LECTURE NOTES

PROF. HARDY CROSS PROF. PAUL BRENNAN

YALE 1952-1953

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Class Notes from Hardy Cross's Last Course

Hardy Cross should need little introduction. He was born February 10, 1885 in Nansemond, Virginia, received B.A. and B.S. degrees from Hampden-Sydney College at the ages of 17 and 18, and taught briefly at Norfolk Academy before receiving a B.S. in Civil Engineering from MIT in 2 years. Following another short stint of teaching at Norfolk Academy and work with the Missouri and Pacific railroads, he received an M.S. from Harvard. He then taught at Brown University for 7 years and worked professionally in New York and Boston before joining the faculty at the University of Illinois in 1921. He left Illinois for Yale in 1937 where he stayed until his retirement in 1953.

He authored many important technical papers on column analogy, moment distribution, network theory, the civil engineering curriculum and the relation of analysis to structural design. He received numerous awards, the most notable of which may be the Gold Medal of the Institution of Structural Engineers of Great Britain. He was the first American and only the fifth recipient of the award in the first 50 years of the institute. Hardy Cross died in 1959.

I first saw these notes in 1980 when I began work with Robert A. Heller, PE in Westport, Connecticut. At that time Bob's practice was geotechnical engineering including the design of bulkheads and timber pile foundations. In the early 1950's many engineers felt that soils and foundation engineering was simply another subject that a structural engineer should learn and practice. So Bob obtained two Master's degrees – one in structural engineering from Yale and one in soils and foundation from Princeton. Within a few years of graduation he concluded that the fields were becoming too complicated for an engineer to master both and he decided to focus on soils and foundation engineering.

Bob remained very proud of these notes. He told me that he typed and drafted his notes after each class with the intention of some day publishing them as a book. Sadly, he passed away unexpectedly in 1995 before he could achieve his goal.

I wish to thank Professor Emeritus William J. Hall for his thoughtful review comments and encouragement.

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Prof. Cross
Sept. 2?, 1952

The purpose of this first meeting of graduate students was to present a "broad picture" of civil engineering, and more precisely structural engineering as it will be presented this year. This broad picture should be kept in mind throughout the course. Reference will often be made to it.

There are three obvious phases into which any civil engineering project may be divided:

1. Planning
   Consideration is given here to locating railroad and highway connections, to determining need and positioning of bridges, buildings, and the like.

2. Building
   This can be divided into three phases:
   1. Design or layout
   2. Dimensions
   3. Proper Structure

   These will be discussed further below.

3. Operating
   Self-explanatory

Number 1, above, Planning, cannot be considered a separate entity but must be combined with 2 and 3 above. To plan one must know how to operate, and what difficulties may arise in dimensioning, and so on.

As stated above, "Building" can be divided into three phases.

1. Design
   In this phase the building is laid out or, if a bridge is considered, its length determined.

2. Dimensions
   (The following refers to a bridge, but similar examples may be used for building.) If a span is a certain length will the pieces be too large?--can it be erected in a reasonable time at a reasonable cost? These questions will be answered in this phase.

3. Proper Structure
   This phase will determine if the building has satisfied the following three conditions:
   a. Strength and Stability
   b. Stiffness
      We know very little about this. We may be able to estimate how much it will deflect but we do not know if it is too much or not. This is an excellent research problem.
   c. Satisfactoriness
      Even though the other two points are satisfied, there are many other considerations. For example, ceilings

may crack, plastics and certain concrete have bad reputations and may deteriorate without warning some years after the building has been constructed.

The following questions summarize the above:

1. What do you want?
   The answer may be a bridge, or a bridge 1000 feet long, or a bridge with three spans 1000 feet long.
2. Can you get it?
   This answer requires the services of a structural engineer.
3. Have you got it?
   We will have it if we have carried dimensioning so far so that we know it can be built with available labor and material and if it is assured it is a "proper structure."

"Strength" will be discussed separately now. This phase is not over-emphasized but rather the other phases are under-emphasized. Failure does not necessarily mean that the structure will fall down, but, more likely, that it will break, bust, or crack. Determining strength involves generally a computation of stress assuming that a proportionality exists between stress and strain. Ideally, therefore, stress can be computed exactly. This is accomplished by multiplying strain by the ratio of stress to strain (E) and getting an equivalent stress. However, it should be noted that when adding strains, because the ratio E changes after the yield point is reached, it is possible for the sum 20 KSI and 30KSI to equal 35KSI. However, if stresses are added 20KSI and 30KSI will equal 50KSI.

The reason for the above computation is to determine quickly and easily if there is a possibility of failure, if the size of the members are practicable, and if the structure can be laid out and erected. These standard methods of analysis are probably not right but, nevertheless, their purpose served by obtaining the scale (the size of the forces, and of the members) quickly and easily.

These standard methods are written into codes which often become laws or conditions in contracts. Codes serve to protect the public and to reduce strife caused by competition between these standard methods. Since codes vary from one locale to another, ultimately it is required that figures be put down that can be checked anywhere. This is also protection against lawsuits and the like. It is good practice to compute first, using standard methods and then to run an analysis in the lab and measure the result. The two generally will not agree. It may then pay to investigate the cause of the disagreement, to go into more precise theory to find what is wrong. Therefore, compute; measure; and investigate conflicting results.

Other sources of evidence to compute strength will be discussed Thursday.

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Prof. Cross
Sept. 25, 1952

This lecture is a continuation of the lecture of September 23, 1952 in which a "broad picture" was presented. It is essential we do not become bogged down in computations. As mentioned before, there are three phases into which a project may be divided: Plan, Build, and Operate. Using house building as an illustration, we layout the house, build the house, and live in the house.

In each of these phases use caution so as not to offend the public.

Although the relative order of importance varies, the following are the elements of building:

Construction
Appearance
Use
Structural
Economy

Appearance and architecture may be used synonymously. The structural element can be subdivided into:

1. Strength and stability
2. Stiffness
3. Structural satisfactoriness

A stress and strain on a material may cause a failure. Evidence is required to show how a stress and a strain will cause the failure. The following are sources of evidence:

1. Analysis
   Models, which are a mechanical means of making an analysis, are often used. They do not, however, have the validity of reality.

2. Experiments
   a. Laboratory
   b. Field

3. Experience
   This as often includes information obtained from failures as from successful ventures. However, after failure it is difficult to get data. Experience acquired secondhand is as reliable as the person or perons through whom the experience is transmitted.

4. Commonsense
   Although intuitive, it can be increased or suppressed.

5. Authority
   Engineers (good ones!) do not follow blindly what they read.

Analysis is the most common source of evidence because it is the quickest and the easiest. Sometimes it is only necessary to figure simple stresses and strains. However, experiments may take years. There are two reasons for making analyses:

1. To quickly and easily obtain the scale.
   It is important to know how big a member will be, how many bars will be in a concrete beam...10?...100??

2. Because it involves a common and routine procedure.
   Each field has their own particular custom. So, for ordinary highway and railroad bridges, buildings, etc., standard specifications are used.

There are three types of specifications:

1. Building codes
   These are laws.

2. General Specifications
   Special groups in most fields correlate available knowledge and make specifications. These are not law but, nevertheless, are usually adhered to very closely.

3. General Recommendations
   This is a European custom coming about because there is no separation between contractor and designer as there is in this country.

Codes tell:

1. What loads or conditions to assume.
   H-15 truck loadings and E-60 locomotive loadings do not really occur, but are satisfactory assumptions.

2. What assumptions to make in analysis.
   A usual one is the assumption that Hooke's law applies.

3. What stresses to permit for the above assumptions.
   Assuming greater loads and higher permissible stresses will produce the same result as small loads and smaller permissible stresses.

4. What details are to be regulated.

There are six kinds of stresses: deadload, live load, deformation, parasitic, locked, and additional. The following explains these stresses:

1. Dead Loads

(Self
(Superimposed

2. Live Loads

(Static
(Dynamic
(Impact is used synonymously
(with dynamic

3. Deformation
   This is not a standard term. This stress will occur as a result of the motion of an adjoining member.

4. Parasitic
   This is a common term. Something preys on the structure causing stresses without a useful contribution.

(Temperature change
(Settlement of supports
(This is the main objection
(to continuous structures.

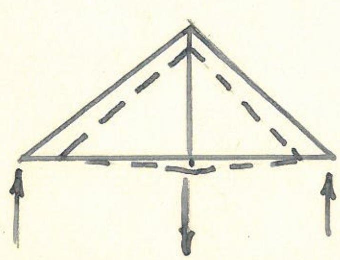
5. Locked
   This is caused by an inequality of internal stresses.

(Unintentional
(As a result of welding,
(the yield point stress is
(put in. Due to the differ-
(ence of thickness of the web
(of a girder and its flange,
(the web will cool quicker
(and cause a locked in stress.
(In reinforced concrete, the
(concrete will shrink but the
(steel will not.

(Intentional
(Stresses, opposite to
(those expected are locked
(into a member or structure
(deliberately. This is called
(often prestressing.

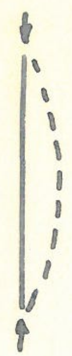
6. Additional
   This is not a standard term. In most cases these stresses are very small. The following diagrams will illustrate this stress.

a.)



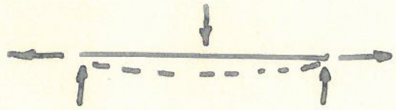
Simple truss.

b.)



Buckling of a column.

c.)



One set of forces tend to straighten out the member while the other set tends to bend it.

We have outlined here in this lecture and in the preceding one a broad picture of one phase of civil engineering. We should not, however, belittle the other phases. For example, consider water. Rivers have undermined bridge abutments, wave action has destroyed concrete, etc.

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Prof. Cross
Sept. 30, 1952

As stated in the previous lectures, any project may be divided into three phases: Planning, Building, and Operating. The same man or organization will probably never do all these operations. However, the engineer doing the planning must know how to build and how to operate. He cannot help but be thrown into contact with all the phases.

Building may also be divided into three obvious phases as was covered in the previous lectures: Design, Dimension, Construction. Design here refers to the layout, or length of span, or type of truss, etc. Design necessarily depends upon the kind of dimensions encountered.

This lecture will be devoted to the second phase of building, dimensioning. Dimensioning may in turn be divided into three phases:

1. Kind and approximate size of member to use.
   As much duplication without undue waste is a good rule. It may be more economical to use the same size members throughout.

2. Preliminary computations to see whether stresses are too high or too low.
   If the stress is too high, the question to ask is what is to be done about it. There are a great many answers, but the best one must be selected from them. If the stresses are too low, the question arises as to whether advantage can be made of it. If not it may not matter if you want the same size member originally selected.

   There are four essentials of approximate computations:

   1. Obvious
   2. Easy to learn
   3. Easy to do
   4. Hard to forget

   Examples of approximate solutions will be shown below.

3. Final computations that someone else can duplicate.

   These will be performed on the member that already has been laid out and dimensioned.

An outstanding characteristic of the engineer is that he tries to get the <u>scale</u> as quickly as he can. He must know how big it is in comparison. Scale! scale! scale! must be continually emphasized. So, all sketches should be drawn to scale.

The following pages are examples of how preliminary computations are made. This will show how to get approximate sizes, and how to know at once the number of bars in concrete or the maximum stress in the steel.

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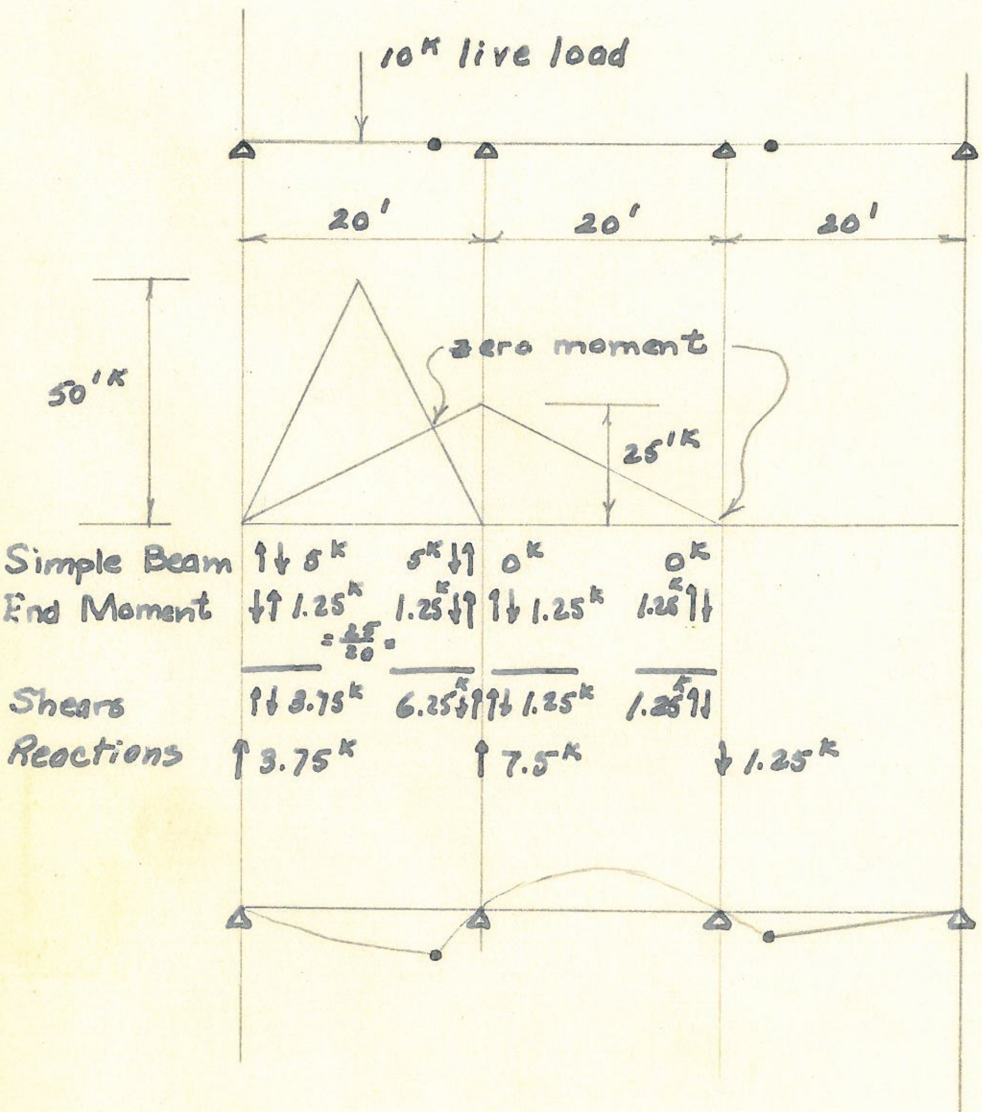
Prof. Cross
Sept. 30, 1952

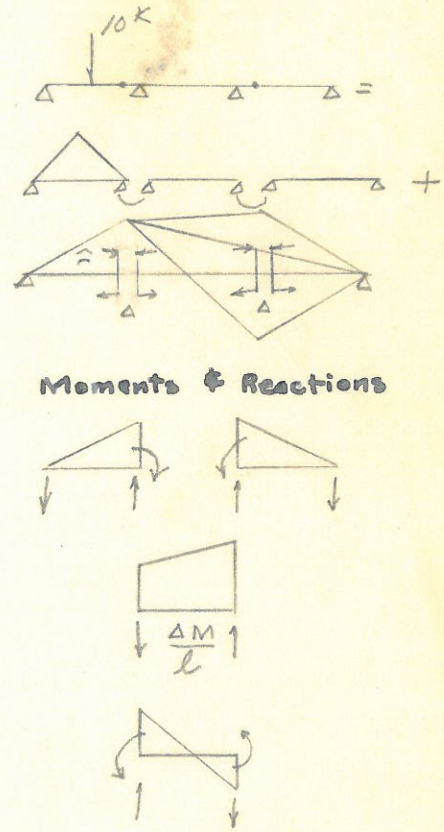
As an illustration, a cantilever beam, which is very uncommon, will be used. The beam will have concentrated loads coming from a floor system.

10^K live load

10^K dead load

When the deflected beam is drawn, it cannot be drawn to scale because the movements are very small.





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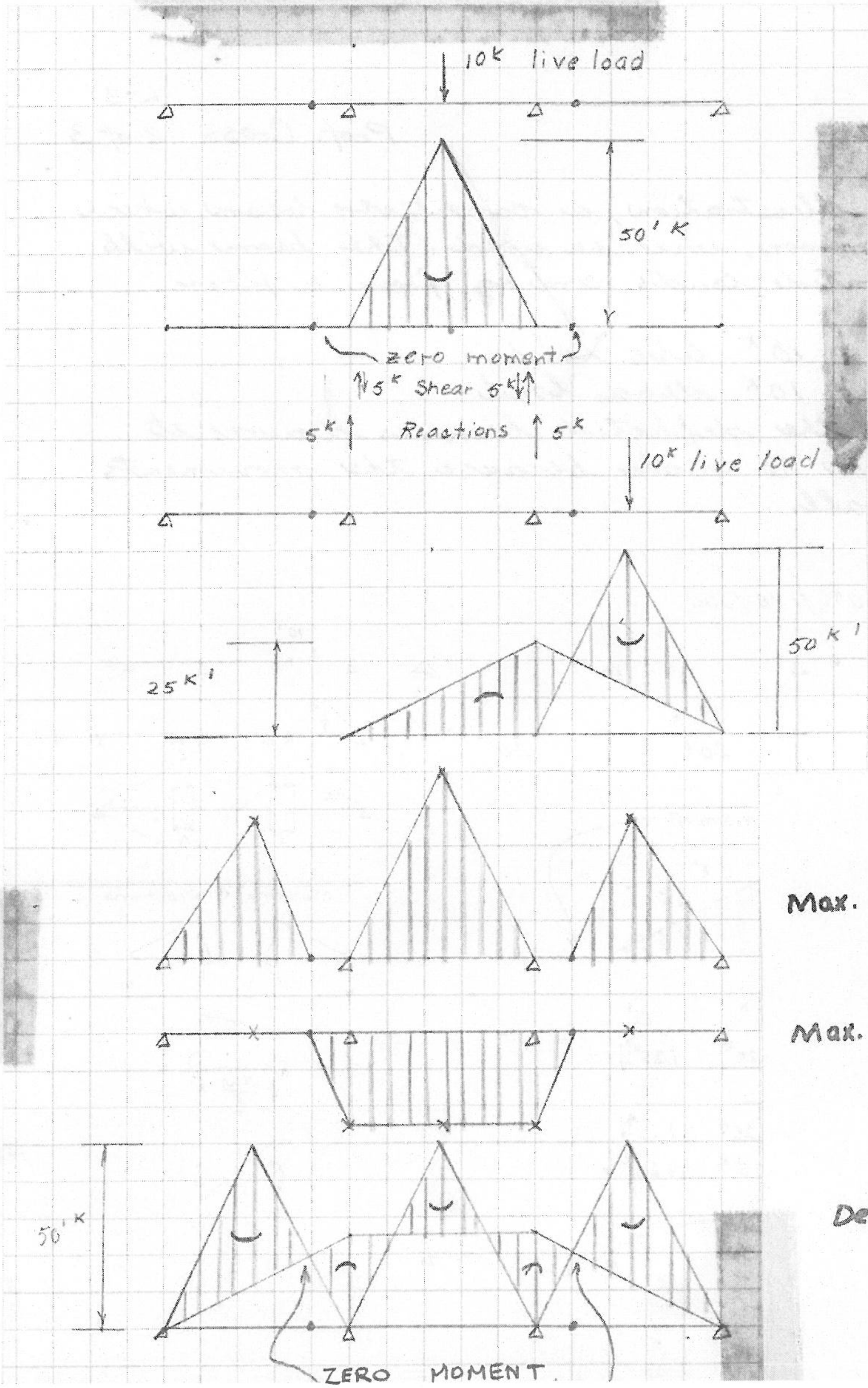

Max. positive moment
live load

Max. negative moment
live load

Dead Load Moments

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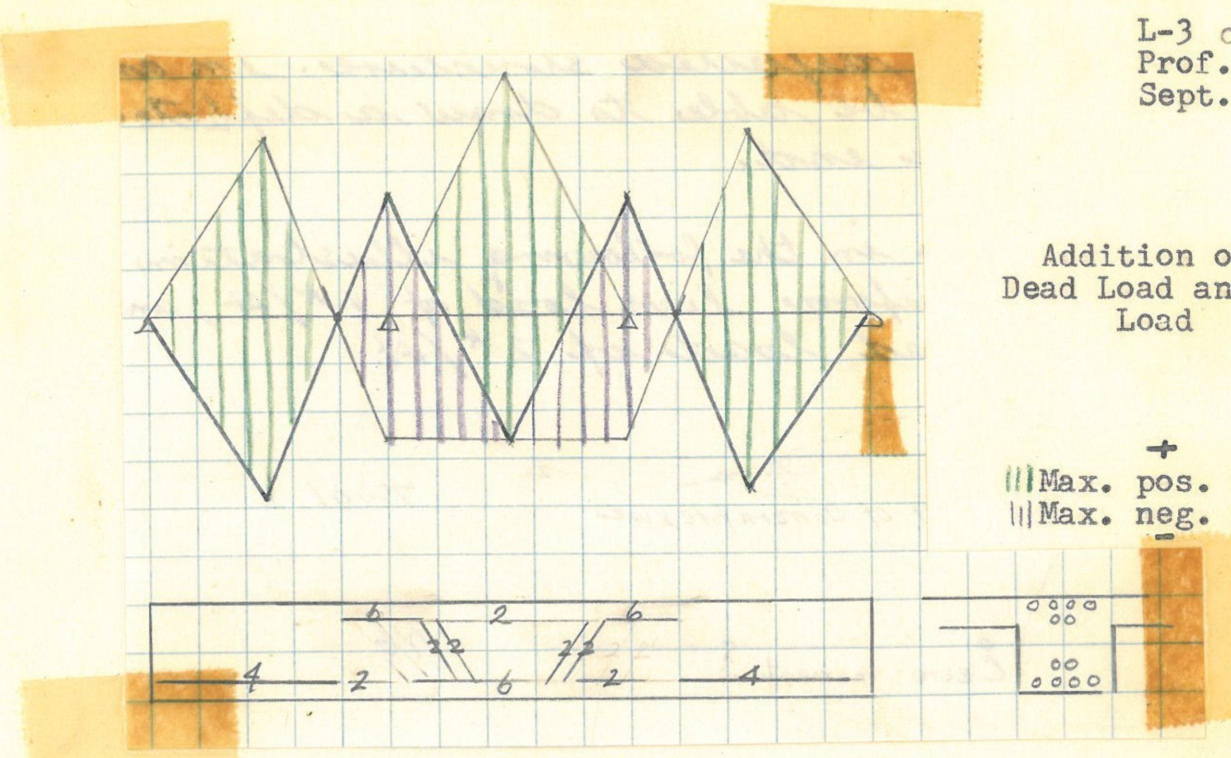


Addition of
Dead Load and Live
Load

+
||| Max. pos. moment
||| Max. neg. moment
−

Add up live load shears. Add up dead load shears. Add the two together. Do the same thing for reactions. See ~~appendix,~~ pg. 5.

The moments of a cantilever beam, such as was just computed, are very different from that of a simply supported beam. However, the reactions and shears will only be slightly different.

The following illustration will show how a completely continuous beam is quickly analized. The method is as follows:

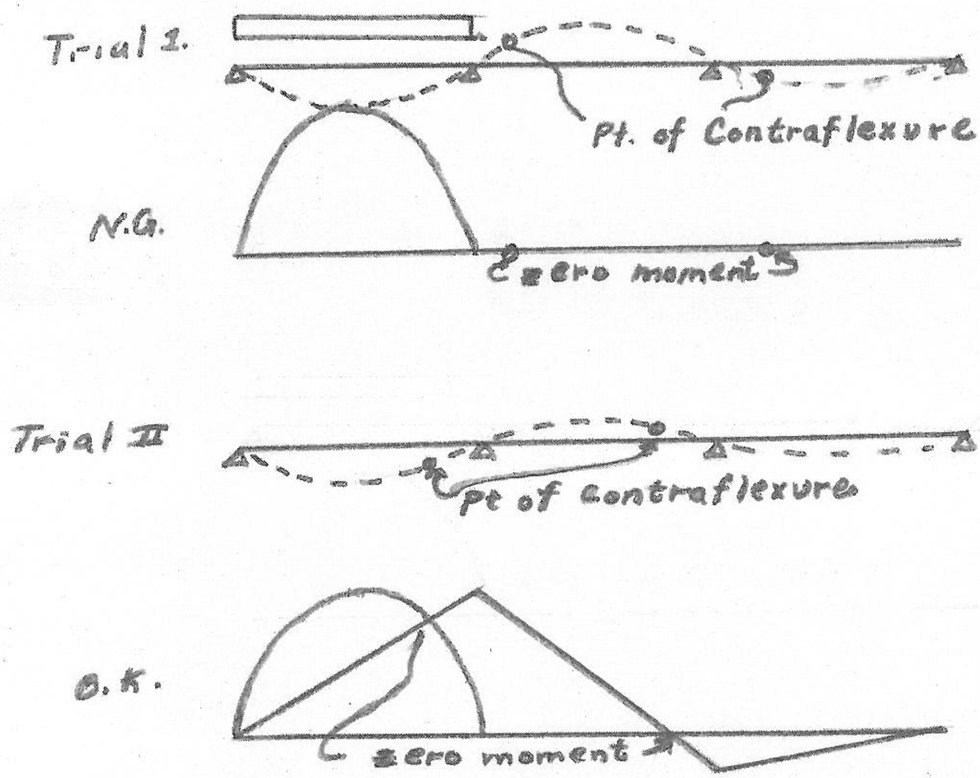
1. Draw deflected structure.
   The deflected structure is continuous.
   There are no sharp breaks.

2. Draw moments for the deflections.
   Curvature must agree with the moments. The bigger the moments, the sharper the curvature. Therefore, moments follow from deflected structure.

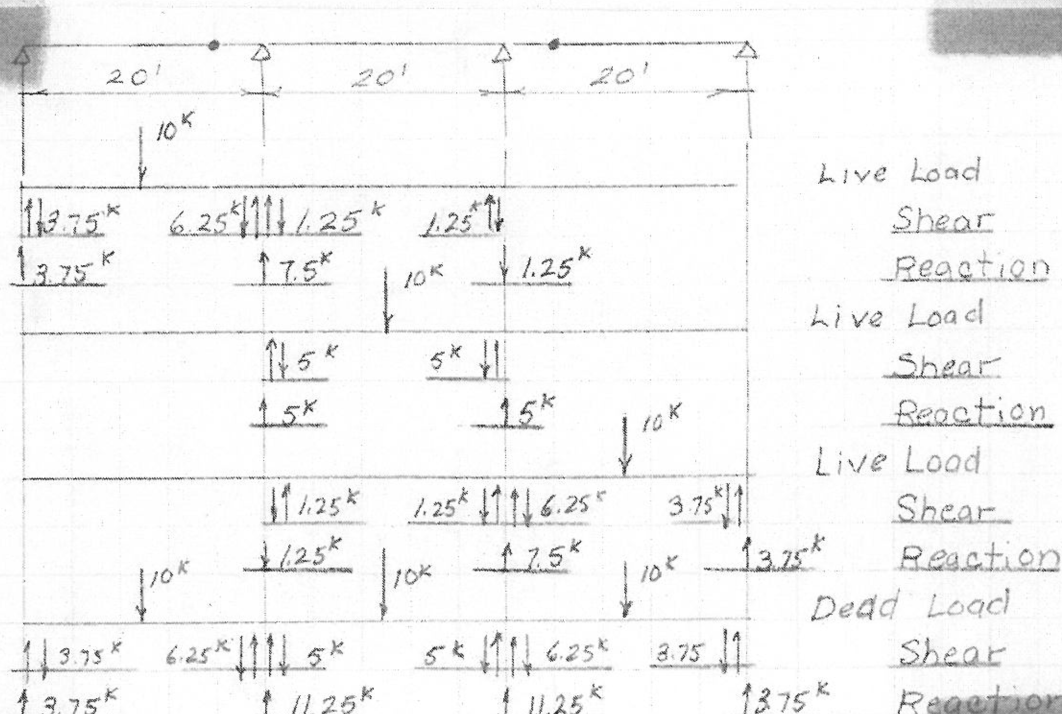
3. Check moments by moment distribution.

4. If it does not check and your calculations are correct, there is likely to be an error in judgement in drawing deflected structure. With practice these errors will be minimized. Therefore, the first thing to do is draw the deflected structure, and the last thing to do is draw the deflected structure. In any case, you must be able to draw a deflected structure at the end.

The beam in the ~~following~~ illustration on pg. 6 will have a uniform live load of 1 ^{K}/ft, and a uniform dead load of 2 ^{K}/ft.



Shears and reactions of cantilever beam, (pg. 2 & 3, this lecture).



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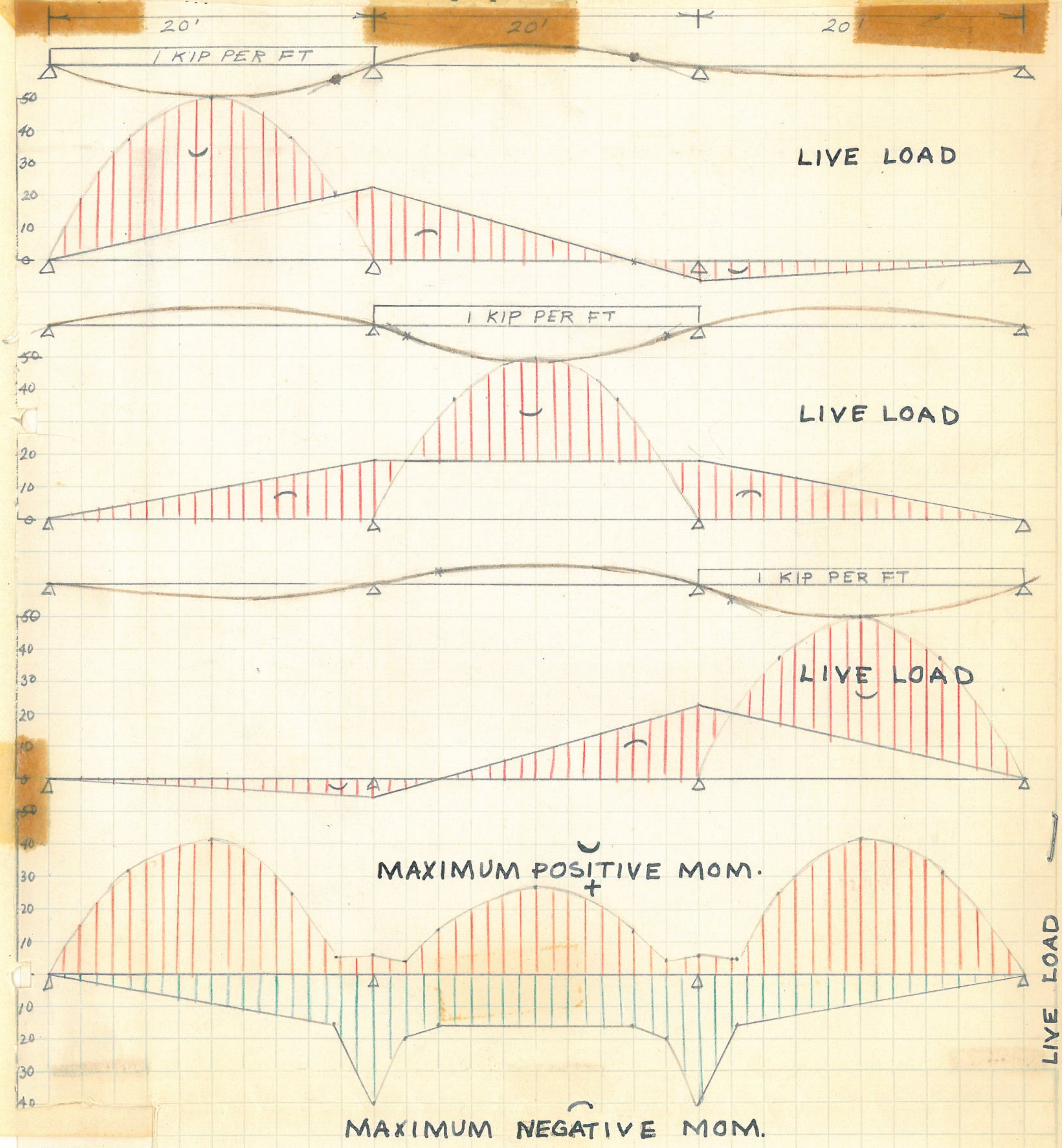
Continuous beam with uniform live load of 1 KIP per ft. and a uniform dead load of 2 Kips per ft.



