3.5.3 *Example 3.4: Deflection of a Pin-Jointed Truss*

A pin-jointed truss ABCD is shown in Figure 3.17 in which both a vertical and a horizontal load are applied at joint B as indicated. Determine the magnitude and direction of the resultant deflection at joint B and the vertical deflection at joint D.

Assume the cross-sectional area of all members is equal to A and all members are made from the same material, i.e. have the same modulus of elasticity E

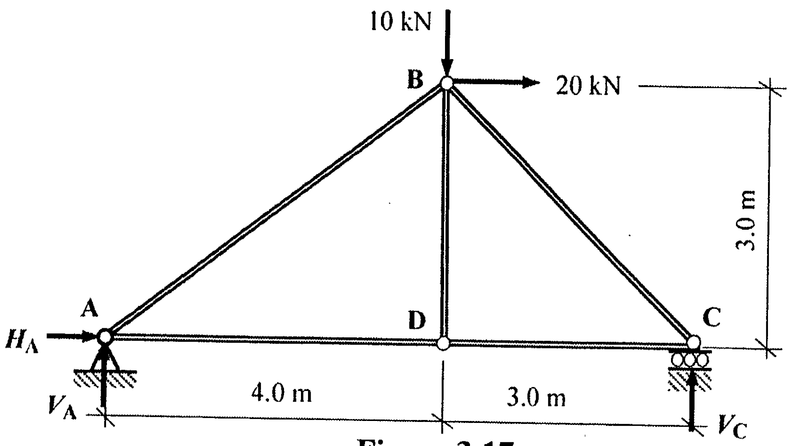


Figure 3.17

Step 1: Evaluate the member forces. The reader should follow the procedure given in Example 3.1 to determine the following results:

Horizontal component of reaction at support A $H_A = -20.0$ kN ←
Vertical component of reaction at support A $V_A = -4.29$ kN ↓
Vertical component of reaction at support C $V_C = +14.29$ kN ↑

Use the *method of sections* or *joint resolution* as indicated in Sections 3.2 and 3.3 respectively to determine the magnitude and sense of the unknown member forces (i.e. the ***P*** forces).

The reader should complete this calculation to determine the member forces as indicated in Figure 3.18.

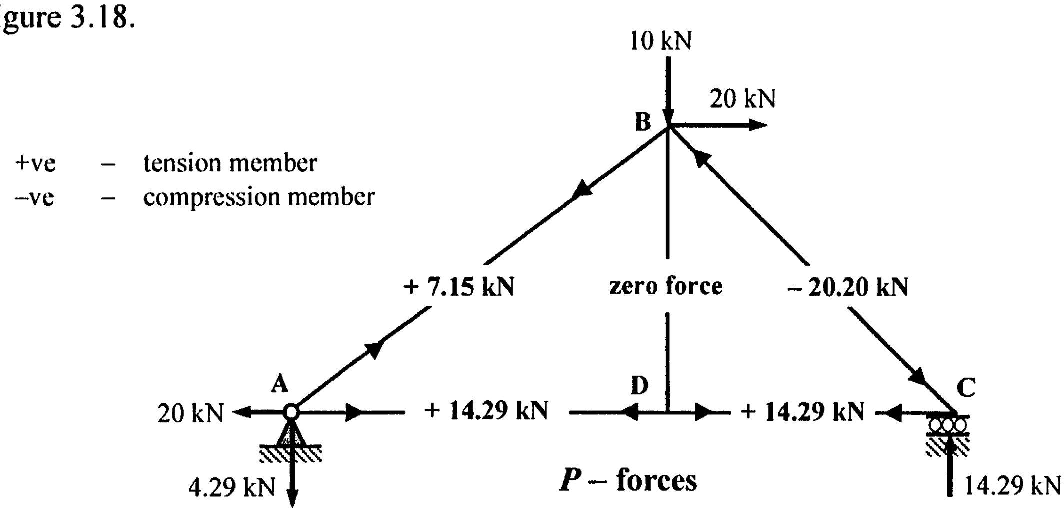


Figure 3.18

Step 2: To determine the vertical deflection at joint B remove the externally applied load system and apply a **unit load only** in a vertical direction at joint B as shown in Figure 3.19. Use the *method of sections* or *joint resolution* as before to determine the magnitude and sense of the unknown member forces (i.e. the ***u*** forces).

The reader should complete this calculation to determine the member forces as indicated in Figure 3.19.

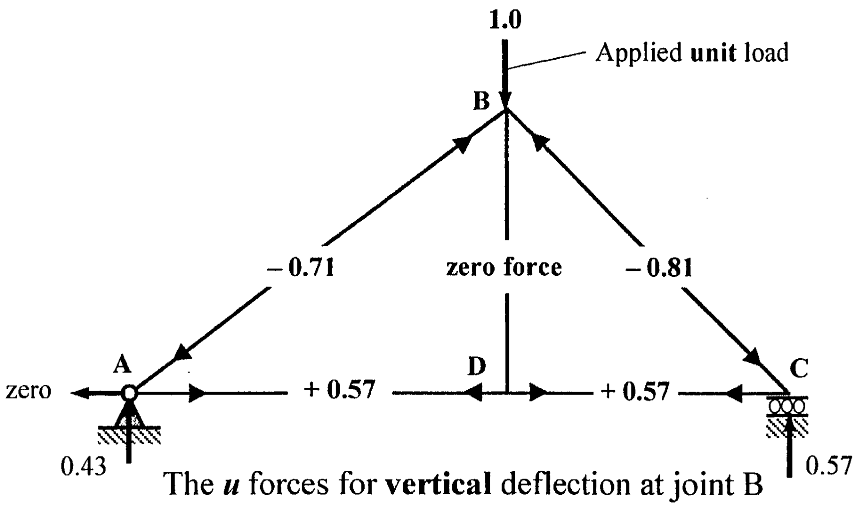


Figure 3.19

The vertical deflection $\delta_{V,B} = \sum \frac{PL}{AE} u$

This is better calculated in tabular form as shown in Table 3.3.

Member	Length (*L*)	Cross-section (*A*)	Modulus (*E*)	*P* forces (kN)	*u* forces	*PL* × *u* (kNm)
AB	5.0 m	*A*	*E*	+ 7.15	− 0.71	**− 25.38**
BC	4.24 m	*A*	*E*	− 20.20	− 0.81	**+ 69.37**
AD	4.0 m	*A*	*E*	+ 14.29	+ 0.57	**+ 32.58**
CD	3.0 m	*A*	*E*	+ 14.29	+ 0.57	**+ 24.44**
BD	3.0 m	*A*	*E*	0.0	0.0	**0.0**
					Σ	**+ 101.01**

Table 3.3

The +ve sign indicates that the deflection is in the same direction as the applied unit load.

Hence the vertical deflection $\delta_{V,B} = \sum \frac{PL}{AE} u = +(101.01/AE)$ ↓

Note: Where the members have different cross-sectional areas and/or modulii of elasticity each entry in the last column of the table should be based on $(PL \times u)/AE$ and not only $(PL \times u)$.

A similar calculation can be carried out to determine the horizontal deflection at joint B. The reader should complete this calculation to determine the member forces as indicated in Figure 3.20.

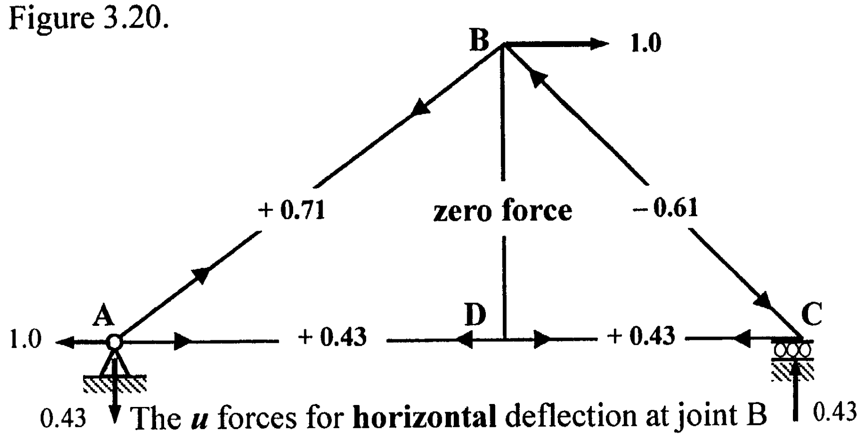


Figure 3.20

The horizontal deflection $\delta_{H,B} = \sum \frac{PL}{AE} u$

Member	Length (*L*)	Cross-section (*A*)	Modulus (*E*)	*P* forces (kN)	*u* forces	*PL* × *u* (kNm)
AB	5.0 m	*A*	*E*	+ 7.15	+ 0.71	+ 25.74
BC	4.24 m	*A*	*E*	− 20.20	− 0.61	+ 52.25
AD	4.0 m	*A*	*E*	+ 14.29	+ 0.43	+ 24.58
CD	3.0 m	*A*	*E*	+ 14.29	+ 0.43	+ 18.43
BD	3.0 m	*A*	*E*	0.0	0.0	0.0
					Σ	**+ 121.00**

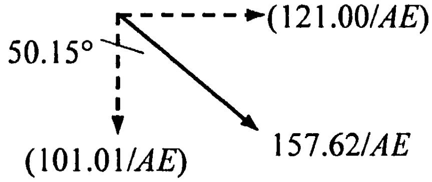
Table 3.4

Hence the horizontal deflection $\delta_{H,B} = \sum \frac{PL}{AE} u = +(121.00/AE)$ →

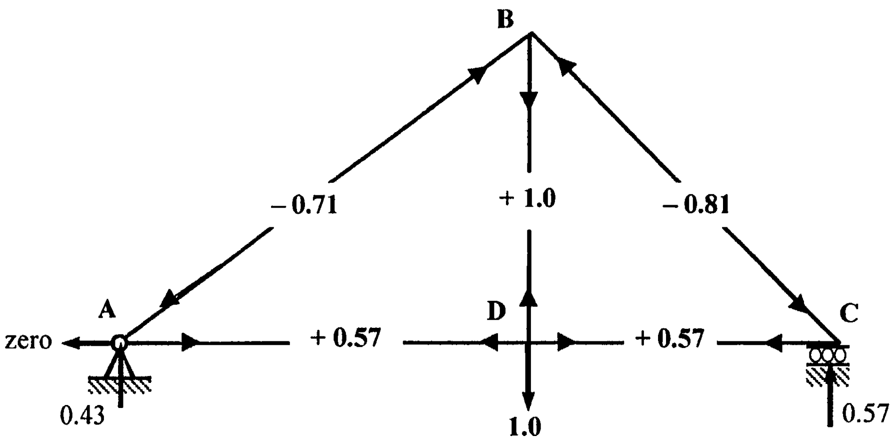
The resultant deflection at joint B can be determined from the horizontal and vertical components evaluated above, i.e.

$$R = \sqrt{(101.01^2 + 121.0^2)}\,/AE = 157.62/AE$$

$$\theta = \text{Tan}^{-1}(121.00/101.01) = 50.15°$$



A similar calculation can be carried out to determine the vertical deflection at joint D. The reader should complete this calculation to determine the member forces as indicated in Figure 3.21.



The member ***u*** forces for **vertical** deflection at joint D

Figure 3.21

The vertical deflection $\delta_{V,D} = \sum \frac{PL}{AE} u$

Member	Length (*L*)	Cross-section (*A*)	Modulus (*E*)	*P* forces (kN)	*u* forces	***PL* × *u* (kNm)**
AB	5.0 m	*A*	*E*	+ 7.15	– 0.71	– 25.38
BC	4.24 m	*A*	*E*	– 20.20	– 0.81	+ 69.37
AD	4.0 m	*A*	*E*	+ 14.29	+ 0.57	+ 32.58
CD	3.0 m	*A*	*E*	+ 14.29	+ 0.57	+ 24.44
BD	3.0 m	*A*	*E*	0.0	+1.0	0.0
					Σ	**+ 101.01**

Table 3.5

Hence the vertical deflection $\delta_{V,D} = \sum \frac{PL}{AE} u = +(101.01/AE)$ ↓

3.5.3.1 Fabrication Errors – (Lack-of-fit)

During fabrication it is not unusual for a member length to be slightly too short or too long and assembly is achieved by forcing members in to place. The effect of this can be accommodated very easily in this method of analysis by adding additional terms relating to each member for which lack-of-fit applies. The δL term for the relevant members is equal to the magnitude of the error in length, i.e. Δ_L where negative values relate to members which are too short and positive values to members which are too long.

(**Note:** under normal applied loading the δL term $= \dfrac{PL}{AE}$)

3.5.3.2 Changes in Temperature

The effects of temperature change in members can also be accommodated in a similar manner; in this case the δL term is related to the coefficient of thermal expansion for the material, the change in temperature and the original length,

i.e. $\delta L = \alpha L \Delta_T$
where
α is the coefficient of thermal expansion,
L is the original length,
Δ_T is the change in temperature – a reduction being considered negative and an increase being positive.

Since this is an elastic analysis the principle of superposition can be used to obtain results when a combination of applied load, lack-of-fit and/or temperature difference occurs. This is illustrated in Example 3.5.

3.5.4 Example 3.5: Lack-of-fit and Temperature Difference

Consider the frame indicated in Example 3.4 and determine the vertical deflection at joint D assuming the existing loading and that member BC is too short by 2.0 mm, member CD is too long by 1.5 mm and that members AD and CD are both subject to an increase in temperature of 5°C. Assume $\alpha = 12.0 \times 10^{-6}$/°C and $AE = 100 \times 10^3$ kN.

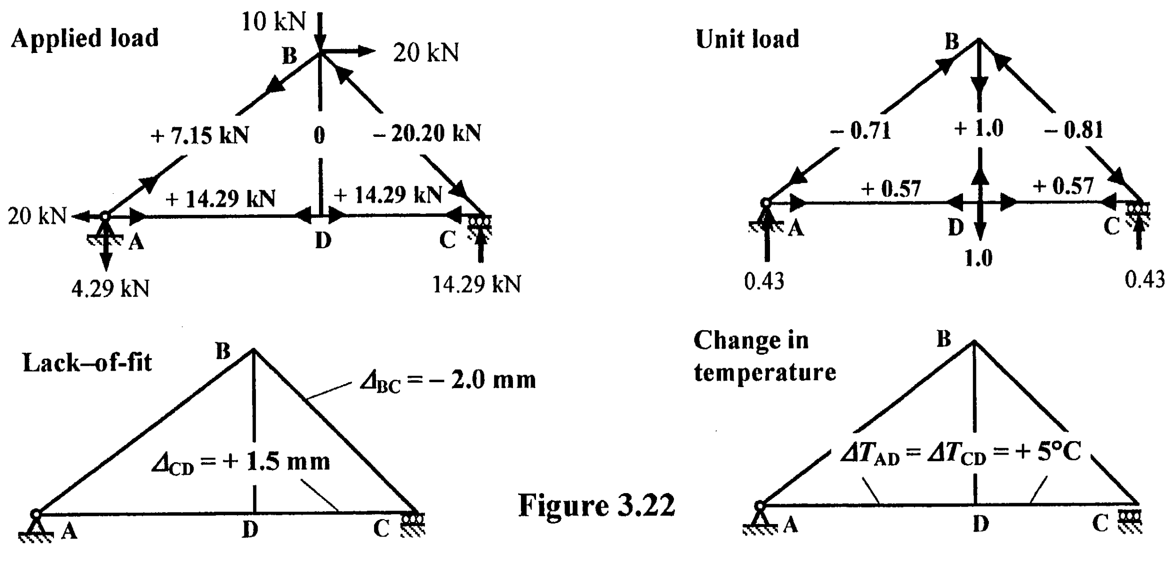


Figure 3.22

The δL value for member BC due to lack-of-fit $\Delta_L = -2.0$ mm
The δL value for member CD due to lack-of-fit $\Delta_L = +1.5$ mm

The δL value for member AD due to temperature change $= +\alpha L_{AD} \Delta_{T,AD}$

$$= +(12 \times 10^{-6} \times 4000 \times 5.0)$$

$$\Delta_T = +0.24 \text{ mm}$$

The δL value for member CD due to temperature change $= +\alpha L_{CD} \Delta_{T,CD}$

$$= +(12 \times 10^{-6} \times 3000 \times 5.0)$$

$$\Delta_T = +0.18 \text{ mm}$$

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	Δ_L (mm)	Δ_T (mm)	*u*	$(PL/AE + \Delta_L + \Delta_T) \times u$ (mm)
AB	5000	100×10^3	+ 7.15	+ 0.36	0	0	– 0.71	– 0.26
BC	4243	100×10^3	– 20.20	– 0.86	– 2.0	0	– 0.81	+ 2.32
AD	4000	100×10^3	+ 14.29	+ 0.57	0	+ 0.24	+ 0.57	+ 0.46
CD	3000	100×10^3	+ 14.29	+ 0.43	+ 1.5	+ 0.18	+ 0.57	+ 1.20
BD	3000	100×10^3	0	0	0	0	1.0	0
								Σ = + 3.72

Table 3.6

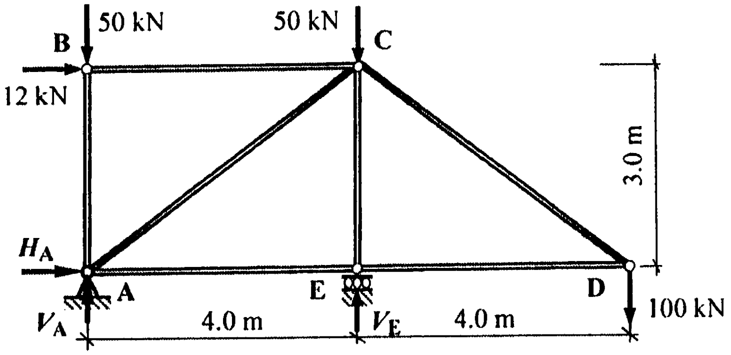
The vertical deflection at joint D due to combined loading, lack-of-fit and temperature change is given by:

$$\delta_{V,D} = \sum\left(\frac{PL}{AE} + \Delta_l + \Delta_t\right) \times u = +3.72 \text{ mm} \downarrow$$

Note: Statically determinate, pin-jointed frames can accommodate small changes in geometry without any significant effect on the member forces induced by the applied load system, i.e. the member forces in Example 3.5 are the same as those in Example 3.4.

