8.4 Problems: Plastic Analysis – Continuous Beams

A series of continuous beams are indicated in which the relative $M_p$ values and the applied collapse loadings are given in Problems 8.1 to 8.5. Determine the required value of $M_p$ to ensure a minimum load factor $\lambda = 1.7$.

### Problem 8.1

### Problem 8.2

### Problem 8.3

### Problem 8.4

### Problem 8.5
8.5 Solutions: Plastic Analysis – Continuous Beams

Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.1 – Kinematic Method

\[ \lambda = 1.7 \]

Factored loads: Beam ABC = (1.7 \times 20) = 34 kN, Beam CDE = (1.7 \times 15) = 25.5 kN

**Kinematic Method:**

**Span ABC**

\[ \delta = 2\beta = 2\theta \quad \therefore \beta = \theta \]

Internal Work = External Work

\[ M_p(\theta) + M_p(\theta + \beta) + M_p(\beta) = (34 \times 2\theta) \]

\[ 4M_p\theta = 68\theta \]

\[ \therefore M_p = 17.0 \text{kNm} \]

**Span CDE**

\[ \delta = 2\beta = 2\theta \quad \therefore \beta = \theta \]

Internal Work = External Work

\[ M_p(\theta) + M_p(\theta + \beta) + M_p(\beta) = (25.5 \times 2\theta) \]

\[ 4M_p\theta = 51.0\theta \]

\[ \therefore M_p = 12.75 \text{kNm} \]

Critical value of \( M_p = 17.0 \text{kNm} \)
**Solution**

**Topic:** Plastic Analysis – Continuous Beams  
**Problem Number:** 8.1 – Static Method

**Static Method:**

**Span ABC**

\[ 34 \text{ kN} \]

\[ 2.0 \text{ m} \quad 2.0 \text{ m} \quad 4.0 \text{ m} \]

\[ M_p \]

\[ M_p \]

\[ 17.0 \text{ kN} \]

\[ (17.0 \times 2.0) = 34.0 \text{ kNm} \]

Free Bending Moment Diagram

\[ M_p \]

\[ M_p \]

\[ 34.0 \text{ kNm} \]

Combined Bending Moment Diagram

\[ (M_p + M_p) = 2M_p = 34.0 \text{ kNm} \]

\[ \therefore M_p = 17.0 \text{ kNm} \]

**Span CDE**

\[ 25.5 \text{ kN} \]

\[ 2.0 \text{ m} \quad 2.0 \text{ m} \quad 4.0 \text{ m} \]

\[ M_p \]

\[ M_p \]

\[ 12.75 \text{ kN} \]

\[ (12.75 \times 2.0) = 25.5 \text{ kNm} \]

Free Bending Moment Diagram

\[ M_p \]

\[ M_p \]

\[ 25.5 \text{ kNm} \]

Combined Bending Moment Diagram

\[ (M_p + M_p) = 2M_p = 25.5 \text{ kNm} \]

\[ \therefore M_p = 12.75 \text{ kNm} \]

As before the critical value of \( M_p = 17.0 \text{ kNm} \)
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.2 – Kinematic Method

\[ \lambda = 1.7 \]

Factored loads: Beam ABC = \((1.7 \times 20) = 34 \text{ kN}, \quad \text{Beam CDE} = (1.7 \times 15) = 25.5 \text{ kN} \]

**Kinematic Method:**

**Span ABC**

\[ \delta = 2\beta = 2\theta \quad \therefore \beta = \theta \]

\[
\text{Internal Work} = \text{External Work} \\
M_p(\theta + \beta) + M_p(\beta) = (34 \times 2\theta) \\
3M_p\theta = 68\theta 
\]

\[
\therefore M_p = 22.67 \text{ kNm} 
\]

**Span CDE**

\[ \delta = 2\beta = 2\theta \quad \therefore \beta = \theta \]

\[
\text{Internal Work} = \text{External Work} \\
M_p(\theta) + M_p(\theta + \beta) = (25.5 \times 2\theta) \\
3M_p\theta = 51.0\theta 
\]

\[
\therefore M_p = 17.0 \text{ kNm} 
\]