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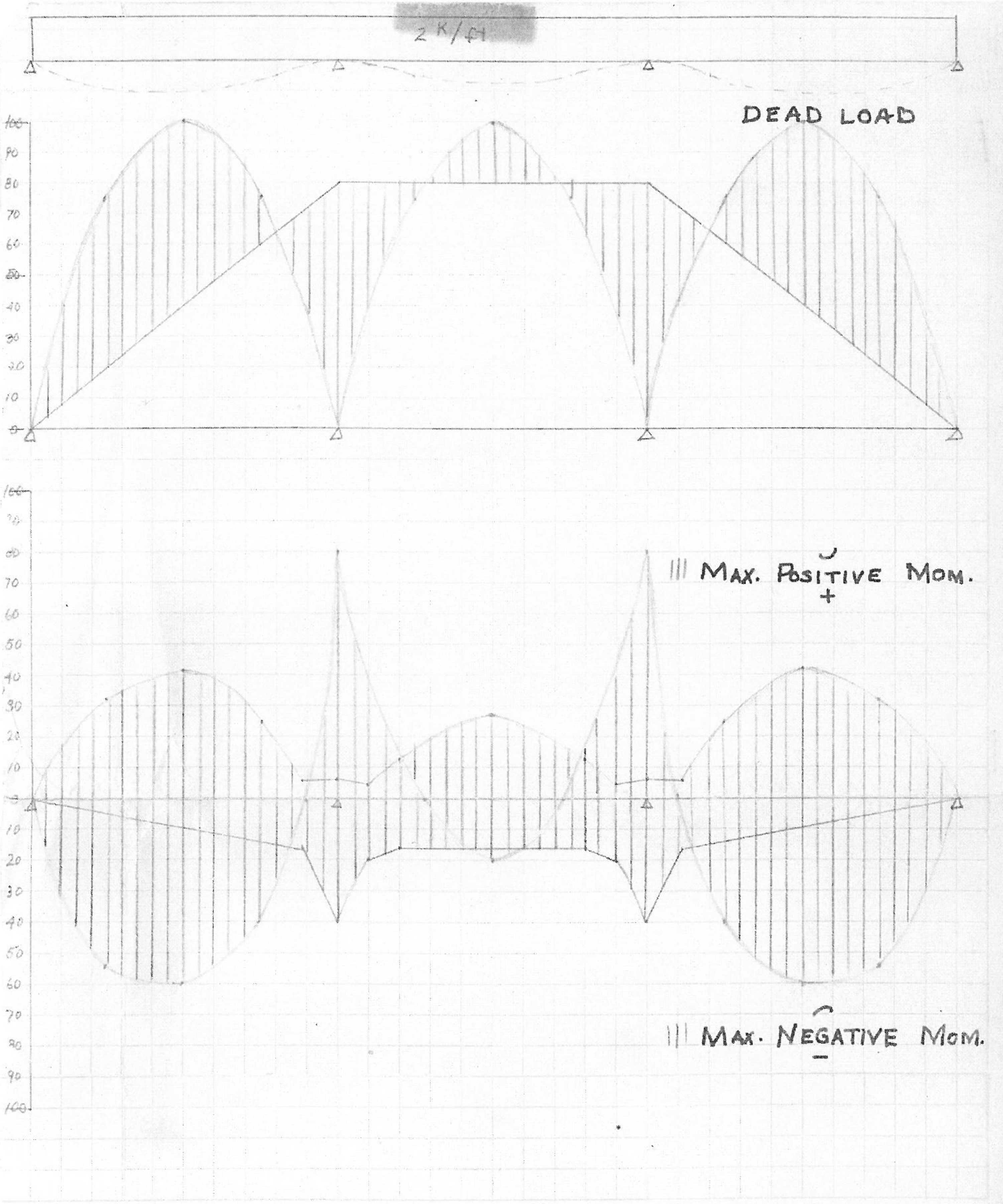

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Continuous beam... by moment distribution.

0 100 $1^K/ft$ 50 | 50 K=1 50 | 50 K=1 100 0

+33	-33	0	0	0	0
+9	+17	+16	+8		
		+2	-4	-4	-2
-42	-21				
+5	+11	+12	+6		
		-1	-3	-3	-1
-5	-3				
+1	+2	+2	+1		
				+1	+3
			-1	-1	
0	-24	+24	+7	-7	0

0 100 50 | 50 50 | 50 100 0

0	0	+33	-33	0	0
-9	-17	-16	-8		
		+10	+21	+21	+10
+9	+4			-5	-10
-4	-7	-7	-4		
+4	+2	+2	+4	+4	+2
-1	-2	-2	-1	-1	-2
+1			+1	+1	
0	-20	+20	-20	+20	0

0 100 50 | 50 $2^K/ft$ 50 | 50 100 0

+67	-67	+67	-67	+67	-67
-67	-33			+33	+67
+8	+16	+17	+8		
-8	-4	-10	-21	-21	-10
+3	+7	+7	+3		
				+5	+10
-3	-2	-2	-4	-4	-2
+1	+2	+2	+1	+1	+2
-1			-1	-1	
0	-81	+81	-81	+80	0

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Prof. Cross
Oct. 2, 1952

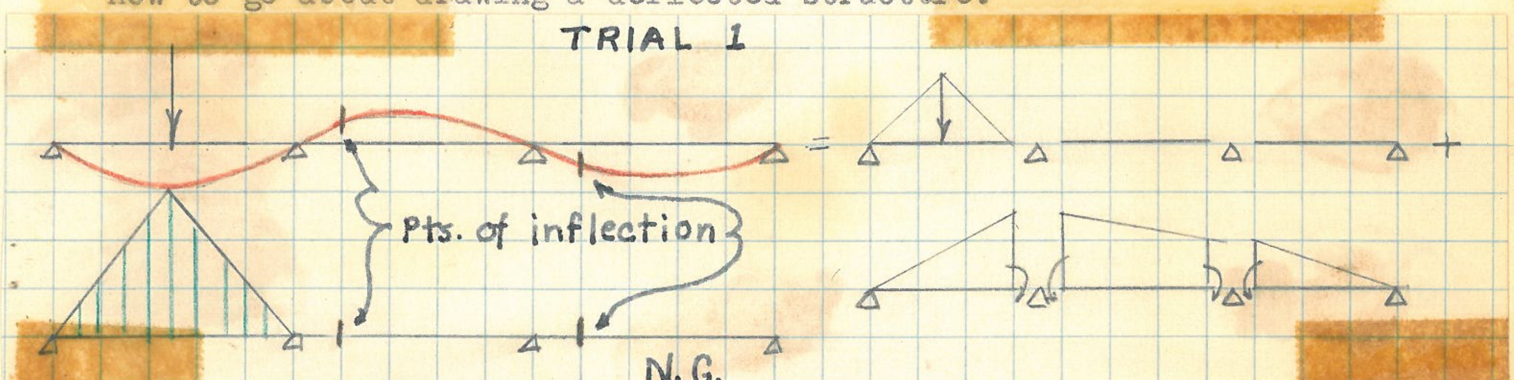
DEFLECTED STRUCTURES

The first thing to do is to draw the deflected structure. It may be necessary to break up the structure to make it less complicated. After making all the necessary calculations, the last thing to do is to draw a deflected structure. If the calculator cannot draw a deflected structure then he does not know what he is talking about.

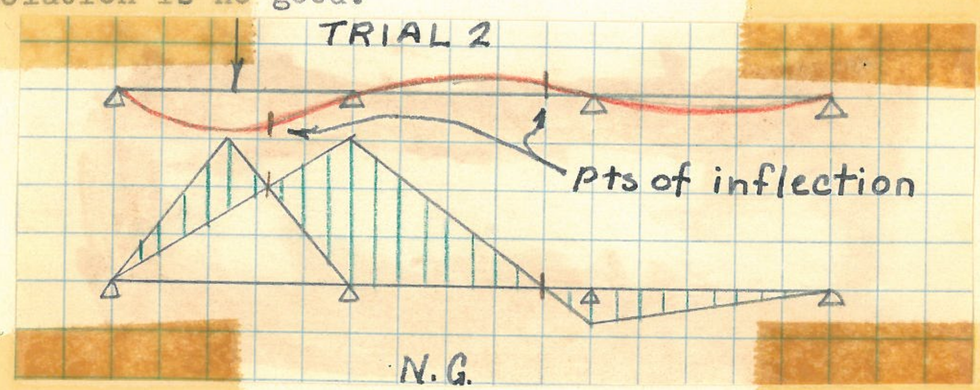
Sometime next spring we will get a problem in an exam that requires only the drawing of a deflected structure for the solution.

When drawing a deflected structure, draw it as a cartoon, that is, exaggerate the vertical scale. Caution: where it is necessary to exaggerate in two or even three directions, be careful to know what you are about.

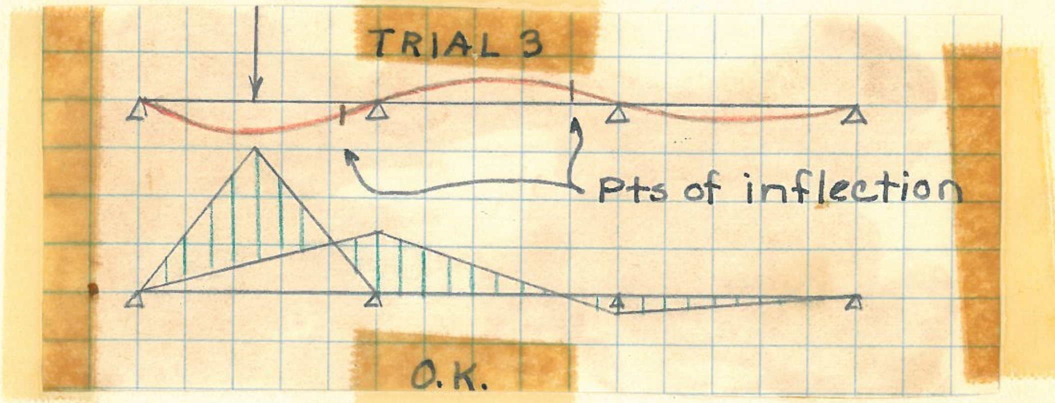
The reason for drawing deflected structures is to get the scale quickly and easily. Forget about precision. If it is not apparent how the structure is deflected, it is wise to draw the deflected structure even incorrectly. The following illustration will show how to go about drawing a deflected structure.



The deflected structure indicates there is bending in every span. Yet, because of the location of the points of inflection, or points of zero moment or curvature, only zero bending moments can be drawn in the unloaded span. This is an impossible situation and, therefore, the solution is no good.



The deflection diagram shows that the curvature under the load is much greater than the curvature at any other point on the beam. The moment diagram, therefore, is not possible since the moment under the load should be greater than the moment at the support. The inflection point needs to be closer to the support. The location of the inflection points cannot be determined exactly by this method. Curvature depends upon the depth of the beam and the stiffness, etc.



The above diagram is not strictly accurate but it is all right for our purposes.

The following illustration shows how to tackle a concrete beam problem.

Remember that structures act in three dimensions. Assume columns have no effect.

LOADS: Uniform Live Load = 1 KIP Per Ft.

Uniform Dead Load = 2 KIP Per Ft.

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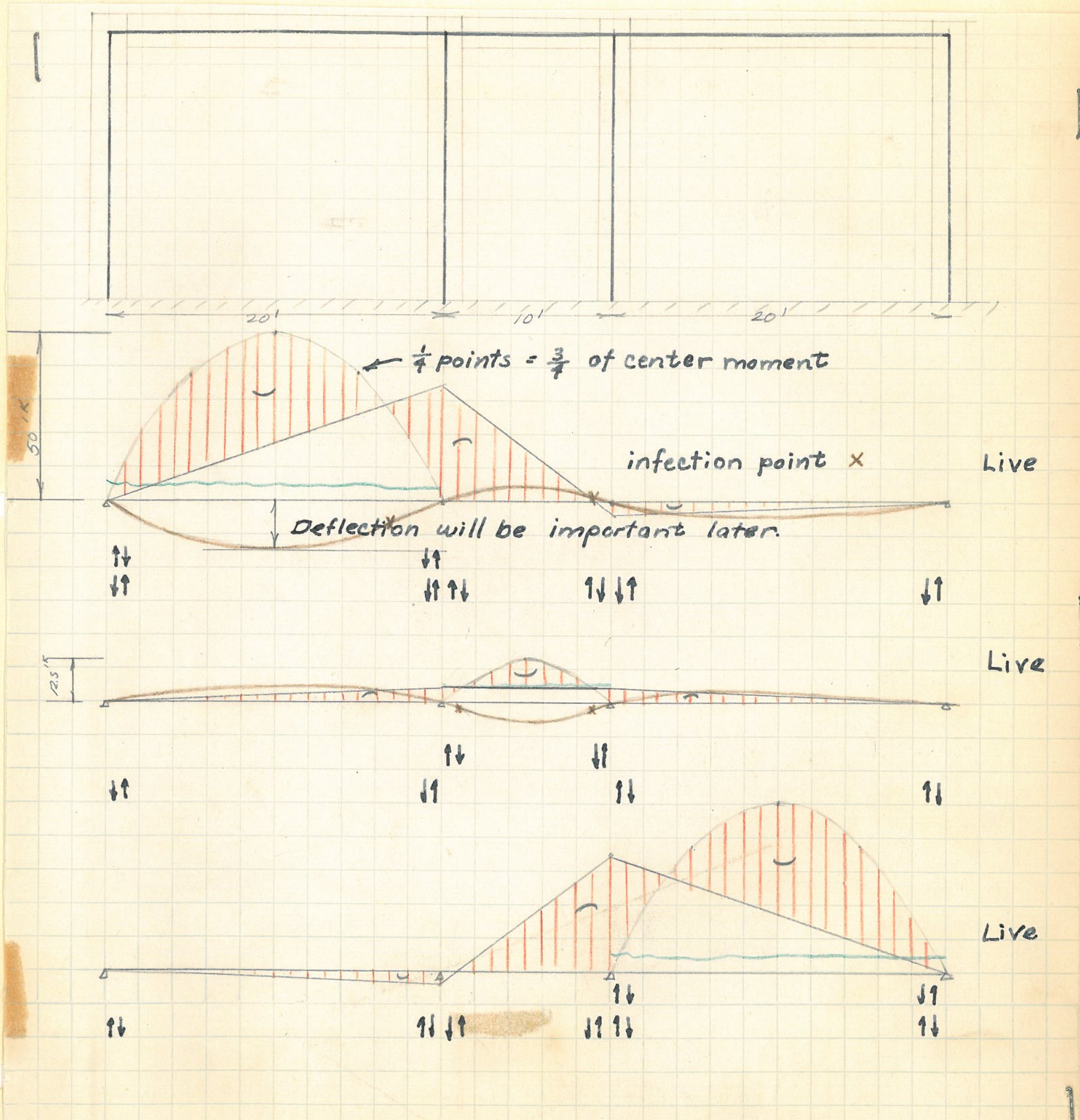
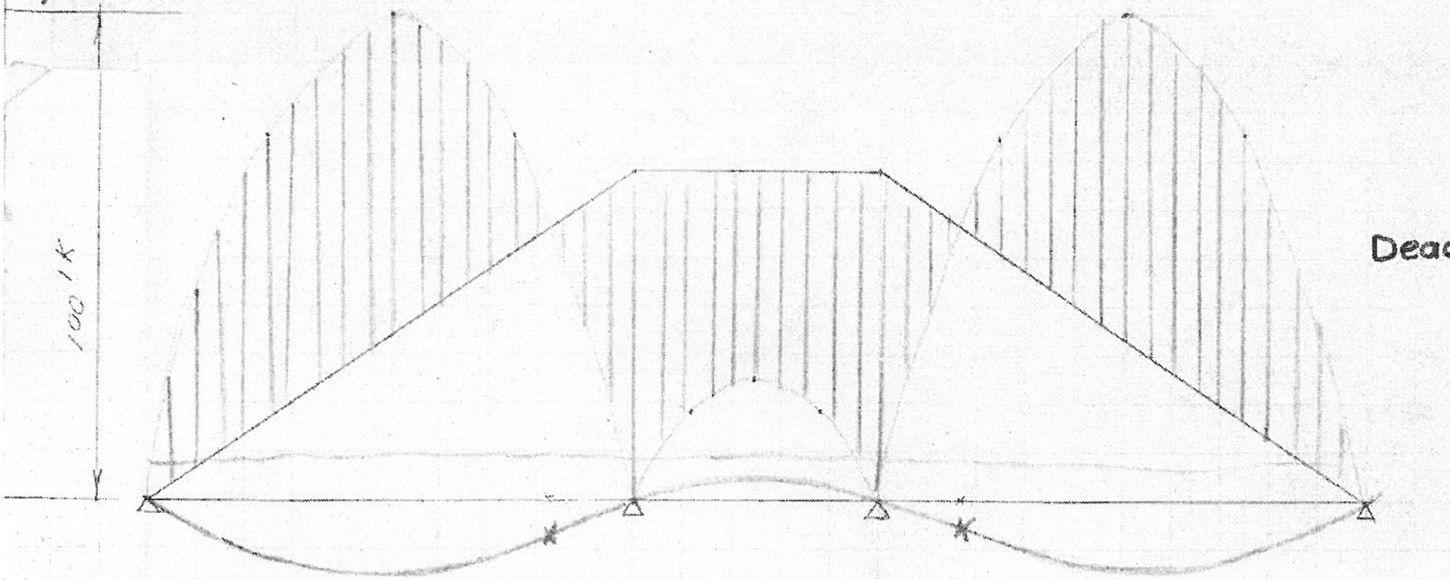
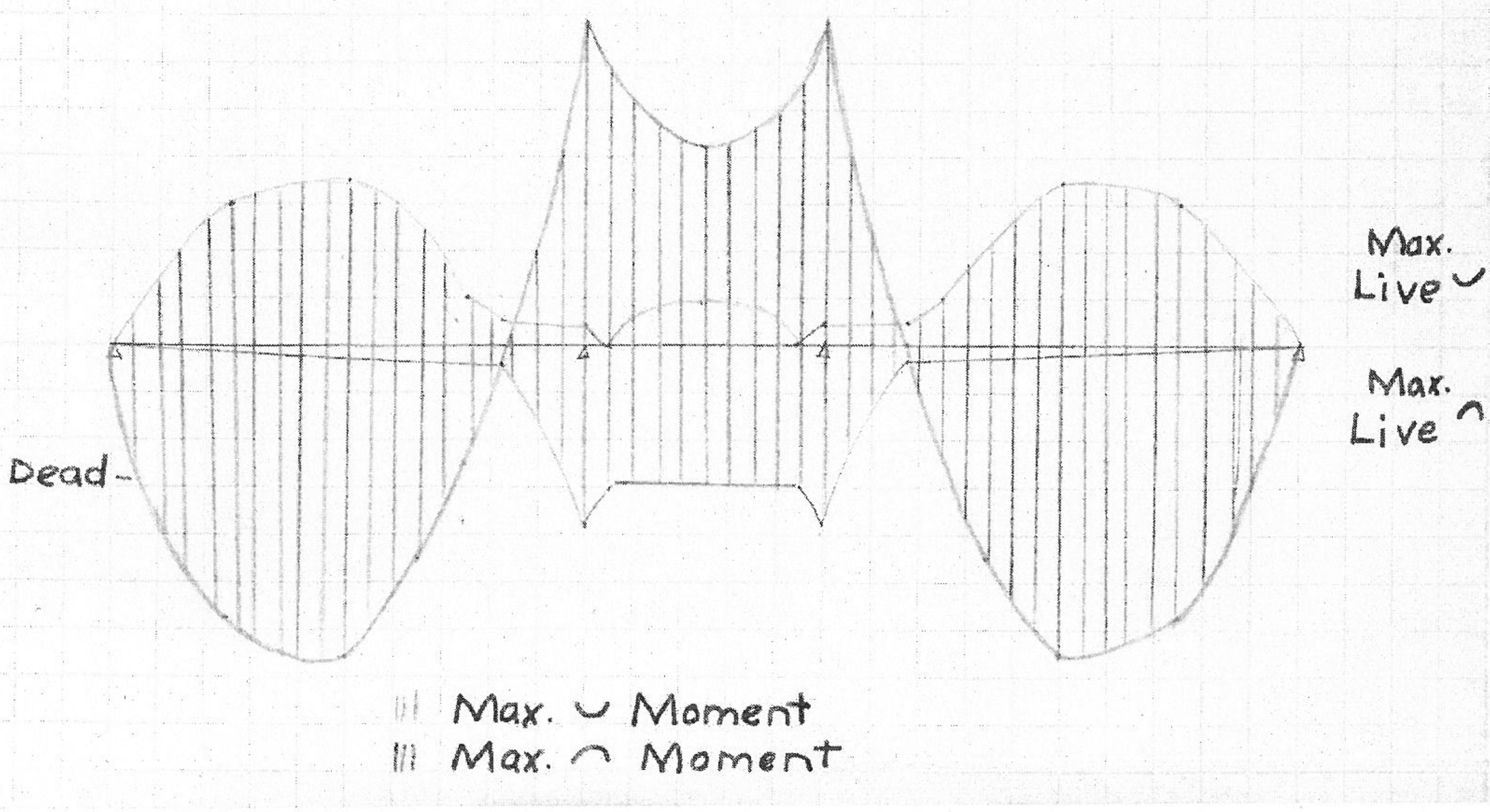
Oct. 2, 1952







||| Max. ∪ Moment
||| Max. ∩ Moment

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Although knowledge of the end shears may be of value, the intermediate shears rarely are worth fussing about. At a later date influence lines will be sketched and used.

How is the depth of beam obtained? The depth may be selected by assuming it to be 1/20th or 1/18th as deep as it is long. On the other hand, the depth may have been predetermined by making it fill the space between the top of a window to the floor above. The carpentry or ease of fitting things together may dictate the depth of the beam. Also, the depth may be chosen so as to make duplication possible.

The width of beam may be selected by assuming it to be about one half of the depth. Carpentry and duplication are important here, too.

The slab thickness was chosen before the beam was designed. It may be four, six, eight inches. Four inches is usually too thin, and likely to shatter if a heavy load is dropped on it.

The following formula will determine the number of bars needed.

$$\text{Number of bars} = \frac{M}{jd\ A_s\ fs}$$

Big bars have an advantage over small bars in that fewer are needed. However, the number of small bars can be selected so as to more nearly approach the area desired. A compromise is usually required.

It is important to keep in mind the order in which the steel will be put into the forms since it will have an effect on the location of the center of gravity of the steel, and, therefore, on d. In ordinary construction with most concrete it is not necessary to figure j.

It is good practice to always put in positive steel for continuity as well as safety. Because of the relatively small cost of concrete, it is not wise to fuss with varying the width of a concrete beam in the hope of saving concrete.

It is "complete nonsense" to compute the compressive strength in concrete. Rather, it is wise to ask the following questions:

a) Is the concrete overstressed?
b) If it is understressed, can you take advantage of it? Since the beam must be of the size selected so that the bars will fit and for other reasons, it is not likely advantage can be made of it.
c) If it is overstressed, what can be done? Many things can be done although not all of them are very good.

1. Make the beam deeper.
2. Make the beam wider.
3. Put in compression steel.
4. Extend tension steel. This is the most practical operation.

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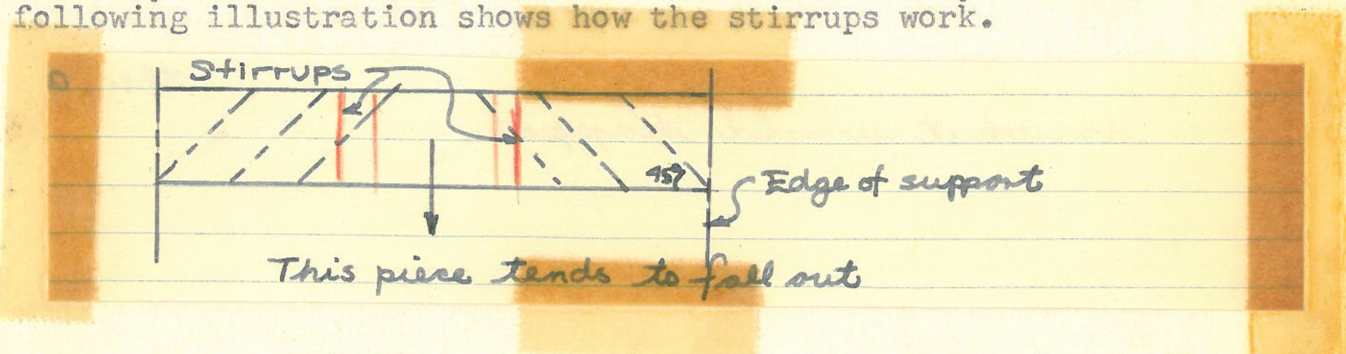
Prof. Cross
Oct. 2, 1952

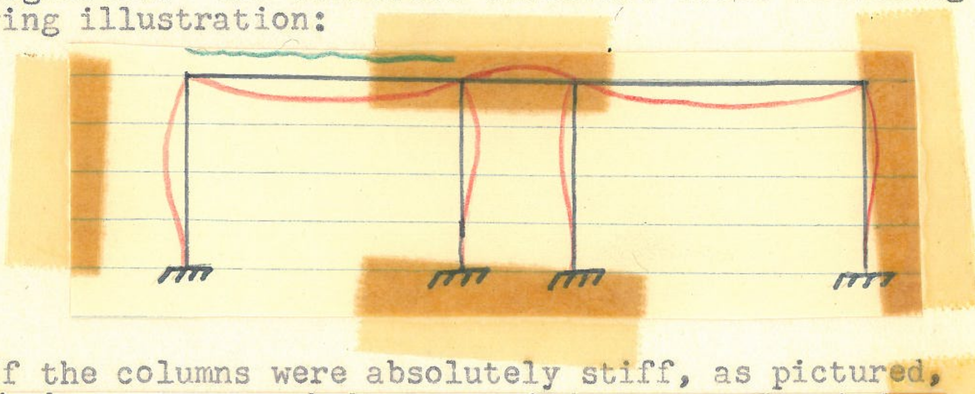
Although there is seldom trouble with bond nowadays, the only trouble will be if the stress is too high. Then, it will be necessary to change the size of the bars and to have more of them.

Diagonal tensile stress is largely "monkey business." However, there will be a tendency to crack on every 45° line starting from the edge of the support towards the center. As suggested, this was verified in Dunham. There must be, therefore, enough vertical stirrups to carry the total shear less the shear carried by the concrete. The following illustration shows how the stirrups work.

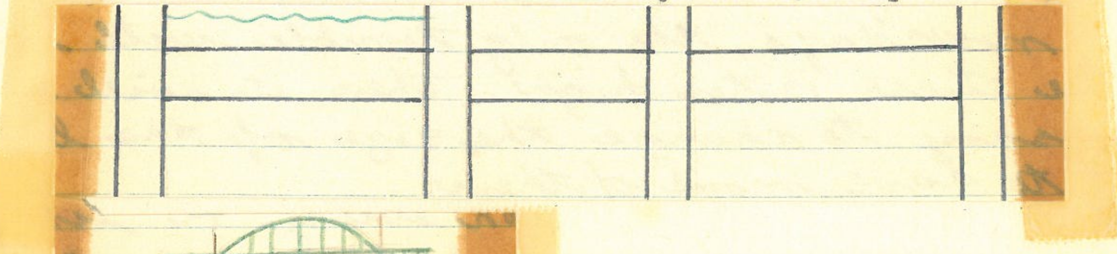


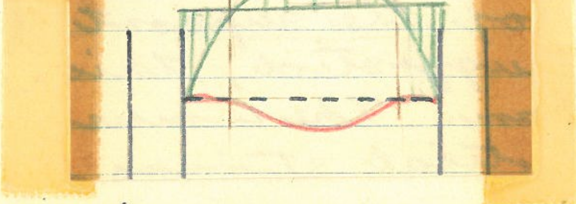
Stirrups spaced farther apart than jd do not resist shear. The maximum stirrup size is 5/8" while the minimum is 3/8". 1/4" bars are too flimsy, while 3/4" bars are too difficult to bend into shape. Inclined stirrups cross the 45° crack so that their vertical component will resist the shear.