3.5.5 Problems: Unit Load Method for Deflection of Pin-Jointed Frames

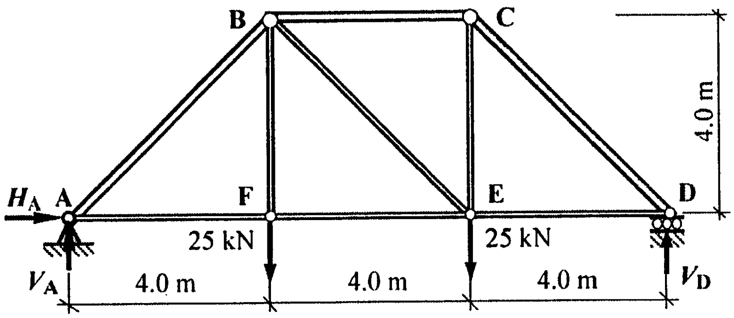
A series of pin-jointed frames are shown in Problems 3.17 to 3.20. Using the applied load systems and data given in each case, determine the value of the deflections indicated. Assume $E = 205$ kN/mm^2 and $\alpha = 12 \times 10^{-6}$/°C where required.



The cross-sectional area of all members is equal to 1500 mm^2.

Determine the value of the resultant deflection at joint D.

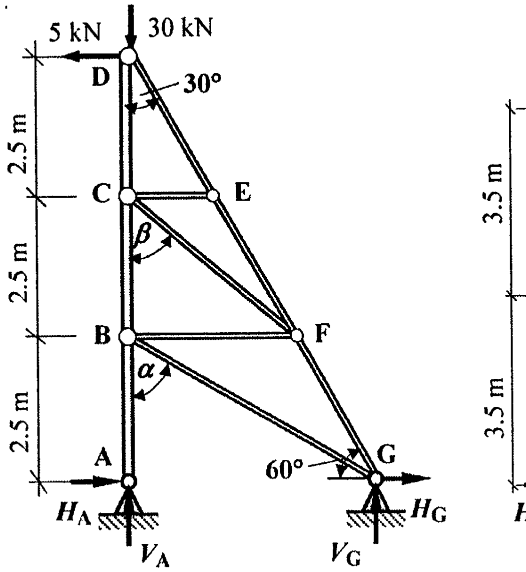
Problem 3.17



Problem 3.18

The cross-sectional area of members AB, BC and CD is equal to 500 mm^2.
The cross-sectional area of all other members is equal to 250 mm^2.
Member BE is too short by 3.0 mm.

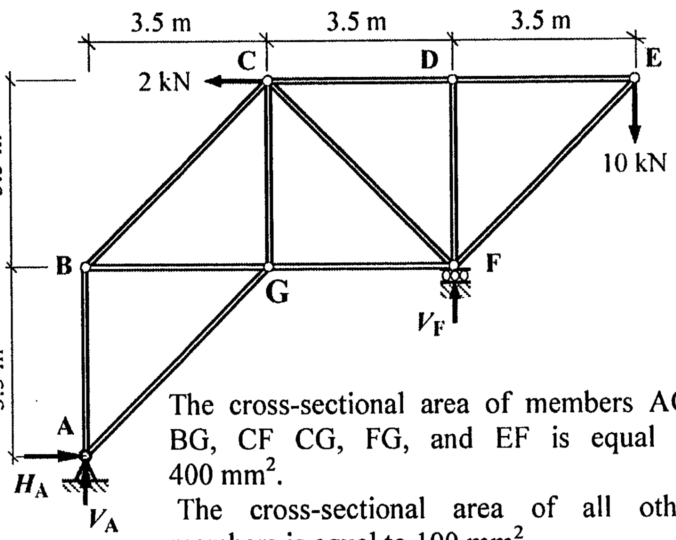
Determine the value of the vertical deflection at joint F and the horizontal deflection at joint B.



The cross-sectional area of all members is equal to 1200 mm^2.

Determine the value of the horizontal deflection at joint D.

Problem 3.19



The cross-sectional area of members AG, BG, CF CG, FG, and EF is equal to 400 mm^2.
The cross-sectional area of all other members is equal to 100 mm^2.
All members are subjected to a decrease in temperature equal to 20°C.

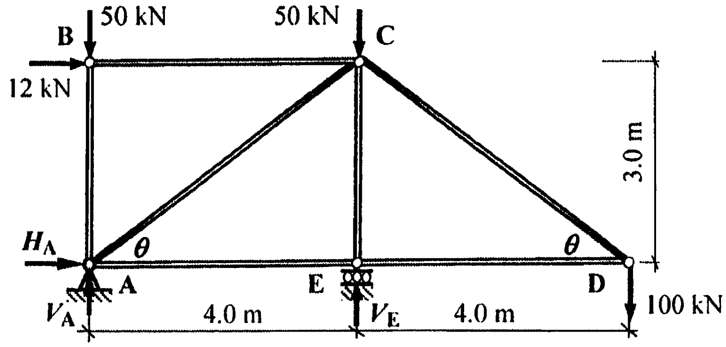
Determine the horizontal deflection at joint F.

Problem 3.20

3.5.6 Solutions: Unit Load Method for Deflection of Pin-Jointed Frames

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.17 **Page No. 1**



The cross-sectional area of all members is equal to 1500 mm^2.
Determine the value of the resultant deflection at joint D.
$E = 205$ kN/mm^2

$\text{Sin}\theta = (3.0/5.0) = 0.6 \qquad \text{Cos}\theta = (4.0/5.0) = 0.8$
$AE_{1500} = (1500 \times 205) = 307.5 \times 10^3$ kN

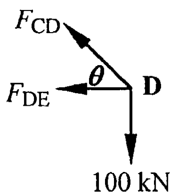
Determine the Support Reactions
Consider the rotational equilibrium of the frame:
+ve ↻ $\Sigma M_A = 0 \qquad + (12.0 \times 3.0) + (50.0 \times 4.0) + (100.0 \times 8.0) - (V_E \times 4.0) = 0$
$\therefore$ $\mathbf{V_E = +\ 259.0\ kN}$ ↑

Consider the horizontal equilibrium of the frame:
+ve → $\Sigma F_x = 0 \qquad + H_A + 12.0 = 0$ $\therefore$ $\mathbf{H_A = -\ 12.0\ kN}$ ←

Consider the vertical equilibrium of the frame:
+ve ↑ $\Sigma F_y = 0 \qquad + V_A - 50.0 - 50.0 - 100.0 + V_E = 0$ $\therefore$ $V_A = 200.0 - V_E$
$V_A = 200.0 - 259.0$ $\therefore$ $\mathbf{V_A = -\ 59.0\ kN}$ ↓

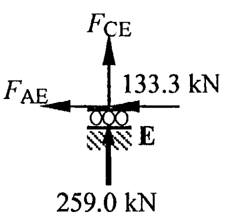
Assume all unknown member forces to be tension and use joint resolution to determine the *P*–forces in the frame.

Consider joint D:



+ve ↑ $\Sigma F_y = 0 \qquad -100.0 + F_{CD}\text{Sin}\theta = 0$ Equation (a)
+ve → $\Sigma F_x = 0 \qquad - F_{DE} - F_{CD}\text{Cos}\theta = 0$ Equation (b)
From Equation (a): $\mathbf{F_{CD} = +\ 166.7\ kN}$ **(Tie)**
From Equation (b): $\mathbf{F_{DE} = -\ 133.3\ kN}$ **(Strut)**

Consider joint E:

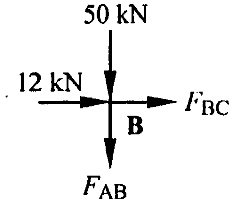


+ve → $\Sigma F_x = 0 \qquad -133.3 - F_{AE} = 0$ Equation (a)
+ve ↑ $\Sigma F_y = 0 \qquad + F_{CE} + 259.0 = 0$ Equation (b)
From Equation (a): $\mathbf{F_{AE} = -\ 133.3\ kN}$ **(Strut)**
From Equation (b): $\mathbf{F_{CE} = -\ 259.0\ kN}$ **(Strut)**

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.17 **Page No. 2**

Consider joint B:



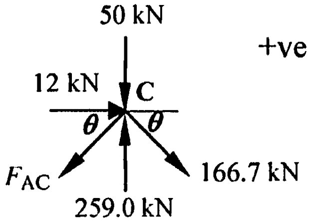
+ve → $\Sigma F_x = 0$ $+12.0 + F_{BC} = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $-50.0 - F_{AB} = 0$ Equation (b)

From Equation (a): $F_{BC} = -12.0$ **kN (Strut)**

From Equation (b): $F_{AB} = -50.0$ **kN (Strut)**

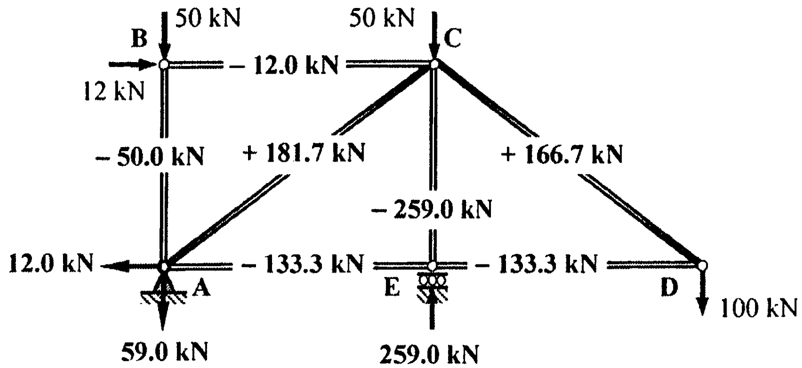
Consider joint C:



+ve → $\Sigma F_x = 0$ $+12.0 + 166.7\text{Cos}\theta - F_{AC}\text{Cos}\theta = 0$

$F_{AC} = +181.7$ **kN (Tie)**

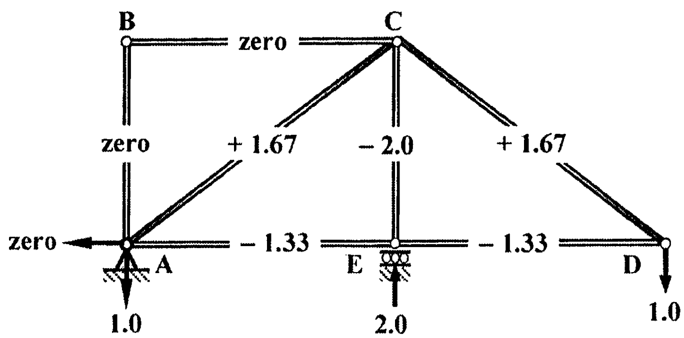
***P* - forces**



Vertical deflection at joint D:

Apply a Unit Load in the vertical direction at joint D and determine the values of the *u*-forces using joint resolution as before.

***u* - forces**



Complete the Unit Load table to determine the value of $\delta_{V,D}$

Solution

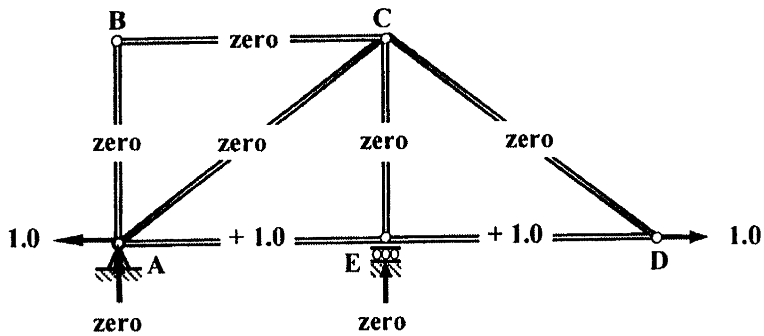
Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.17 **Page No. 3**

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	*u*	(*PL/AE*) × *u*
AB	3000	307.5×10^3	− 50.0	− 0.49	0	0
AC	5000	307.5×10^3	+ 181.7	+ 2.95	+ 1.67	+ 4.93
AE	4000	307.5×10^3	− 133.3	− 1.73	− 1.33	+ 2.31
BC	4000	307.5×10^3	− 12.0	− 0.16	0	0
CD	5000	307.5×10^3	+ 166.7	+ 2.71	+ 1.67	+ 4.53
CE	3000	307.5×10^3	− 259.0	− 2.53	− 2.0	+ 5.05
DE	4000	307.5×10^3	− 133.3	− 1.73	− 1.33	+ 2.31
						Σ = + 19.13

$$\delta_{V,D} = \sum\left(\frac{PL}{AE}\right) \times u = +\,19.13 \text{ mm} \downarrow$$

Horizontal deflection at joint D:
Apply a Unit Load in the horizontal direction at joint D and determine the values of the *u*-forces using joint resolution as before.

***u* - forces**



Complete the Unit Load table to determine the value of $\delta_{H,D}$

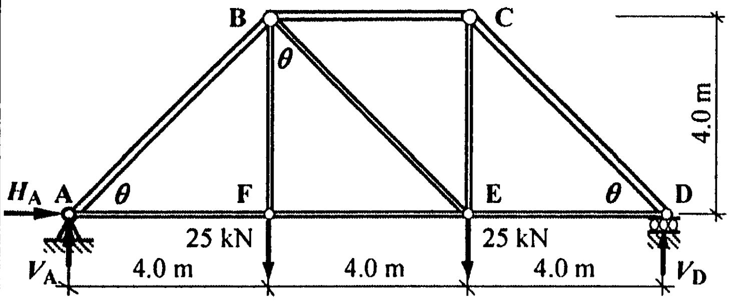
Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	*u*	(*PL/AE*) × *u*
AB	3000	307.5×10^3	− 50.0	− 0.49	0	0
AC	5000	307.5×10^3	+ 181.7	+ 2.95	0	0
AE	4000	307.5×10^3	− 133.3	− 1.73	+ 1.0	− 1.73
BC	4000	307.5×10^3	− 12.0	− 0.16	0	0
CD	5000	307.5×10^3	+ 166.7	+ 2.71	0	0
CE	3000	307.5×10^3	− 259.0	− 2.53	0	0
DE	4000	307.5×10^3	− 133.3	− 1.73	+ 1.0	− 1.73
						Σ = − 3.46

$$\delta_{H,D} = \sum\left(\frac{PL}{AE}\right) \times u = -\,3.46 \text{ mm} \leftarrow$$

Resultant deflection at joint D = $\delta_{R,D} = \sqrt{(19.13^2 + 3.46^2)} = 19.44$ mm 10.3^0

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.18 **Page No. 1**



The cross-sectional area of members AB, BC and CD equals 500 mm^2.
The cross-sectional area of all other members is equal to 250 mm^2.
Member BE is too short by 3.0 mm.

Determine the value of the vertical deflection at joint F and the horizontal deflection at joint B.

$E = 205$ kN/mm^2 and $\alpha = 12 \times 10^{-6}$/°C; $\theta = 45°$ Sin$\theta = 0.707$, Cos$\theta = 0.707$

Length of members AB, BE and CD $L_{AB,BE,CD} = \sqrt{4.0^2 + 4.0^2} = 5.657$ m

$AE_{500} = (500 \times 205) = 102.5 \times 10^3$ kN, $AE_{250} = (250 \times 205) = 51.25 \times 10^3$ kN

Determine the Support Reactions

Consider the rotational equilibrium of the frame:

+ve ↻ $\Sigma M_A = 0$ $+ (25.0 \times 4.0) + (25.0 \times 8.0) - (V_D \times 12.0) = 0$

$\therefore$ $\mathbf{V_D = +25.0}$ **kN** ↑

Consider the horizontal equilibrium of the frame:

+ve → $\Sigma F_x = 0$ $\therefore$ $\mathbf{H_A}$ **= zero**

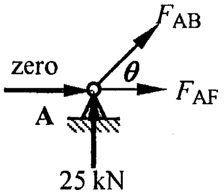
Consider the vertical equilibrium of the frame:

+ve ↑ $\Sigma F_y = 0$ $+ V_A - 25.0 - 25.0 + V_D = 0$ $\therefore$ $V_A = 50.0 - 25.0$

$\therefore$ $\mathbf{V_A = +25.0}$ **kN** ↑

Assume all unknown member forces to be tension and use joint resolution to determine the *P*-forces in the frame.