Critical value of \(M_p = 22.67 \text{ kNm}\)
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.2 – Static Method

Static Method:

Span ABC

\[ 34 \text{ kN} \]
\[ 2.0 \text{ m} \]
\[ 2.0 \text{ m} \]
\[ 4.0 \text{ m} \]

\[ 17.0 \text{ kN} \]
\[ (17.0 \times 2.0) = 34.0 \text{ kNm} \]

Free Bending Moment Diagram

\[ 0.5M_p \]
\[ M_p \]
\[ 34.0 \text{ kNm} \]
\[ M_p \]

Combined Bending Moment Diagram

\[ (M_p + 0.5M_p) = 1.5M_p = 34.0 \text{ kNm} \]

\[ \therefore M_p = 22.67 \text{ kNm} \]

Span CDE

\[ 25.5 \text{ kN} \]
\[ 2.0 \text{ m} \]
\[ 2.0 \text{ m} \]
\[ 4.0 \text{ m} \]

\[ 12.75 \text{ kN} \]
\[ (12.75 \times 2.0) = 25.5 \text{ kNm} \]

Free Bending Moment Diagram

\[ 0.5M_p \]
\[ M_p \]
\[ 25.5 \text{ kNm} \]

Combined Bending Moment Diagram

\[ (M_p + 0.5M_p) = 1.5M_p = 25.5 \]

\[ \therefore M_p = 17.0 \text{ kNm} \]

As before the critical value of \( M_p = 22.67 \text{ kNm} \)
Solution

**Topic:** Plastic Analysis – Continuous Beams  
**Problem Number:** 8.3 – Kinematic Method

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\[ \lambda = 1.7 \]

Factored loads: 
- \((1.7 \times 10) = 17.0\) kN/m
- \((1.7 \times 15) = 25.5\) kN
- \((1.7 \times 20) = 34.0\) kN
- \((1.7 \times 30) = 51.0\) kN

**Kinematic Method:**

**Span AB**

*Note:* Span AB is effectively a propped cantilever and the bending moment diagram is asymmetric. The hinge between A and B does not develop at the mid-span point and should be evaluated in a manner similar to that indicated in Section 8.2.3. The reader should carry-out this calculation to show that the hinge develops at a position equal to 2.582 m from the free support at A as shown below, (see page 3 of this solution).

\[ \delta = 3.418 \beta = 2.582 \theta \quad \therefore \beta = 0.755 \theta \]

Internal Work = External Work

\[ [2.0M_p(\theta + \beta) + (1.5M_p\theta)] = [(17 \times 6.0) \times (0.5 \times \delta)] = (102 \times 0.5 \times 2.582 \theta) \]

\[ 4.643M_p\theta = 131.682\theta \]

\[ \therefore \quad M_p = 28.36\text{ kNm} \]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.3 – Kinematic Method

Span BCDE

\[ \delta_1 = 4\beta = 2\theta \quad \therefore \beta = 0.5\theta \quad \delta_2 = 2\beta = \theta \]

Internal Work
\[1.5M_p (\theta) + 1.5M_p (\theta + \beta) + M_p (\beta) = 4.25M_p \theta\]

External Work
\[ (51.0 \times \delta_1) + (25.5 \times \delta_2) = (51.0 \times 2\theta) + (25.5 \times 2\beta) = 127.5 \theta\]

\[4.25M_p \theta = 127.5 \theta\]

\[\therefore M_p = 30.0 \text{ kNm}\]

\[\delta_1 = 2\theta \quad \delta_2 = 2\beta = 4\theta \quad \therefore \beta = 2\theta\]

Internal Work
\[1.5M_p (\theta) + 1.5M_p (\theta + \beta) + M_p (\beta) = 8.0M_p \theta\]

External Work
\[ (51.0 \times \delta_1) + (25.5 \times \delta_2) = (51.0 \times 2\theta) + (25.5 \times 4\theta) = 204.0 \theta\]

\[8.0M_p \theta = 204 \theta\]

\[\therefore M_p = 25.5 \text{ kNm}\]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.3 – Static Method

Span EFG

Internal Work = External Work
\[ M_p(\theta + \beta) + M_p(\beta) = (34 \times 2 \theta) \]
\[ 3M_p \theta = 68 \theta \]