In the illustration in this lecture, the action of the columns were neglected. The deflected structure looks something like the following illustration:



If the columns were absolutely stiff, as pictured,

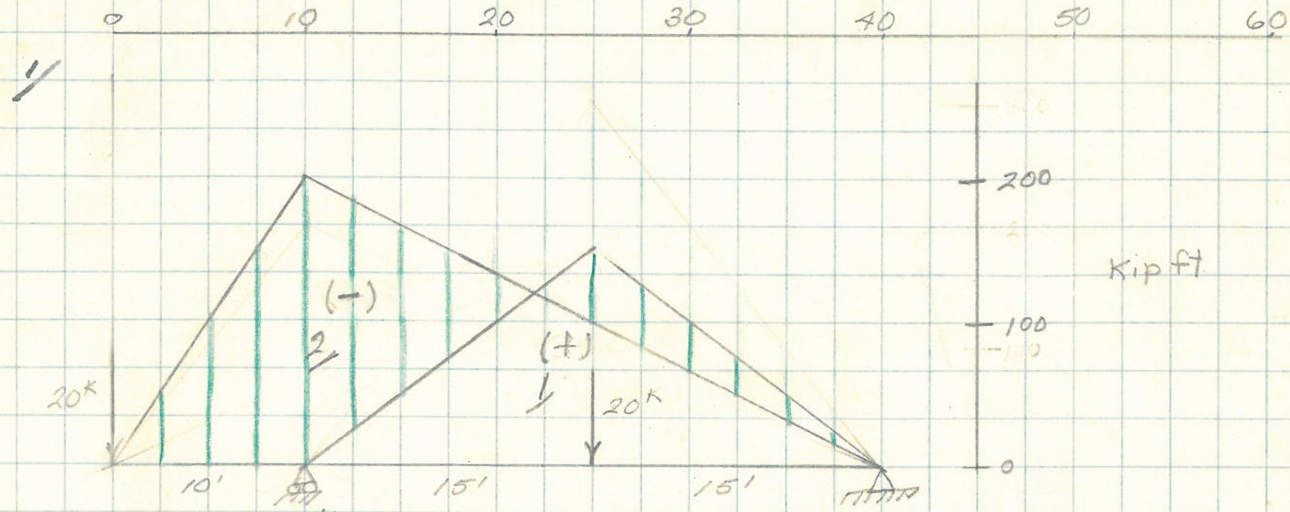




The above is what would happen.

DRAW MOMENT DIAGRAM WITH SCALE
COMPUTE END SHEARS AND REACTIONS

1/



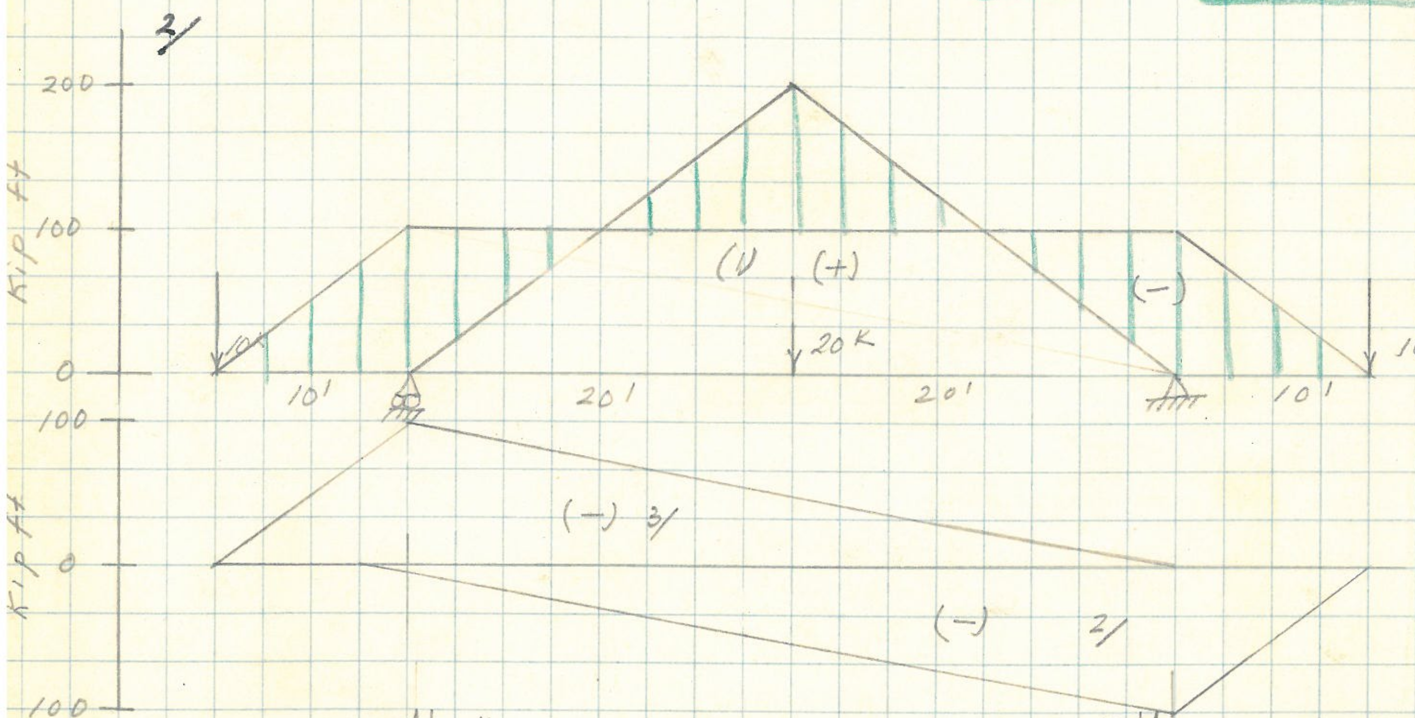
10^K
20^K 6.67^K
20^K 16.67^K
36.67^K

10^K
6.67^K
3.33^K
3.33^K

Shear 1
" 2
" 1+2

Reactions 1+2

2/



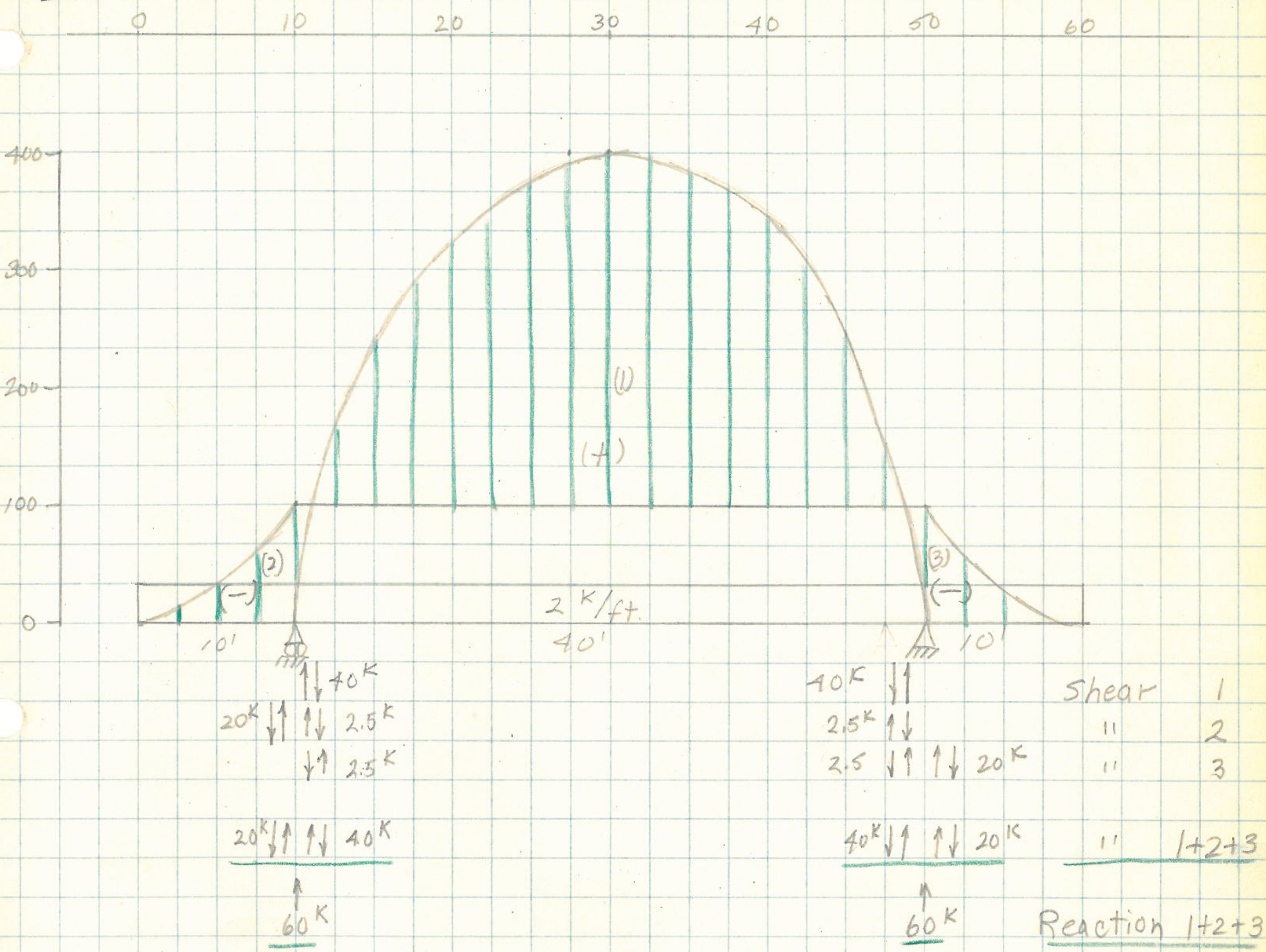
10^K
10^K 2.5^K
2.5^K
10^K 10^K
20^K

10^K
2.5^K
2.5^K 10^K
10^K 10^K
20^K

Shear 1
" 3
" 2
" 1+2+3

Reaction 1+2+3

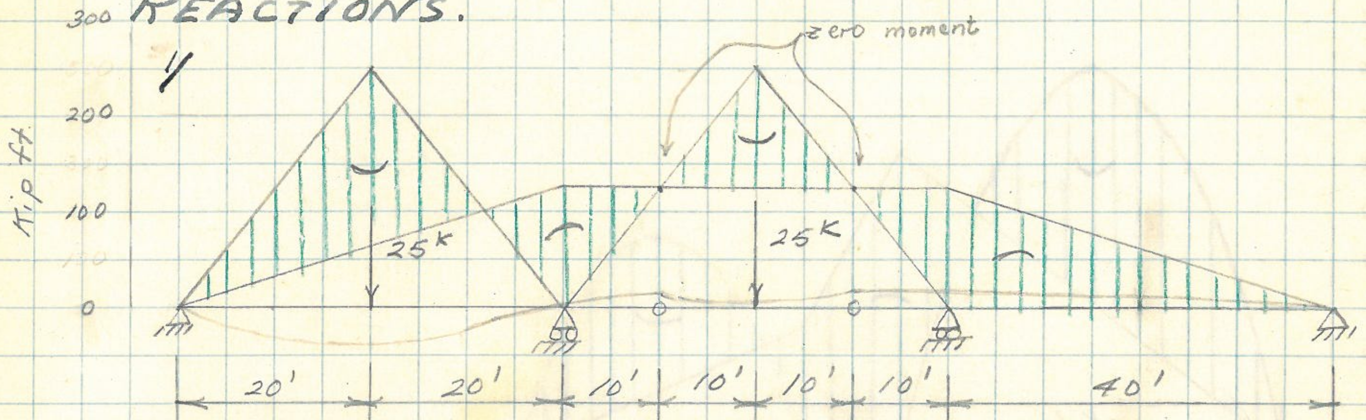
3.


0
10
20
30
40
50
60
400
300
200
100
0
(1)
(+)
(2)
(−)
(3)
(−)
2 K/ft
10'
40'
10'
40K
20K 2.5K
2.5K
20K 40K
60K
40K
2.5K
2.5 20K
40K 20K
60K
Shear 1
" 2
" 3
" 1+2+3
Reaction 1+2+3

Sept. 29, 1952 H'W'K Prof. Cross
Senior Class

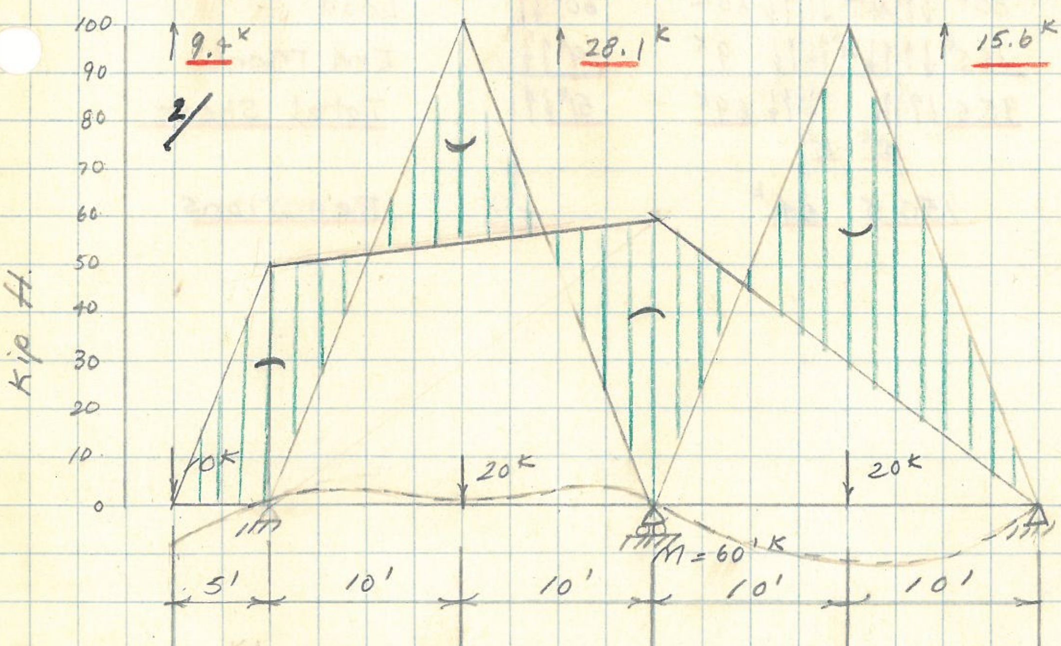
DRAW MOMENT DIAGRAM WITH SCALE: COMPUTE ALL END SHEARS AND ALL REACTIONS.

1/



↑↓12.5^k	12.5^k ↓↑ ↑↓ 12.5^k	12.5^k ↓↑		Load
↓↑3.1^k = 125/40 =	3.1^k ↓↑ 0	0 ↑↓ 3.1^k	3.1^k ↑↓	End Moment
↑↓9.4^k	15.6^k ↓↑ ↑↓ 12.5^k	12.5^k ↓↑ ↑↓ 3.1^k	3.1^k ↑↓	Total Shear
↑ 9.4^k	↑ 28.1^k	↑ 15.6^k	3.1^k ↓	Reactions

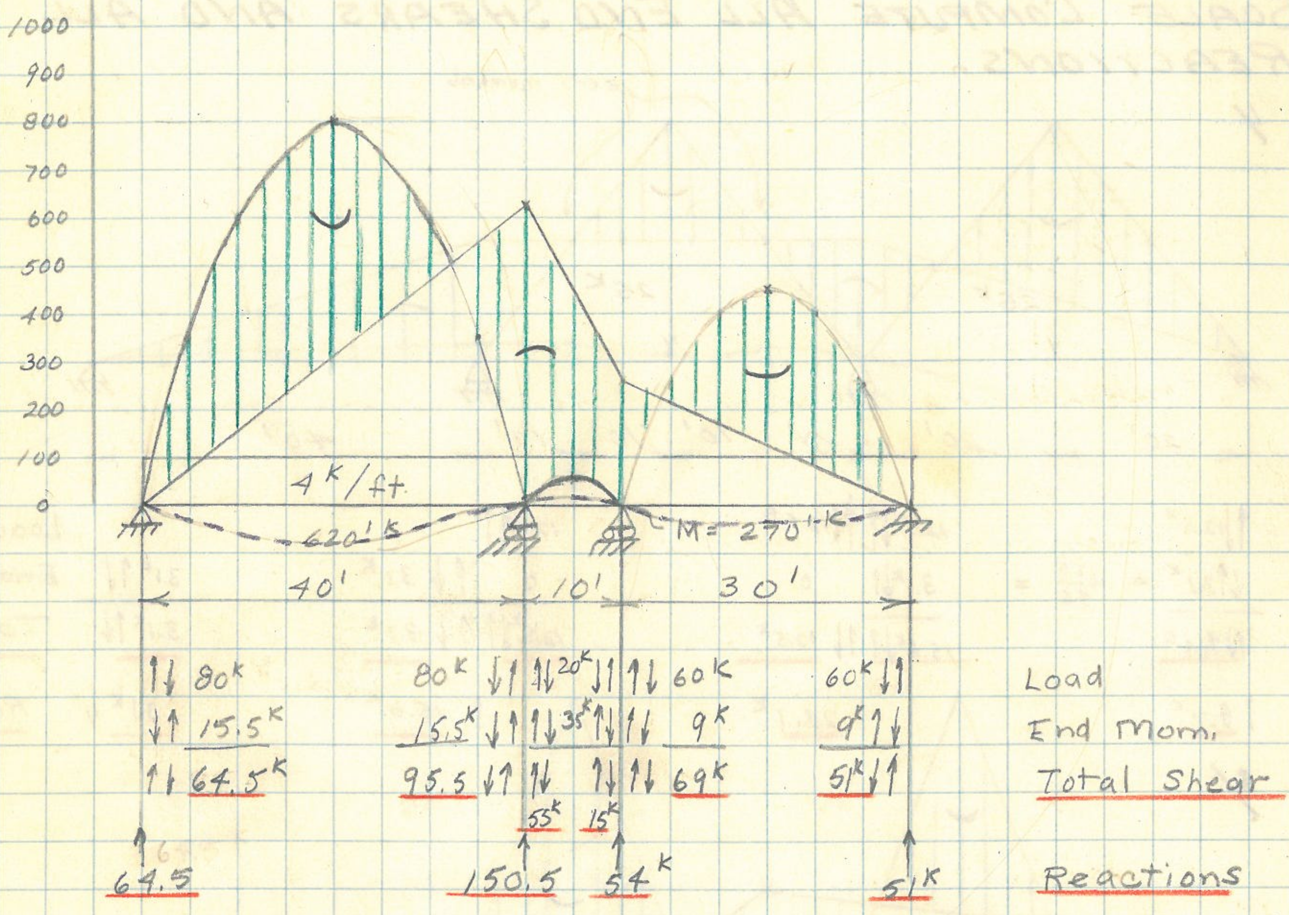
2/



↑↓10^k	10^k ↓↑ ↑↓ 10^k	10^k ↓↑	Load
10^k ↓↑ ↓↑ 0.5^k = 10/20 =	0.5^k ↓↑ ↑↓ 3^k = 60/20 =	3^k ↑↓	End Moment
10^k ↓↑ ↑↓ 9.5^k	10.5^k ↓↑ ↑↓ 13^k	7^k ↓↑	Total Shear
↑ 19.5^k	↑ 23.5^k	7^k ↑	Reactions

(over)

3/



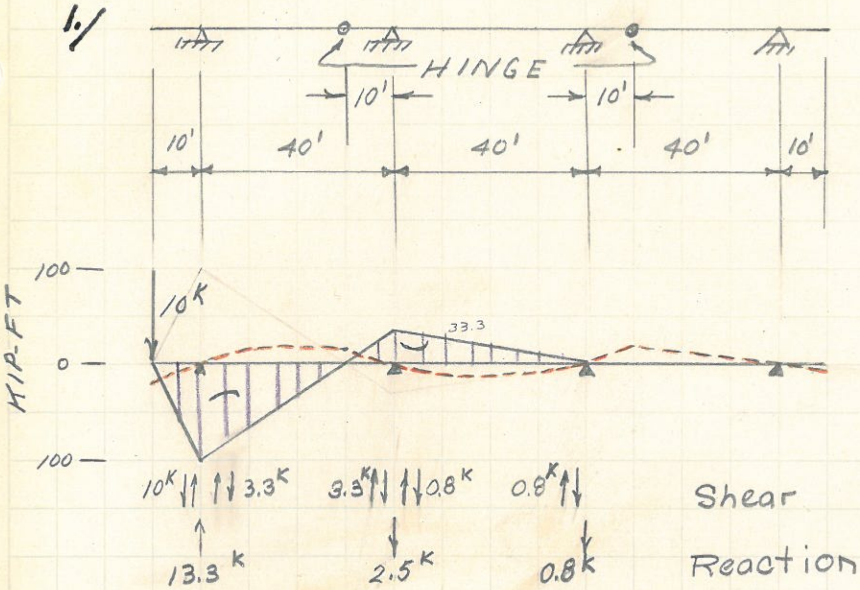
1000
900
800
700
600
500
400
300
200
100
0
4 k/ft
620'k
M= 270'k
40'
10'
30'
80k
80k
20k
60k
60k
15.5k
15.5k
35k
9k
9k
64.5k
95.5
69k
51k
55k
15k
64.5
150.5
54k
51k
Load
End Mom.
Total Shear
Reactions

PROF. CROSS : Senior Class

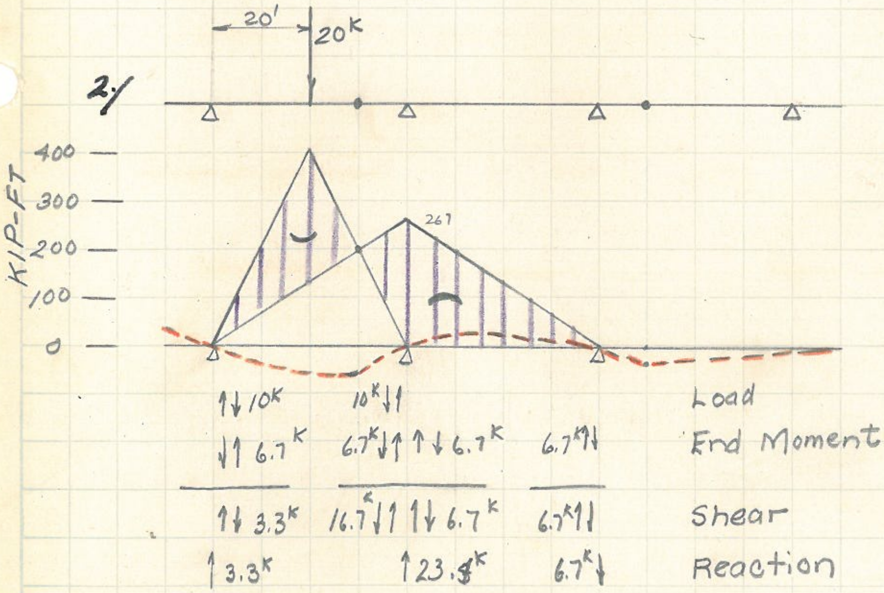
H.W.K. OCT 1, 1952

DRAW MOMENT DIAGRAM WITH SCALE
COMPUTE END SHEARS AND REACTIONS.

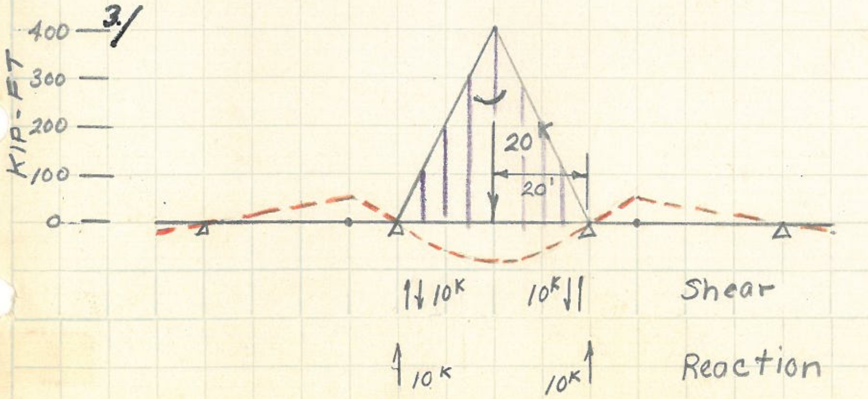
1/



2/



3/



PROF. CROSS: Senior Class

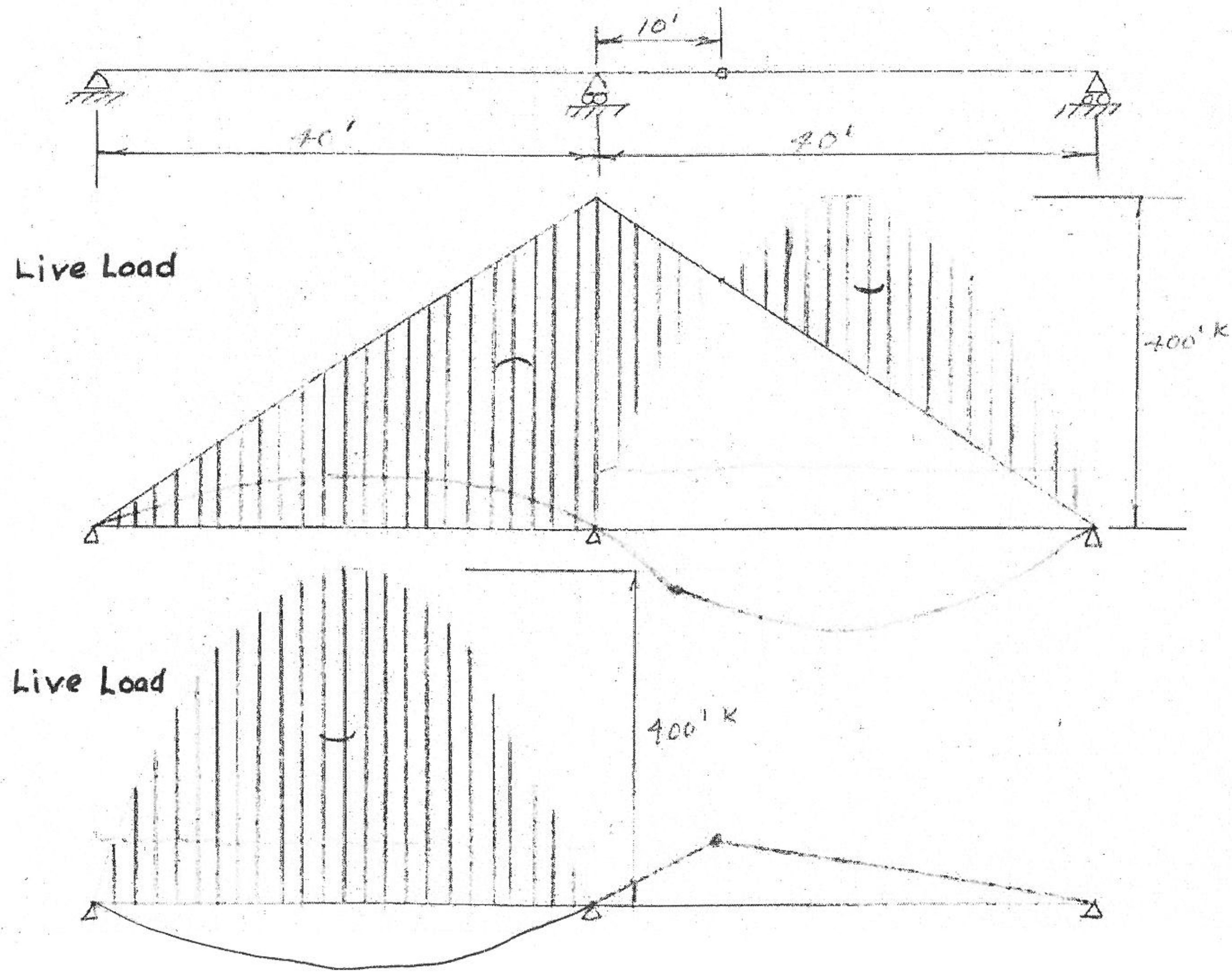
OCT 3, 1952

H'WK

DRAW CURVES OF MAXIMUM MOMENT WITH SCALE

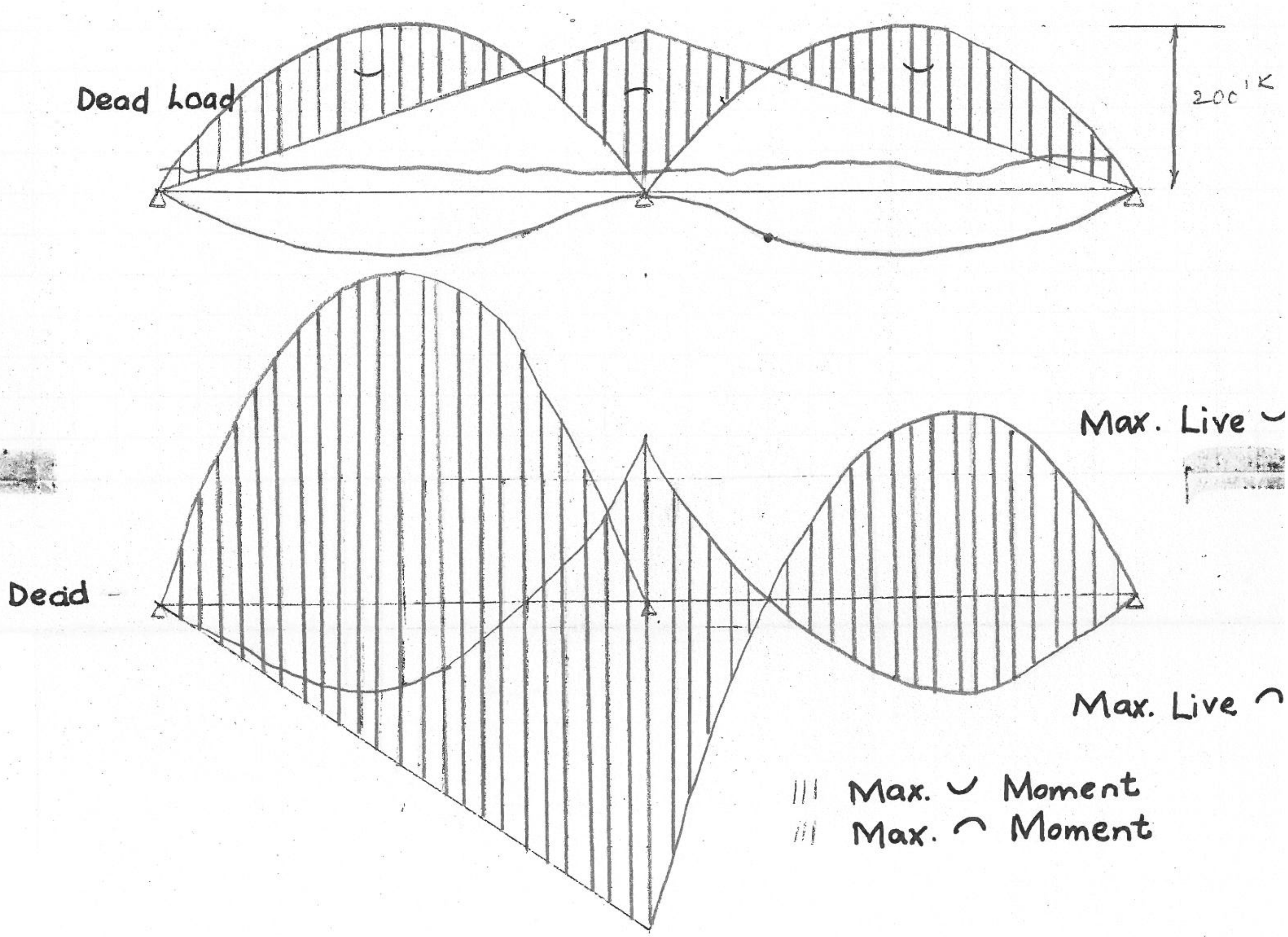
LIVE LOAD = 2 KIPS PER FT.