Consider joint A:



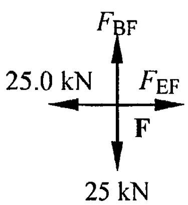
+ve ↑ $\Sigma F_y = 0$ $+ 25.0 + F_{AB}\text{Sin}\theta = 0$ Equation (a)

+ve → $\Sigma F_x = 0$ $+ F_{AF} + F_{AB}\text{Cos}\theta = 0$ Equation (b)

From Equation (a): $\mathbf{F_{AB} = -35.36}$ **kN (Strut)**

From Equation (b): $\mathbf{F_{AF} = +25.0}$ **kN (Tie)**

Consider joint F:



+ve → $\Sigma F_x = 0$ $- 25.0 + F_{EF} = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $+ F_{BF} - 25.0 = 0$ Equation (b)

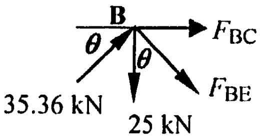
From Equation (a): $\mathbf{F_{EF} = +25.0}$ **kN (Tie)**

From Equation (b): $\mathbf{F_{BF} = +25.0}$ **kN (Tie)**

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.18 **Page No. 2**

Consider joint B:



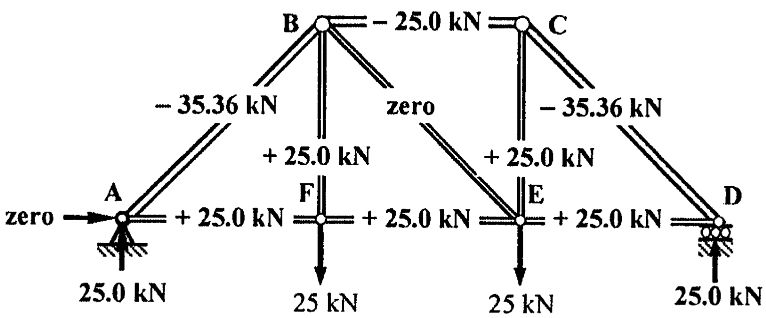
+ve ↑ $\Sigma F_y = 0$ $+ 35.36\text{Sin}\theta - 25.0 - F_{BE}\text{Cos}\theta = 0$
Equation (a)

+ve → $\Sigma F_x = 0$ $+ 35.36\text{Cos}\theta + F_{BC} + F_{BE}\text{Sin}\theta = 0$
Equation (b)

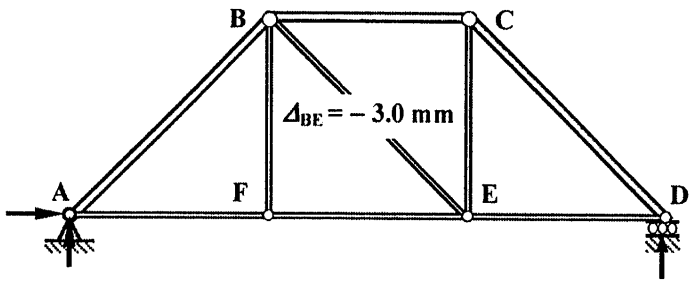
From Equation (a): $\boldsymbol{F_{BE}}$ **= zero**
From Equation (b): $\boldsymbol{F_{BC}}$ **= – 25.0 kN (Strut)**

By symmetry:
$\boldsymbol{F_{CD}}$ **= – 35.36 kN (Strut),** $\boldsymbol{F_{DE}}$ **= + 25.0 kN (Tie),** $\boldsymbol{F_{CE}}$ **= + 25.0 kN (Tie)**

P **- forces**

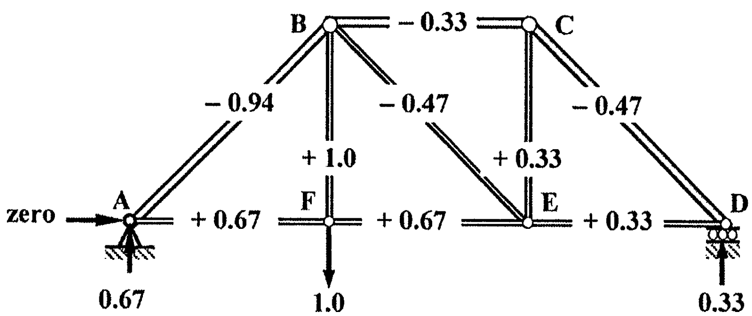


Lack–of–fit (Δ_L)



Vertical deflection at joint F:
Apply a Unit Load in the vertical direction at joint F and determine the values of the *u*-forces using joint resolution as before.

u **- forces**



Complete the Unit Load table to determine the value of $\delta_{V,F}$

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.18 **Page No. 3**

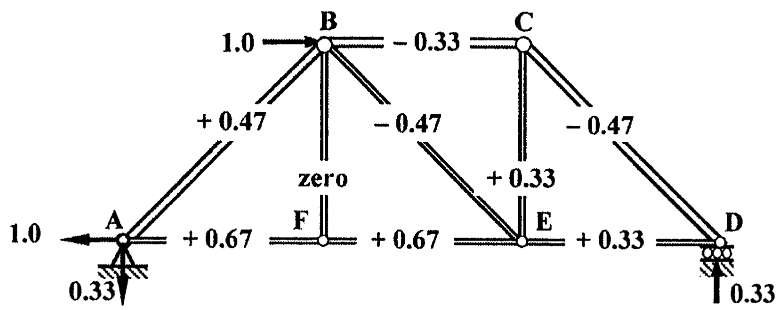
Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	Δ_L (mm)	*u*	$(PL/AE + \Delta_L) \times u$ (mm)
AB	5657	102.5×10^3	− 35.36	−1.95	0	− 0.94	+ 1.83
AF	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.67	+ 1.31
BC	4000	102.5×10^3	− 25.0	− 0.98	0	− 0.33	+ 0.32
BE	5657	51.25×10^3	0	0	− 3.0	− 0.47	+ 1.41
BF	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 1.0	+ 1.95
CD	5657	102.5×10^3	− 35.36	−1.95	0	− 0.47	+ 0.92
CE	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.33	+ 0.64
DE	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.33	+ 0.64
EF	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.67	+ 1.31
							Σ = + 10.33

$$\delta_{V,F} = \sum\left(\frac{PL}{AE}\right) \times u = +\ 10.33 \text{ mm} \downarrow$$

Horizontal deflection at joint B:

Apply a Unit Load in the horizontal direction at joint B and determine the values of the *u*-forces using joint resolution as before.

***u* - forces**



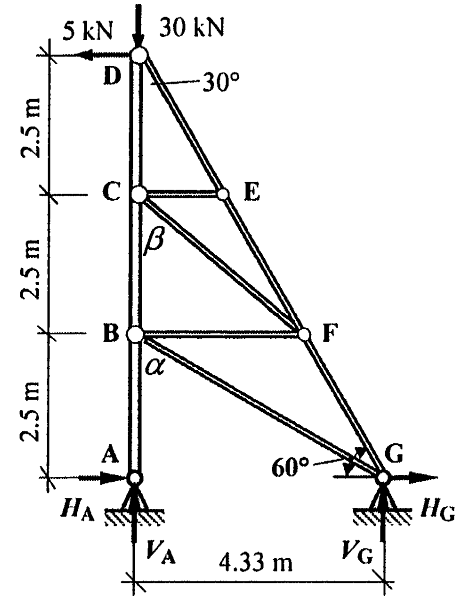
Complete the Unit Load table to determine the value of $\delta_{H,B}$

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	Δ_L (mm)	*u*	$(PL/AE + \Delta_L) \times u$ (mm)
AB	5657	102.5×10^3	− 35.36	−1.95	0	+ 0.47	− 0.92
AF	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.67	+ 1.31
BC	4000	102.5×10^3	− 25.0	−0.98	0	− 0.33	+ 0.32
BE	5657	51.25×10^3	0	0	− 3.0	− 0.47	+ 1.41
BF	4000	51.25×10^3	+ 25.0	+ 1.95	0	0	0
CD	5657	102.5×10^3	− 35.36	−1.95	0	− 0.47	+ 0.92
CE	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.33	+ 0.64
DE	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.33	+ 0.64
EF	4000	51.25×10^3	+ 25.0	+ 1.95	0	+ 0.67	+ 1.31
							Σ = + 5.63

$$\delta_{H,B} = \sum\left(\frac{PL}{AE}\right) \times u = +\ 5.63 \text{ mm} \longrightarrow$$

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.19 **Page No. 1**



The cross-sectional area of all members is equal to 1200 mm^2.
Determine the value of the horizontal deflection at joint D.
$E = 205$ kN/mm^2
$L_{DE} = L_{EF} = L_{FG} = 2.887$ m
$L_{BF} = 2.887$ m $L_{CF} = 3.819$ m
$L_{CE} = 1.443$ m $L_{BG} = 5.0$ m

$\alpha = \tan^{-1}(4.33/2.5) = 60°$
$\beta = \tan^{-1}(2.887/2.5) = 49.11°$
$\sin\alpha = 0.866$ $\sin\beta = 0.756$
$\cos\alpha = 0.5$ $\cos\beta = 0.655$
$\tan\alpha = 1.732$ $\tan\beta = 1.155$

$AE_{1200} = (1200 \times 205) = 246.0 \times 10^3$ kN

Determine the Support Reactions

Consider the rotational equilibrium of the frame:
+ve ↻ $\Sigma M_A = 0$ $-(5.0 \times 7.5) - (V_G \times 4.33) = 0$

$\therefore$ **$V_G = -8.66$ kN** ↓

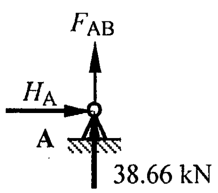
Consider the horizontal equilibrium of the frame:
+ve → $\Sigma F_x = 0$ $+H_A + H_G - 5.0 = 0$ $\therefore H_G = 5.0 - H_A$

Consider the vertical equilibrium of the frame:
+ve ↑ $\Sigma F_y = 0$ $+V_A - 30.0 + V_G = 0$ $\therefore V_A = 30.0 + 8.66$

$\therefore$ **$V_A = +38.66$ kN** ↑

Assume all unknown member forces to be tension and use joint resolution to determine the *P*-forces in the frame.

Consider joint A:



+ve ↑ $\Sigma F_y = 0$ $+38.66 + F_{AB} = 0$ Equation (a)
+ve → $\Sigma F_x = 0$ $+H_A = 0$ Equation (b)
From Equation (a): **$F_{AB} = -38.66$ kN (Strut)**
From Equation (b): **H_A = zero**
$\therefore$ **$H_G = 5.0$ kN** →

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
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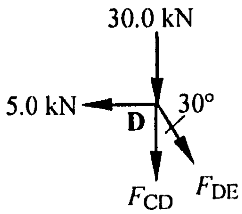
Consider joint D:



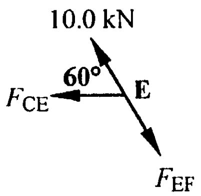
+ve → $\Sigma F_x = 0$ $-5.0 + F_{DE}\text{Sin}30° = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $-30.0 - F_{CD} - F_{DE}\text{Cos}30° = 0$ Equation (b)

From Equation (a): **$F_{DE} = +10.0$ kN (Tie)**

From Equation (b): **$F_{CD} = -38.66$ kN (Strut)**

Consider joint E: Resolve forces perpendicular and parallel to F_{DE} and F_{EF}



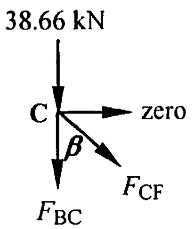
+ve ↗ $\Sigma F_{perpendicular} = 0$ $-F_{CE}\text{Sin}60° = 0$ Equation (a)

+ve ↖ $\Sigma F_{parallel} = 0$ $+F_{DE} - F_{EF} + F_{CE}\text{Cos}60° = 0$ Equation (b)

From Equation (a): **F_{EC} = zero**

From Equation (b): **$F_{EF} = +10.0$ kN (Tie)**

Consider joint C:



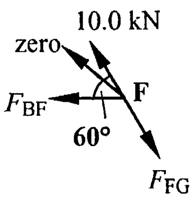
+ve → $\Sigma F_x = 0$ $+F_{CF}\text{Sin}\beta = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $-38.66 - F_{BC} - F_{CF}\text{Cos}\beta = 0$ Equation (b)

From Equation (a): **F_{CF} = zero**

From Equation (b): **$F_{BC} = -38.66$ kN (Strut)**

Consider joint F: Resolve forces perpendicular and parallel to F_{FG}



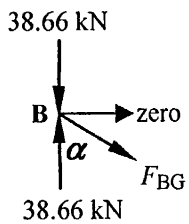
+ve ↗ $\Sigma F_{perpendicular} = 0$ $-F_{BF}\text{Sin}60° = 0$ Equation (a)

+ve ↖ $\Sigma F_{parallel} = 0$ $+10.0 - F_{FG} + F_{BF}\text{Cos}60° = 0$ Equation (b)

From Equation (a): **F_{BF} = zero**

From Equation (b): **$F_{FG} = +10.0$ kN (Tie)**

Consider joint B:

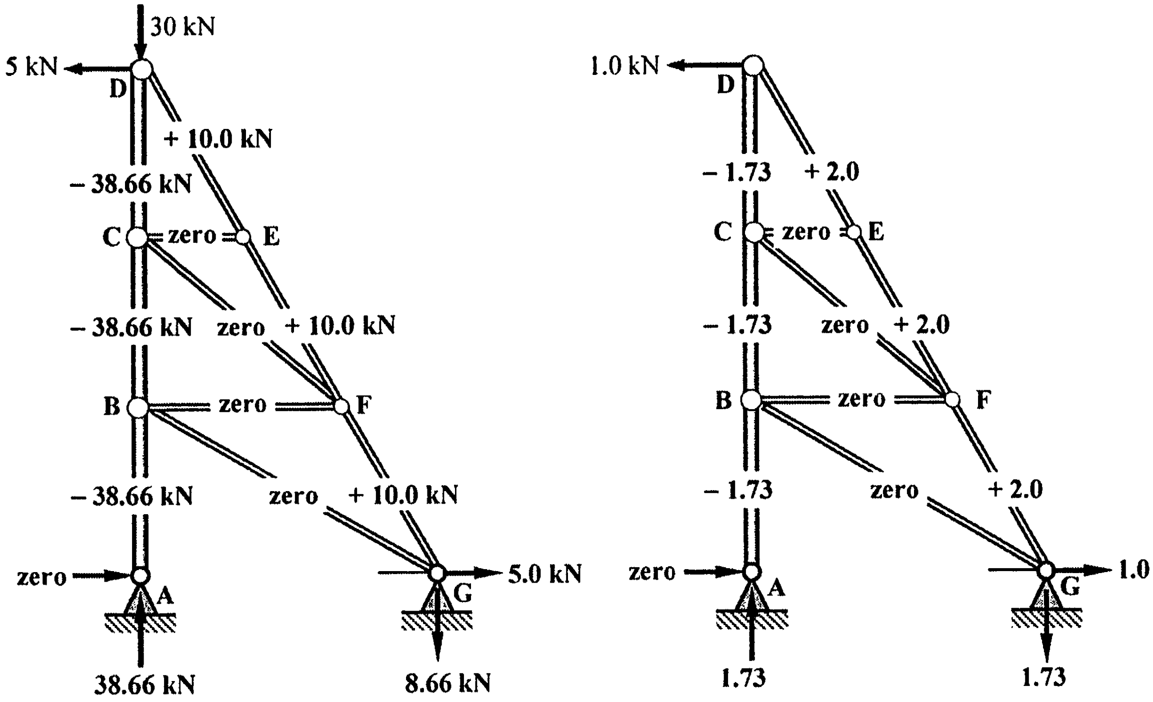


+ve → $\Sigma F_x = 0$ $+F_{BG}\text{Sin}\alpha = 0$ **F_{BG} = zero**

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.19 **Page No. 3**

Horizontal deflection at joint D:
Apply a Unit Load in the horizontal direction at joint D and determine the values of the *u*-forces using joint resolution as before.



***P* - forces** ***u* - forces**

Complete the Unit Load table to determine the value of $\delta_{H,D}$

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	*u*	(*PL/AE*) × *u* (mm)
AB	2500	246.0×10^3	– 38.66	– 0.39	– 1.73	+ 0.68
BC	2500	246.0×10^3	– 38.66	– 0.39	– 1.73	+ 0.68
BF	2887	246.0×10^3	0	0	0	0
BG	5000	246.0×10^3	0	0	0	0
CD	2500	246.0×10^3	– 38.66	– 0.39	– 1.73	+ 0.68
CE	1443	246.0×10^3	0	0	0	0
CF	3819	246.0×10^3	0	0	0	0
DE	2887	246.0×10^3	+ 10.0	+ 0.12	+ 2.0	+ 0.23
EF	2887	246.0×10^3	+ 10.0	+ 0.12	+ 2.0	+ 0.23
FG	2887	246.0×10^3	+ 10.0	+ 0.12	+ 2.0	+ 0.23
						Σ = + 2.73

$$\delta_{H,D} = \sum \left(\frac{PL}{AE} \right) \times u = +2.73 \text{ mm} \quad \leftarrow$$

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames

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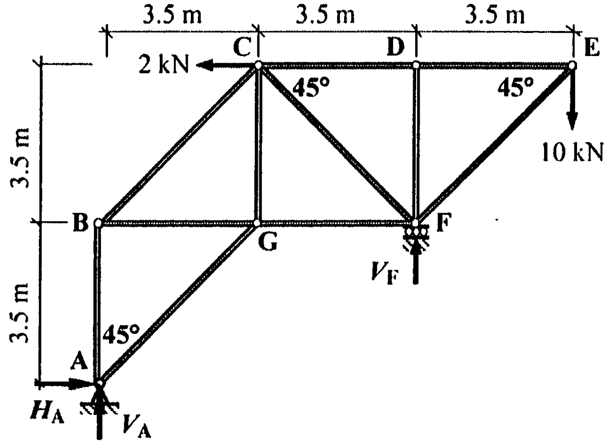


The cross-sectional area of members AG, BG, CF, CG, EF, and FG is equal to 400 mm^2.

The cross-sectional area of all other members is equal to 100 mm^2.