Static Method:
Span AB

\[ +\text{ve} \sum M_A = 0 \]
\[ (17.0x^2)/2 - 2M_p = 0 \]
\[ 8.5x^2 - 2M_p = 0 \]
\[ \therefore M_p = 4.25x^2 \]

Equate the \( M_p \) values to determine \( x \):
\[ 4.25x^2 = 2.429(36.0 - 12x + x^2) \]
\[ \therefore 1.821x^2 + 29.148x - 87.44 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-29.148 \pm \sqrt{29.148^2 + (4 \times 1.821 \times 87.44)}}{2 \times 1.821} = +2.582 \text{ m} \]

\[ M_p = 4.25x^2 = (4.25 \times 2.582^2) \]
\[ \therefore M_p = 28.33 \text{ kNm} \]

The critical value of \( M_p = 30.0 \text{ kNm} \)
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.3 – Static Method

Span BCDE

\[ \begin{align*}
51.0 \text{ kN} & \quad 25.5 \text{ kN} \\
2.0 \text{ m} & \quad 2.0 \text{ m} & \quad 2.0 \text{ m} & \quad 6.0 \text{ m}
\end{align*} \]

\[ \begin{align*}
1.5M_p & \quad 1.33M_p \\
M_p & \quad 85.0 \text{ kNm}
\end{align*} \]

Combined Bending Moment Diagram

\[ (1.5M_p + 1.33M_p) = 85.0 \text{ kNm} \]
\[ 2.83M_p = 85.0 \text{ kNm} \]

\[ \therefore M_p = 30.0 \text{ kNm} \]

\[ \begin{align*}
51.0 \text{ kN} & \quad 25.5 \text{ kN} \\
2.0 \text{ m} & \quad 2.0 \text{ m} & \quad 2.0 \text{ m} & \quad 6.0 \text{ m}
\end{align*} \]

\[ \begin{align*}
42.5 \text{ kN} & \quad 34.0 \text{ kNm} \\
(42.5 \times 2.0) & = 85.0 \text{ kNm}
\end{align*} \]

Free Bending Moment Diagram

\[ \begin{align*}
1.5M_p & \quad 1.33M_p \\
1.3M_p & \quad M_p
\end{align*} \]

Fixed Bending Moment Diagram

\[ \begin{align*}
1.5M_p & \quad 1.33M_p \\
M_p & \quad 68.0 \text{ kNm}
\end{align*} \]

Combined Bending Moment Diagram

\[ (1.5M_p + 1.17M_p) = 68.0 \text{ kNm} \]
\[ 2.67M_p = 68.0 \text{ kNm} \]

\[ \therefore M_p = 25.5 \text{ kNm} \]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.3 – Static Method

Span EFG

![Free Bending Moment Diagram](image)

\[(17.0 \times 2.0) = 34.0 \text{ kNm}\]

Combined Bending Moment Diagram

\[(M_p + 0.5M_p) = 25.5 \quad \therefore M_p = 22.67 \text{ kNm}\]

As before the critical value of \(M_p = 30.0 \text{ kNm}\)
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.4 – Kinematic Method

\[ \lambda = 1.7 \]

Factored loads:
\[ (1.7 \times 10) = 17.0 \text{ kN} \quad (1.7 \times 20) = 34.0 \text{ kN} \]
\[ (1.7 \times 15) = 25.5 \text{ kN} \quad (1.7 \times 30) = 51.0 \text{ kN} \]

Kinematic Method:
Span ABC
Note: The bending moment diagram on span ABC is asymmetric and in this case the hinge between A and C does not necessarily develop under the point load.
The position should be evaluated in a manner similar to that indicated in Section 8.2.3. The reader should carry-out this calculation to show that the hinge develops at a position equal to 2.333 m from the support at A as shown below, (see page 3 of this solution).

\[ \delta_i = 3.667 \beta = 2.333 \theta \quad \therefore \beta = 0.635 \theta \quad \delta_i = 2.0 \theta \]

Internal Work:
\[ = [1.5M_p (\theta) + 1.5M_p (\theta + \beta) + (1.5M_p \beta)] = 4.91M_p \theta \]