DEAD LOAD = 1 KIP PER FT.



H-4 1 of 2 (2)
2 of 2

Oct 3, 1952



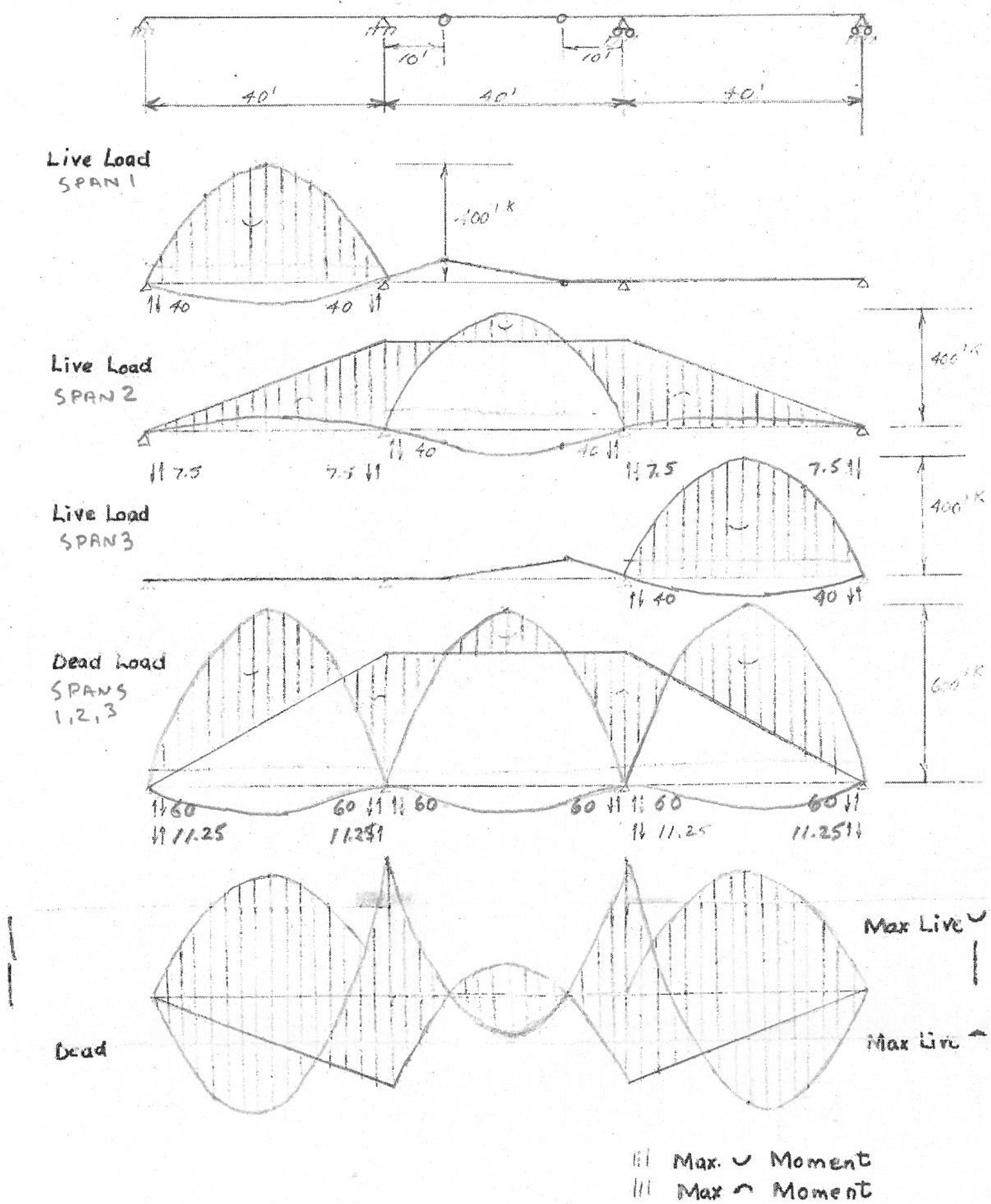
H-5 1 of 2 / 2 of 2

PROF. CROSS: Senior Class

OCT 6, 1952

DRAW CURVES OF MAXIMUM MOMENTS WITH SCALE.
COMPUTE MAXIMUM END SHEARS AND MAXIMUM REACTIONS

LIVE LOAD = 2 KIPS PER FT.
DEAD LOAD = 3 KIPS PER FT.



D.L. SHEAR	48.75	71.25	60	60	71.25	48.75
MAX SHEAR	88.75	118.75	100	100	118.75	88.75
MAX REACT.	88.75	218.75			218.75	88.75

PROF. CROSS: Senior Class

NAME

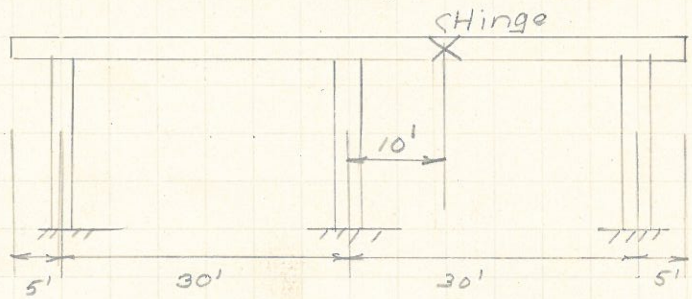
DATE Oct 8, 1952

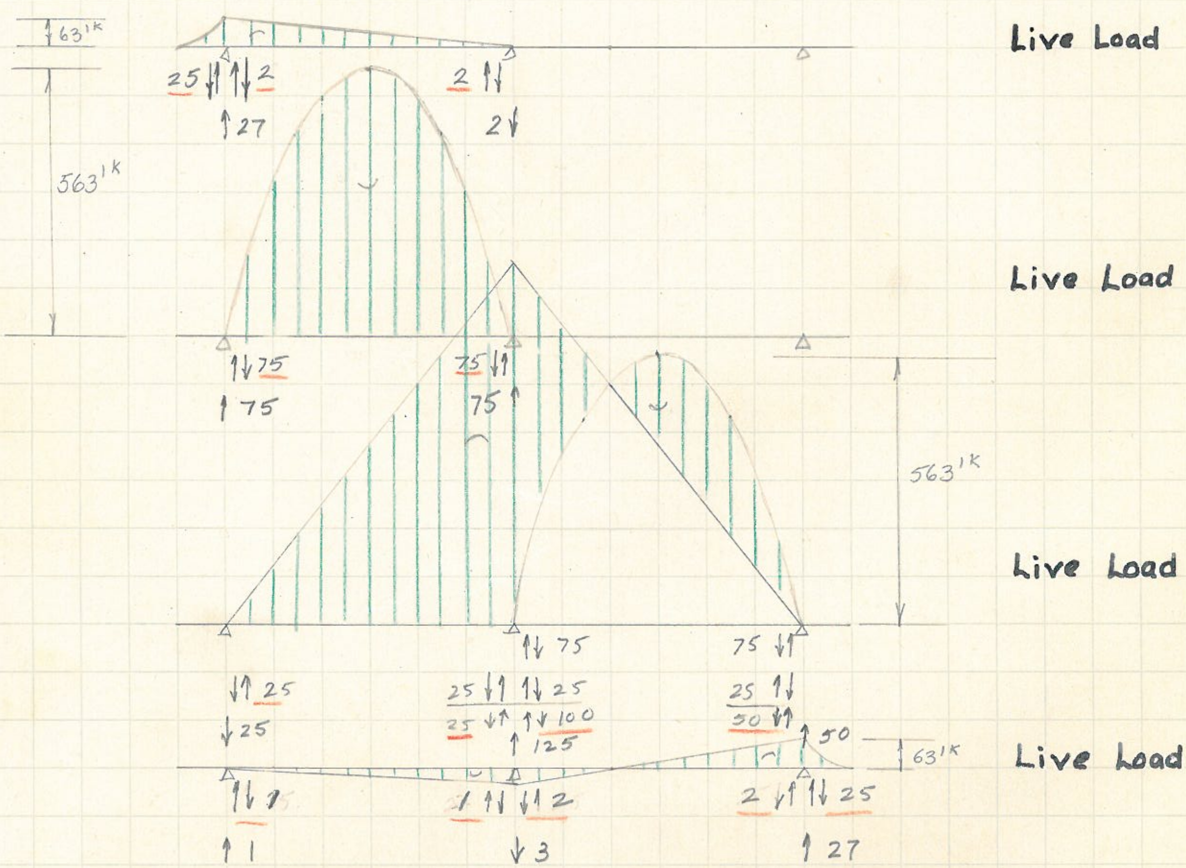
DRAW CURVES OF MAXIMUM MOMENTS FOR THE GIRDER, WITH SCALE.
COMPUTE MAXIMUM END SHEARS AND MAXIMUM REATIONS

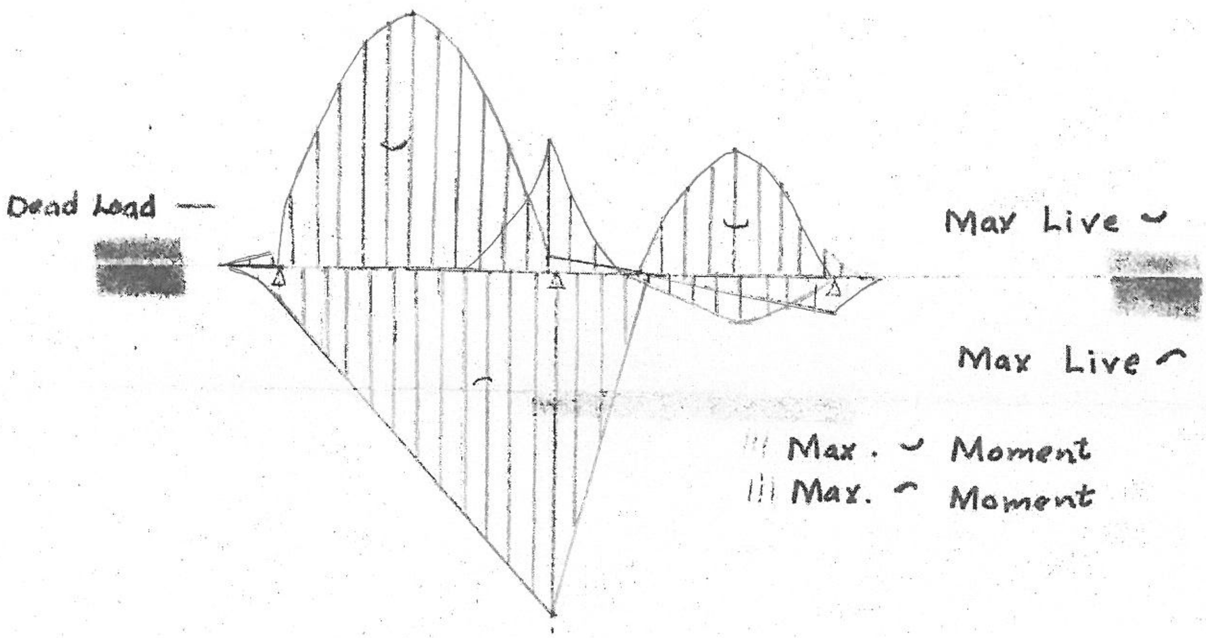
LIVE LOAD = 5 KIPS PER FT.

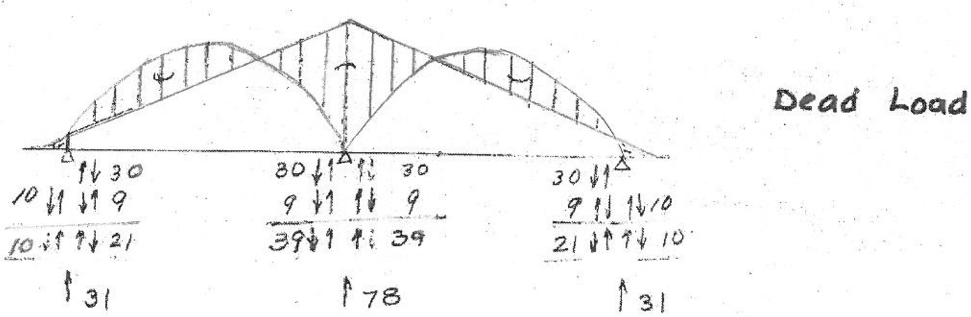
DEAD LOAD = 2 KIPS PER FT

ASSUME COLUMNS VERY FLEXIBLE






Dead Load —
Max Live
Max Live
Max. Moment
Max. Moment

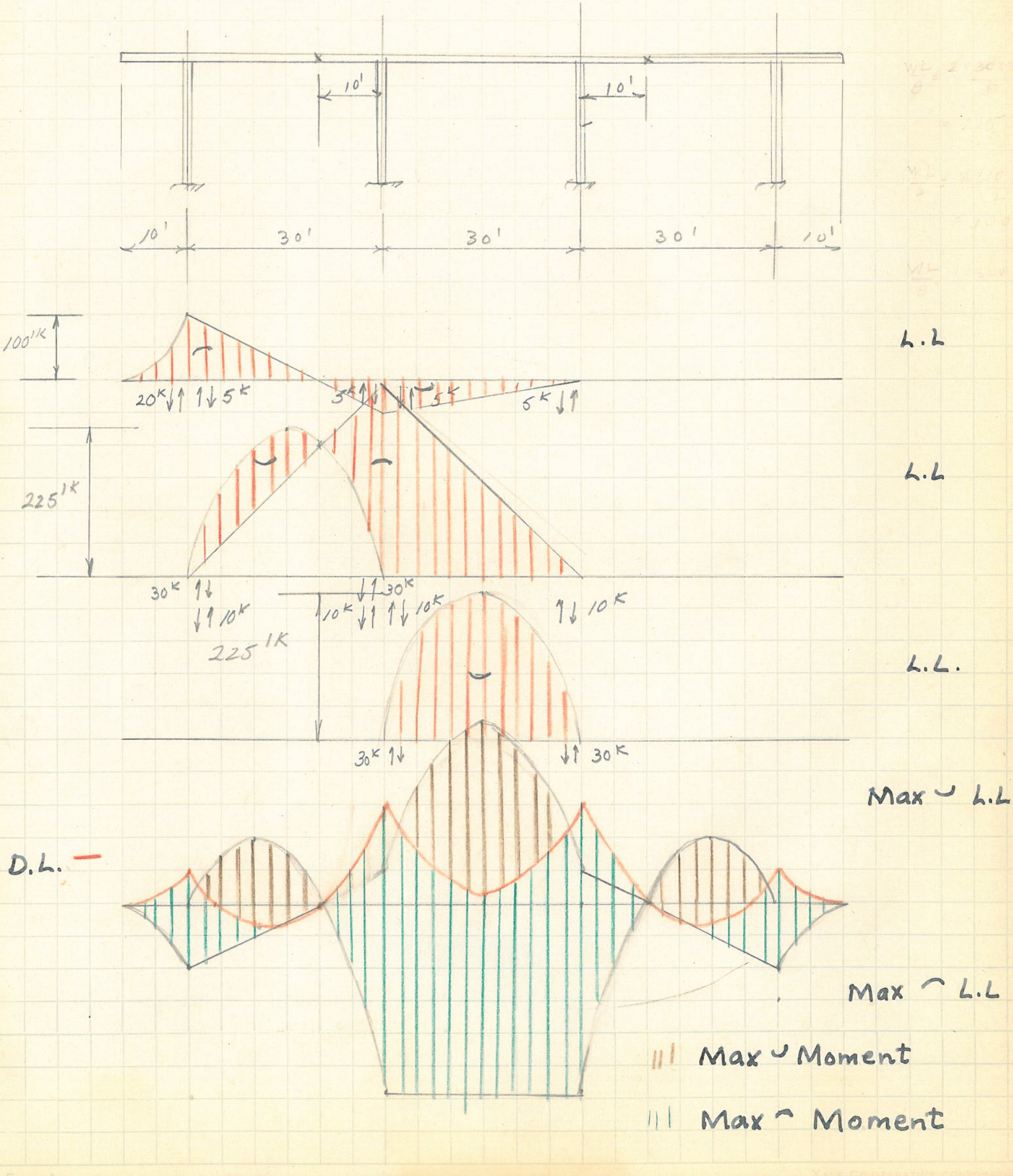

Dead Load
30 30 30 30
10 9 9 9 9 10
10 21 39 39 21 10
31
78
31

PROF CROSS - Senior Class

NAME

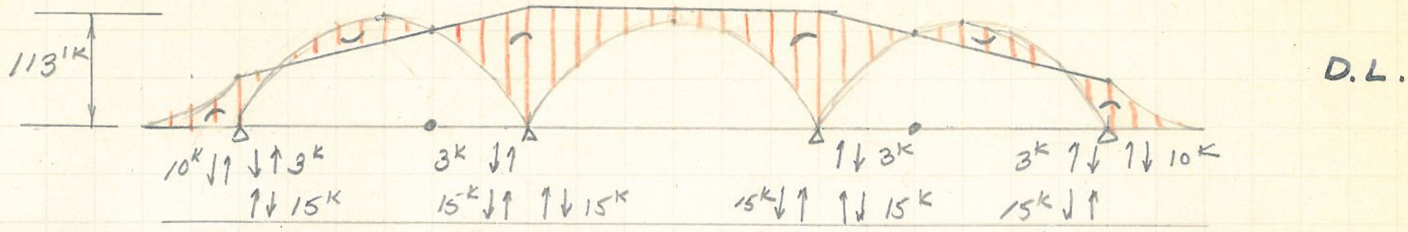
DATE OCT 10, 1952

DRAW CURVES OF MAXIMUM MOMENTS WITH SCALE
COMPUTE MAXIMUM END SHEARS AND MAX. REACTIONS
ASSUME NO STIFFNESS OF COLUMNS $L.L. = 2^k/FT$
$D.L. = 1^k/FT$



PROF. CROSS: Senior Class

DATE OCT 10, 1952



D.L. Shear 10K ↓↑ ↑↓ 12K 18K ↓↑ ↑↓ 15K 15K ↓↑ ↑↓ 18K 12K ↓↑ ↑↓ 10K

D.L. React. ↑ 22K ↑ 33K ↑ 33K ↑ 22K

PROF. CROSS : Senior Class

H-8

1.

NAME

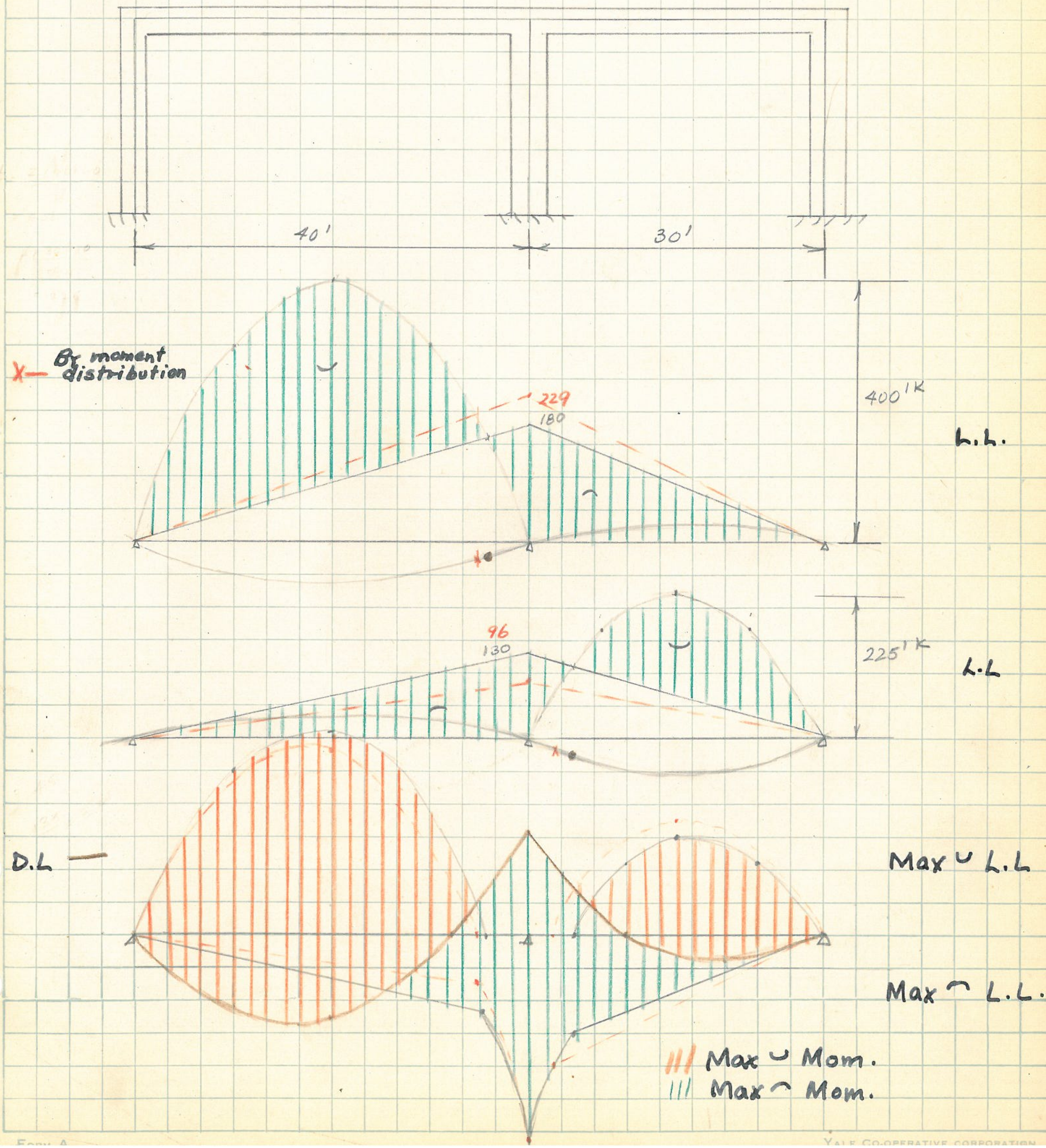
DATE Oct 13, 1952

HWK

USE DEFLECTED STRUCTURES TO DRAW CURVES OF MAXIMUM MOMENTS WITH SCALE. COMPUTE MAXIMUM END SHEARS & MAXIMUM REACTIONS. ASSUME COLUMNS COMPLETELY FLEXIBLE.

L.L = 2 K/ft

D.L. = 1 K/ft.



PROF. CROSS: Senior Class

NAME

DATE OCT. 13, 1952

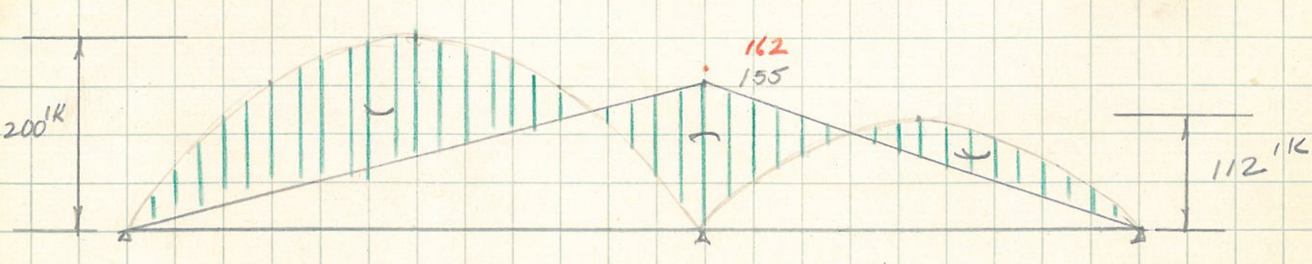
HWK

3 | 4 — L.L.

+267		−267	0		0
−267		+114	+153		
+57		−134			+76
−57		+58	+76		−76
+29		−28	−38		+38
−29		+28	+38		−38
+14		−14	−19		+19
	etc.	+14	+19	etc.	
0		−229	+229		0

3 | 4 — L.L.

			+150		−150
		−64	−86		+150
−32			+75		−43
+32		−32	−43		+43
−16		+16	+21		−21
+16		−16	−21		+21
0		−96	+96		0



D.L.

H. 9

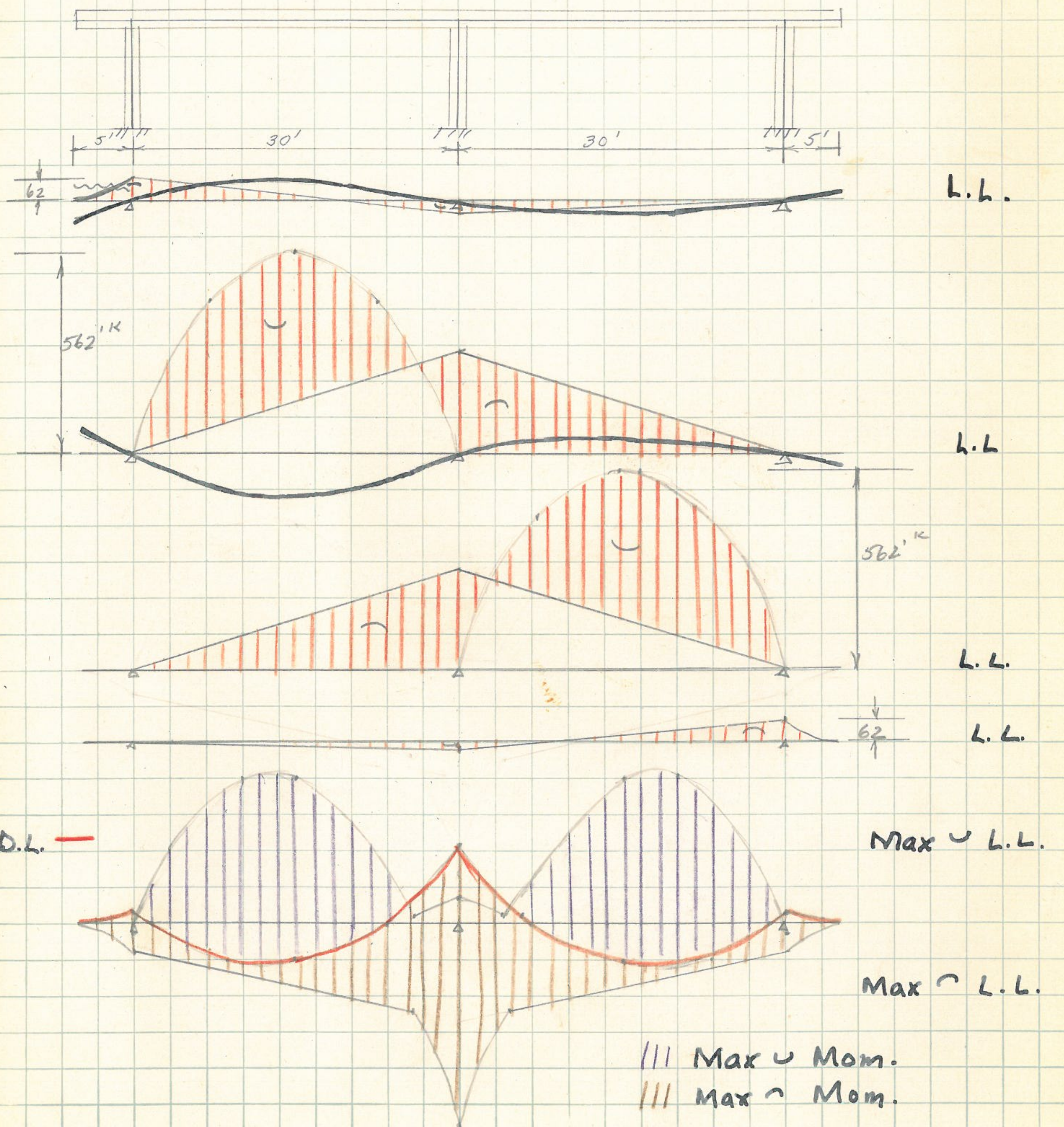
PROF. CROSS: Senior Class

NAME

DATE Oct. 15, 1952

DRAW CURVES OF MAXIMUM MOMENTS WITH SCALE. COMPUTE MAXIMUM END SHEARS & MAXIMUM REACTIONS. ASSUME COLUMNS COMPLETELY FLEXIBLE.

L.L. = 5 K/FT. D.L. 2 K/FT



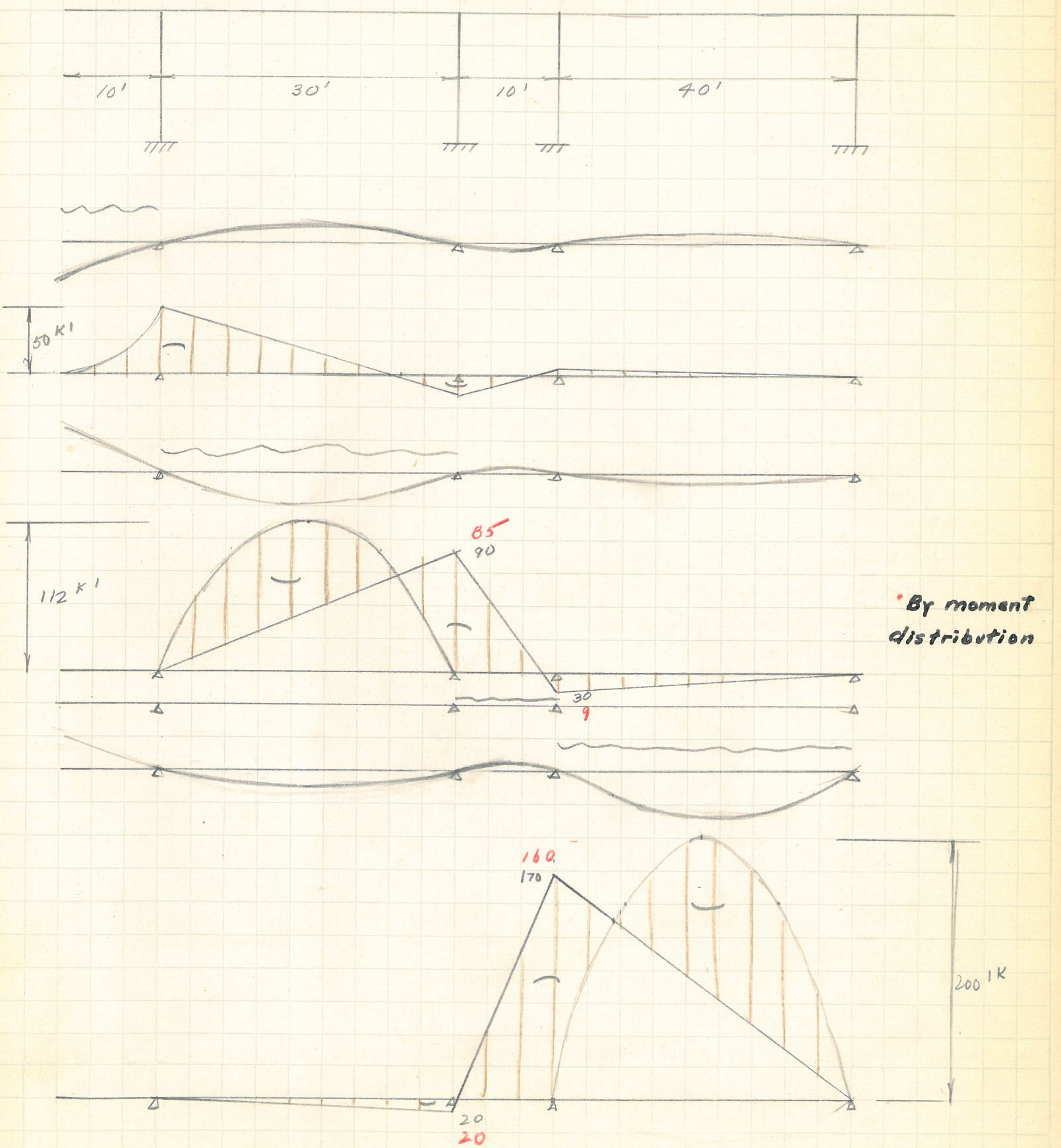
PROF. CROSS : Senior Class

NAME

DATE Oct. 17, 1952

QUIZ

DRAW CURVES OF MAXIMUM MOMENT WITH SCALE. COMPUTE MAXIMUM END SHEARS AND MAXIMUM REACTIONS. ASSUME COLUMNS COMPLETELY FLEXIBLE. L.L = 1 k/ft D.L = 2 k/ft.



