm and $EI = 100.0 \times 10^3$ kNm^2
Determine the value of the support reactions and sketch the shear force and bending moment diagrams.
Assume that the reaction at C is the redundant reaction.-

Note: B.M. diagrams not to scale



Solution

Topic: Unit Load – Singly-Redundant Beams
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$$\int_0^L \frac{Mm}{EI}dx + \left\{ \int_0^L \frac{mm}{EI}dx \right\} \times V_C = -0.004 \qquad V_C = -\left(0.004 + \int_0^L \frac{Mm}{EI}dx\right) \Big/ \int_0^L \frac{m^2}{EI}dx$$

$$\int_0^L \frac{Mm}{EI}dx = \int_A^F \frac{Mm}{EI}dx$$

(**Note:** The reader should check this using the coefficients given in Table 4.1).

$A_1 = +(0.67 \times 7.0 \times 73.5) = +344.72$ kNm^2
y_1 (3.5 m from A) $= -2.14$ m $\therefore A_1y_1 = -737.70$ kNm^3
$A_2 = +(0.5 \times 7.0 \times 179.69) = +628.92$ kNm^2
y_2 (4.67 m from A) $= -2.85$ m $\therefore A_2y_2 = -1792.42$ kNm^3
$A_3 = +(0.5 \times 11.0 \times 179.69) = +988.30$ kNm^2
y_3 (7.33 m from F) $= -2.85$ m $\therefore A_3y_3 = -2816.66$ kNm^3
$A_4 = +(0.5 \times 3.0 \times 25.0) = +37.50$ kNm^2
y_4 (2.0 m from A) $= -1.22$ m $\therefore A_4y_4 = -45.75$ kNm^3
$A_5 = +(4.0 \times 18.33) = +73.32$ kNm^2
y_5 (5.0 m from A) $= -3.05$ m $\therefore A_5y_5 = -223.63$ kNm^3
$A_6 = +(0.5 \times 4.0 \times 6.67) = +13.34$ kNm^2
y_6 (4.33 m from A) $= -2.64$ m $\therefore A_6y_6 = -35.22$ kNm^3
$A_7 = +(0.5 \times 11.0 \times 18.33) = +100.82$ kNm^2
y_7 (7.33 m from F) $= -2.85$ m $\therefore A_7y_7 = -287.34$ kNm^3
$A_8 = +(0.5 \times 7.0 \times 24.92) = +87.22$ kNm^2
y_8 (4.67 m from A) $= -2.85$ m $\therefore A_8y_8 = -248.58$ kNm^3
$A_9 = +(3.0 \times 24.92) = +74.76$ kNm^2
y_9 (9.50 m from F) $= -3.71$ m $\therefore A_9y_9 = -277.36$ kNm^3
$A_{10} = +(0.5 \times 3.0 \times 10.68) = +16.02$ kNm^2
y_{10} (9.0 m from F) $= -3.51$ m $\therefore A_{10}y_{10} = -56.23$ kNm^3
$A_{11} = +(0.5 \times 8.0 \times 35.6) = +142.4$ kNm^2
y_{11} (5.33 m from F) $= -2.08$ m $\therefore A_{11}y_{11} = -296.19$ kNm^3
$A_{12} = +(0.5 \times 7.0 \times 7.0) = +24.50$ kNm^2
y_{12} (4.67 m from A) $= -2.85$ m $\therefore A_{12}y_{12} = -69.83$ kNm^3
$A_{13} = +(8.0 \times 7.0) = +56.0$ kNm^2
y_{13} (7.0 m from F) $= -2.73$ m $\therefore A_{13}y_{13} = -152.88$ kNm^3
$A_{14} = +(0.5 \times 8.0 \times 8.0) = +32.0$ kNm^2
y_{14} (5.67 m from F) $= -2.21$ m $\therefore A_{14}y_{14} = -70.72$ kNm^3
$A_{15} = +(0.5 \times 3.0 \times 15.0) = +22.5$ kNm^2
y_{15} (2.0 m from F) $= -0.78$ m $\therefore A_{15}y_{15} = -17.55$ kNm^3

$$\int_0^L \frac{Mm}{EI}dx = \sum_{n=1}^{n=15} \frac{(A_n y_n)}{EI} = -7128.06/EI \text{ m}$$

Solution

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$$\int_0^L \frac{m^2}{EI}dx = \int_A^F \frac{m^2}{EI}dx$$