External Work:
\[ = [(34 \times \delta_2)] + [(17 \times 6.0) \times (0.5 \times \delta_i)] \]
\[ = [(34 \times 2 \theta)] + [(102.0) \times (0.5 \times 2.333 \theta)] = 186.98 \theta \]

\[ 4.91M_p \theta = 186.98 \theta \]
\[ \therefore \quad M_p = 38.08 \text{ kNm} \]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.4 – Kinematic Method

Span CDEF

\[ \delta_1 = 4\beta = 2\theta \quad \therefore \beta = 0.5\theta \quad \delta_2 = 2\beta = \theta \]

Internal Work
\[ 1.5M_p (\theta) + 2.0M_p (\theta + \beta) + M_p (\beta) = [1.5M_p (\theta) + 2.0M_p (1.5\theta) + M_p (0.5\theta)] = 5.0M_p\theta \]

External Work
\[ (51.0 \times \delta_1) + (25.5 \times \delta_2) = [(51.0 \times 2\theta) + (25.5 \times 2\beta)] = [(102\theta) + (25.5\theta)] = 127.5\theta \]

\[ 5.0M_p\theta = 127.5\theta \quad \therefore M_p = 25.5 \text{ kNm} \]

Internal Work
\[ 1.5M_p (\theta) + 2.0M_p (\theta + \beta) + M_p (\beta) = [1.5M_p (\theta) + 2.0M_p (3.0\theta) + M_p (2.0\theta)] = 9.5M_p\theta \]

External Work
\[ (51.0 \times \delta_1) + (25.5 \times \delta_2) = [(51.0 \times 2\theta) + (25.5 \times 4\theta)] = [(102\theta) + (102\theta)] = 204.0\theta \]

\[ 9.5M_p\theta = 204.0\theta \quad \therefore M_p = 21.47 \text{ kNm} \]
Span FG
Note: Span FG is effectively a propped cantilever and the bending moment diagram is asymmetric. The hinge between F and G develops at a position 0.4142L from the simply supported end as indicated in Section 8.2.3.

\[ M_p \text{ at F} \]

\[-M_p \text{ at } G \]

\[ w \text{ kN/m} \]

\[ M_p = 0.4142L \]

\[ L \]

\[ 34.0 \text{ kN/m} \]

\[ 2.343 \text{ m} \]

\[ 1.657 \text{ m} \]

\[ 4.0 \text{ m} \]

\[ \delta = 1.657\beta = 2.343\theta \quad \therefore \beta = 1.414\theta \]

Internal Work = External Work
\[ [M_p (\theta) + M_p (\theta + \beta)] = [(34.0 \times 4.0) \times (0.5 \times \delta)] \]
\[ [M_p (\theta) + M_p (2.414\theta)] = (136 \times 0.5 \times 2.343\theta) \]
\[ 3.414M_p\theta = 159.32\theta \]
\[ \therefore M_p = 46.67 \text{ kNm} \]

Static Method:
Span ABC

\[ 34.0 \text{ kN} \]

\[ 17.0 \text{ kN/m} \]

\[ x \text{ m} \]

\[ (6.0 - x) \text{ m} \]

\[ 6.0 \text{ m} \]

\[ 1.5M_p \]

\[ x \text{ m} \]

\[ 1.5M_p \]

\[ 1.5M_p \]

\[ 34.0 \text{ kN} \]

\[ 17.0 \text{ kN/m} \]

\[ 2.0 \text{ m} \]

\[ x \text{ m} \]

\[ 1.5M_p \]

\[ V_A \]

\[ 1.5M_p \]

\[ 17.0 \text{ kN/m} \]

\[ 1.5M_p \]

\[ 1.5M_p \]

\[ 1.5M_p \]

\[ 1.5M_p \]

\[ (6.0 - x) \text{ m} \]

\[ V_B \]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.4 – Static Method

\[ +ve \sum M_A = 0 \]
\[ -1.5M_p + (34 \times 2.0) + (17.0x^2)/2 - 1.5M_p = 0 \]
\[ 68.0 + 8.5x^2 - 3.0M_p = 0 \quad \therefore M_p = 22.667 + 2.833x^2 \]

\[ +ve \sum M_C = 0 \]
\[ 1.5M_p - 17.0(6.0 - x)^2/2 + 1.5M_p = 0 \quad \therefore M_p = 2.833(6.0 - x)^2 \]