All members are subjected to a decrease in temperature equal to 20°C.

Determine the horizontal deflection at joint F.

$E = 205$ kN/mm^2 and $\alpha = 12 \times 10^{-6}$/°C

$L_{AG,BC,CF,EF} = \sqrt{3.5^2 + 3.5^2} = 4950$ mm^2

Sin45° = 0.707, Cos45° = 0.707

$AE_{100} = (100 \times 205) = 20.5 \times 10^3$ kN

$AE_{400} = (400 \times 205) = 82.0 \times 10^3$ kN

The δL value for members AG, BC, CF and EF due to temperature change:

$\Delta_T = -\,\alpha L \Delta_T = -\,(12 \times 10^{-6} \times 4950 \times 20.0) = -\,1.19$ mm

The δL value for all other members due to temperature change:

$\Delta_T = -\,\alpha L \Delta_T = -\,(12 \times 10^{-6} \times 3500 \times 20.0) = -\,0.84$ mm

Determine the Support Reactions

Consider the rotational equilibrium of the frame:

+ve ↻ $\Sigma M_A = 0$ $\quad -(2.0 \times 7.0) + (10 \times 10.5) - (V_F \times 7.0) = 0$

$\therefore$ **$V_F = +\,13.0$ kN** ↑

Consider the horizontal equilibrium of the frame:

+ve → $\Sigma F_x = 0$ $\quad +H_A - 2.0 = 0$ $\quad \therefore$ **$H_A = +\,2.0$ kN** →

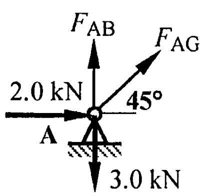
Consider the vertical equilibrium of the frame:

+ve ↑ $\Sigma F_y = 0$ $\quad +V_A - 10.0 + V_F = 0$ $\quad \therefore V_A = 10.0 - 13.0$

$\therefore$ **$V_A = -\,3.0$ kN** ↓

Assume all unknown member forces to be tension and use joint resolution to determine the P-forces in the frame.

Consider joint A:



+ve → $\Sigma F_x = 0$ $\quad +2.0 + F_{AG}\text{Cos}45° = 0$ $\quad$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $\quad -3.0 + F_{AB} + F_{AG}\text{Sin}45° = 0$ $\quad$ Equation (b)

From Equation (a): **$F_{AG} = -\,2.83$ kN (Strut)**

From Equation (b): **$F_{AB} = +\,5.0$ kN (Tie)**

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames

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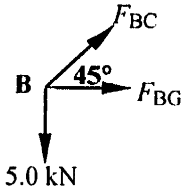
Consider joint B:



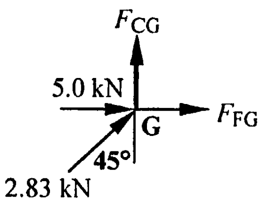
+ve ↑ $\Sigma F_y = 0$ $-5.0 + F_{BC}\text{Sin}45° = 0$ Equation (a)

+ve → $\Sigma F_x = 0$ $+F_{BG} + F_{BC}\text{Cos}45° = 0$ Equation (b)

From Equation (a): **$F_{BC} = +7.07$ kN (Tie)**

From Equation (b): **$F_{BG} = -5.0$ kN (Strut)**

Consider joint G:



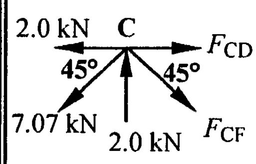
+ve ↑ $\Sigma F_y = 0$ $+2.83\text{Cos}45° + F_{CG} = 0$ Equation (a)

+ve → $\Sigma F_x = 0$ $+5.0 + 2.83\text{Sin}45° + F_{FG} = 0$ Equation (b)

From Equation (a): **$F_{CG} = -2.0$ kN (Strut)**

From Equation (b): **$F_{FG} = -7.0$ kN (Strut)**

Consider joint C:



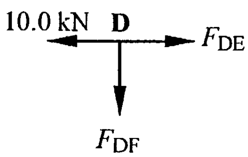
+ve ↑ $\Sigma F_y = 0$ $+2.0 - 7.07\text{Sin}45° - F_{CF}\,\text{Sin}45° = 0$ Equation (a)

+ve → $\Sigma F_x = 0$ $-2.0 - 7.07\text{Cos}45° + F_{CF}\text{Cos}45° + F_{CD} = 0$ Equation (b)

From Equation (a): **$F_{CF} = -4.24$ kN (Strut)**

From Equation (b): **$F_{CD} = +10.0$ kN (Tie)**

Consider joint D:



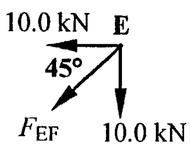
+ve → $\Sigma F_x = 0$ $-10.0 + F_{DE} = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $-F_{DF} = 0$ Equation (b)

From Equation (a): **$F_{DE} = +10.0$ kN (Tie)**

From Equation (b): **F_{DF} = zero**

Consider joint E:



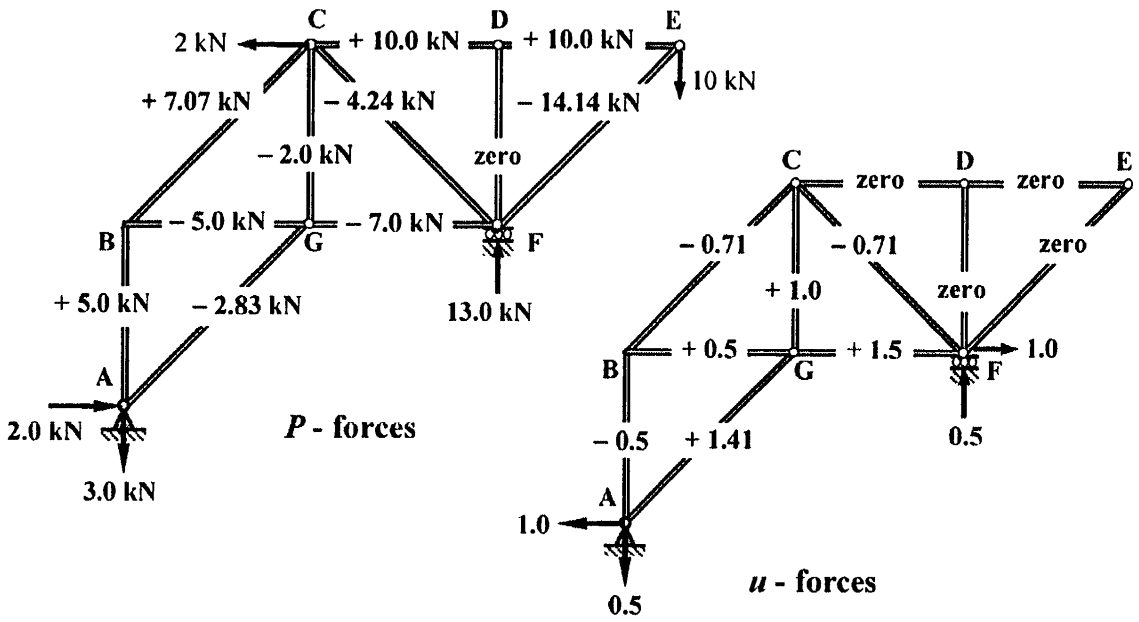
+ve → $\Sigma F_x = 0$ $-10.0 - F_{EF}\text{Cos}45° = 0$

$F_{EF} = -14.14$ kN (Strut)

Solution

Topic: Unit Load Method for Deflection of Pin-Jointed Frames
Problem Number: 3.20 **Page No. 3**

Horizontal deflection at joint F:
Apply a Unit Load in the horizontal direction at joint F and determine the values of the *u*-forces using joint resolution as before.



The δL value for members (AG, BC, CF and EF) due to temperature change: $\Delta_T = -1.19$ mm

The δL value for all other members due to temperature change: $\Delta_T = -0.84$ mm

Complete the Unit Load table to determine the value of $\delta_{H,F}$

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	Δ_T (mm)	*u*	$(PL/AE + \Delta_T) \times u$ (mm)
AB	3500	20.5×10^3	+ 5.0	+ 0.85	– 0.84	– 0.50	– 0.01
AG	4950	82.0×10^3	– 2.83	– 0.17	– 1.19	+ 1.41	– 1.92
BC	4950	20.5×10^3	+ 7.07	+ 1.71	– 1.19	– 0.71	– 0.37
BG	3500	82.0×10^3	– 5.0	– 0.21	– 0.84	+ 0.50	– 0.53
CD	3500	20.5×10^3	+ 10.0	+ 1.71	– 0.84	0	0
CF	4950	82.0×10^3	– 4.24	– 0.26	– 1.19	– 0.71	+ 1.02
CG	3500	82.0×10^3	– 2.0	– 0.09	– 0.84	+ 1.00	– 0.93
DE	3500	20.5×10^3	+ 10.0	+ 1.71	– 0.84	0	0
DF	3500	20.5×10^3	0	0	– 0.84	0	0
EF	4950	82.0×10^3	– 14.14	– 0.85	– 1.19	0	0
FG	3500	82.0×10^3	– 7.0	– 0.30	– 0.84	+ 1.50	– 1.71
							$\Sigma = -4.45$

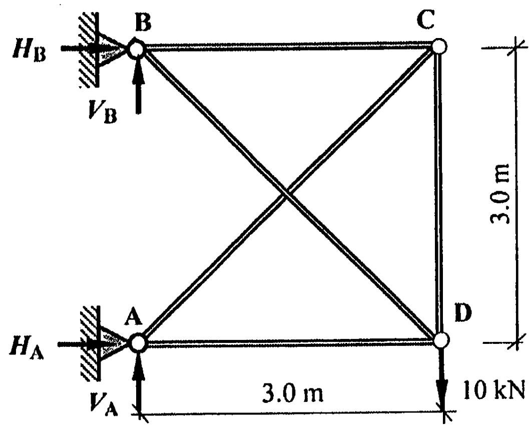
$$\delta_{H,F} = \sum\left(\frac{PL}{AE}\right) \times u = -4.45 \text{ mm} \leftarrow$$

3.6 *Unit Load Method for Singly-Redundant Pin-Jointed Frames*

The method of analysis illustrated in Section 3.5 can also be adopted to determine the member forces in singly-redundant frames. Consider the frame shown in Example 3.6.

3.6.1 Example 3.6: Singly-Redundant Pin-Jointed Frame 1

Using the data given, determine the member forces and support reactions for the pin-jointed frame shown in Figure 3.23.



The cross-sectional area of all members is equal to 175 mm^2.

$E = 205$ kN/mm^2

Figure 3.23

The degree–of–indeterminacy $I_D = (m + r) - 2n = (5 + 4) - (2 \times 4) = 1$

Assume that member BD is a redundant member and consider the original frame to be the superposition of two structures as indicated in Figures 3.24(a) and (b). The frame in Figure 3.24(b) can be represented as shown in Figure 3.25.

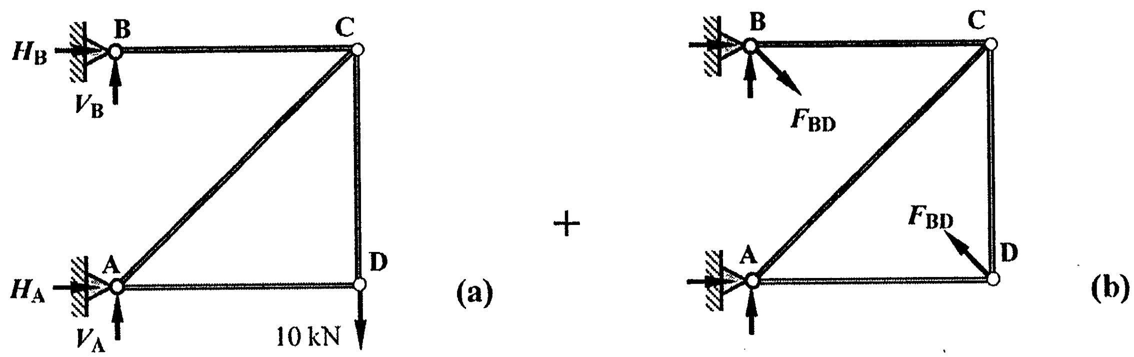


Figure 3.24

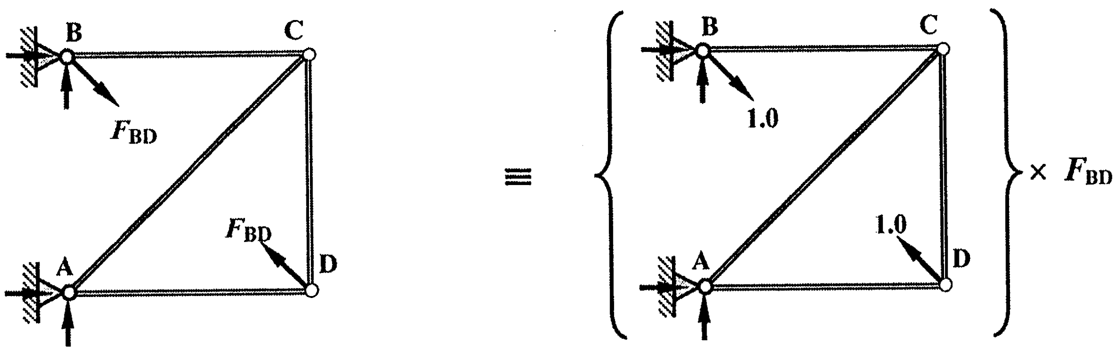


Figure 3.25

To maintain compatibility in the length of member BD in the original frame the change in length of the diagonal BD in Figure 3.24(a) must be equal and opposite to that in Figure 3.24(b) as shown in Figure 3.26.

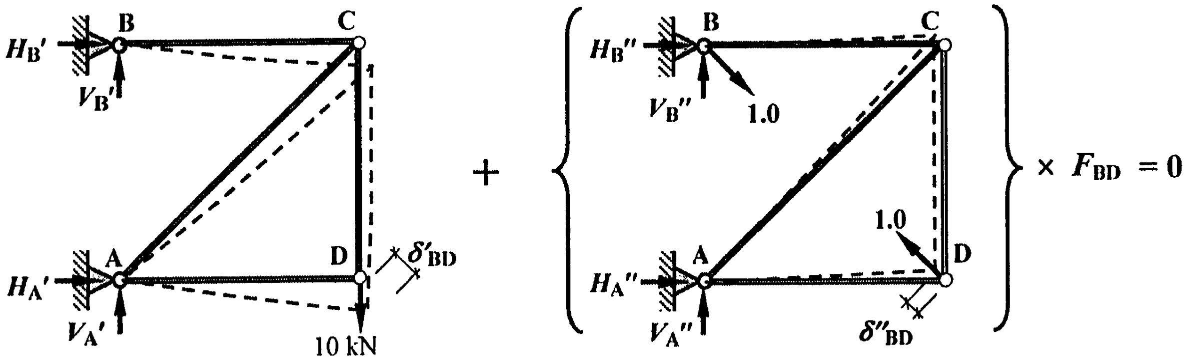


Figure 3.26

(δ'_{BD} due to P–forces) + (δ''_{BD} due to unit load forces) × $F_{BD} = 0$

i.e. $\sum \frac{PL}{AE}u + \left(\sum \frac{uL}{AE}u\right) \times F_{BD} = 0 \quad \therefore \quad F_{BD} = -\sum \frac{PL}{AE}u \Big/ \sum \frac{uL}{AE}u$

Using joint resolution the P-forces and the u-forces can be determined as indicated in Figure 3.27.