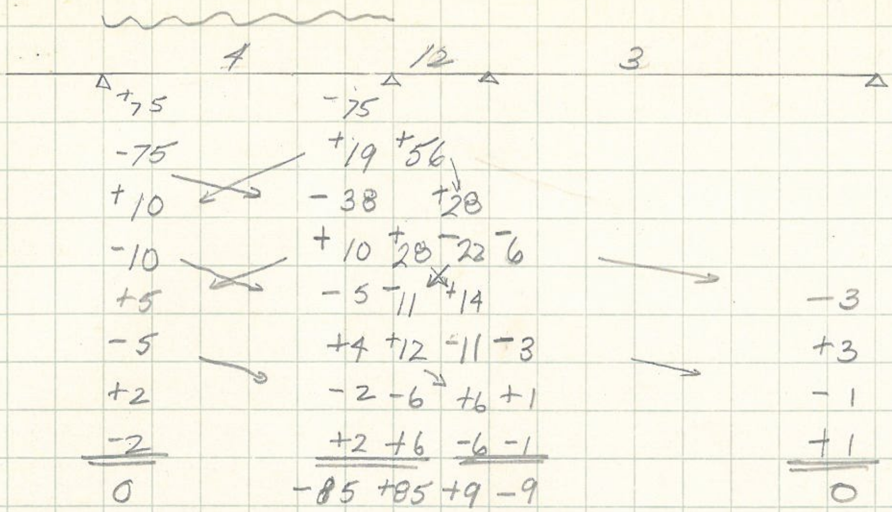
Q 1

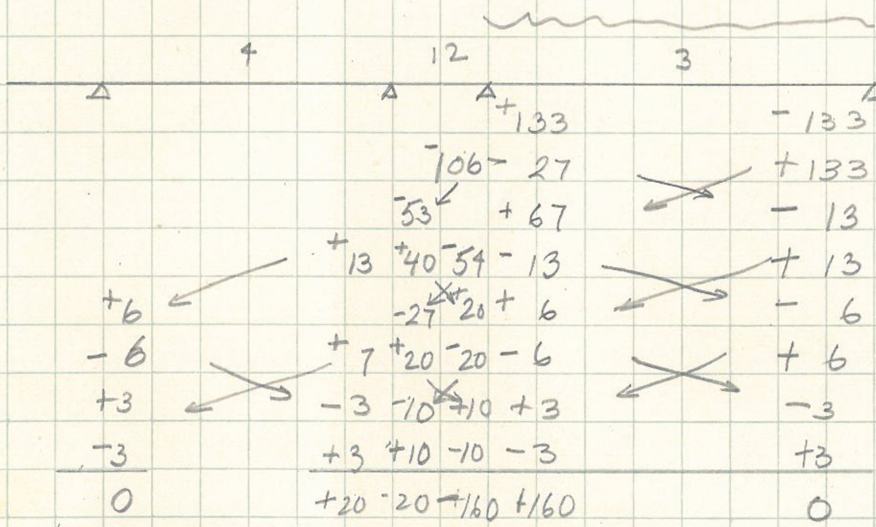
PROF. CROSS : Senior Class

NAME

DATE OCT. 17, 1952

Quiz





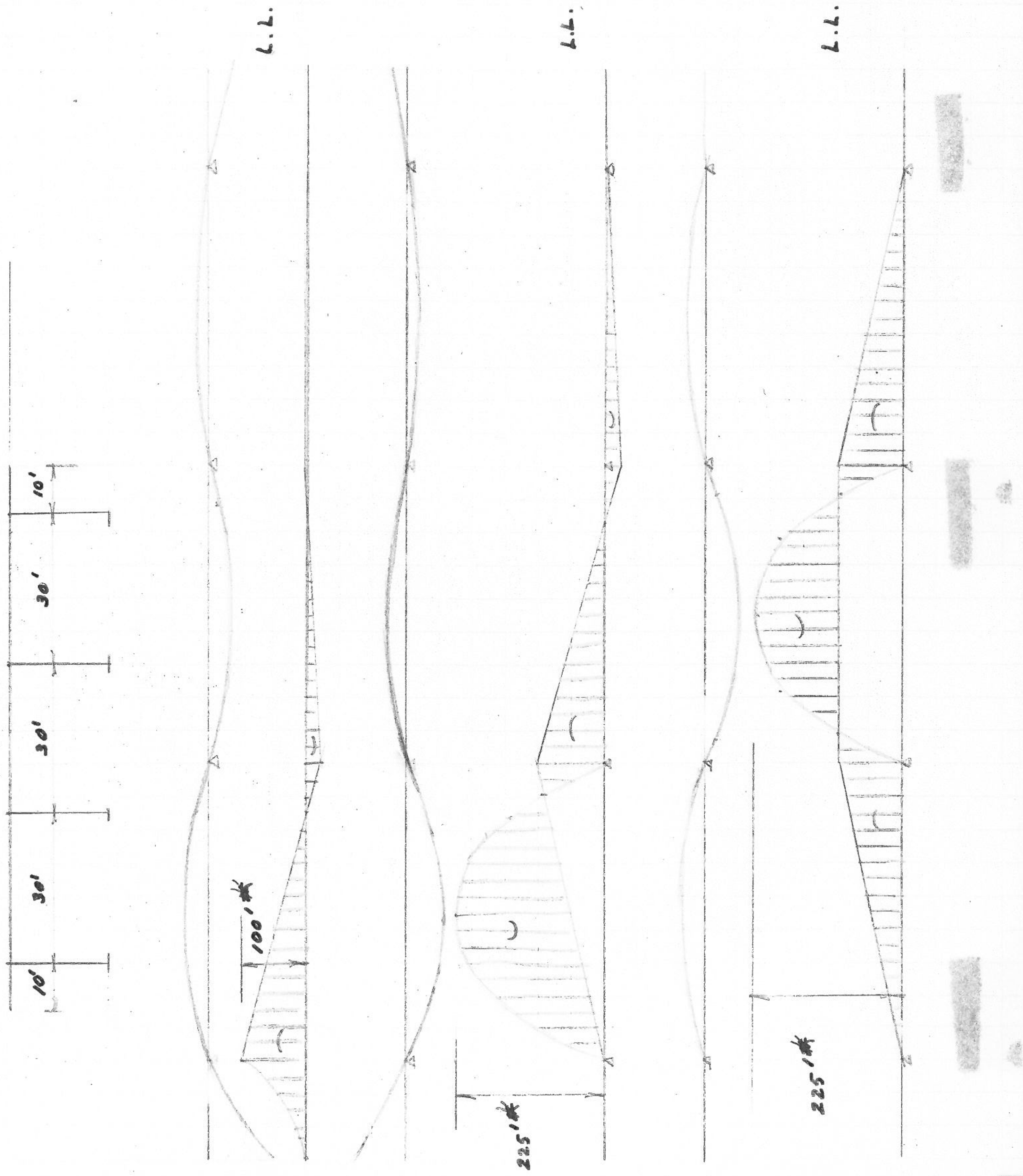
H-10

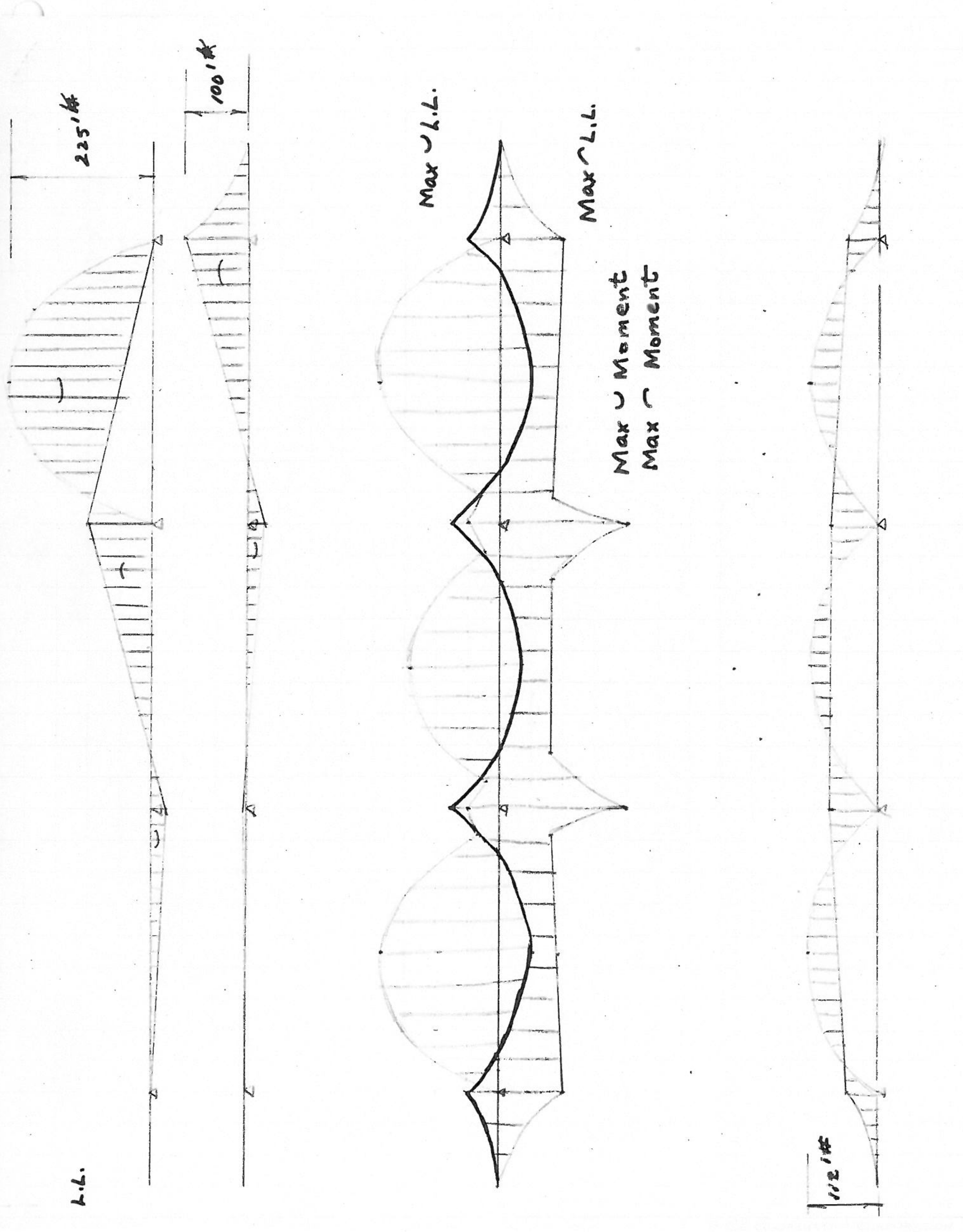
PROF. CROSS : Senior Class

Oct. 17, 1952

HWK

DRAW CURVES OF MAXIMUM MOMENTS WITH SCALE. COMPUTE MAXIMUM END SHEARS AND MAXIMUM REACTIONS. ASSUME COLUMNS COMPLETELY FLEXIBLE $L.L = 2^{K}/ft$ $D.L = 1^{K}/ft$




L.L.
225'#
100'#
Max ∪ L.L.
Max ∩ L.L.
Max ∪ Moment
Max ∩ Moment
112'#

H - 11

PROF. CROSS : Senior Class

NAME

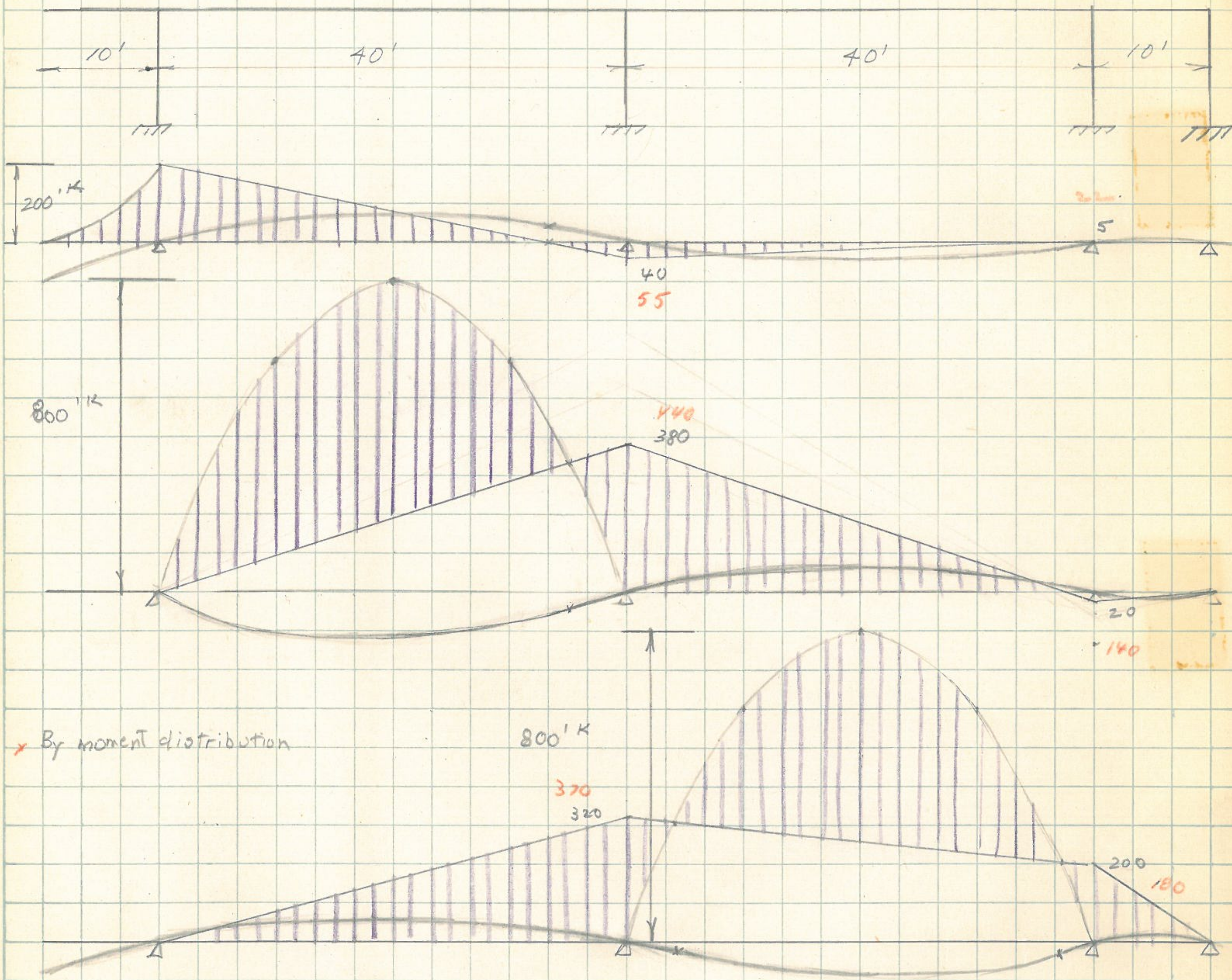
DATE Oct. 20, 1952

HWK

DRAW CURVES OF MAXIMUM MOMENTS - WITH SCALE. COMPUTE MAXIMUM END SHEARS & MAXIMUM REACTIONS.

L.L = 4 k/ft

D.L. = 2 k/ft.



H-12

PROF. CROSS: Senior Class

NAME

DATE OCT. 22, 1952

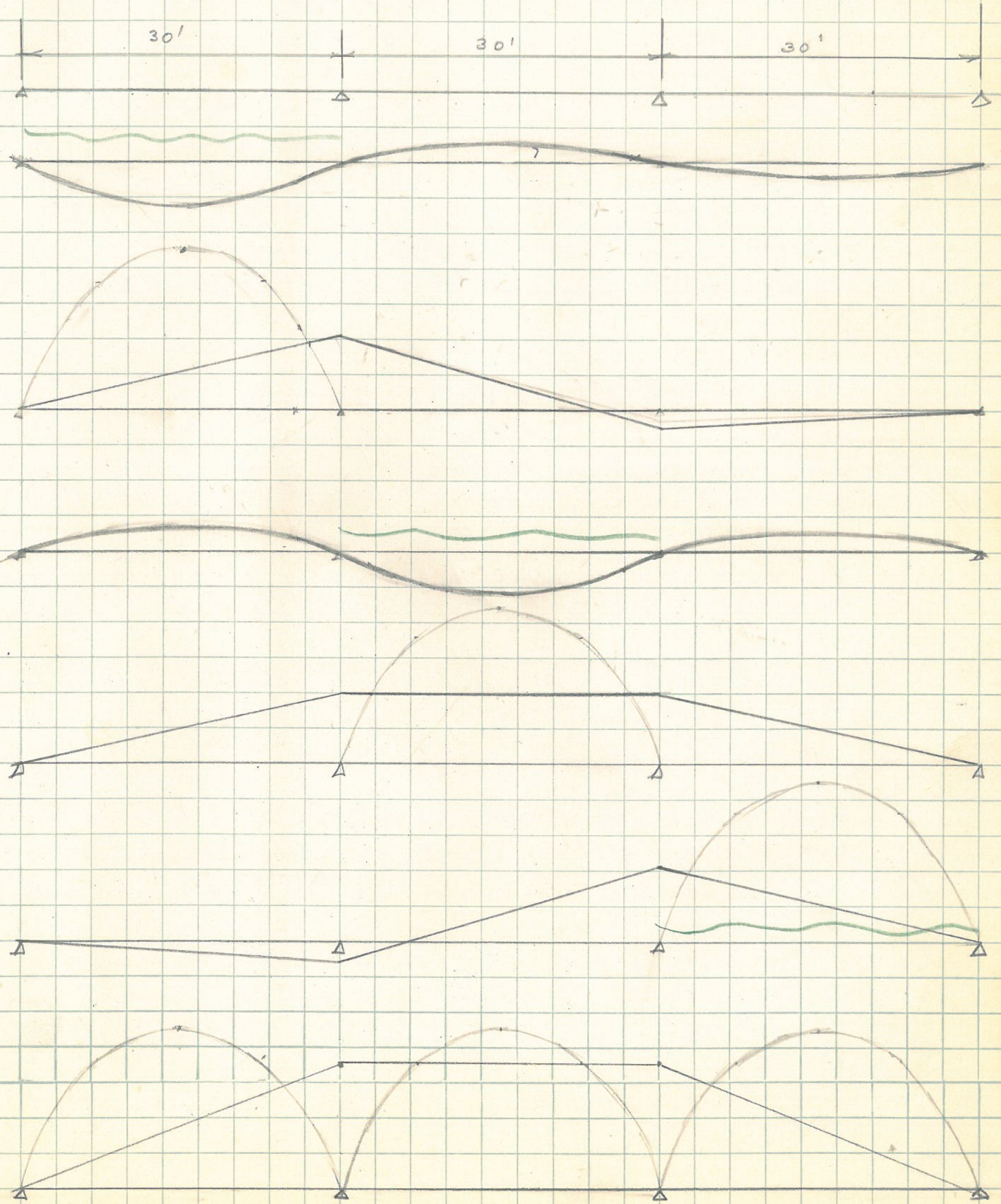
HWK

COMPARE CURVES OF MAXIMUM MOMENTS WITH SCALE FOR EACH CASE BY DRAWING ONE CURVE OVER THE OTHER. COMPARE VALUES OF MAXIMUM END SHEARS AND MAXIMUM REACTIONS.

L.L = 2^k/ft.

D.L = 2^k/ft.

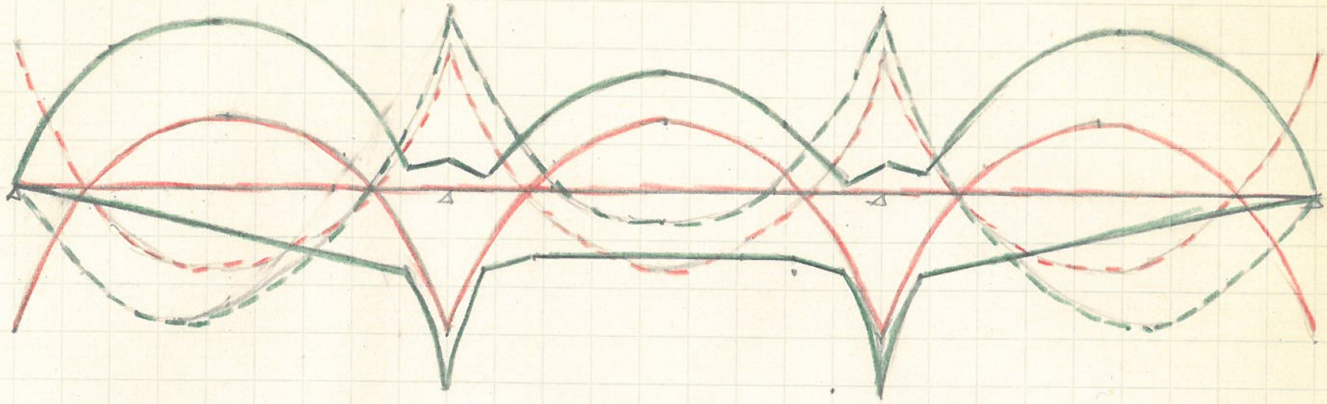
a) Columns completely flexible. b.) Columns completely rigid.



H-12

NAME

DATE



FORM A

YALE CO-OPERATIVE CORPORATION

C-1

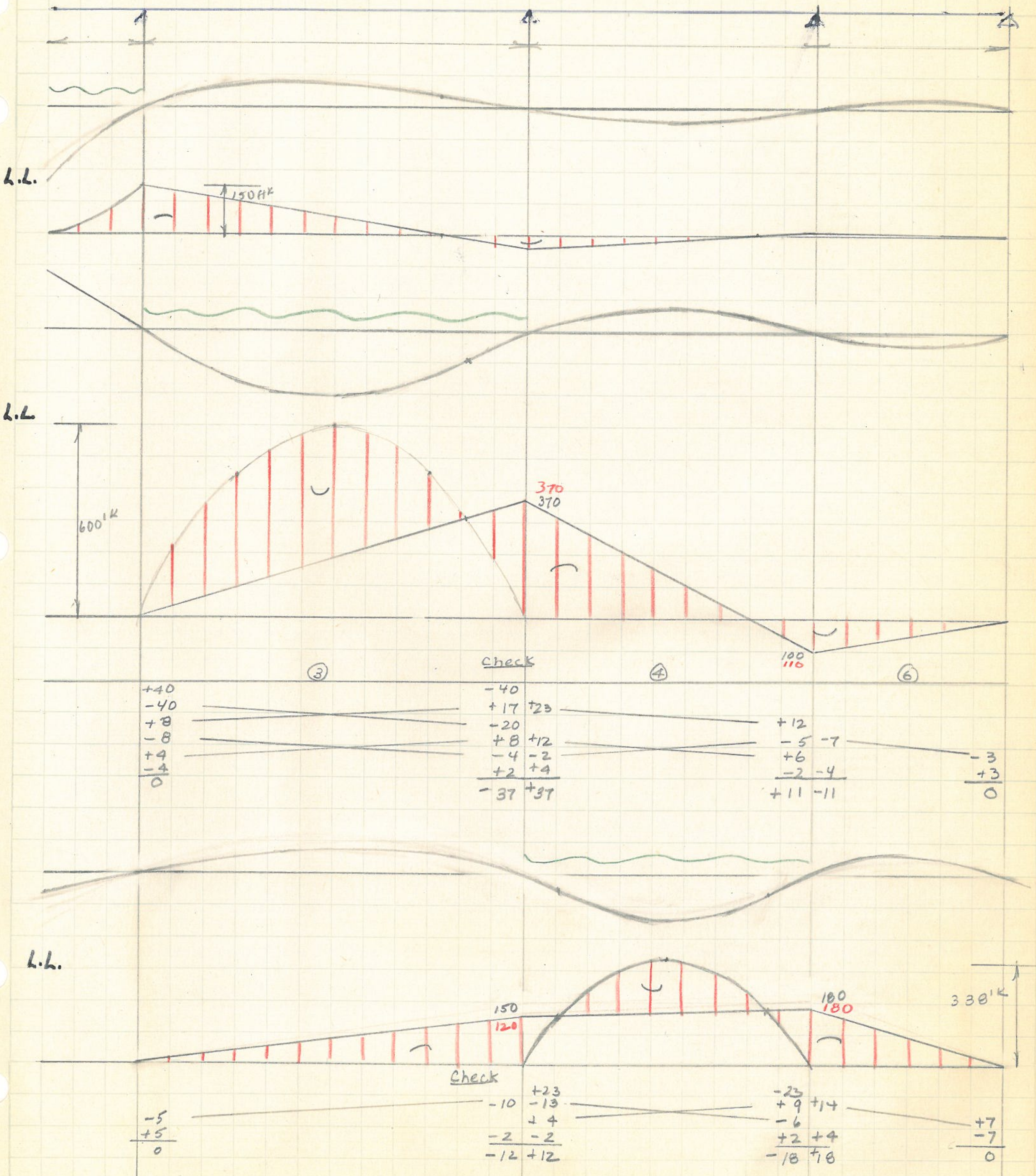
PROF CROSS: Senior Class

NAME

DATE Oct. 27, 1952

CWK

$L.L = 3^k/ft.$ $D.L. = 3^k/ft.$



x By moment distribution

FORM A

YALE CO-OPERATIVE CORPORATION

C-4

PROF. CROSS: Senior Class

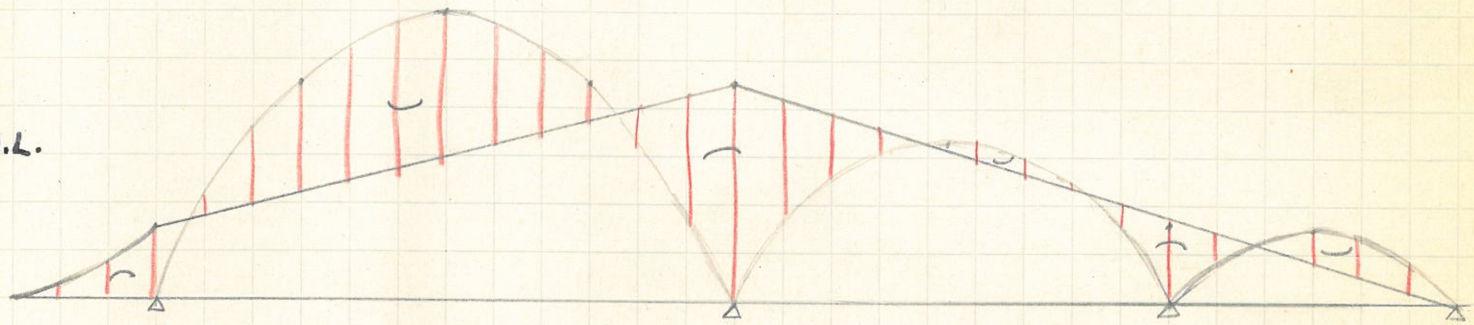
NAME

DATE Oct. 27, 1952

CWK

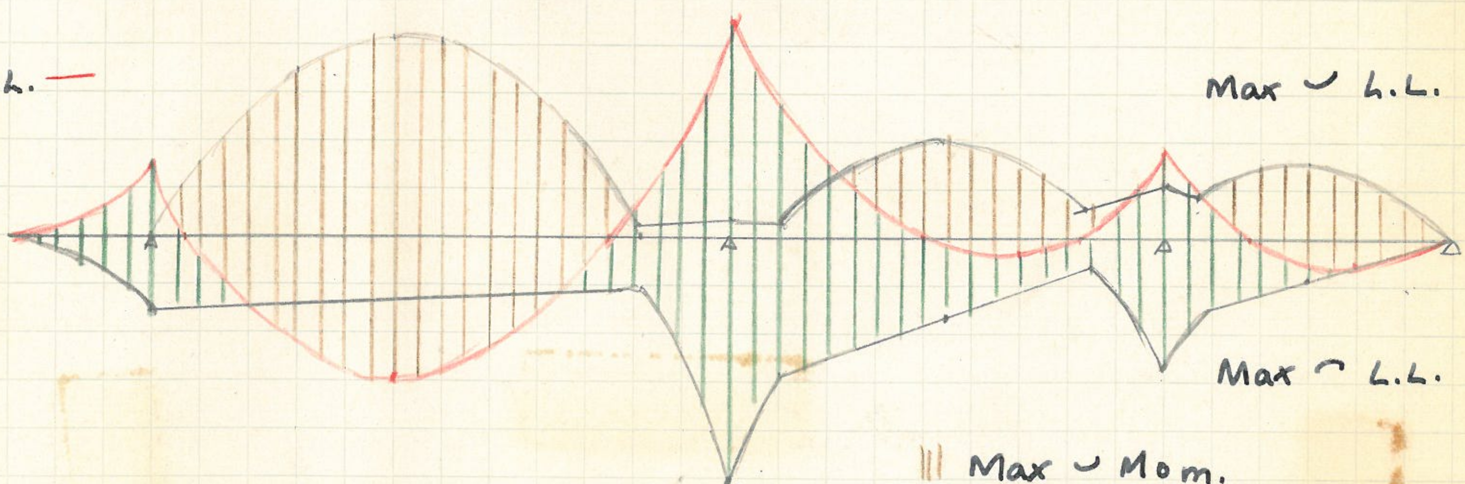
L.L.

D.L.