Equate the \( M_p \) values to determine \( x \):
\[ 22.667 + 2.833x^2 = 2.833(36.0 - 12x + x^2) \]
\[ \therefore 33.996x - 79.321 = 0 \]
\[ x = 2.333 \text{ m} \]

\[ M_p = 2.833(6.0 - x)^2 = 2.833(6.0 - 2.333)^2 \]
\[ \therefore M_p = 38.09 \text{ kNm} \]

Span CDEF

\[ 1.5M_p \quad 1.33M_p \quad M_p \]
\[ 2.0M_p \quad 85.0 \text{ kNm} \]

Combined Bending Moment Diagram

\[ (2.0M_p + 1.33M_p) = 85.0 \text{ kNm} \]
\[ 3.33M_p = 85.0 \text{ kNm} \]
\[ \therefore M_p = 25.5 \text{ kNm} \]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.4 – Static Method

\[ 51.0 \text{kN} \quad 25.5 \text{kN} \]
\[ \begin{array}{c}
\text{C} \quad \text{D} \quad \text{E} \quad \text{F} \\
2.0 \text{m} \quad 2.0 \text{m} \quad 2.0 \text{m} \\
6.0 \text{m}
\end{array} \]

\[ (34.0 \times 2.0) = 68.0 \text{kNm} \]

\[ \begin{array}{c}
\text{Free Bending Moment Diagram} \\
\text{Combined Bending Moment Diagram} \\
\text{Fixed Bending Moment Diagram}
\end{array} \]

\[ (2.0M_p + 1.17M_p) = 68.0 \text{kNm} \]
\[ 3.17M_p = 68.0 \text{kNm} \]

\[ \therefore M_p = 21.47 \text{kNm} \]

Span FG

\[ \begin{array}{c}
\text{F} \quad \text{G} \\
34.0 \text{kN/m} \quad \text{G}
\end{array} \]

\[ x \text{m} \quad (4.0 - x) \quad 4.0 \text{m} \]

\[ +ve \quad \sum M_F = 0 \]
\[ (34.0x^2)/2 - M_p - M_p = 0 \]
\[ 17.0x^2 - 2.0 M_p = 0 \]
\[ M_p = 8.5x^2 \]

\[ +ve \quad \sum M_G = 0 \]
\[ M_p - 34.0(4.0 - x)^2 / 2 = 0 \]
\[ M_p = 17.0(4.0 - x)^2 \]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.4 – Static Method

Equate the \( M_p \) values to determine \( x \):

\[
8.5x^2 = 17.0(16.0 - 8x + x^2) \quad \therefore 8.5x^2 - 136x + 272 = 0
\]

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{136 \pm \sqrt{136^2 - (4 \times 8.5 \times 272)}}{(2 \times 8.5)} = +2.343 \text{ m}
\]

\[
M_p = 8.5x^2 = (8.5 \times 2.343)^2
\]

\( M_p = 46.67 \text{ kNm} \)

As before the critical value of \( M_p = 46.67 \text{ kNm} \)

Note: Span FG is the same as the standard propped cantilever in Example 8.3 in which the hinge develops at a point \( 0.414L \) from the simply supported end and the \( M_p \) value equals \( 0.0858wL^2 \), i.e.

Distance of hinge from support F = \([4.0 - 0.414L] = [4.0 - (0.414 \times 4.0)] = 2.344 \text{ m}\)

\( \therefore M_p = (0.0858 \times 34.0 \times 4.0^2) = 46.67 \text{ kNm} \)
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.5 – Kinematic Method

\[ \lambda = 1.7 \]

Factored loads: \( (1.7 \times 15) = 25.5 \text{kN} \)  \( (1.7 \times 20) = 34.0 \text{kN} \)
\( (1.7 \times 30) = 51.0 \text{kN} \)

Kinematic Method:

Span ABC

Note: The bending moment diagram on span ABC is asymmetric and in this case the hinge between A and C does not necessarily develop under the point load and its position should be evaluated in a manner similar to that indicated in Section 8.2.3. The reader should carry-out this calculation to show that the hinge develops at a position equal to 3.725 m from the support at A as shown below, (see page 2 of this solution).

\[ \delta_1 = 4.275\beta = 3.725\theta \quad \therefore \beta = 0.871\theta; \quad \delta_2 = 2.0\beta \]

Internal Work = \[ 2.0M_p (\theta + \beta + (1.5M_p\beta)) = 5.05M_p\theta \]

External Work = \[ ([51 \times \delta_2]) + ([34 \times 8.0] \times (0.5 \times \delta_1)) \]
= \[ ([51 \times 1.742\theta]) + ([272.0] \times (0.5 \times 3.725\theta)) = 595.44\theta \]

\[ 5.05M_p\theta = 595.44\theta \]

\[ \therefore M_p = 117.91 \text{kNm} \]
Solution

Topic: Plastic Analysis – Continuous Beams
Problem Number: 8.5 – Kinematic Method

Span CDE

\[ \delta_l = 4\beta = 2\theta \quad \therefore \beta = 0.5\theta \]

Internal Work = External Work
\[ 1.5M_p(\theta) + 1.5M_p(\theta + \beta) + 1.5M_p(\beta) = (51.0 \times \delta_l) = (51.0 \times 2\theta) \]
\[ 4.5M_p\theta = 102\theta \]
\[ \therefore M_p = 22.67 \text{ kNm} \]

Span EF

\[ \delta_l = 2\theta \]

Internal Work = External Work
\[ 1.5M_p(\theta) = (25.5 \times \delta_l) = (25.5 \times 2\theta) = 51.0\theta \]
\[ \therefore M_p = 34.0 \text{ kNm} \]

The critical value of \( M_p \) = 117.91 kNm

Static Method:
Span ABC

\[ 34.0 \text{ kN/m} \]
\[ 51.0 \text{ kN} \]
\[ x \text{ m} \]
\[ (6.0 - x) \text{ m} \]
\[ 8.0 \text{ m} \]

\[ 2M_p \]
\[ V_A \]
\[ x \text{ m} \]

\[ 34.0 \text{ kN/m} \]
\[ 1.5M_p \]
\[ 2M_p \]
\[ (8.0 - x) \text{ m} \]
\[ V_C \]
Solution

**Problem Number:** 8.5 - Static Method

**Topic:** Plastic Analysis – Continuous Beams

\[ +\text{ve} \sum M_\Lambda = 0 \]
\[ 34.0x^2/2 - 2M_\Lambda = 0 \]
\[ 17.0x^2 - 2M_p = 0 \]
\[ M_p = 8.5x \]

\[ +\text{ve} \sum M_C = 0 \]
\[ 2M_p - 34.0(8.0 - x)^2/2 - (51.0 \times 2.0) + 1.5M_p = 0 \]
\[ M_p = 4.857(8.0 - x)^2 - 29.143 \]

\[ M_p = 8.5x^2 = (8.5 \times 3.725^2) \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ = \frac{-77.712 \pm \sqrt{77.712^2 + (4 \times 3.643 \times 339.991)}}{2 \times 3.643} = +3.725 \text{ m} \]

\[ \therefore M_p = 117.94 \text{ kNm} \]

**Span CDE**

\(51.0 \text{ kN} \)
\(2.0 \text{ m} \)
\(4.0 \text{ m} \)
\(6.0 \text{ m} \)

\(1.5M_p \)
\(1.5M_p \)

\(68.0 \text{ kNm} \)

\(34.0 \text{ kN} \)
\((34.0 \times 2.0) = 68.0 \text{ kNm} \)

\(17.0 \text{ kN} \)

**Combined Bending Moment Diagram**

\((1.5M_p + 1.5M_p) = 68.0 \text{ kNm} \)

**Span EF**

\(25.5 \text{ kN} \)
\(2.0 \text{ m} \)

\(1.5M_p = PL = (25.5 \times 2.0) = 51.0 \text{ kNm} \)

\[ \therefore M_p = 34.0 \text{ kNm} \]

**Critical value of** \(M_p = 117.94 \text{ kNm} \)