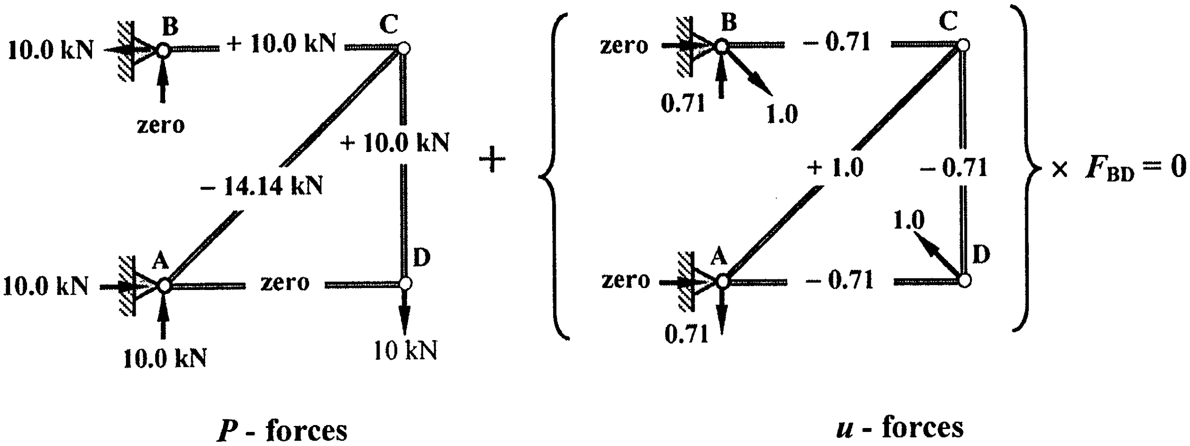


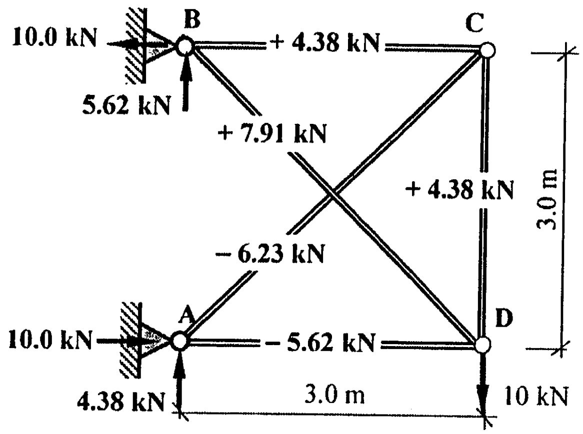
Figure 3.27

Member	Length (mm)	AE (kN)	P-force (kN)	PL/AE (mm)	u	$(PL/AE) \times u$ (mm)	$(uL/AE) \times u$ (mm)	Member forces
BC	3000	**35.88 ×10³**	+ 10.00	+ 0.84	– 0.71	– 0.59	0.04	**+ 4.38**
CD	3000	**35.88 ×10³**	+ 10.00	+ 0.84	– 0.71	– 0.59	0.04	**+ 4.38**
DA	3000	**35.88 ×10³**	0	0	– 0.71	0	0.04	**– 5.62**
AC	4243	**35.88 ×10³**	– 14.14	– 1.67	+ 1.00	– 1.67	0.12	**– 6.23**
BD	4243	**35.88 ×10³**	0	0	+ 1.00	0	0.12	**+ 7.91**
						Σ = – 2.85	**Σ = + 0.36**	

$F_{BD} = -\sum \frac{PL}{AE}u \Big/ \sum \frac{uL}{AE}u = +2.85/0.36 = +7.91$ kN **(Tie)**

The final member forces = [P-forces + (u-forces × 7.91)] and are given in the last column of the table

V_A = + 10.0 – (0.71 × 7.91) = + 4.38 kN ↑
H_A = + 10.0 + zero = + 10.0 kN →
V_B = zero + (0.71 × 7.91) = + 5.62 kN ↑
H_B = – 10.0 + zero = – 10.0 kN ←



Final member forces and support reactions

Figure 3.28

3.6.2 Example 3.7: Singly-Redundant Pin-Jointed Frame 2

Using the data given, determine the member forces and support reactions for the pin-jointed frame shown in Figure 3.29.
The cross–sectional area of all members is equal to 140 mm^2. Assume E = 205 kN/mm^2

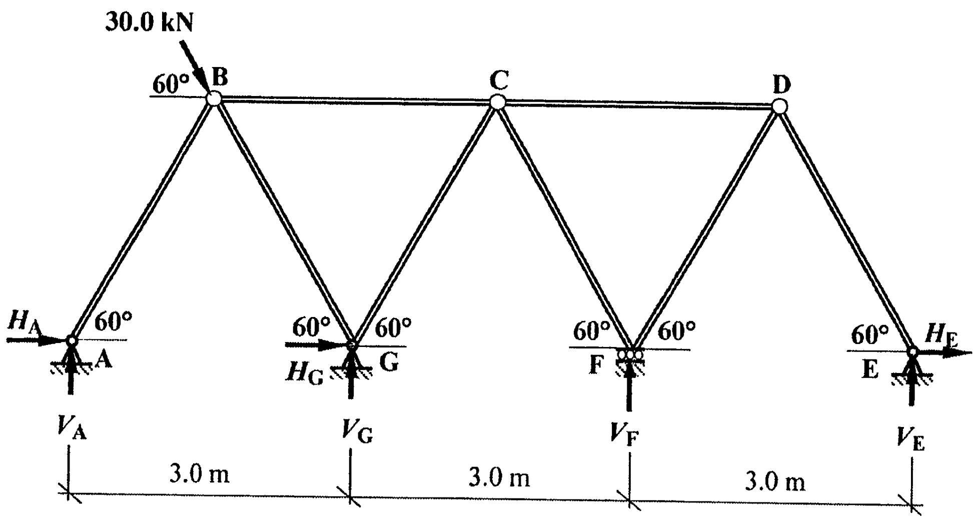


Figure 3.29

All member lengths L = 3.0 m
AE = (140 × 205) = 28.7 × 10^3 kN
Sin60° = 0.866 Cos60° = 0.5
Consider the applied load as two components 30.0Sin60° = 25.98 kN ↓
30.0Cos60° = 15.0 kN →
The degree of indeterminacy $I_D = (m + r) - 2n = (8 + 7) - (2 \times 7) = 1$

Consider the vertical reaction at support F to be redundant. The equivalent system is the superposition of the statically determinate frame and the (unit load frame × V_F) as shown in Figures 3.30 and 3.31.

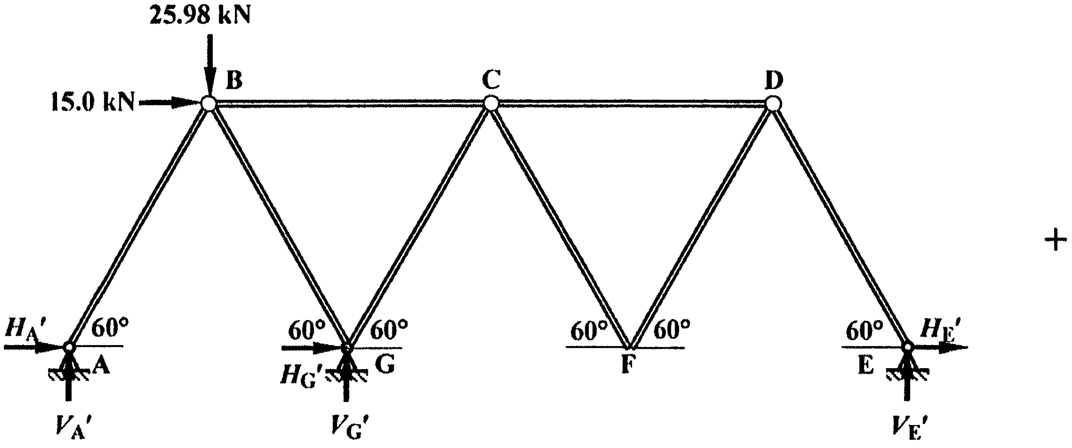


Figure 3.30

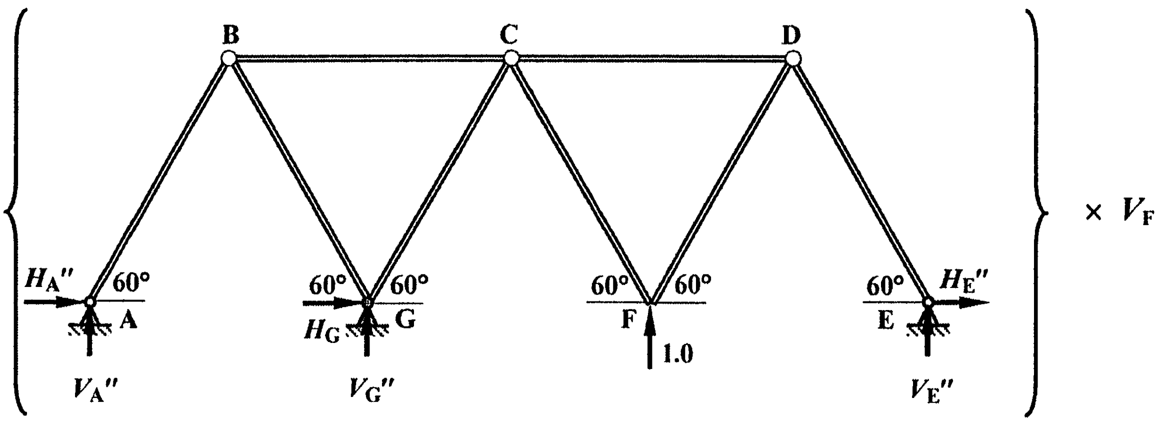


Figure 3.31

Using joint resolution the P-forces and the u-forces can be determined as indicated in Figures 3.32 and 3.33.

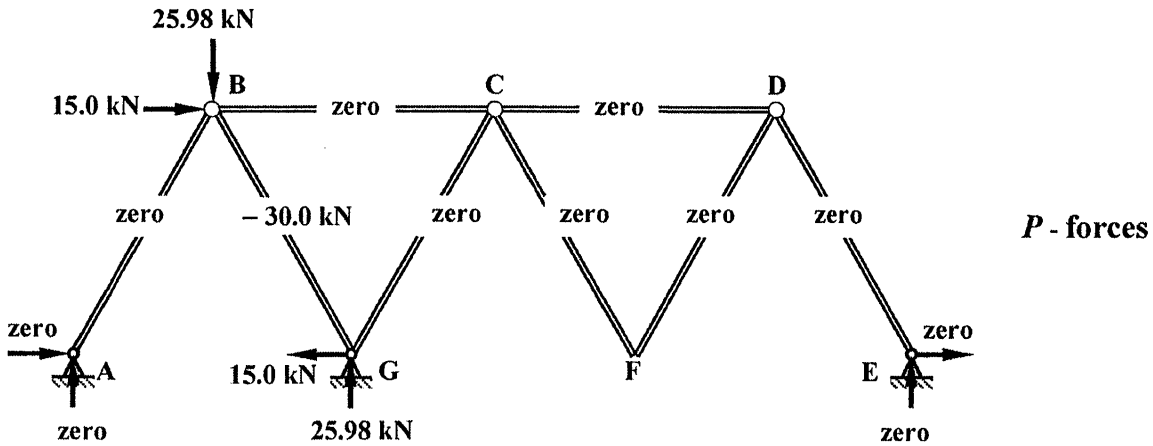


Figure 3.32

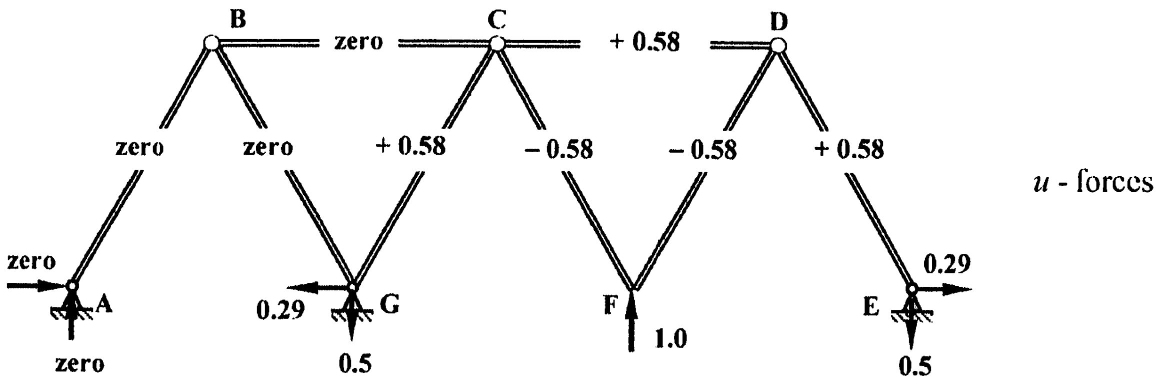


Figure 3.33

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	*u*	(*PL/AE*) × *u* (mm)	(*uL/AE*) × *u* (mm)	Member forces
AB	3000	28.7×10^3	0	0	0	0	0	**0**
BC	3000	28.7×10^3	0	0	0	0	0	**0**
CD	3000	28.7×10^3	0	0	+ 0.58	0	0.035	**0**
DE	3000	28.7×10^3	0	0	+ 0.58	0	0.035	**0**
DF	3000	28.7×10^3	0	0	– 0.58	0	0.035	**0**
CF	3000	28.7×10^3	0	0	– 0.58	0	0.035	**0**
CG	3000	28.7×10^3	0	0	+ 0.58	0	0.035	**0**
BG	3000	28.7×10^3	– 30.00	– 3.14	0	0	0	**– 30.00**
						Σ = zero	**Σ = + 0.18**	

i.e. $\sum \frac{PL}{AE}u + \left(\sum \frac{uL}{AE}u\right) \times V_F = 0$

$V_F = -\sum \frac{PL}{AE}u \Big/ \sum \frac{uL}{AE}u = 0/0.18 = \text{zero}$

The final member forces = [*P*-forces + (*u*-forces × 0)] and are given in the last column of the table.

$V_G = + 25.98$ kN ↑
$H_G = - 15.0$ kN ←

All other reactions are equal to zero.

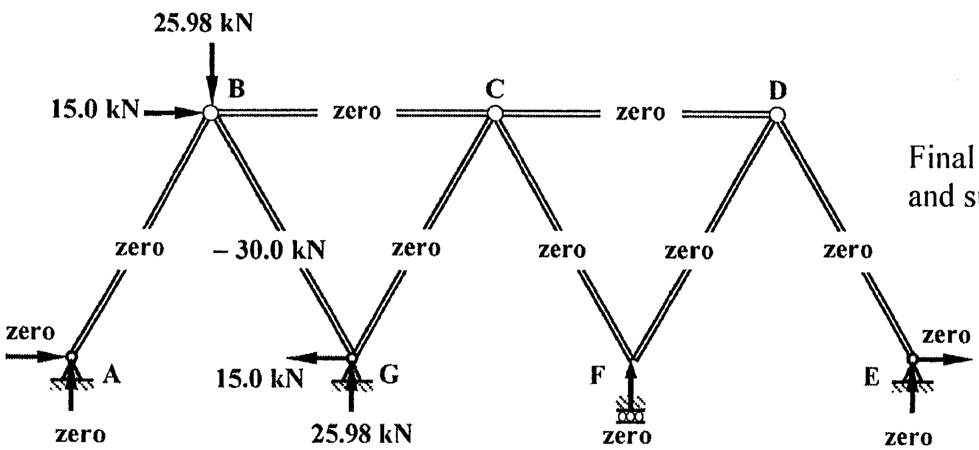
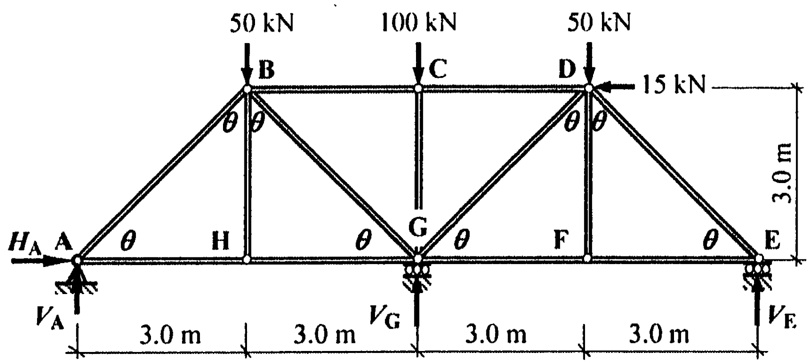


Figure 3.34

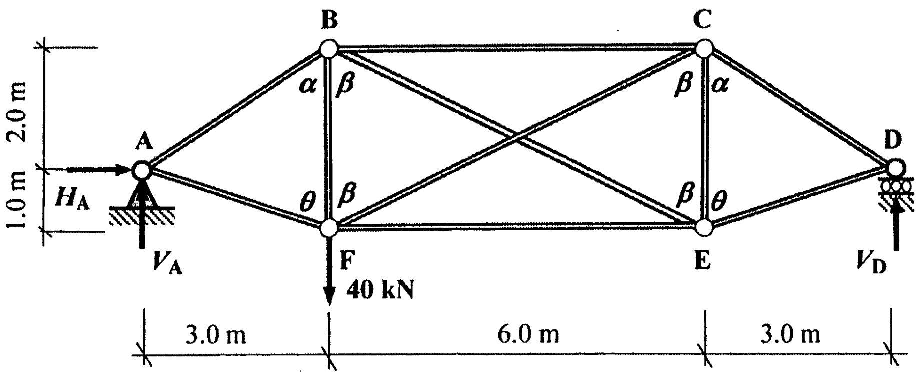
3.6.3 Problems: Unit Load Method for Singly-Redundant Pin-Jointed Frames

Using the data given in the singly-redundant, pin-jointed frames shown in Problems 3.21 to 3.24, determine the support reactions and the member forces due to the applied loads. Assume $E = 205$ kN/mm^2 and $\alpha = 12 \times 10^{-6}$/°C where required.



The cross-sectional area of members AH, GH, EF and FG is equal to 200 mm^2.
The cross-sectional area of all other members is equal to 500 mm^2.
The support at G settles by 12 mm.

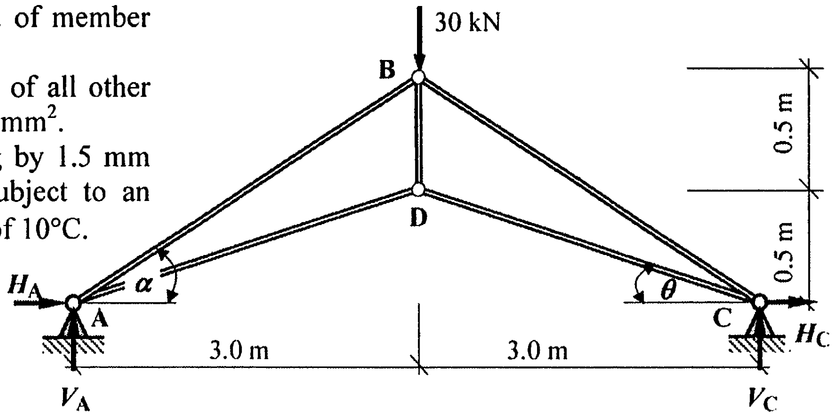
Problem 3.21



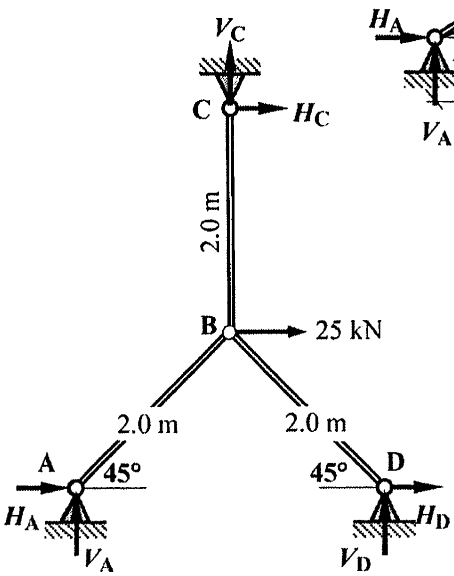
The cross-sectional area of all members is equal to 180 mm^2.