D.L. —

Max ⌣ L.L.



Max ⌢ L.L.

||| Max ⌣ Mom.

||| Max ⌢ Mom.

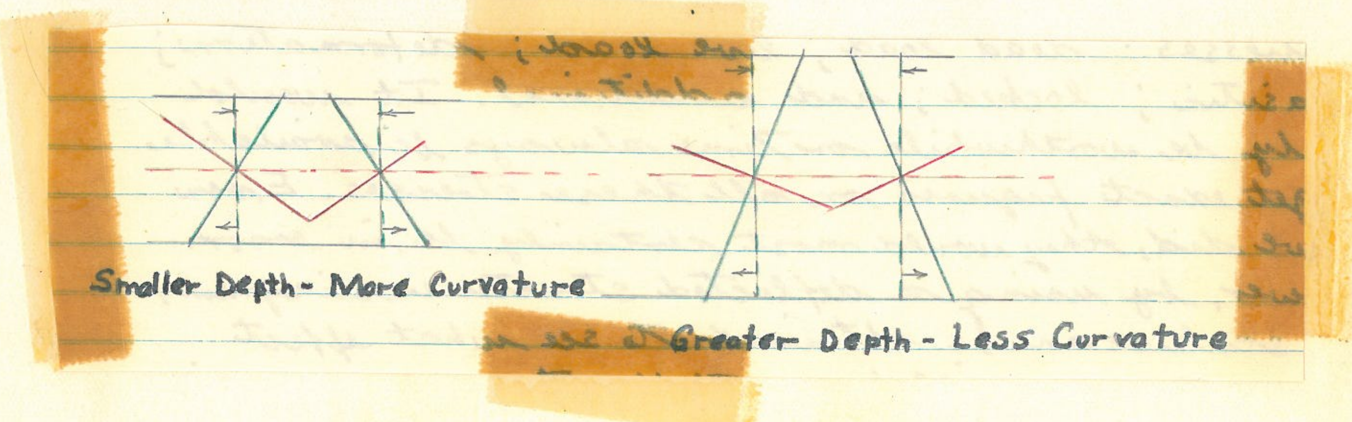
This lecture, and that of October 9, 1952 will continue with the discussion of deflected structures. This is of the <u>greatest</u> possible <u>importance</u>. The ability to draw deflected, and, later, deformed and displaced structures must be acquired. These drawings give a picture. From this picture it can be seen which side to put reinforcement. At this time, the scale of moments can quickly and easily be determined. Later, the same will be done for the scale of deflections.

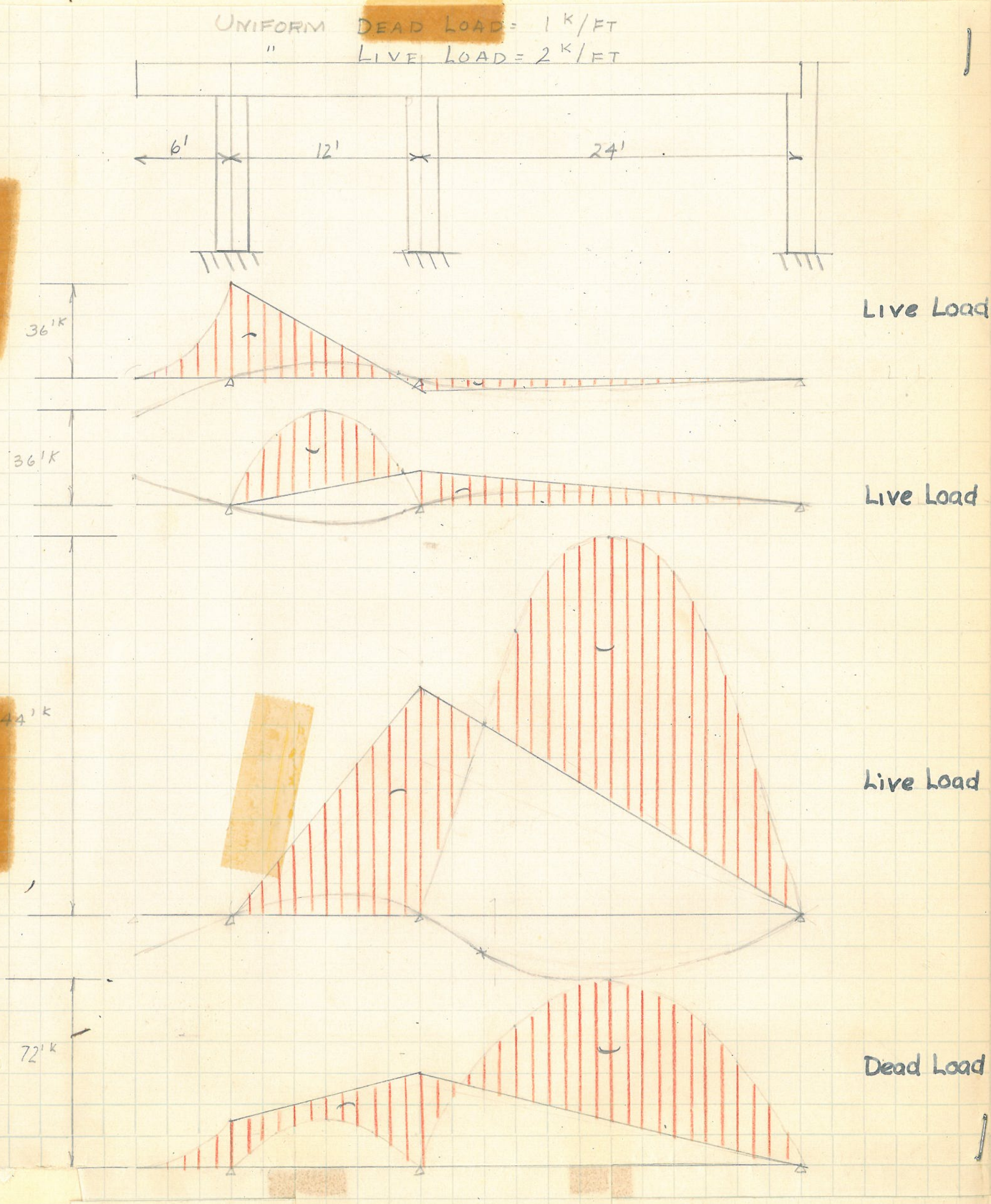
As were listed before, there are six kinds of stresses: dead load; live load; deformation; parasitic; locked; and additional. It would rarely be worthwhile or time always be available to get exact figures on all these stresses. Even if we did, they would most certainly be in error. However, by using a deflected structure diagram, the scale can be obtained to see what effect these stresses will have. If the stresses seem serious, then it may be necessary to have exact computations. The question is not how big the stresses are, but how to get rid of them. The answer is not necessarily to make the structure stronger. The function of the structure is to support the load.

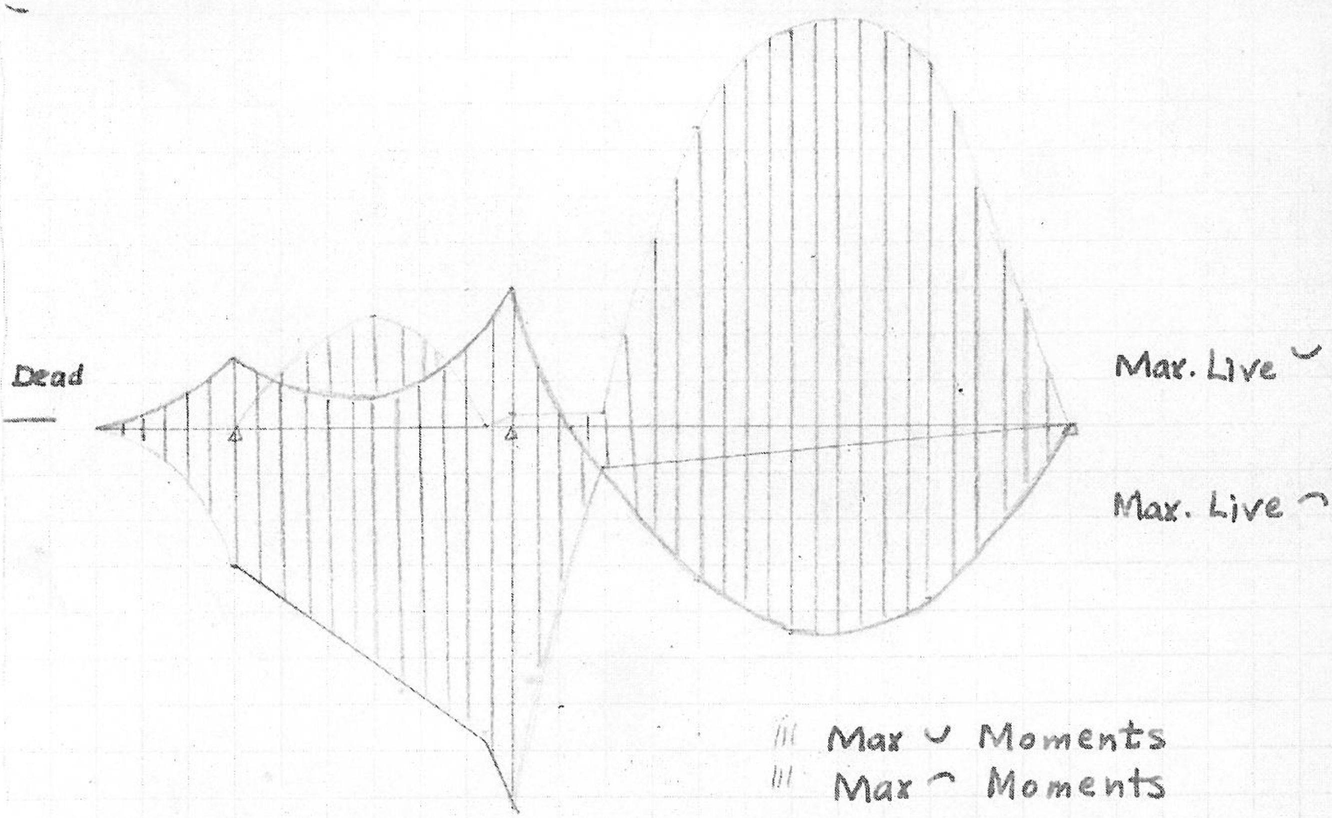
The gravity loads include live loads and dead loads. Although the loads may be concentrated loads, like locomotive wheel loads on a bridge, an equivalent uniform load can be used. It must have the same reactions and shears at the support and the same bending moment at the center.

Later in this course, the concept of influence lines will be introduced. When this phase is reached, the ability to draw deflected diagrams is enormously valuable. This is true, also, when analyzing parasitic stresses.

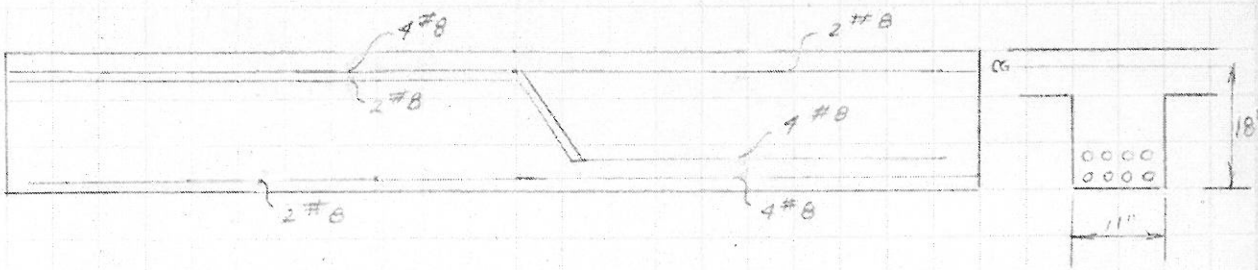
The following page illustrates a solution for a continuous concrete structure. The effect of the columns bending while the beam bends was neglected. The effect of the columns will be discussed later. Also, the concrete was of a constant depth. It is not necessary that it be constant at all unless because of architectural requirements. An increase of depth would reduce the curvature, as the following diagram illustrates, and, therefore, reduce the moment since the bending moments are proportional to the curvature.




UNIFORM DEAD LOAD = 1 K/FT
" LIVE LOAD = 2 K/FT
6'
12'
24'
36'K
Live Load
36'K
Live Load
144'K
Live Load
72'K
Dead Load



No. of bars $= \frac{M}{jd\,A_s f_s} = \frac{162 \times 12 \times 1000}{.9 \times 18 \times 0.79 \times 20{,}000} \approx 8$ bars

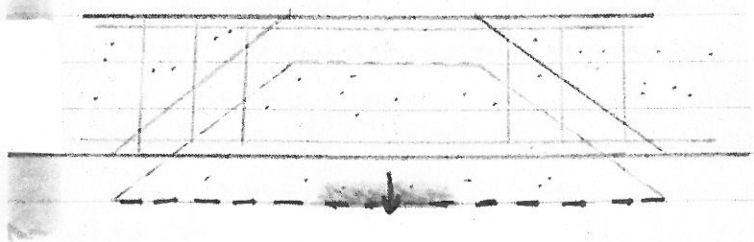


The load on the structure consists of dead load and live load. The dead load includes the self-imposed load, i.e., the weight of the beam, and the super-imposed load. When dealing with long-span bridges and big structures, one method is used to reduce the self-imposed load, and another to reduce the super-imposed load.

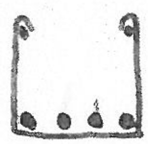
After the moments have been quickly determined as have been done in the sample problem, the number of bars are calculated. The size of the beam and the size of the bars should have already been determined.

STIRRUPS

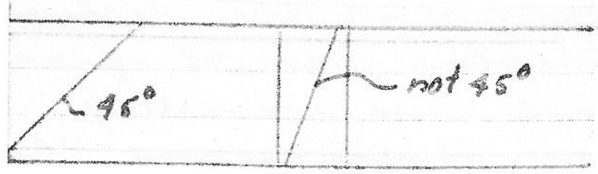
Stirrups do not stop cracks, which is a common misconception; they stop the cracks from opening. The following diagram shows the stirrups resisting the chunk of concrete which is tending to fall out.



It should be remembered that the stirrups must be tied at both ends.



The cracks are assumed to be in a 45° plane. Therefore, stirrups theoretically should be placed at a maximum spacing of jd. By custom or law, stirrups are placed somewhat closer, 3/4 d, 2/3 d, or 1/2 d. However, because the beam has no respect for education, custom, or law, ~~the cracks are not at a 45° angle or law,~~ the cracks are not at a 45° angle and even 1/2 d spacing may not be adequate.



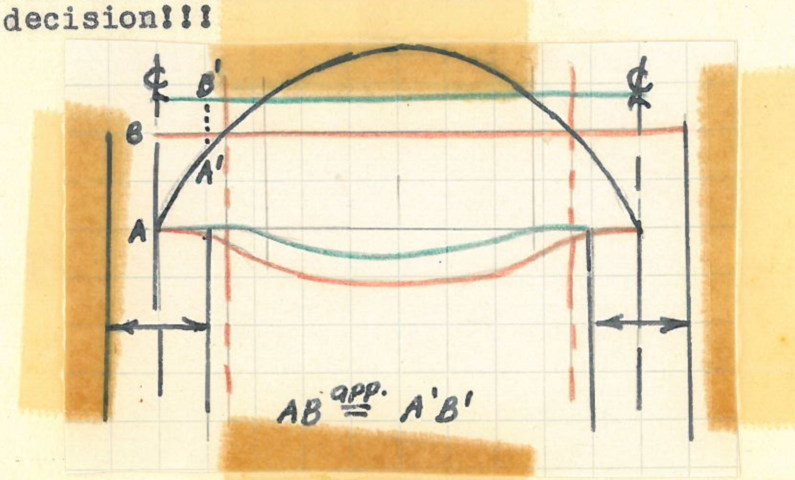
Only common sense will dictate where to place the stirrups. Here, as in other phases of structural engineering, it is important to form a clear visual picture of what is happening.

The methods of teaching in preparatory school have the student doing the problem first and then looking at the answer in the back of the book. On the other hand, in engineering it is far better practice to find the answer if you can and then to do the problem. The illustration in this lecture is an example of how to get an answer quickly. If the results indicate a serious condition exists, or should calculations which may be duplicated be required, the beam should be checked by moment distribution. If the moment distribution does not check, there is either an error in calculations, the most likely situation, or, an error in judgement. The results by the approximate method of drawing deflected structures is as good or better than results gotten any other way. However, because of the need of duplicating calculations, the "exact" figures usually must be put down.

COLUMNS

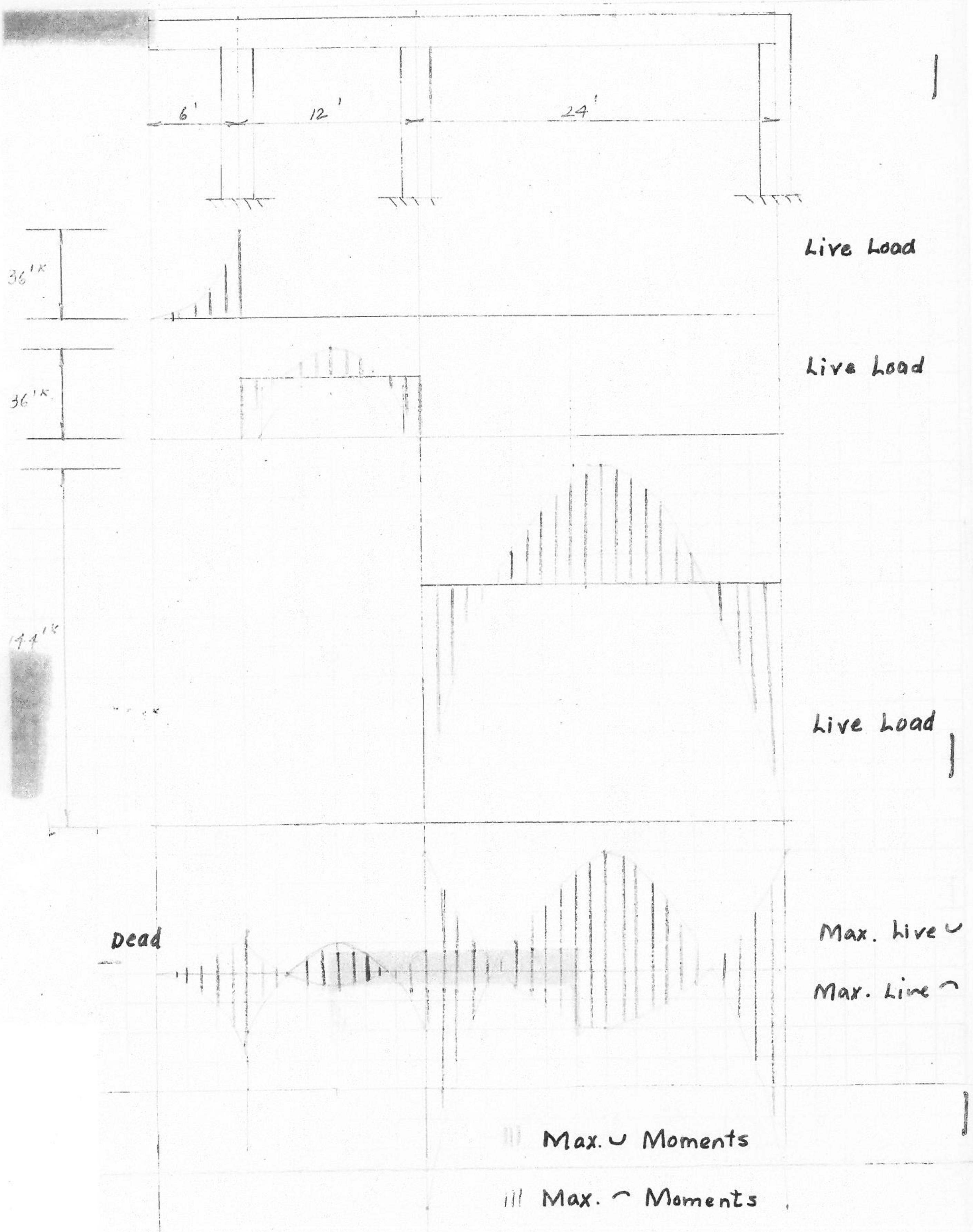
Up to this point the effect of columns has been neglected. This has the same effect as assuming the columns to be infinitely flexural. The moment diagrams for that case are drawn in the illustrative problem in this lecture. However, the same problem can be quickly solved if the columns were infinitely stiff as will be shown. If you compare the results if the column were infinitely stiff with the results if the column were infinitely flexible you would be able to see if the difference is major. You may then decide if you need to stiffen the columns or, perhaps, to let them be more flexible. There is no way known to estimate relative stiffness, but there are methods to arrive at a figure of relative stiffness that can be duplicated.

Is there justification in reducing moment by taking centerline of column to draw deflected structure? Don't hang around too long making a decision!!!

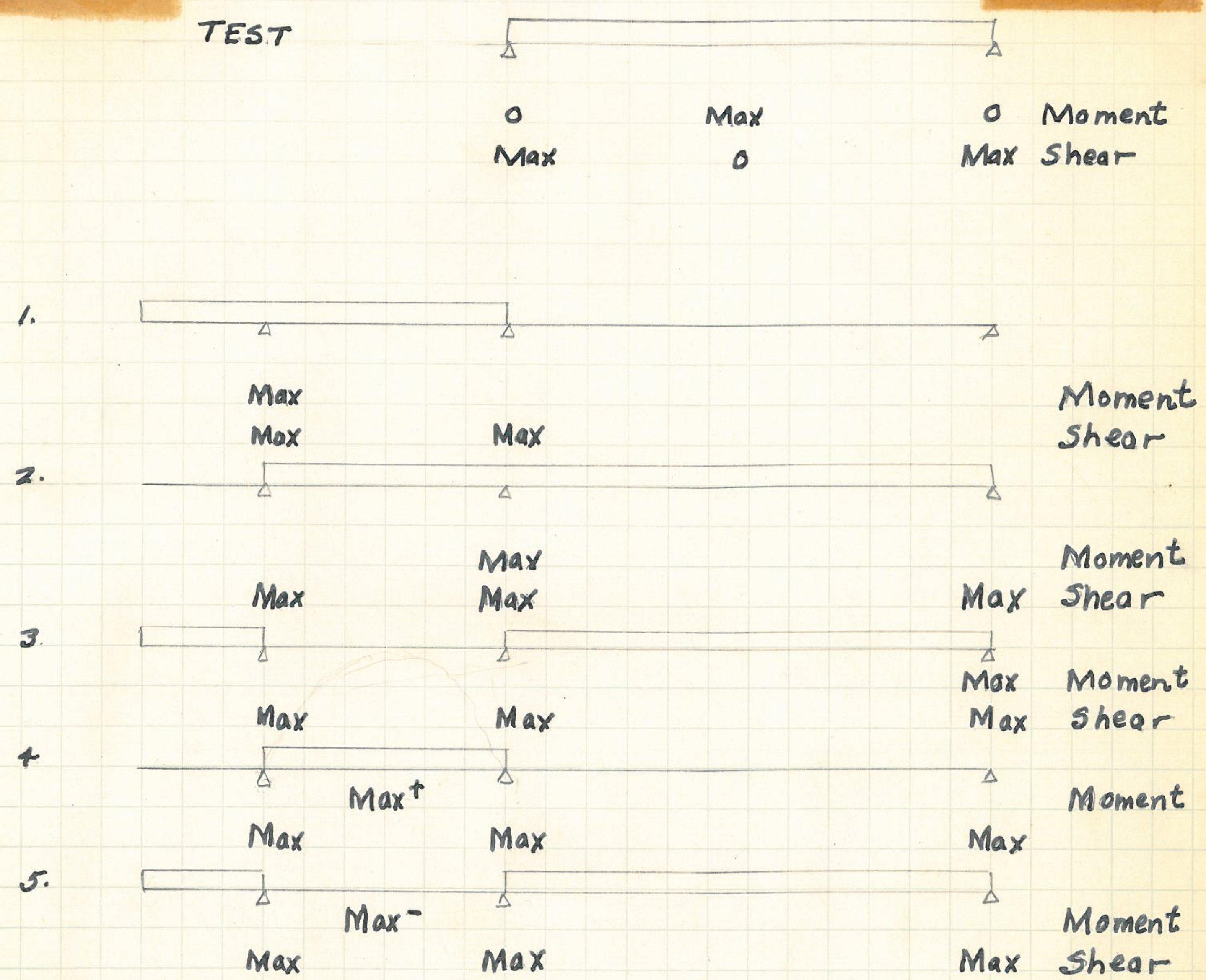


Haunches tend to increase the moment of the centerline of the column and reduce the moment at midspan. The moment is less at the edge of the haunch than at the centerline of the column.

Prof. Cross
Oct. 7, 1952



The following example illustrates why tests results should not be applied indiscriminately to actual structures.



In 1, 2, & 3 above the maximum moment and maximum shear occur at the same point.

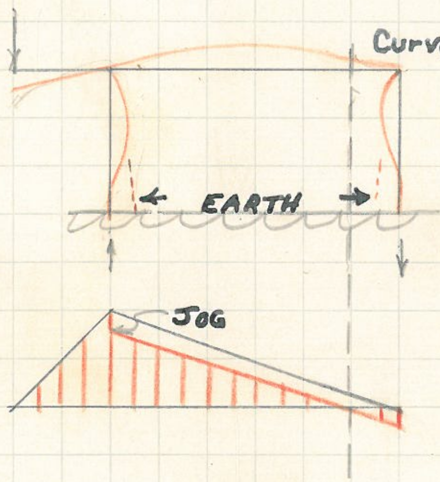
DEFLECTION OF STRUCTURES

Although we need a guardian probably to help us with deflected structures, we will not be able to pull out a professor like a pack of cigarettes.

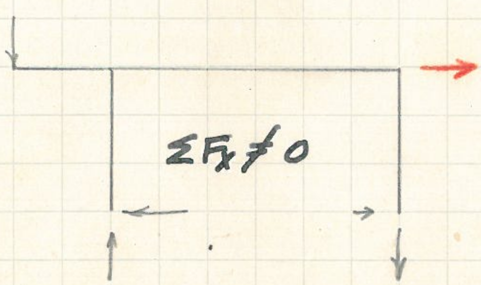
When columns deflect they will remain at right angles to the girder if constructed in a proper manner. This is not always done and should be given consideration. The following diagrams will illustrate the way columns <u>must</u> act and the way in which they would be unstable.

Right Wrong

Since the earth resists the movement of the column, the direction of the reaction is known.



Curvature must change direction

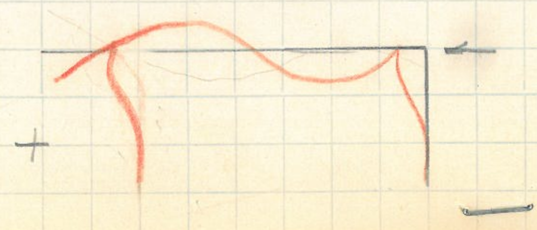


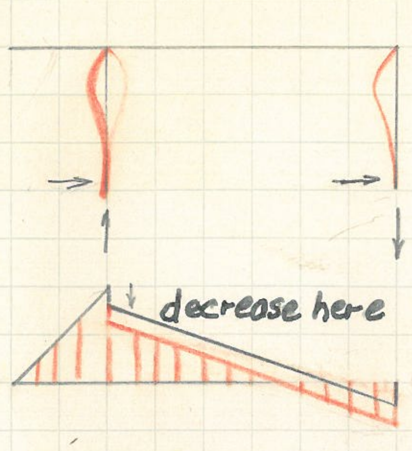
Balancing force

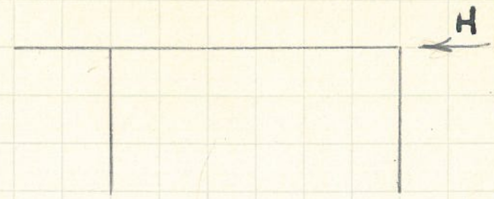
$\Sigma F_x \neq 0$

Although it is known that the structure is not free to move since the slab or something will keep it there, we will assume it is free to move.

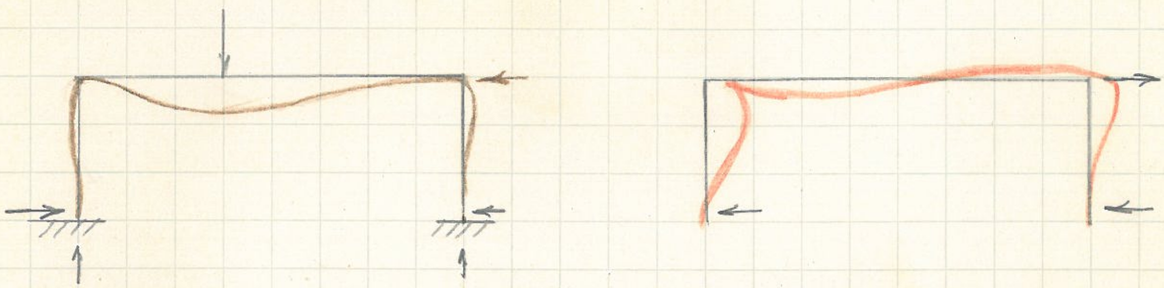
Therefore,

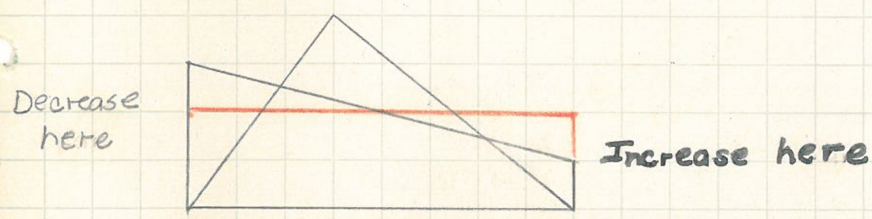




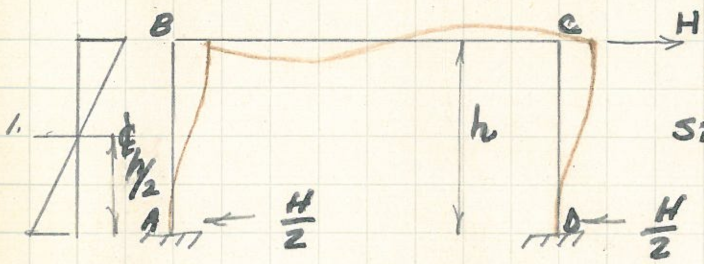


Due to load H





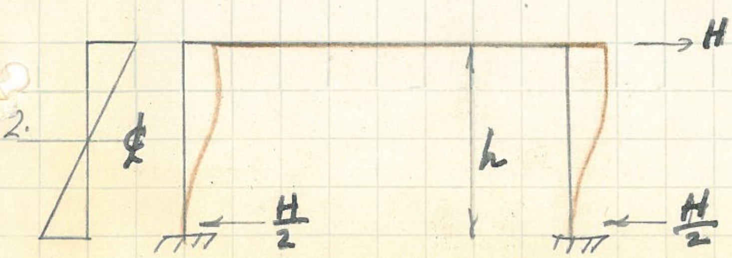
EFFECTS OF STIFFNESS



Columns and girder of same relative stiffness.