$A_1 = +(0.5 \times 7.0 \times 4.28) = -14.98 \text{ kNm}^2$

y_1 (4.67 m from A) $= -2.85$ m $\therefore A_1y_1 = +42.69 \text{ kNm}^3$

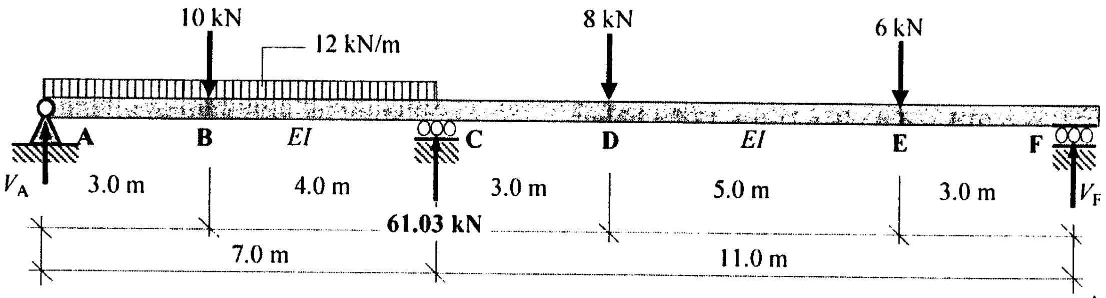
$A_2 = +(0.5 \times 11.0 \times 4.28) = -23.57 \text{ kNm}^2$

y_2 (7.33 m from A) $= -2.85$ m $\therefore A_2y_2 = +67.09 \text{ kNm}^3$

$$\int_0^L \frac{m^2}{EI}dx = \sum_{n=1}^{n=2}\frac{(A_n y_n)}{EI} = [+(42.69/EI) + (67.09/EI)] = +109.78/EI$$

$$V_C = -\left(0.004 + \int_0^L \frac{Mm}{EI}dx\right) \Big/ \int_0^L \frac{m^2}{EI}dx = -(0.004 - 7128.06/EI)/\ 109.78/EI$$

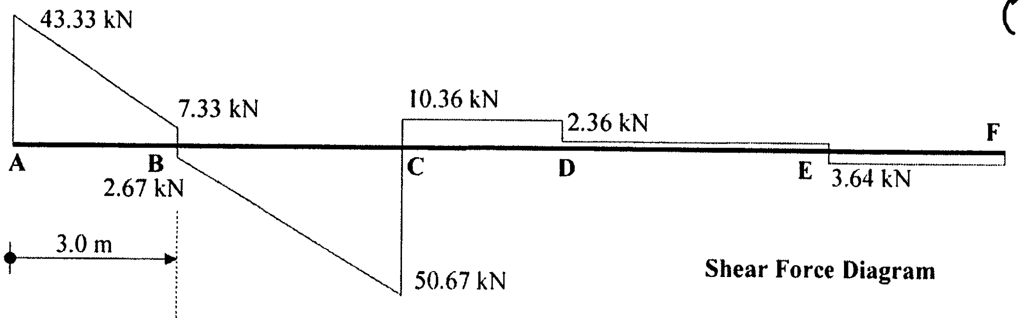
$= +61.03$ kN



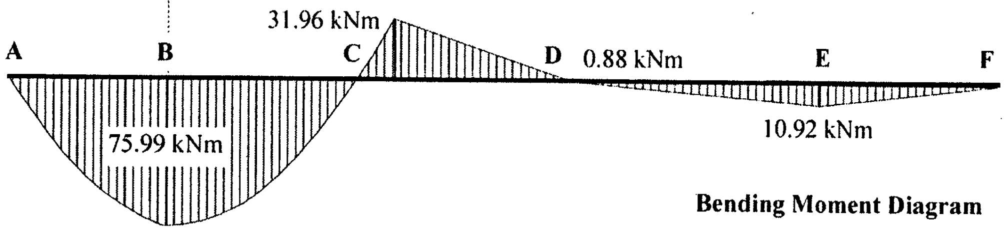
$V_A = +42.0 + 25.67 + 8.33 + 3.56 + 1.0 - (0.61 \times 61.03)$ $\therefore$ $\mathbf{V_A = +43.33}$ **kN**

$V_F = +16.33 + 1.67 + 4.44 + 5.0 - (0.39 \times 61.03)$ $\therefore$ $\mathbf{V_F = +3.64}$ **kN**

$M_C = +179.69 + 18.33 + 24.92 + 7.0 - (4.28 \times 61.03)$ $\therefore$ $\mathbf{M_C = +31.27}$ **kNm**



Shear Force Diagram



Bending Moment Diagram