Problem 3.22

The cross-sectional area of member BD is equal to 100 mm^2.
The cross-sectional area of all other members is equal to 300 mm^2.
Member AD is too long by 1.5 mm and all members are subject to an increase in temperature of 10°C.



Problem 3.23



The cross-sectional area of all members is equal to 150 mm^2.

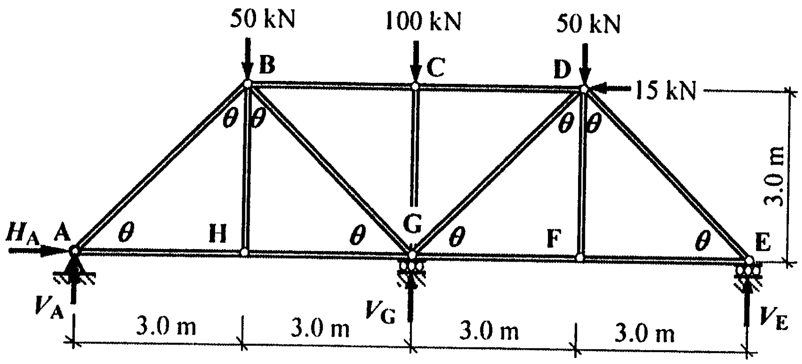
Member BD is 2.0 mm too short

Problem 3.24

3.6.4 Solutions: Unit Load Method for Singly-Redundant Pin-Jointed Frames

Solution

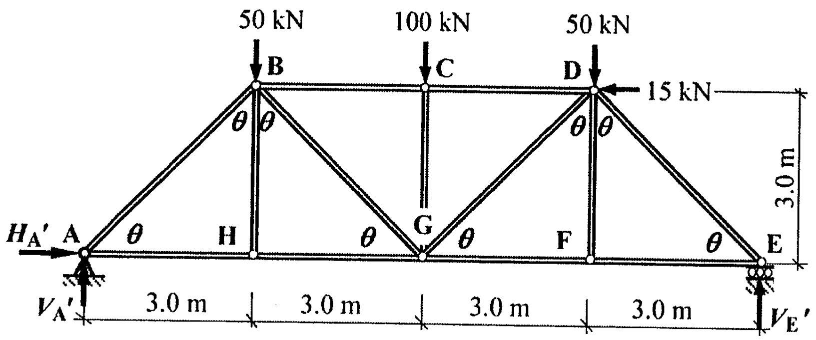
Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.21 **Page No. 1**



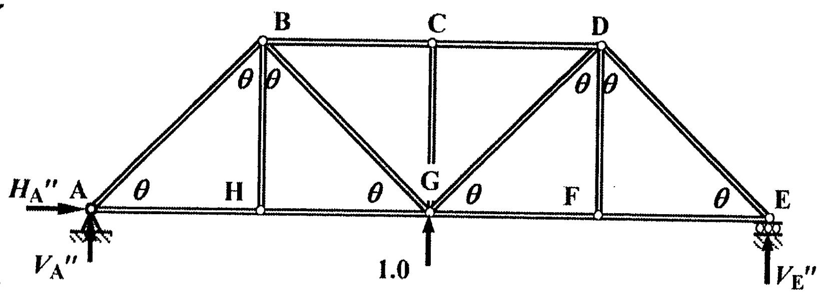
The cross–sectional area of members AH, GH, EF and FG is equal to 200 mm^2
The cross-sectional area of all other members is equal to 500 mm^2.
The support at G settles by 12 mm.
E = 205 kN/mm^2

$L_{AB,\,BG,\,DG,\,DE} = \sqrt{3.0^2 + 3.0^2} = 4.243$ m
$\sin\theta = (3.0/4.243) = 0.707$ $\cos\theta = (3.0/4.243) = 0.707$
$AE_{200} = (200 \times 205) = 41.0 \times 10^3$ kN
$AE_{500} = (500 \times 205) = 102.5 \times 10^3$ kN

Consider the vertical reaction at support G to be redundant.
The equivalent system is the superposition of the statically determinate frame and the (unit load frame × V_G) as shown:



***P* - forces**



***u* - forces** × V_G

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.21 **Page No. 2**

Determine the Support Reactions for the statically determinate frame.

Consider the rotational equilibrium of the frame:

+ve ↻ $\Sigma M_A = 0$ $+ (50.0 \times 3.0) + (100 \times 6.0) + (50.0 \times 9.0) - (15.0 \times 3.0)$
$- (V_E' \times 12.0) = 0$ $\therefore$ $V_E' = + 96.25$ **kN** ↑

Consider the horizontal equilibrium of the frame:

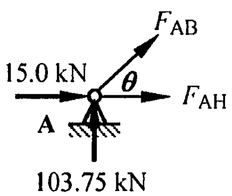
+ve → $\Sigma F_x = 0$ $+ H_A' - 15.0 = 0$ $\therefore$ $H_A' = + 15.0$ **kN** →

Consider the vertical equilibrium of the frame:

+ve ↑ $\Sigma F_y = 0$ $+ V_A' - 50.0 - 100.0 - 50.0 + V_E' = 0$ $\therefore$ $V_A' = 200.0 - 96.25$
$\therefore$ $V_A' = + 103.75$ **kN** ↑

Assume all unknown member forces to be tension and use joint resolution to determine the *P*-forces in the frame.

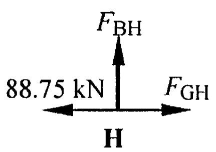
Consider joint A:



+ve ↑ $\Sigma F_y = 0$ $+ 103.75 + F_{AB}\mathrm{Sin}\theta = 0$ Equation (a)
+ve → $\Sigma F_x = 0$ $+ 15.0 + F_{AH} + F_{AB}\mathrm{Cos}\theta = 0$ Equation (b)

From Equation (a): $F_{AB} = - 146.70$ **kN (Strut)**
From Equation (b): $F_{AH} = + 88.75$ **kN (Tie)**

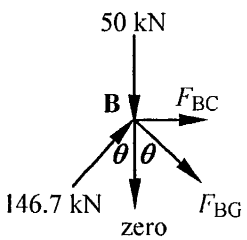
Consider joint H:



+ve ↑ $\Sigma F_y = 0$ $+ F_{BH} = 0$ Equation (a)
+ve → $\Sigma F_x = 0$ $- 88.75 + F_{GH} = 0$ Equation (b)

From Equation (a): $F_{BH} =$ **zero**
From Equation (b): $F_{GH} = + 88.75$ **kN (Tie)**

Consider joint B:



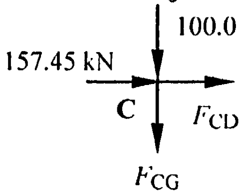
+ve ↑ $\Sigma F_y = 0$ $- 50.0 + 146.7\mathrm{Cos}\theta - F_{BG}\mathrm{Cos}\theta = 0$
Equation (a)
+ve → $\Sigma F_x = 0$ $+ 146.7\mathrm{Sin}\theta + F_{BG}\,\mathrm{Sin}\theta + F_{BC} = 0$
Equation (b)

From Equation (a): $F_{BG} = + 76.0$ **kN (Tie)**
From Equation (b): $F_{BC} = - 157.45$ **kN (Strut)**

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.21 **Page No. 3**

Consider joint C:



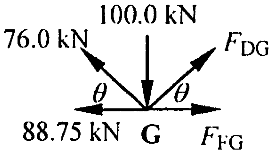
+ve ↑ $\Sigma F_y = 0$ $\quad - 100.0 - F_{CG} = 0$ Equation (a)

+ve → $\Sigma F_x = 0$ $\quad + 157.45 + F_{CD} = 0$ Equation (b)

From Equation (a): $\quad$ **F_{CG} = – 100.0 kN (Strut)**

From Equation (b): $\quad$ **F_{CD} = – 157.45 kN (Strut)**

Consider joint G:



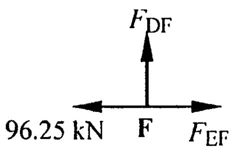
+ve ↑ $\Sigma F_y = 0$ $\quad - 100.0 + 76.0 \sin\theta + F_{DG}\sin\theta = 0$ Equation (a)

+ve → $\Sigma F_x = 0$ $\quad - 88.75 - 76.0\cos\theta + F_{DG}\cos\theta + F_{FG} = 0$ Equation (b)

From Equation (a): $\quad$ **F_{DG} = + 65.42 kN (Tie)**

From Equation (b): $\quad$ **F_{FG} = + 96.25 kN (Tie)**

Consider joint F:



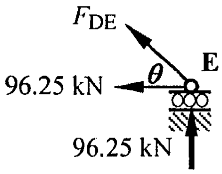
+ve ↑ $\Sigma F_y = 0$ $\quad + F_{DF} = 0$ Equation (a)

+ve → $\Sigma F_x = 0$ $\quad - 96.23 + F_{EF} = 0$ Equation (b)

From Equation (a): $\quad$ **F_{DF} = zero**

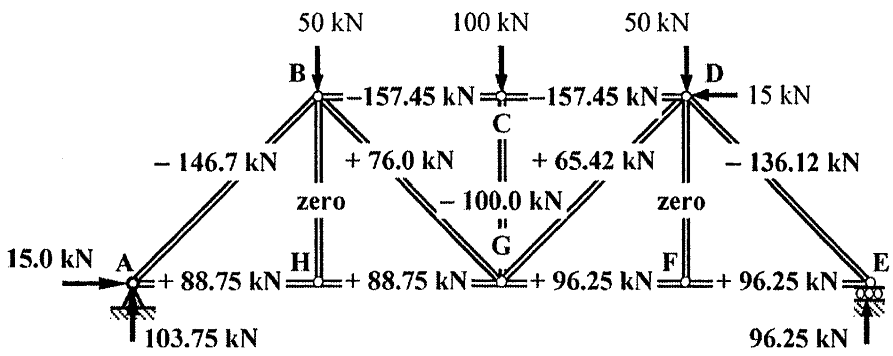
From Equation (b): $\quad$ **F_{EF} = + 96.25 kN (Tie)**

Consider joint E:



+ve → $\Sigma F_x = 0$ $\quad - 96.25 - F_{DE}\cos\theta = 0$

F_{DE} = – 136.12 kN (Strut)

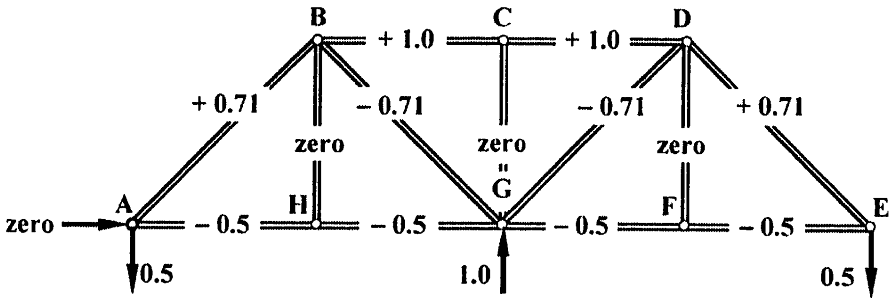


***P* - forces**

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.21 **Page No. 4**

Apply a Unit Load in the vertical direction at support G and determine the values of the u-forces using joint resolution as before.



(δ_{VG} due to P-forces) + (δ_{VG} due to unit forces) × V_G = − 12.0 mm

$$\text{i.e.} \sum \frac{PL}{AE}u + \left(\sum \frac{uL}{AE}u\right) \times V_G = -12.0$$

$$\therefore\ V_G = \left(-12.0 - \sum \frac{PL}{AE}u\right) \Big/ \sum \frac{uL}{AE}u$$

Complete the Unit Load table to determine the value of V_G

Member	Length (mm)	AE (kN)	P-force (kN)	PL/AE (mm)	u	$(PL/AE) \times u$ (mm)	$(uL/AE) \times u$ (mm)	Member forces
AB	4243	**102.5 ×10³**	−146.70	−6.07	+0.71	− 4.293	+ 0.021	**− 70.40**
AH	3000	**41.0 ×10³**	+88.75	+6.49	−0.50	− 3.247	+ 0.018	**+ 34.79**
BC	3000	**102.5 ×10³**	−157.45	−4.61	+1.00	− 4.608	+ 0.029	**− 49.53**
BG	4243	**102.5 ×10³**	+76.00	+3.15	−0.71	− 2.224	+ 0.021	**− 0.30**
BH	3000	**102.5 ×10³**	0	0	0	0	0	**zero**
CD	3000	**102.5 ×10³**	−157.45	−4.61	+1.00	− 4.608	+ 0.029	**− 49.53**
CG	3000	**102.5 ×10³**	−100.00	−2.93	0	0	0	**− 100.00**
DE	4243	**102.5 ×10³**	−136.12	−5.63	+0.71	− 3.984	+ 0.021	**− 59.82**
DG	4243	**102.5 ×10³**	+65.42	+ 2.71	−0.71	− 1.915	+ 0.021	**− 10.88**
DF	3000	**102.5 ×10³**	0	0	0	0	0	**zero**
EF	3000	**41.0 ×10³**	+96.25	+7.04	−0.50	− 3.521	+ 0.018	**+ 42.29**
FG	3000	**41.0 ×10³**	+96.25	+7.04	−0.50	− 3.521	+ 0.018	**+ 42.29**
GH	3000	**41.0 ×10³**	+88.75	+6.49	−0.50	− 3.247	+ 0.018	**+ 34.79**
						Σ = − 35.169	**Σ = + 0.215**	

$$V_G = \left(-12.0 - \sum \frac{PL}{AE}u\right) \Big/ \sum \frac{uL}{AE}u = [-12.0 - (-35.169)]/0.215 = +\mathbf{107.76\ kN} \uparrow$$

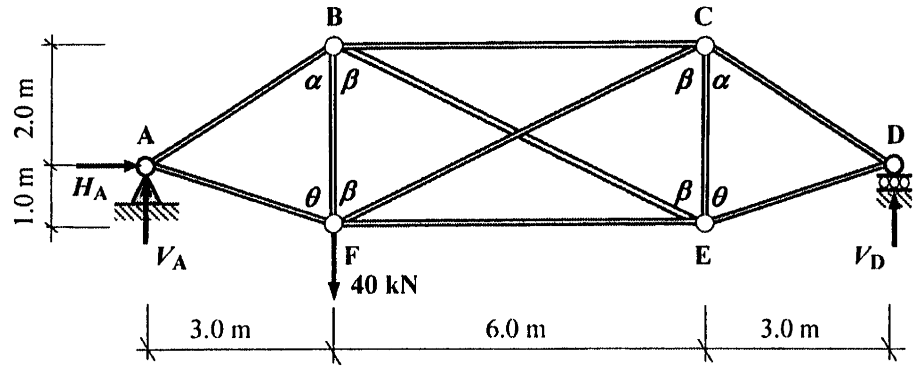
The final member forces = [P-forces + (u-forces × 107.76)] and are given in the last column of the table

V_A = 103.75 − (0.5 × 107.76) = **+ 49.87 kN** ↑ H_A = **+ 15.0 kN** →

V_E = 96.25 − (0.5 × 107.76) = **+ 42.37 kN** ↑

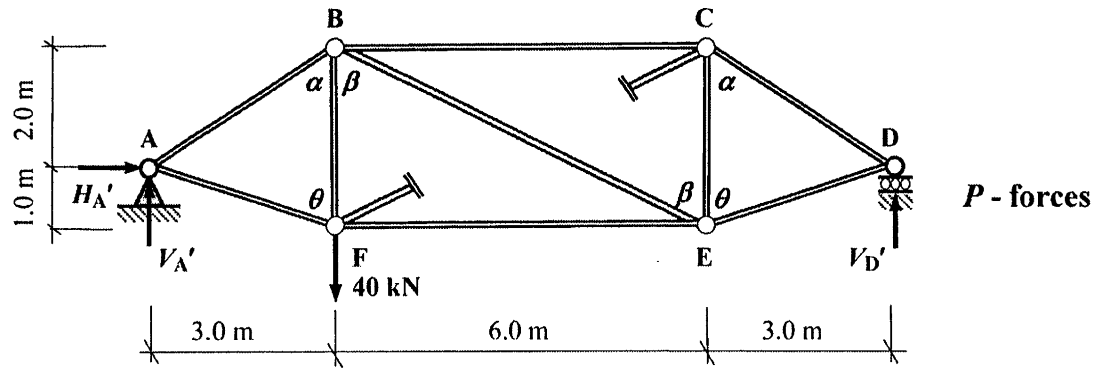
Solution

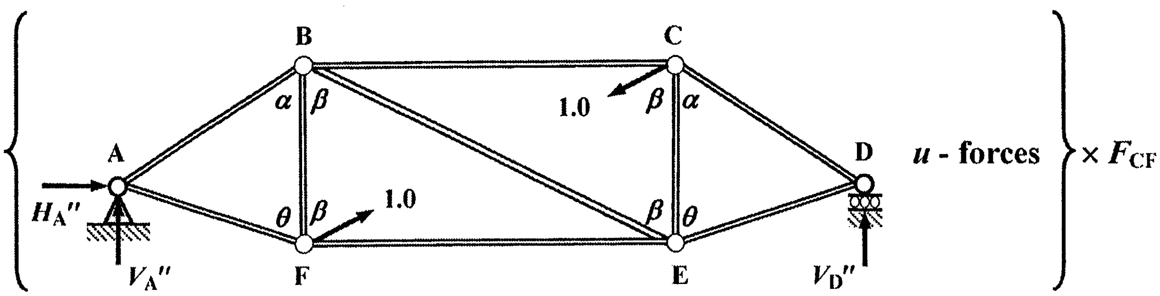
Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.22 **Page No. 1**



The cross-sectional area of all members is equal to 180 mm^2.
$E = 205$ kN/mm^2
$L_{AB,\,CD} = 3.606$ m $\quad L_{AF,\,DE} = 3.162$ m $\quad L_{BE,\,CF} = 6.708$ m
$\text{Sin}\alpha = (3.0/3.606) = 0.832 \quad \text{Cos}\alpha = (2.0/3.606) = 0.555$
$\text{Sin}\beta = (6.0/6.708) = 0.894 \quad \text{Cos}\beta = (3.0/6.708) = 0.447$
$\text{Sin}\theta = (3.0/3.162) = 0.949 \quad \text{Cos}\theta = (1.0/3.162) = 0.316$
$AE_{180} = (180 \times 205) = 36.9 \times 10^3$ kN

Consider member CF to be redundant.
The equivalent system is the superposition of the statically determinate frame and the (unit load frame × F_{CF}) as shown:





Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.22 **Page No. 2**

Determine the Support Reactions for the statically determinate frame.