Moments at B and C are the same. As for reactions, divide the load equally. The movement is also equal.

$$\text{Moment} = \frac{H}{2} \times \frac{h}{2} = \frac{Hh}{4}$$



Girder much stiffer than column

$$\text{Moment} = \frac{H}{2} \times \frac{h}{2} = \frac{Hh}{4}$$

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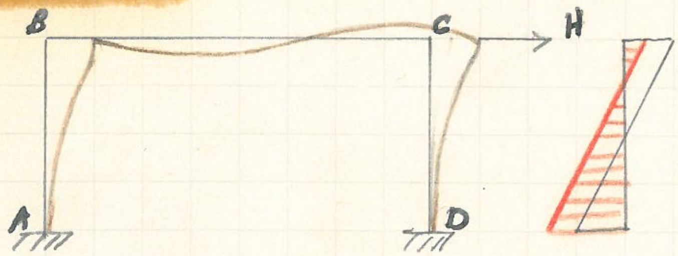
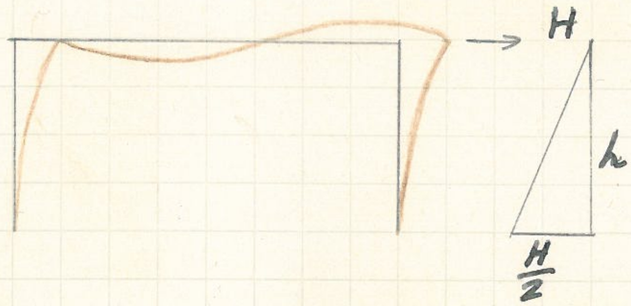
3.



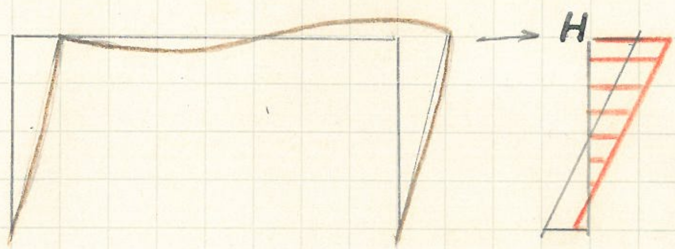
Girder more flexible than column

As girder becomes more flexible relative to the column the inflection points move up the columns until the moment is zero.



Moment = $\frac{Hh}{2}$

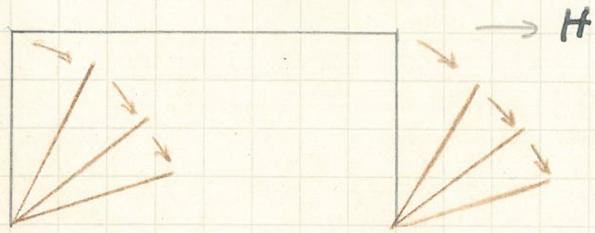
4.



Ground weak, girder and column equally stiff or girder stiffer than column.

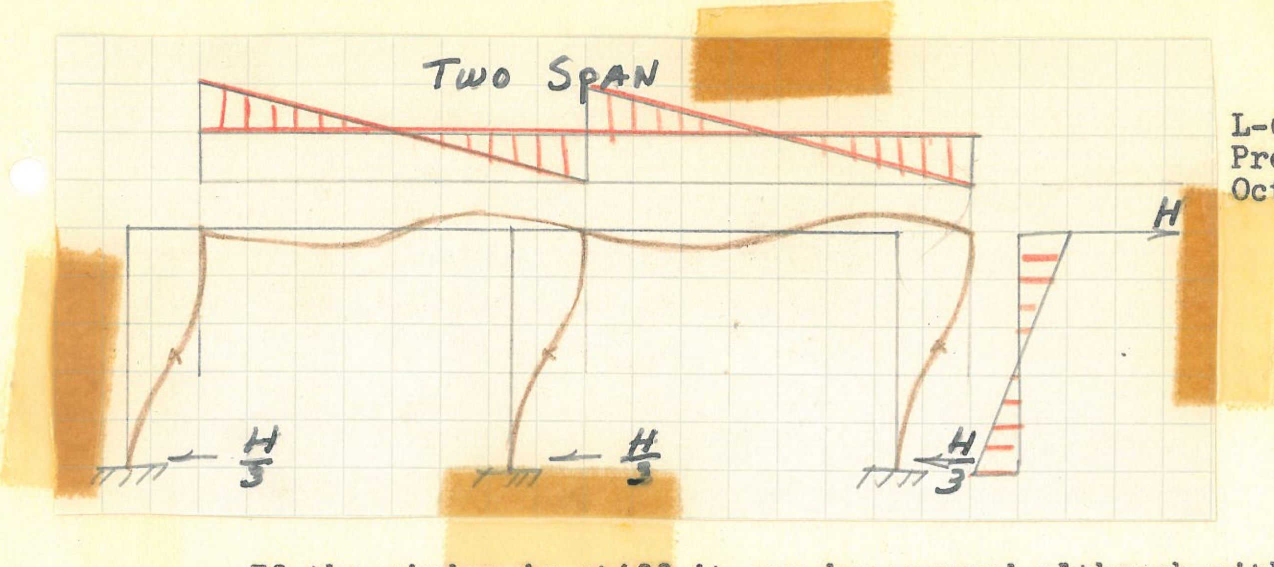
As ground becomes weaker the inflection points moves down the column.

5.

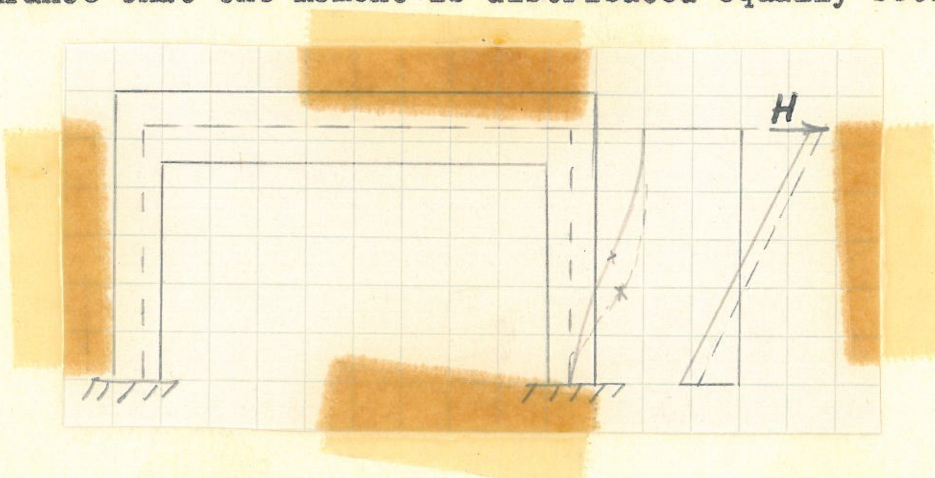


Girder flexible, ground weak.

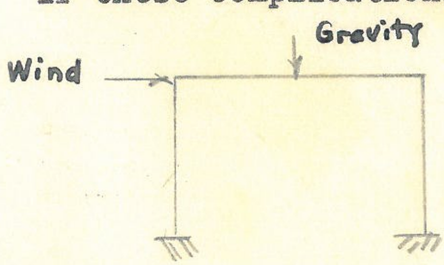
A flexible girder tends to move the inflection points up while the weak ground tends to more the inflection points down. Result: instability,- it will fall down.



If the girder is stiff it can be assumed although without assurance that the moment is distributed equally between the columns.

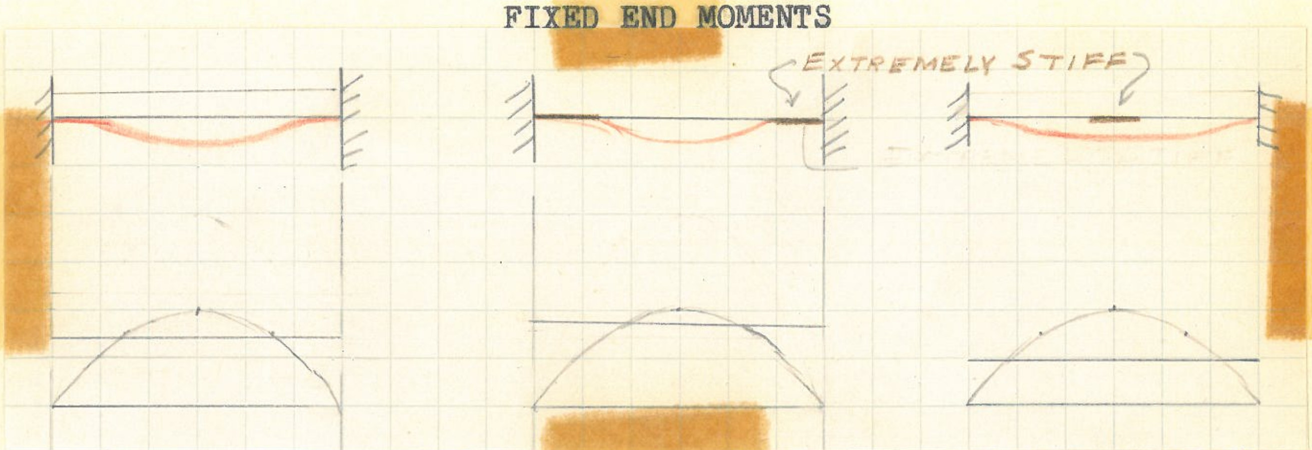


Making line diagrams results in uncertainties. The full figure above illustrates how the moment may vary because the outside edge would start to bend first. It is fairly obvious that such lack of exactness in our analysis is rarely worth worrying about. Other uncertainties like the connection between the column and the girder, etc., are usually of no concern. However, the scale of these will show if these complications may be serious.



The problem illustrated here is impossible. Wind and maximum gravity will not occur at precisely the same time. It is important to establish the origin of loads. Investigation may disclose the advisability of increasing the working stress by 1/3 or even 50% when combining the effects of wind and gravity loads. In the tropics where wind storms occur often and wind pressures are intense, this may not be advisable.

FIXED END MOMENTS



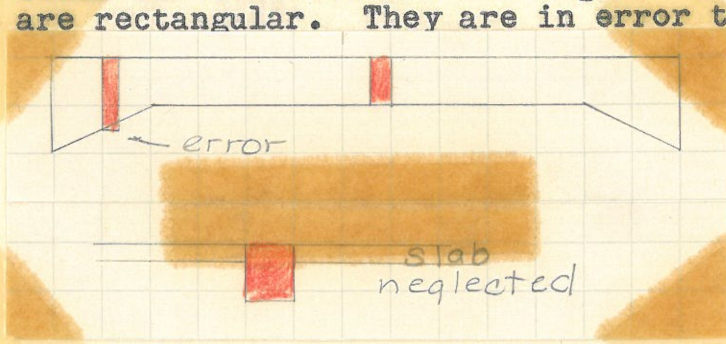
There is a great deal more uncertainty in center moments than in end moments. We must decide if it is worth bothering with. Most likely it is.

It is important to note that a great deal of information is being obtained very rapidly by drawing these figures. At a later time it may be suggested that figures be put down that can be duplicated based on certain assumptions that these approximate methods may advise.

What appears in codes is not always right. For example, the values in ACI codes for haunches are not right. The sections they are based on are rectangular. They are in error two ways:

1.

2.



Therefore, although tables are apparently precise they really are not. It is obvious that a judgement must be based on a quick method to decide if it is worthwhile to consider the discrepancies above.

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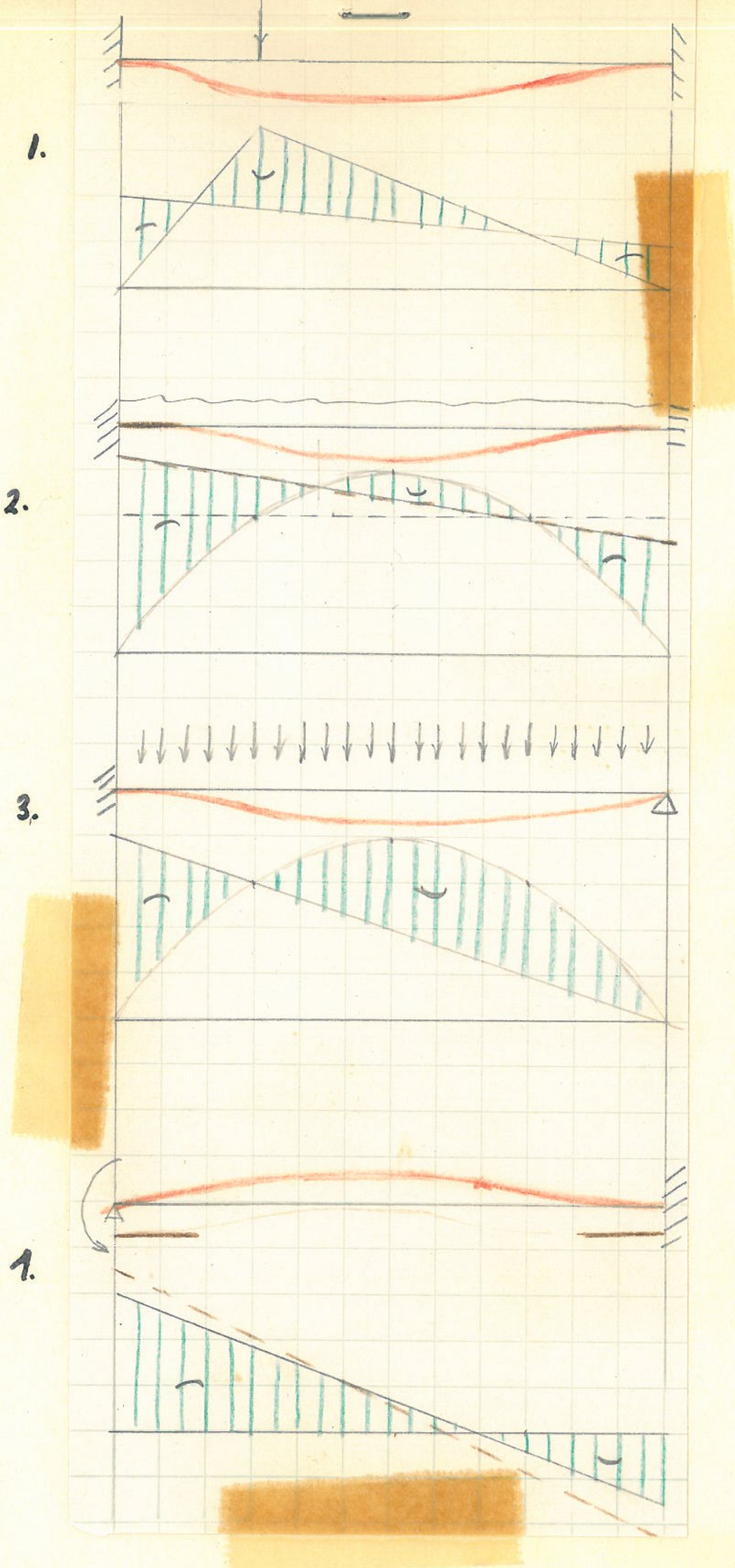


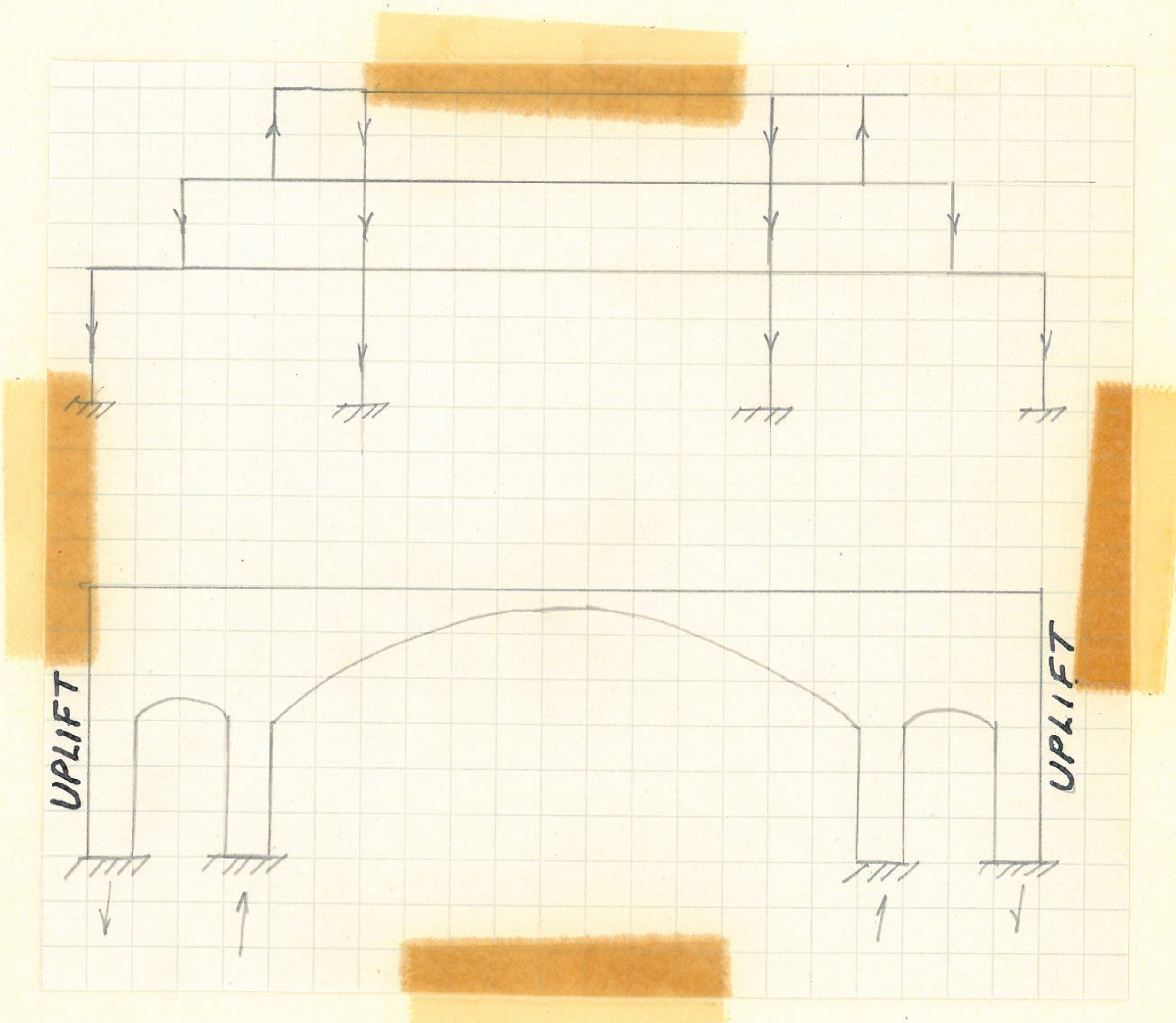
Comparison of stiff and flexible beams give a clear idea of effects. The effect of stiffening the member may be very slight as is illustrated in 4. The changes in end shear is not very great. This calls for judgement to decide if it is worthwhile to consider in analyzing a problem. Physical data is never under control in our field or in others. The E of concrete may vary 30 percent or more over the length of a beam.

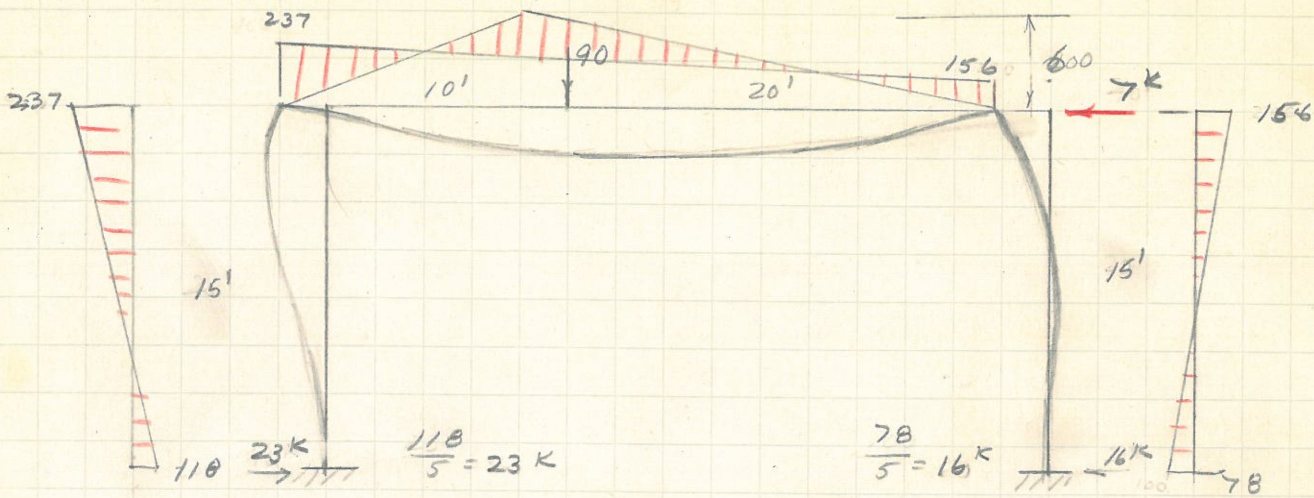
Deflected beam of 4 similar to 3 above.

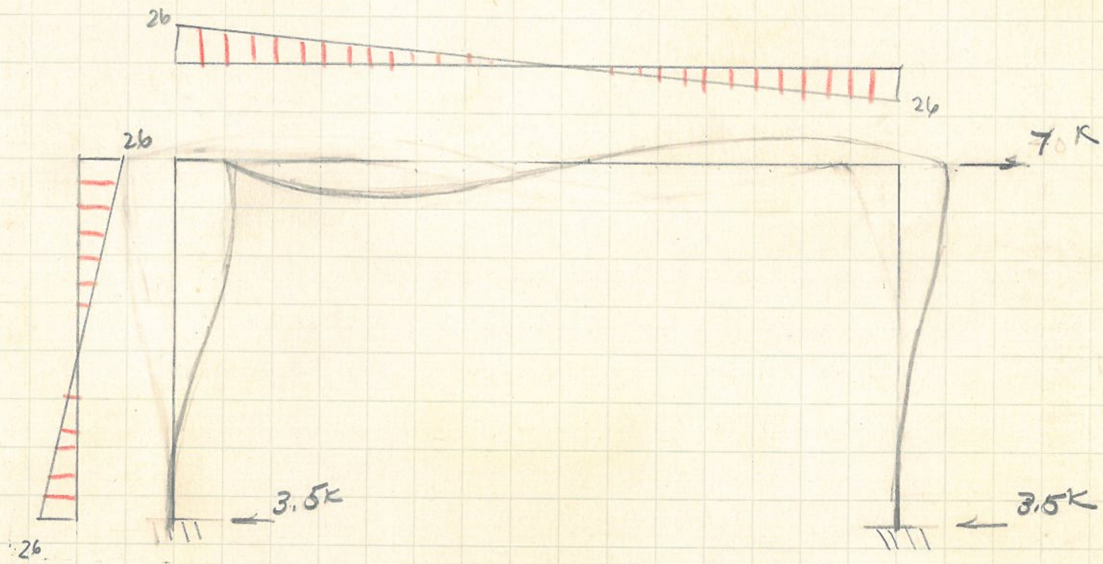
The moment this end exactly one half the other end.

The above diagrams lead to moment distribution and carry over factors.

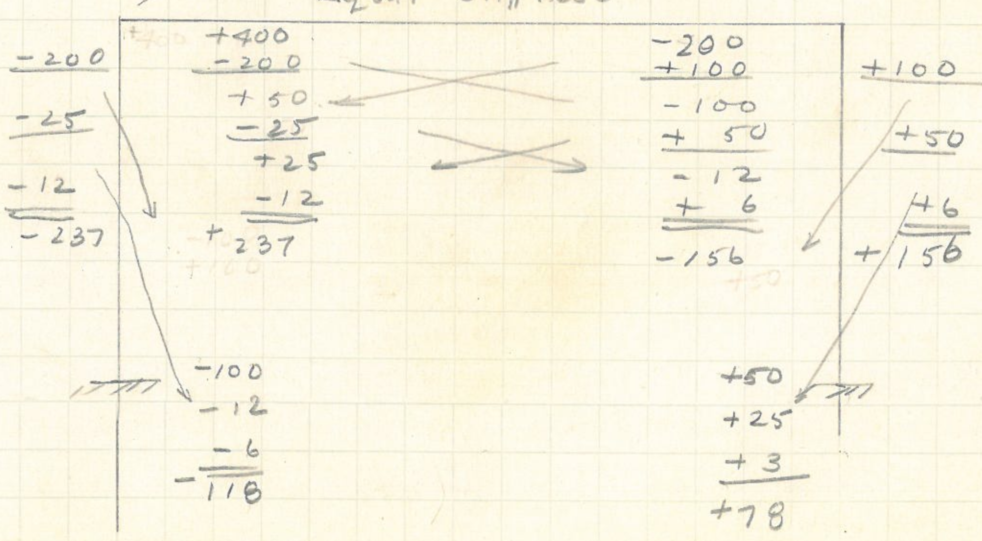
It is not advisable to make long spans continuous with short spans. The reactions of the center span is increased drastically while the reactions of the outside columns are reduced or even reversed. It has the effect of a crow bar.

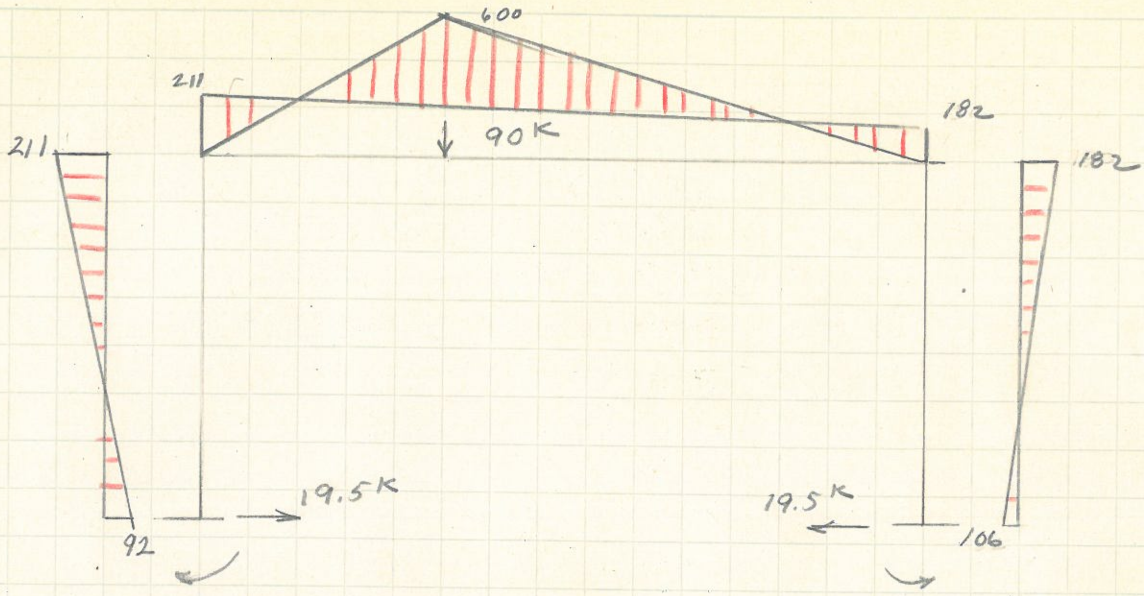






Equal stiffness





60.5 ↑ ↑ 29.5

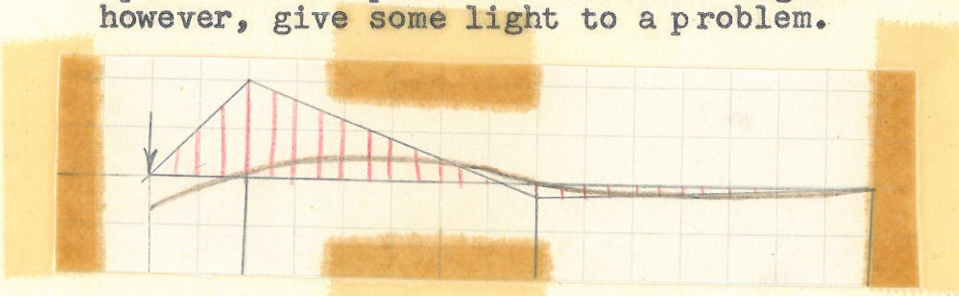
$$\frac{900-14}{30} = \frac{886}{30} = 29.5$$

$$\frac{1800+14}{30} = \frac{1814}{30} = 60.5$$

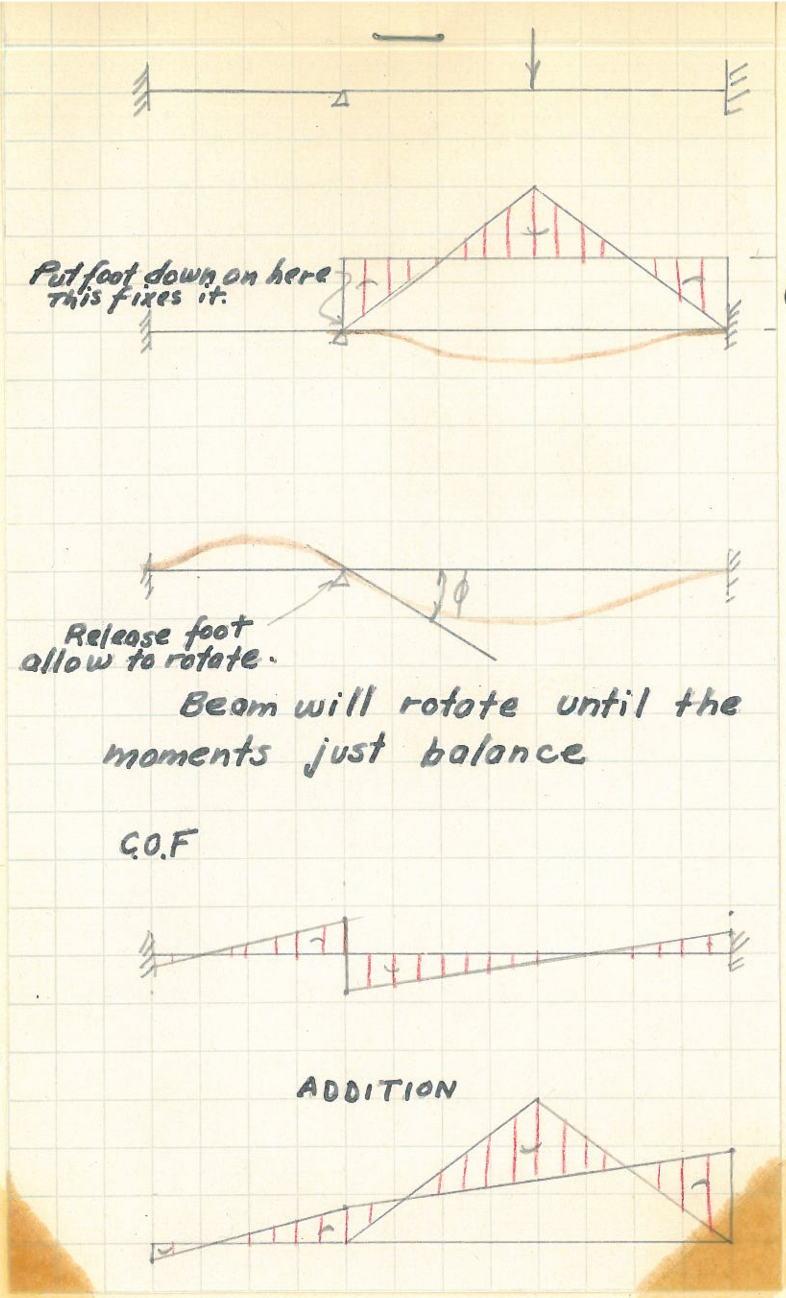
There are two purposes of analysis. This will be a repetition of what has been said many times in many other lectures.

