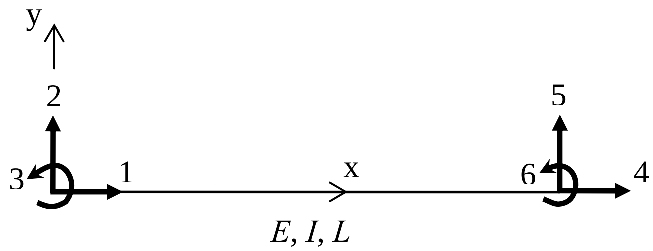
**BEAM ELEMENT**




Member Stiffness Matrix in Local Coordinates

$$
\left[K_{local}\right] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
$$

Rotation Matrix

$$
\left[R\right] = \begin{bmatrix}
\cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\
-\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\
0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

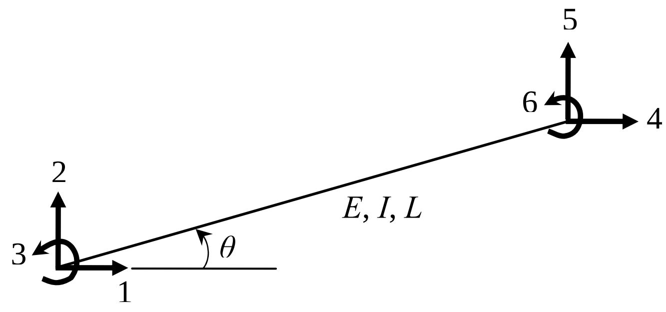
**BEAM ELEMENT**

Member Stiffness Matrix in Global Coordinates (i.e. member is rotated by $\theta$ degrees)

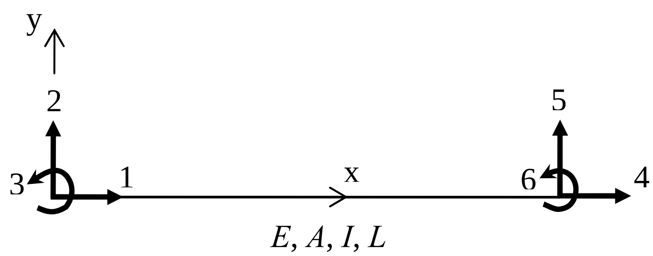
$$\left[K_{global}\right]=\left[R\right]^T\left[K_{local}\right]\left[R\right]$$

let $C=\cos\theta$ and $S=\sin\theta$

$$\left[K_{global}\right]=\begin{bmatrix} S^2\frac{12EI}{L^3} & -CS\frac{12EI}{L^3} & -S\frac{6EI}{L^2} & -S^2\frac{12EI}{L^3} & CS\frac{12EI}{L^3} & -S\frac{6EI}{L^2} \\ -CS\frac{12EI}{L^3} & C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} & CS\frac{12EI}{L^3} & -C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} \\ -S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{4EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{2EI}{L} \\ -S^2\frac{12EI}{L^3} & CS\frac{12EI}{L^3} & S\frac{6EI}{L^2} & S^2\frac{12EI}{L^3} & -CS\frac{12EI}{L^3} & S\frac{6EI}{L^2} \\ CS\frac{12EI}{L^3} & -C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} & -CS\frac{12EI}{L^3} & C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} \\ -S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{2EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$



**FRAME ELEMENT**




Member Stiffness Matrix in Local Coordinates

$$[K_{local}] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Rotation Matrix

$$[R] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

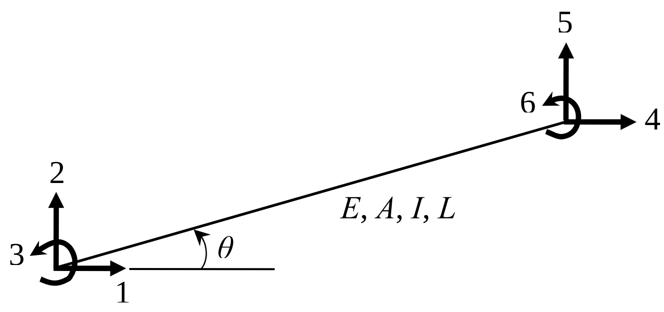
**FRAME ELEMENT**

Member Stiffness Matrix in Global Coordinates (i.e. member is rotated by $\theta$ degrees)

$$\left[K_{global}\right]=\left[R\right]^{T}\left[K_{local}\right]\left[R\right]$$

let $C=\cos\theta$ and $S=\sin\theta$

$$\left[K_{global}\right]=\begin{bmatrix}
C^2\frac{EA}{L}+S^2\frac{12EI}{L^3} & CS\frac{EA}{L}-CS\frac{12EI}{L^3} & -S\frac{6EI}{L^2} & -C^2\frac{EA}{L}-S^2\frac{12EI}{L^3} & -CS\frac{EA}{L}+CS\frac{12EI}{L^3} & -S\frac{6EI}{L^2} \\
CS\frac{EA}{L}-CS\frac{12EI}{L^3} & S^2\frac{EA}{L}+C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} & -CS\frac{EA}{L}+CS\frac{12EI}{L^3} & -S^2\frac{EA}{L}-C^2\frac{12EI}{L^3} & C\frac{6EI}{L^2} \\
-S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{4EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{2EI}{L} \\
-C^2\frac{EA}{L}-S^2\frac{12EI}{L^3} & -CS\frac{EA}{L}+CS\frac{12EI}{L^3} & S\frac{6EI}{L^2} & C^2\frac{EA}{L}+S^2\frac{12EI}{L^3} & CS\frac{EA}{L}-CS\frac{12EI}{L^3} & S\frac{6EI}{L^2} \\
-CS\frac{EA}{L}+CS\frac{12EI}{L^3} & -S^2\frac{EA}{L}-C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} & CS\frac{EA}{L}-CS\frac{12EI}{L^3} & S^2\frac{EA}{L}+C^2\frac{12EI}{L^3} & -C\frac{6EI}{L^2} \\
-S\frac{6EI}{L^2} & C\frac{6EI}{L^2} & \frac{2EI}{L} & S\frac{6EI}{L^2} & -C\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}$$



**TRUSS ELEMENT**

Member Stiffness Matrix in Local Coordinates

$$[K_{local}] = \begin{bmatrix} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rotation Matrix

$$[R] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

Member Stiffness Matrix in Global Coordinates (i.e. member is rotated by $\theta$ degrees)

$$[K_{global}] = [R]^T [K_{local}][R]$$

let $C = \cos\theta$ and $S = \sin\theta$

$$[Kglobal] = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$