Consider the rotational equilibrium of the frame:

+ve ↻ $\Sigma M_A = 0$ $\quad + (40.0 \times 3.0) - (V_D' \times 12.0) = 0$ $\quad \therefore$ $\boldsymbol{V_D' = +\ 10.0\ \text{kN}}$ ↑

Consider the horizontal equilibrium of the frame:

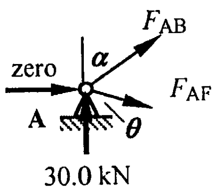
+ve → $\Sigma F_x = 0$ $\quad + H_A' = 0$ $\quad \therefore$ $\boldsymbol{H_A' = \text{zero}}$

Consider the vertical equilibrium of the frame:

+ve ↑ $\Sigma F_y = 0$ $\quad + V_A' - 40.0 + V_D' = 0$ $\quad \therefore V_A' = 40.0 - 10.0$
$\therefore$ $\boldsymbol{V_A' = +\ 30.0\ \text{kN}}$ ↑

Assume all unknown member forces to be tension and use joint resolution to determine the *P*-forces in the frame.

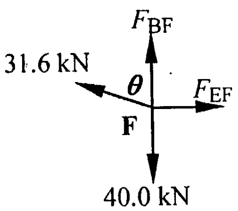
Consider joint A:



+ve ↑ $\Sigma F_y = 0$ $\quad + 30.0 + F_{AB}\,\text{Cos}\alpha - F_{AF}\text{Cos}\theta = 0$ Equation (a)
+ve → $\Sigma F_x = 0$ $\quad + F_{AB}\text{Sin}\alpha + F_{AF}\ \text{Sin}\theta = 0$ Equation (b)

From Equation (a): $\boldsymbol{F_{AB} = -\ 36.06\ \text{kN}}$ **(Strut)**
From Equation (b): $\boldsymbol{F_{AF} = +\ 31.6\ \text{kN}}$ **(Tie)**

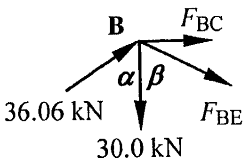
Consider joint F:



+ve ↑ $\Sigma F_y = 0$ $\quad + 31.6\text{Cos}\theta - 40.0 + F_{BF} = 0$ Equation (a)
+ve → $\Sigma F_x = 0$ $\quad - 31.6\text{Sin}\theta + F_{EF} = 0$ Equation (b)

From Equation (a): $\boldsymbol{F_{BF} = +\ 30.0\ \text{kN}}$ **(Tie)**
From Equation (b): $\boldsymbol{F_{EF} = +\ 30.0\ \text{kN}}$ **(Tie)**

Consider joint B:



+ve ↑ $\Sigma F_y = 0$ $\quad - 30.0 + 36.06\ \text{Cos}\alpha - F_{BE}\text{Cos}\beta = 0$
Equation (a)
+ve → $\Sigma F_x = 0$ $\quad + 36.06\text{Sin}\alpha + F_{BE}\ \text{Sin}\beta + F_{BC} = 0$
Equation (b)

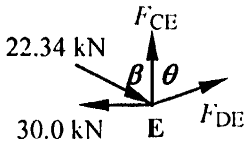
From Equation (a): $\boldsymbol{F_{BE} = -\ 22.34\ \text{kN}}$ **(Strut)**
From Equation (b): $\boldsymbol{F_{BC} = -\ 10.0\ \text{kN}}$ **(Strut)**

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames

Problem Number: 3.22 **Page No. 3**

Consider joint E:



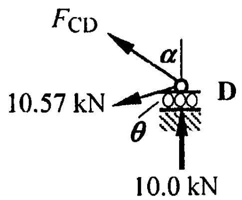
+ve → $\Sigma F_x = 0$ $-30.0 + 22.34\mathrm{Sin}\beta + F_{DE}\mathrm{Sin}\theta = 0$ Equation (a)

+ve ↑ $\Sigma F_y = 0$ $-22.34\mathrm{Cos}\beta + F_{DE}\mathrm{Cos}\theta + F_{CE} = 0$ Equation (b)

From Equation (a): $\boldsymbol{F_{DE} = +10.57}$ **kN (Tie)**

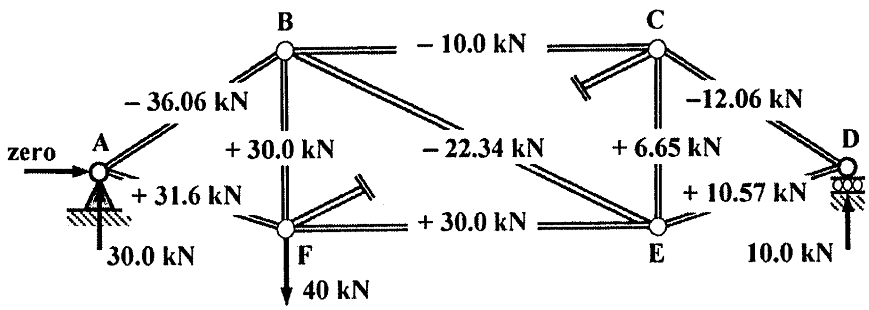
From Equation (b): $\boldsymbol{F_{CE} = +6.65}$ **kN (Tie)**

Consider joint D:



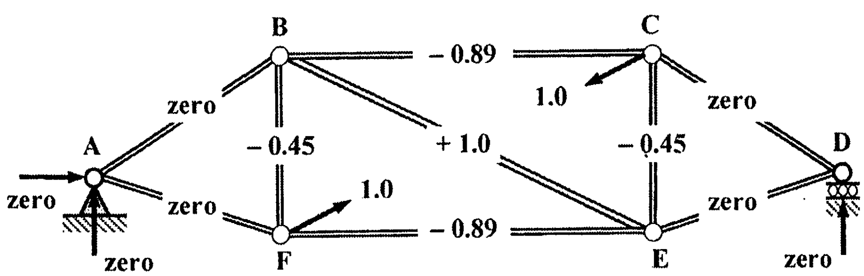
+ve → $\Sigma F_x = 0$ $-10.57\mathrm{Sin}\theta - F_{CD}\mathrm{Sin}\alpha = 0$

$\boldsymbol{F_{CD} = -12.06}$ **kN (Strut)**



***P* - forces**

Apply a Unit Load at joints F and C in the direction of member FC and determine the values of the *u*-forces using joint resolution as before.



***u* - forces**

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.22 **Page No. 4**

(δ_{FC} due to P–forces) + (δ_{FC} due to unit forces) × $F_{CF} = 0$

i.e. $\sum \frac{PL}{AE}u + \left(\sum \frac{uL}{AE}u\right) \times F_{CF} = 0$

$\therefore\ F_{CF} = -\sum \frac{PL}{AE}u \Big/ \sum \frac{uL}{AE}u$

Complete the Unit Load table to determine the value of F_{CF}

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	*PL/AE* (mm)	*u*	(*PL/AE*) × *u* (mm)	(*uL/AE*) × *u* (mm)	Member forces
AB	3606	**36.9×10^3**	–36.06	– 3.52	0	0	0	**– 36.06**
AF	3162	**36.9×10^3**	+ 31.60	+ 2.71	0	0	0	**+ 31.60**
BC	6000	**36.9×10^3**	–10.00	– 1.63	– 0.89	+ 1.454	+ 0.130	**– 21.31**
BE	6708	**36.9×10^3**	–22.34	– 4.06	1.00	– 4.061	+ 0.182	**– 9.69**
BF	3000	**36.9×10^3**	+ 30.00	+ 2.44	– 0.45	– 1.090	+ 0.016	**+ 24.35**
CD	3606	**36.9×10^3**	–12.06	– 1.18	0	0	0	**– 12.06**
CE	3000	**36.9×10^3**	+ 6.65	+ 0.54	– 0.45	– 0.242	+ 0.016	**+ 1.00**
CF	6708	**36.9×10^3**	0	0	1.00	0	+ 0.182	**+ 12.65**
DE	3162	**36.9×10^3**	+ 10.57	+ 0.91	0	0	0	**+ 10.57**
EF	6000	**36.9×10^3**	+ 30.00	+ 4.88	– 0.89	– 4.361	+ 0.130	**+ 18.69**
						Σ = – 8.30	**Σ = + 0.656**	

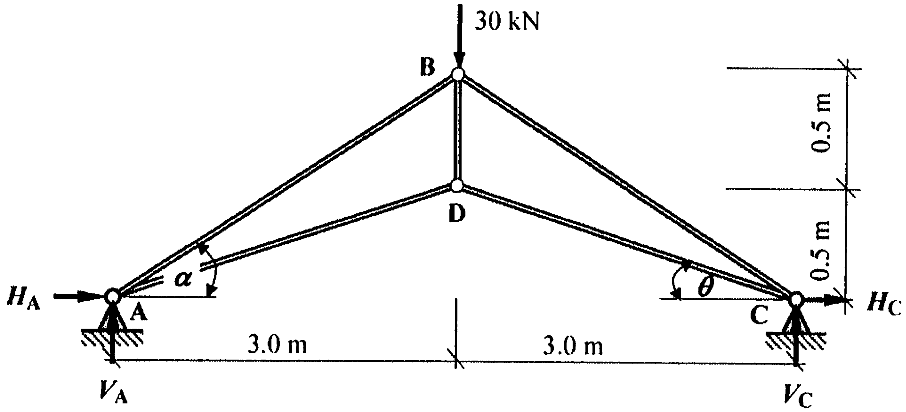
$F_{CF} = -\sum \frac{PL}{AE}u \Big/ \sum \frac{uL}{AE}u = -(-8.30)/0.656 = +12.65$ kN **(Tie)**

The final member forces = [P-forces + (u-forces × 12.65)] and are given in the last column of the table

$V_A = +$ **30.0 kN** ↑ $H_A =$ **zero** $V_D = +$ **10.0 kN** ↑

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.23 **Page No. 1**



The cross-sectional area of member BD is equal to 100 mm^2.
The cross-sectional area of all other members is equal to 300 mm^2.
Member AD is too long by 1.5 mm and all members are subject to an increase in temperature of 10°C.

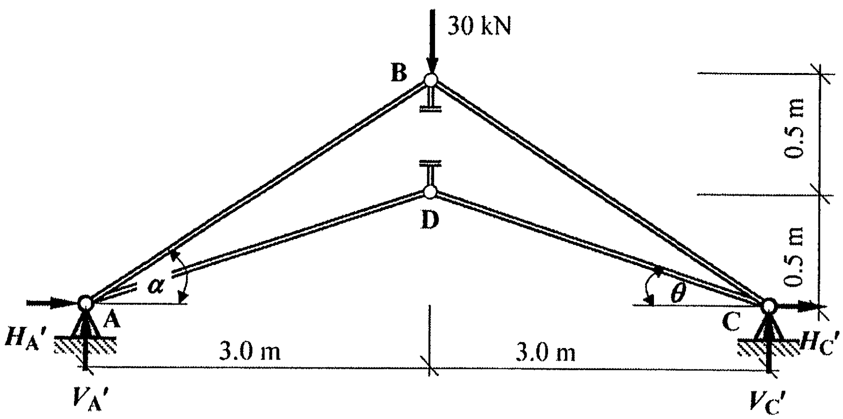
$E = 205$ kN/mm^2 $\alpha = 12 \times 10^{-6}$/°C
$L_{AB,BC} = 3.162$ m $L_{AD,CD} = 3.041$ m $L_{BD} = 0.5$ m

The δL value for all members due to temperature change:

$\Delta_{T,AB,BC} = -\alpha L \Delta_T = -(12 \times 10^{-6} \times 3162 \times 10.0) = +0.38$ mm
$\Delta_{T,AD,CD} = -\alpha L \Delta_T = -(12 \times 10^{-6} \times 3041 \times 10.0) = +0.36$ mm
$\Delta_{T,BD} = -\alpha L \Delta_T = -(12 \times 10^{-6} \times 500 \times 10.0) = +0.06$ mm

$\text{Sin}\alpha = (1.0/3.162) = 0.316$ $\text{Cos}\alpha = (3.0/3.162) = 0.949$
$\text{Sin}\theta = (0.5/3.041) = 0.164$ $\text{Cos}\theta = (3.0/3.041) = 0.987$
$AE_{100} = (100 \times 205) = 20.5 \times 10^3$ kN $AE_{300} = (300 \times 205) = 61.5 \times 10^3$ kN

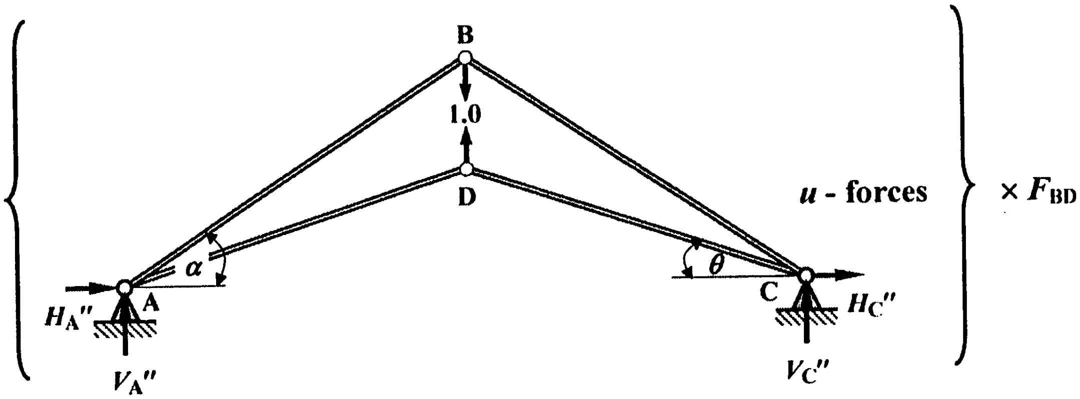
Consider member BD to be redundant.
The equivalent system is the superposition of the statically determinate frame and the (unit load frame × F_{BD}) as shown:



***P* - forces**

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.23 **Page No. 2**



Determine the Support Reactions for the statically determinate frame.

Consider the rotational equilibrium of the frame:

+ve ↻ $\Sigma M_A = 0 \quad + (30.0 \times 3.0) - (V_C' \times 6.0) = 0 \qquad \therefore\ V_C' = +\mathbf{15.0\ kN}$ ↑

Consider the horizontal equilibrium of the frame:

+ve → $\Sigma F_x = 0 \quad + H_A' + H_C' = 0 \qquad H_C' = - H_A'$

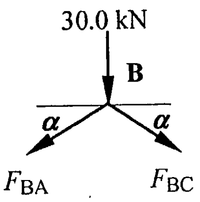
Consider the vertical equilibrium of the frame:

+ve ↑ $\Sigma F_y = 0 \quad + V_A' - 30.0 + V_C' = 0 \qquad V_A' = 30.0 - 15.0$

$\therefore\ V_A' = +\mathbf{15.0\ kN}$ ↑

Assume all unknown member forces to be tension and use joint resolution to determine the *P*-forces in the frame.

