BEAM ELEMENT

Member Stiffness Matrix in Local Coordinates

\[
[K_{\text{local}}] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Rotation Matrix

\[
[R] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 & 0 & 0 & 0 & 0 \\
-\sin \theta & \cos \theta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \theta & \sin \theta & 0 & 0 \\
0 & 0 & 0 & -\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]
BEAM ELEMENT

Member Stiffness Matrix in Global Coordinates (i.e. member is rotated by $\theta$ degrees)

$$\begin{bmatrix} K_{\text{global}} \end{bmatrix} = [R]^T [K_{\text{local}}] [R]$$

let $C = \cos \theta$ and $S = \sin \theta$

$$\begin{bmatrix} K_{\text{global}} \end{bmatrix} = \begin{bmatrix} S^2 \frac{12EI}{L^3} & -CS \frac{12EI}{L^3} & -S \frac{6EI}{L^2} & -S^2 \frac{12EI}{L^3} & CS \frac{12EI}{L^3} & -S \frac{6EI}{L^2} \\ -CS \frac{12EI}{L^3} & C^2 \frac{12EI}{L^3} & C \frac{6EI}{L^2} & CS \frac{12EI}{L^3} & -C^2 \frac{12EI}{L^3} & C \frac{6EI}{L^2} \\ -S \frac{6EI}{L^2} & C \frac{6EI}{L^2} & 4EI \frac{1}{L} & S \frac{6EI}{L^2} & -C \frac{6EI}{L^2} & 2EI \frac{1}{L} \\ -S^2 \frac{12EI}{L^3} & C \frac{12EI}{L^3} & S \frac{6EI}{L^2} & S^2 \frac{12EI}{L^3} & -CS \frac{12EI}{L^3} & S \frac{6EI}{L^2} \\ CS \frac{12EI}{L^3} & -C^2 \frac{12EI}{L^3} & -C \frac{6EI}{L^2} & -CS \frac{12EI}{L^3} & C^2 \frac{12EI}{L^3} & -C \frac{6EI}{L^2} \\ -S \frac{6EI}{L^2} & C \frac{6EI}{L^2} & 2EI \frac{1}{L} & S \frac{6EI}{L^2} & -C \frac{6EI}{L^2} & 4EI \frac{1}{L} \end{bmatrix}$$
FRAME ELEMENT

Member Stiffness Matrix in Local Coordinates

\[
[K_{\text{local}}] = \begin{bmatrix}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
0 & 0 & \frac{EA}{L} & 0 & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\]

Rotation Matrix

\[
[R] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 & 0 & 0 & 0 \\
-\sin \theta & \cos \theta & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \theta & \sin \theta & 0 \\
0 & 0 & 0 & -\sin \theta & \cos \theta & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
FRAME ELEMENT

Member Stiffness Matrix in Global Coordinates (i.e. member is rotated by $\theta$ degrees)

$$\left[ K_{\text{global}} \right] = [R]^T \left[ K_{\text{local}} \right] [R]$$

let $C = \cos \theta$ and $S = \sin \theta$

$$\left[ K_{\text{global}} \right] = \begin{bmatrix}
C^2 \frac{EA}{L} + S^2 \frac{12EI}{L^3} & CS \frac{EA}{L} - CS^2 \frac{12EI}{L^3} & -S \frac{6EI}{L^2} & -C^2 \frac{EA}{L} - S^2 \frac{12EI}{L^3} & -CS \frac{EA}{L} + CS \frac{12EI}{L^3} & -S \frac{6EI}{L^2} \\
CS \frac{EA}{L} - CS^2 \frac{12EI}{L^3} & S^2 \frac{EA}{L} + C^2 \frac{12EI}{L^3} & C \frac{6EI}{L^2} & CS \frac{EA}{L} + CS \frac{12EI}{L^3} & -S \frac{6EI}{L^2} & -C \frac{6EI}{L^2} \\
-S \frac{6EI}{L^2} & C \frac{6EI}{L^2} & 4EI/L & S \frac{6EI}{L^2} & 2EI/L & 2EI/L \\
-C^2 \frac{EA}{L} - S^2 \frac{12EI}{L^3} & -CS \frac{EA}{L} + CS \frac{12EI}{L^3} & S \frac{6EI}{L^2} & C^2 \frac{EA}{L} + S^2 \frac{12EI}{L^3} & CS \frac{EA}{L} - CS \frac{12EI}{L^3} & S \frac{6EI}{L^2} \\
CS \frac{EA}{L} + CS \frac{12EI}{L^3} & -S^2 \frac{EA}{L} - C^2 \frac{12EI}{L^3} & -C \frac{6EI}{L^2} & CS \frac{EA}{L} - CS \frac{12EI}{L^3} & S \frac{6EI}{L^2} & -C \frac{6EI}{L^2} \\
-S \frac{6EI}{L^2} & C \frac{6EI}{L^2} & 2EI/L & S \frac{6EI}{L^2} & -C \frac{6EI}{L^2} & 4EI/L \\
\end{bmatrix}$$
**TRUSS ELEMENT**

Member Stiffness Matrix in Local Coordinates

\[
[K_{\text{local}}] = \begin{bmatrix}
\frac{EA}{L} & 0 & -\frac{EA}{L} & 0 \\
0 & 0 & 0 & 0 \\
-\frac{EA}{L} & 0 & \frac{EA}{L} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} = \frac{EA}{L} \begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Rotation Matrix

\[
[R] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 & 0 \\
-\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & \cos \theta & \sin \theta \\
0 & 0 & -\sin \theta & \cos \theta
\end{bmatrix}
\]

Member Stiffness Matrix in Global Coordinates (i.e. member is rotated by \( \theta \) degrees)

\[
[K_{\text{global}}] = [R]^T [K_{\text{local}}] [R]
\]

let \( C = \cos \theta \) and \( S = \sin \theta \)

\[
[K_{\text{global}}] = \frac{EA}{L} \begin{bmatrix}
C^2 & CS & -C^2 & -CS \\
CS & S^2 & -CS & -S^2 \\
-C^2 & -CS & C^2 & CS \\
-CS & -S^2 & CS & S^2
\end{bmatrix}
\]