Consider joint B:



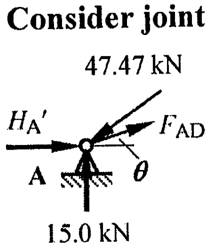
+ve ↑ $\Sigma F_y = 0 \quad -30.0 - F_{BA}\text{Sin}\alpha - F_{BC}\text{Sin}\alpha = 0$ Equation (a)

+ve → $\Sigma F_x = 0 \quad - F_{BA}\text{Cos}\alpha + F_{BC}\text{Cos}\alpha = 0$ Equation (b)

From Equation (a): $\mathbf{F_{BA} = -47.47\ kN}$ **(Strut)**

From Equation (b): $\mathbf{F_{BC} = -47.47\ kN}$ **(Strut)**

Consider joint A:



+ve ↑ $\Sigma F_y = 0 \quad + 15.0 - 47.47\text{Sin}\alpha + F_{AD}\text{Sin}\theta = 0$ Equation (a)

+ve → $\Sigma F_x = 0 \quad H_A - 47.47\text{Cos}\alpha + F_{AD}\text{Cos}\theta = 0$ Equation (b)

From Equations (a): $\mathbf{F_{AD}}$ **= zero**

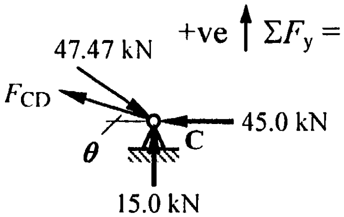
From Equation (b): $\mathbf{H_A = +45.0\ kN}$

$\mathbf{H_C = -45.0\ kN}$

Solution

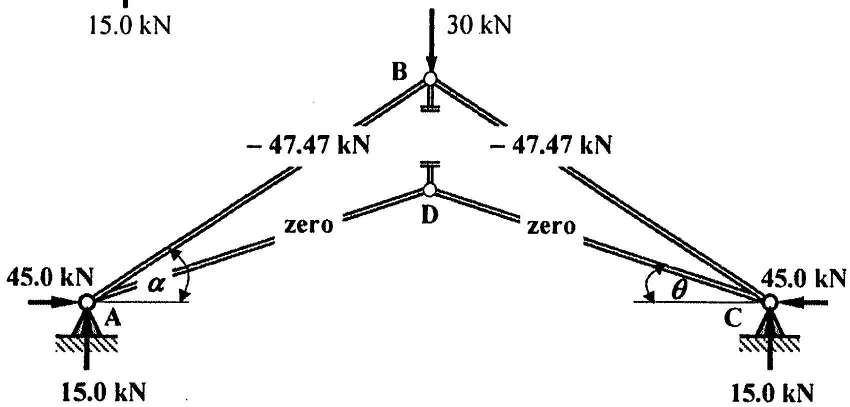
Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.23 **Page No. 3**

Consider joint C:



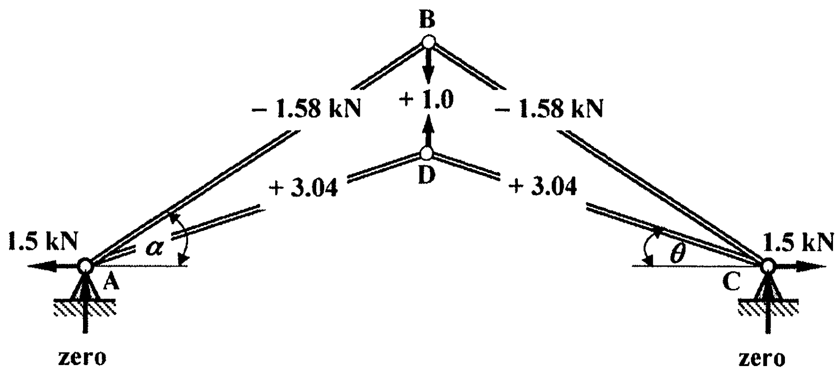
+ve ↑ $\Sigma F_y = 0$ $+ 15.0 - 47.47\text{Sin}\alpha + F_{CD}\text{Sin}\theta = 0$ Equation (a)

F_{CD} = **zero**



***P* - forces**

Apply a Unit Load at joints B and D in the direction of member BD and determine the values of the *u*-forces using joint resolution as before.



***u* - forces**

(δ_{BD} due to P-forces) + (δ_{BD} due to unit forces) × F_{BD} = 0

$$\text{i.e.} \sum\left(\frac{PL}{AE} + \Delta_L + \Delta_T\right)u + \left(\sum\frac{uL}{AE}u\right) \times F_{BD} = 0$$

$$\therefore\ F_{BD} = -\sum\left(\frac{PL}{AE} + \Delta_L + \Delta_T\right)u \Big/ \sum\frac{uL}{AE}u$$

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.23 **Page No. 4**

The term $\sum\left(\frac{PL}{AE}+\Delta_L+\Delta_T\right)$ is evaluated separately here for convenience, normally this would be incorporated in one table.

Member	Length (mm)	*AE* (kN)	*P*-force (kN)	(*PL*/*AE*) (mm)	Δ_L	Temp. change	Δ_T	($PL/AE+\Delta_L+\Delta_T$) (mm)
AB	3162	**61.5 ×10³**	– 47.47	– 2.44	0	+ 10	+ 0.38	– 2.06
BC	3162	**61.5 ×10³**	– 47.47	– 2.44	0	+ 10	+ 0.38	– 2.06
BD	500	**20.5 ×10³**	0	0	0	+ 10	+ 0.06	+ 0.06
CD	3041	**61.5 ×10³**	0	0	0	+ 10	+ 0.36	+ 0.36
DA	3041	**61.5 ×10³**	0	0	+ 1.5	+ 10	+ 0.36	+ 1.86

Complete the Unit Load table to determine the value of F_{BD}

Member	Length (mm)	*AE* (kN)	($PL/AE+\Delta_L+\Delta_T$) (mm)	*u*	($PL/AE+\Delta_L+\Delta_T$) × *u* (mm)	(*uL*/*AE*) × *u* (mm)	Member forces
AB	3162	**61.5 ×10³**	– 2.06	–1.58	+ 3.261	+ 0.129	**– 29.80**
BC	3162	**61.5 ×10³**	– 2.06	–1.58	+ 3.261	+ 0.129	**– 29.80**
BD	500	**20.5 ×10³**	+ 0.06	+ 1.00	+ 0.060	+ 0.024	**– 11.17**
CD	3041	**61.5 ×10³**	+ 0.36	+ 3.04	+ 1.110	+ 0.457	**– 33.97**
DA	3041	**61.5 ×10³**	+ 1.86	+ 3.04	+ 5.671	+ 0.457	**– 33.97**
					Σ = + 13.363	**Σ = + 1.196**	

$$F_{BD} = -\sum\left(\frac{PL}{AE}+\Delta_L+\Delta_T\right)u\Big/\sum\frac{uL}{AE}u = -13.363/1.196 = -11.17 \text{ kN } \textbf{(Strut)}$$

The final member forces = [*P*-forces + (*u*-forces × (–)11.17)] and are given in the last column of the table

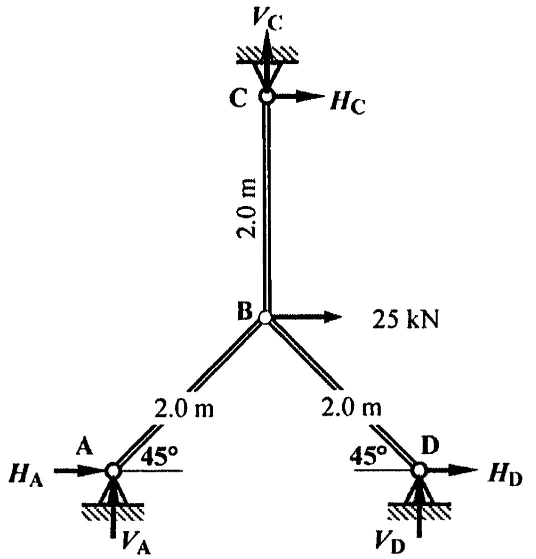
V_A **= + 15.0 + zero = + 15.0 kN** ↑
H_A **= + 45.0 – (1.5 × (–)11.17) = + 61.76 kN** →

V_C **= + 15.0 + zero = + 15.0 kN** ↑
H_C **= – 45.0 + (1.5 × (–)11.17) = – 61.76 kN** ←

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames

Problem Number: 3.24 **Page No. 1**



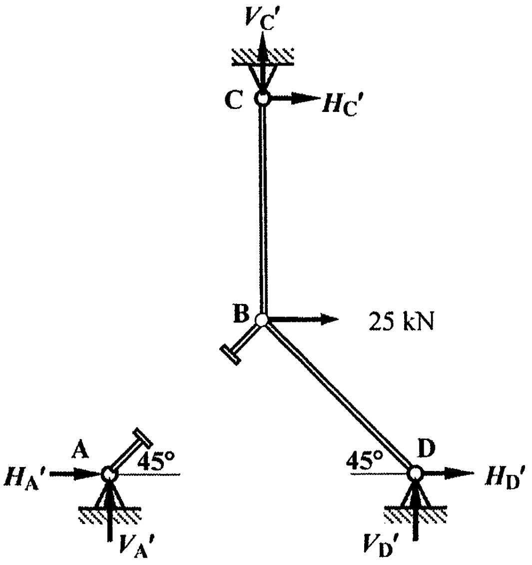
The cross-sectional area of all members is equal to 150 mm^2.

Member BD is 2.0 mm too short.
$E = 205$ kN/mm^2

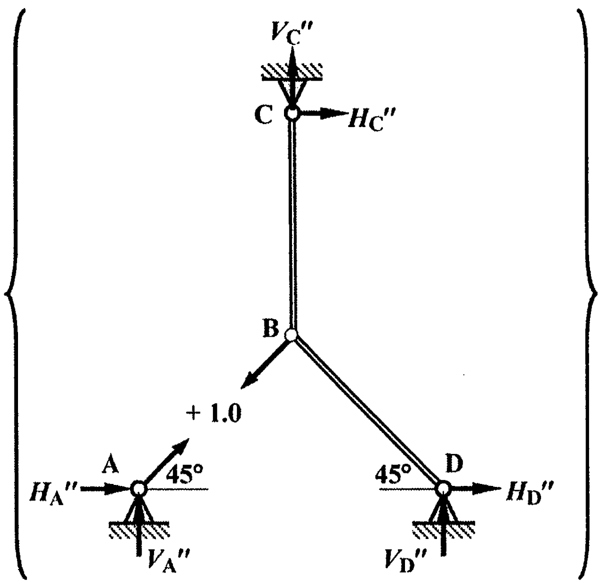
$AE_{150} = (150 \times 205) = 30.75 \times 10^3$ kN

Sin 45° = 0.707
Cos 45° = 0.707

Consider member AB to be redundant.
The equivalent system is the superposition of the statically determinate frame and the (unit load frame × F_{AB}) as shown:



***P* – forces**



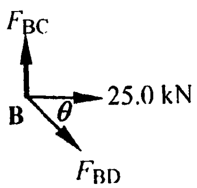
(*u* – forces) × F_{AB}

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.24 **Page No. 2**

Assume all unknown member forces to be tension and use joint resolution to determine the P-forces in the frame.

Consider joint B:



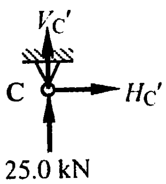
+ve $\rightarrow$ $\Sigma F_x = 0$ $\quad + 25.0 + F_{BD}\,\mathrm{Cos}\theta = 0$ Equation (a)

+ve $\uparrow$ $\Sigma F_y = 0$ $\quad + F_{BC} - F_{BD}\,\mathrm{Sin}\theta = 0$ Equation (b)

From Equation (a): **$F_{BD} = -\,35.36$ kN (Strut)**

From Equation (b): **$F_{BC} = -\,25.0$ kN (Strut)**

Consider joint C:



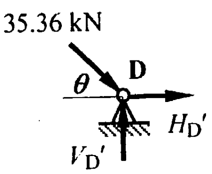
+ve $\uparrow$ $\Sigma F_y = 0$ $\quad + 25.0 + V_C' = 0$ Equation (a)

+ve $\rightarrow$ $\Sigma F_x = 0$ $\quad + H_C' = 0$ Equation (b)

From Equation (a): **$V_C' = -\,25.0$ kN** $\downarrow$

From Equation (b): **H_C' = zero**

Consider joint D:

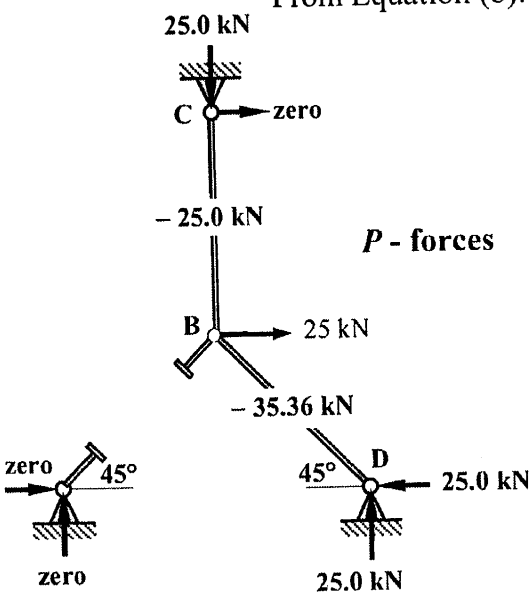


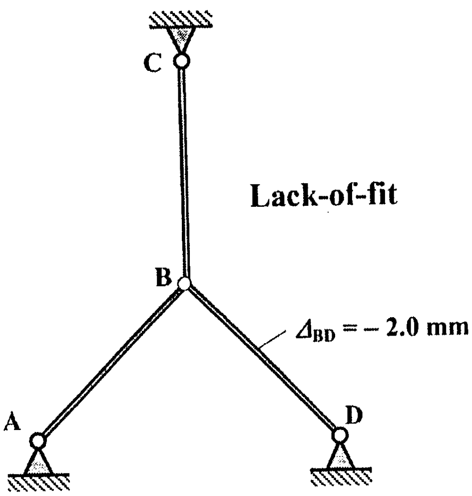
+ve $\uparrow$ $\Sigma F_y = 0$ $\quad - 35.36\mathrm{Sin}\theta + V_D' = 0$ Equation (a)

+ve $\rightarrow$ $\Sigma F_x = 0$ $\quad + 35.36\mathrm{Cos}\theta + H_D' = 0$ Equation (b)

From Equation (a): **$V_D' = +\,25.0$ kN** $\uparrow$

From Equation (b): **$H_D' = -\,25.0$ kN** $\leftarrow$

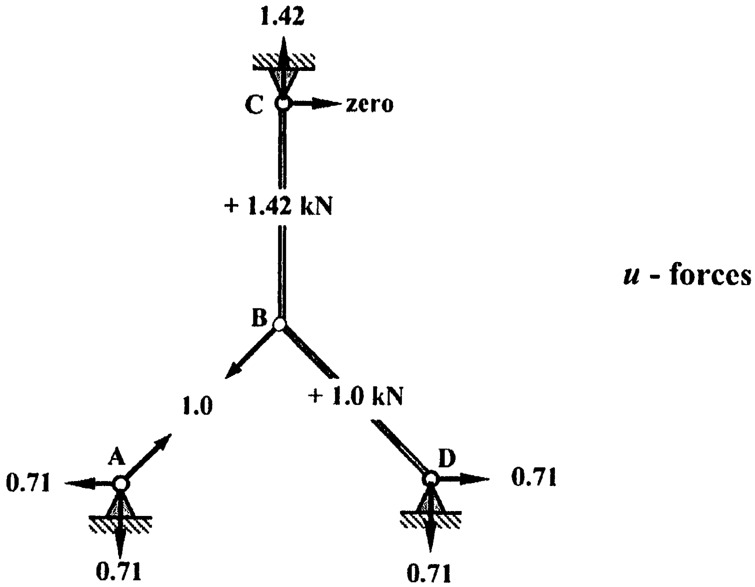




Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.24 **Page No. 3**

Apply a Unit Load at joints A and B in the direction of member AB and determine the values of the *u*-forces using joint resolution as before.



(δ_{AB} due to P-forces) + (δ_{AB} due to unit forces) $\times F_{AB} = 0$

i.e. $\sum\left(\frac{PL}{AE}+\Delta_L\right)u + \left(\sum\frac{uL}{AE}u\right) \times F_{AB} = 0$

$\therefore\ F_{AB} = -\sum\left(\frac{PL}{AE}+\Delta_L\right)u \Big/ \sum\frac{uL}{AE}u$

The term $\sum\left(\frac{PL}{AE}+\Delta_L\right)$ is evaluated separately here for convenience, normally this would be incorporated in one table.

Member	Length (mm)	AE (kN)	P-force (kN)	(PL/AE) (mm)	Δ_L	$(PL/AE + \Delta_L)$ (mm)
AB	2000	30.75×10^3	0	0	0	0
BC	2000	30.75×10^3	– 25.00	– 1.63	0	– 1.63
BD	2000	30.75×10^3	– 35.36	– 2.30	– 2.0	– 4.30

Solution

Topic: Unit Load Method for Singly-Redundant Pin-Jointed Frames
Problem Number: 3.24 **Page No. 4**

Complete the Unit Load table to determine the value of F_{AB}

Member	Length (mm)	AE (kN)	$(PL/AE+\Delta_L)$ (mm)	u	$(PL/AE+\Delta_L)\times u$ (mm)	$(uL/AE)\times u$ (mm)	Member forces
AB	2000	**30.75 ×10³**	0	+ 1.00	0	+ 0.065	**+ 25.38**
BC	2000	**30.75 ×10³**	– 1.63	+ 1.42	– 2.315	+ 0.131	**+ 10.87**
BD	2000	**30.75 ×10³**	– 4.30	+ 1.00	– 4.300	+ 0.065	**– 9.99**
					Σ = – 6.615	**Σ = + 0.261**	

$$F_{AB} = -\sum\left(\frac{PL}{AE}+\Delta_L\right)u\Big/\sum\frac{uL}{AE}u = +6.615/0.261 = 25.34\text{ kN}$$ **(Tie)**

The final member forces = [P-forces + (u-forces × 25.37)] and are given in the last column of the table

V_A = zero – (0.71 × 25.34) = – 17.99 kN ↓
H_A = zero – (0.71 × 25.34) = – 17.99 kN ←

V_C = – 25.0 + (1.42 × 25.34) = + 10.98 kN ↑
H_C = zero

V_D = + 25.0 – (0.71 × 25.34) = + 7.01 kN ↑
H_D = – 25.0 + (0.71 × 25.34) = – 7.01 kN ←