1. To get the scale of the structure.
   We will see if the structure is satisfactory, if the elements of strength and stability, and stiffness are met. To obtain the scale the loads and working stress must be more or less assumed. These assumptions are made on some basis or evidence which will be looked into later. When the scale is obtained, it will be shown if there is danger of a column crushing or a girder fracturing, etc. The classic question of the "practical man" is how does the structure? It will act differently on wet days than on dry days, after it has been cracked than before cracking, when the structure is new than when it is old, etc., etc. Don't ask how does it act, but under certain conditions what happens.

2. For psychological reasons.
   a.) Figures can be put down here today that can be duplicated a month from now in Los Angeles by a different person. Difference will occur because of a corresponding difference in "the rules of the game" between here and there.
   b.) The desire we all have for a "scientific," "precise" procedure makes exact computation seem important. We like to take out of our pocket a professor and squeeze him and "poof" we can put down exact!??! figures. These may, however, give some light to a problem.



In the above problem the action of the column can vary the moment diagram. Further, slabs and other contingencies will alter the position of the points of inflection. Of course, it may be very small, but it may be necessary to satisfy "some old lady of either sex." A systematic procedure can be developed which will give the old lady figures to twenty decimal places.



F.E.M.

To obtain "exact" figures three things must be known:

1. Fixed End Moment - FEM
   How to find FEM another matter.

2. Carry-Over Factor - C.O.F.
   Moment produced at end of beam that cannot rotate by a rotation of the other end.

3. Stiffness, S
   What is the ratio of positive and negative moments, or how much is distributed over supports? If one end is fixed how much moment is needed to bend the other end?

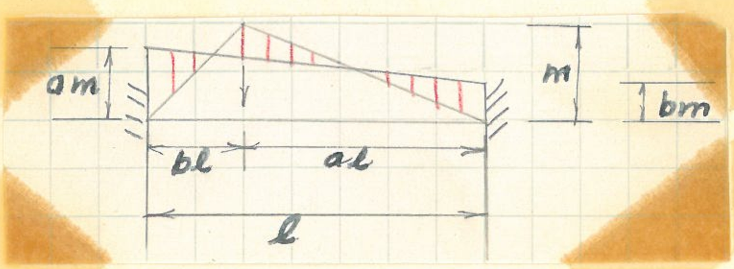
Stiffness $\propto M/\phi$

M/ϕ is the moment per unit rotation.

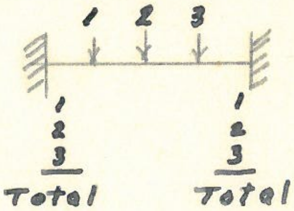
FIXED END MOMENTS

If it is assumed that the beam is of uniform thickness, a constant section, throughout its length (it may very likely not be!) the <u>fixed end moment</u> with a uniform load is $\frac{1}{12}$ WL.

The following diagram will illustrate the fixed end moments under similar conditions but with a single concentrated load.



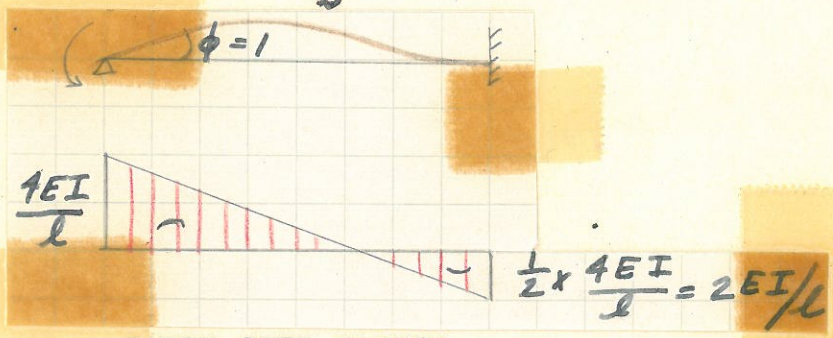
For more than one load work each load out separately and add them up.



STIFFNESS

Stiffness $= \frac{4EI}{l}$

Beam rotated a unit angle.



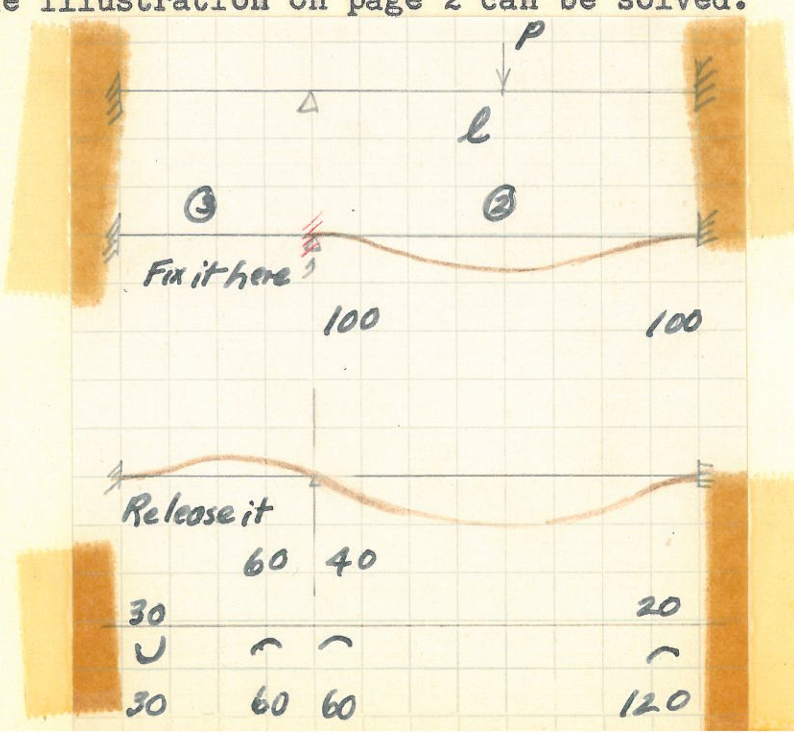
CARRY-OVER FACTOR

C.O.F. = 1/2

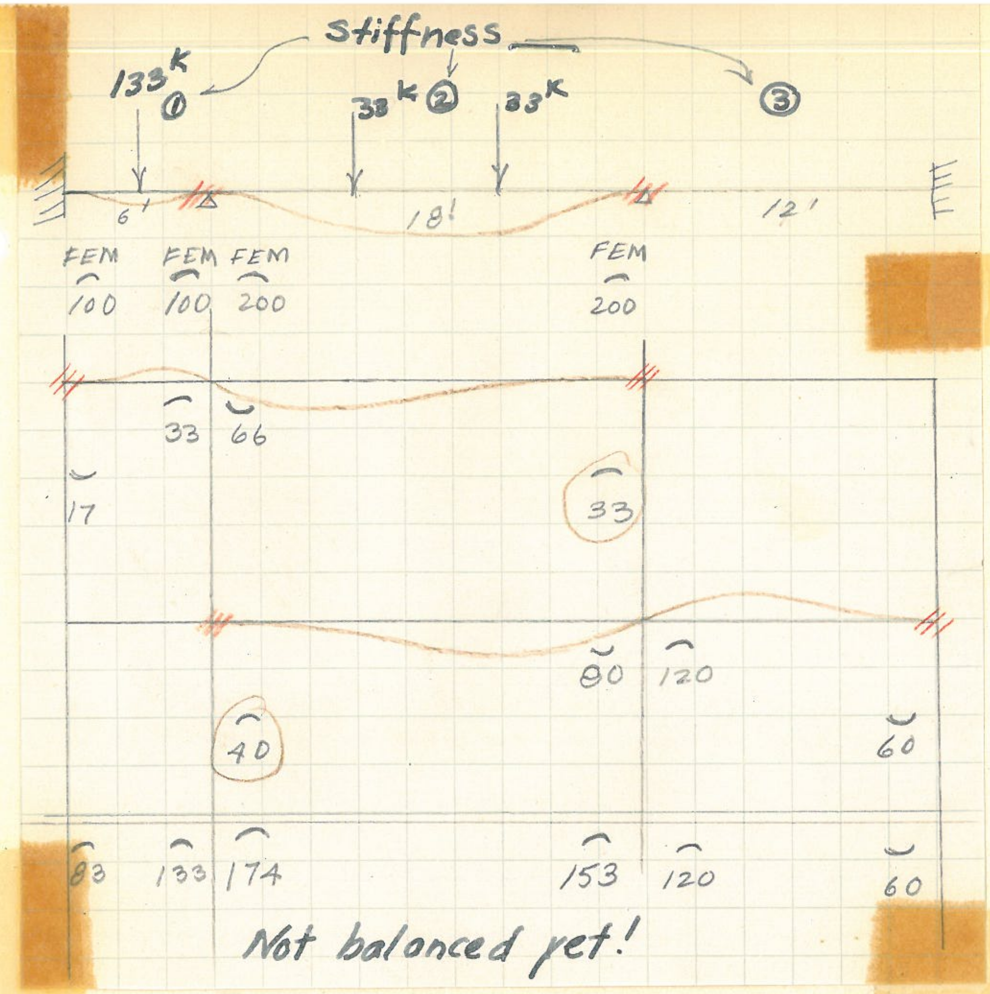
If we assume the following things:

1. The joints do not move.
   If they do an allowance can be made for them.
2. The members are of constant section.
3. We can compute I.
4. That the carry-over factor is 1/2.
5. And we have evidence to support everything else.

The illustration on page 2 can be solved.



if $P = 50$ ¢ $l = 16$
FEM = 100

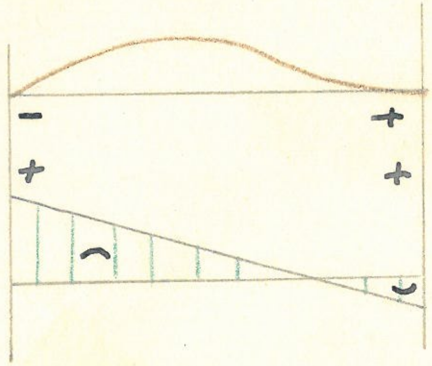


As done in this example is very clumsy. The distribution is not completed since those circled must be distributed, too. The distribution should be continued until the precision that is required is reached, i.e., until some "old lady" is satisfied.

There are two conventions, the structural engineer's convention and the mechanics' method. In the structural engineer's method, negative is , and positive is . This is particularly messy with columns. With the mechanics' method, positive tends to rotate the joint clockwise, negative tends to rotate the joint counter-clockwise.

Structural Engineer's Method

Mechanics' Method



Structural Engineer's
Mechanics'

With the mechanics' method moment diagrams cause difficulty.

If a problem is complicated, use the mechanics' convention. A joint is balanced when the total moment is zero (+ and -)

ROUTINE:

1. Find the F.E.M.

2. Balance all the joints.

3. Carry-over.

4. Balance all the joints.

5. Carry-over, etc.

There are three possibilities for mistakes:

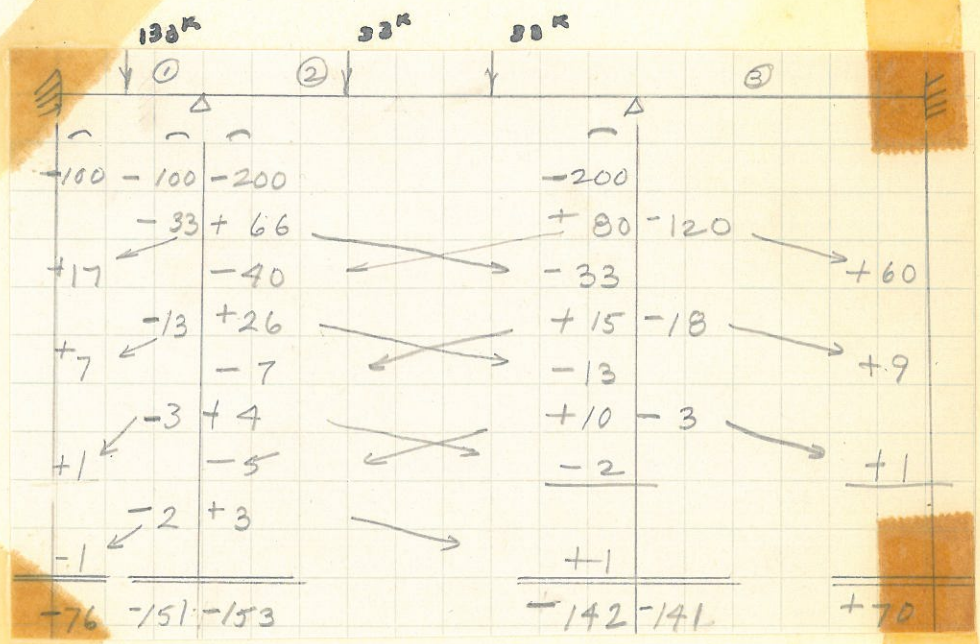
1. Forget something
   F.E.M., C.O.F.

2. Decimals wrong
   Omit a zero

3. Signs

+100	-200
-150	+150
-50	-50

Both sides go the same way.



A correction can be made if the joints move.

MOMENT DISTRIBUTION

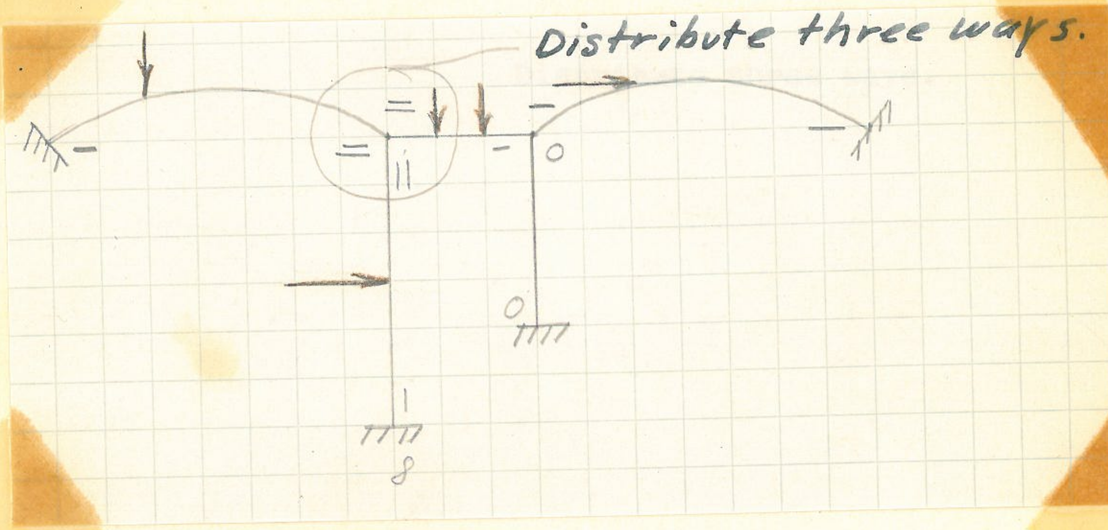
1. Write moments that exist if the joints could not rotate.

2. Distribute at each joint one by one the unbalanced moment to the members connecting to that joint in proportion to their stiffness.

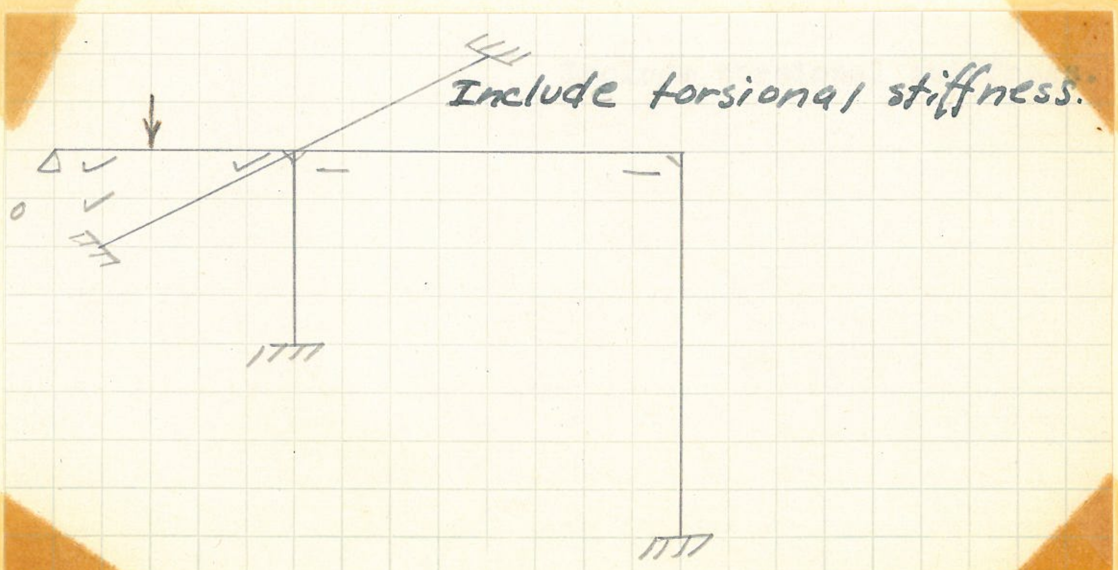
3. Carry-over one half.

4. Repeat, again and again.

5. Add up the moments.

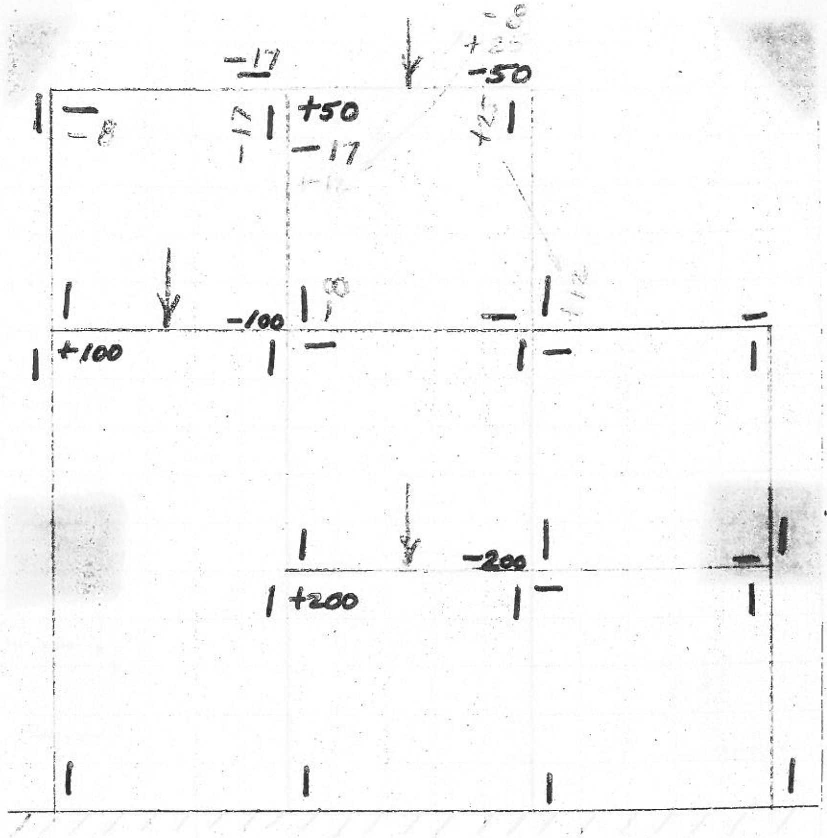


In doing the above problem we assume the joints do not move. They move but is it of a serious nature. Get the scale. Probably only one partin a million.



The following problem is practically never worth working. However, it is worthwhile to know how to work it and to have a reason why it is not worth working. Notice how the figures are kept. It is a matter of good bookkeeping.

Assume equal stiffness of all members.



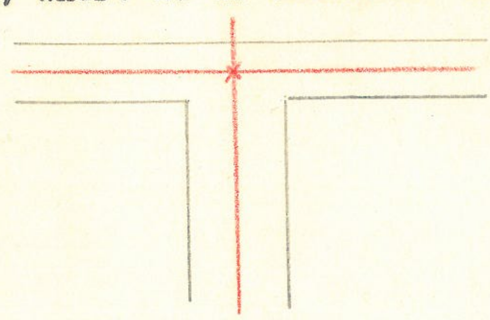
There are short cuts which will be commented upon later.

Generally speaking, problems such as these will not be worked out every day. If we are engaged in work that requires much of this, we still, probably, will not be called upon to do moment distribution more than once or twice a week. Since we are going to use them but not often moment distribution must satisfy the criteria that it be easy to learn and hard to forget. However, we should be able by deflected structures or some approximate method to know the answer before the calculations are started. In this manner many errors will be avoided or can be corrected when discovered.

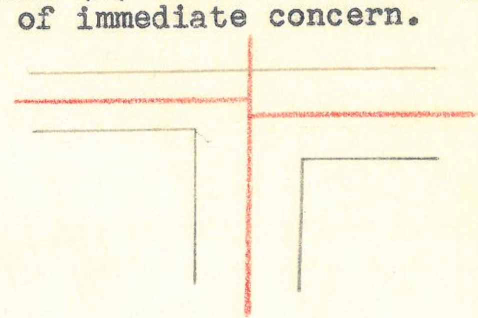
First, we will briefly summarize what was said in the last lecture. We did not talk then about what would happen if the joints move. We knew that they did move, and we saw what happened when they did not move. Undoubtedly, the matter would be greatly complicated if the effects of lateral loads were analyzed. Assuming the joints do not move, the moments were found that would exist at the ends of the member if the ends were not free to rotate. These are fixed end moments (F.E.M.). How to get them will be discussed later. Next, the unbalanced moment at that joint is distributed successively to the joint on each member, if the members are continuous, that frames into the joint in proportion to how hard the member resists rotation if one end is fixed. A fraction of the moment is carried over to the end that is not free to rotate. This is repeated over and over again until the "precision" satisfies you, the design, or "some old lady".

What we are after is a structure, not an analysis. If our interest is reinforced concrete, then where the most bars are needed, and how many bars will we need is what we are after. It can be, as far as the concrete is concerned, a matter of looking and seeing if the stresses in the concrete are exceeded or not. If they are then how to get out of trouble is the next problem. It may be better not to build out of concrete but with another material, or, perhaps, better not to build at all. The purpose of this method is to see all the stresses by which the structure may be proportioned all in one picture.

In using F.E.M. it is customary to assume certain things. Primarily, that the beam is at constant cross-section. Further, that the point of rotation is known. Diagram (1) shows this: diagram (2) where it is unknown. This is not of immediate concern.



(1)



(2)

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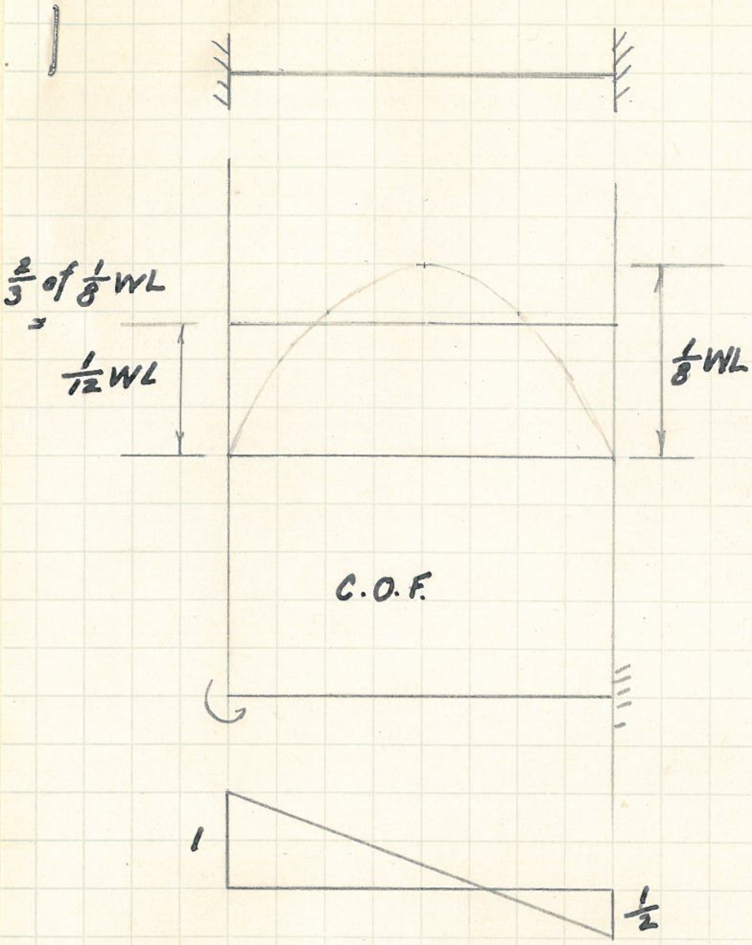
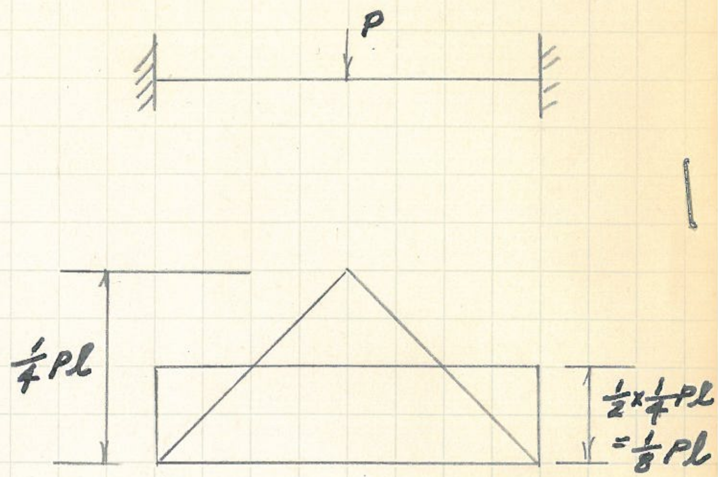
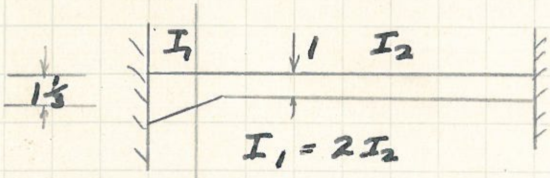


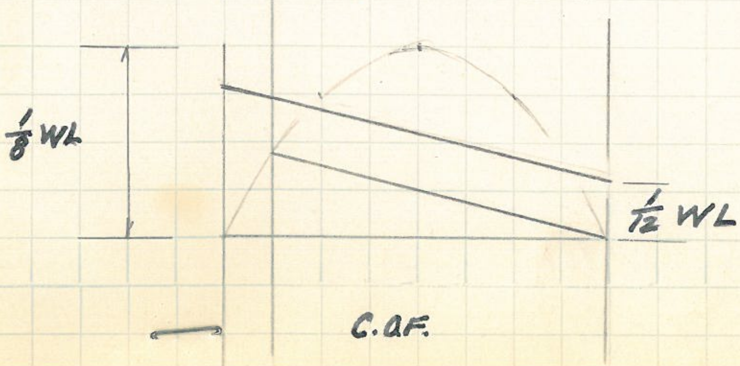


Theorem will be derived later.

Effect of Haunching



I varies as the cube of the depth



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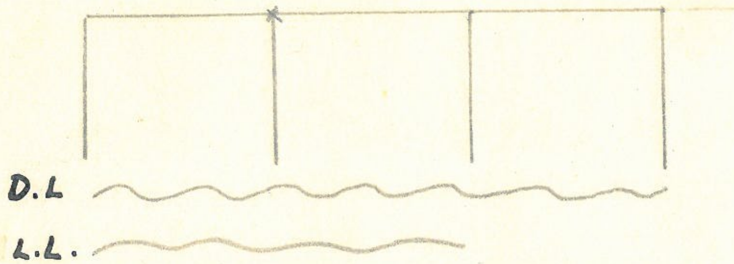
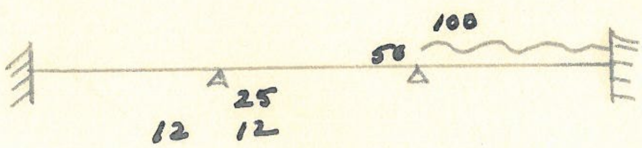
The following problems should be attempted to see the effect stiffness has on the maximum moment.

The curve of maximum moment not very different 1 thru 3. 4. much more.

The effect on the column is much greater than the effect on the girder.



There is not much tendency for joint to move if loaded on both spans. It will not make much difference if the last span is loaded, too. A quick approximation is that it is near fixed ended.



Moment will vary 12%

INFLUENCE LINES

Now we will enter a new phase. Influence lines are an artificial idea. They tell, if drawn to scale, when to load for maximum. Although this can be scaled off, Prof. Cross does not favor this practice. Nevertheless, it shows other things very quickly, particularly when several influence lines are compared, the effects of certain variations between members or beams can be observed.

It has been proven that if a model is made of a structure and it is assumed the deformation is proportional to the stress then an influence line can be drawn. However, while "some Yankee" said let's make a model of bakelite or some other material and deform it, we will make only a mental model. An influence line can be drawn for any stress effect, bending moment, shear, reactions. In the structure the deformation will be unity. This is an entirely artificial concept.

"The principle may be summarized with a general statement. If any stress function of an indeterminate structure--reaction, thrust, tension, bending moment, shear, fibre stress, (for an assumed position of the neutral axis) produce freely and alone a unit corresponding displacement, the structure remaining otherwise as before, then the displacements of the deflected structure are, to scale, ordinates for loads in the direction of these displacements, of one influence line for the stress function. Rotations either across or around the axis of the structure (change of slope or torsional rotation) at various points represent ordinates to influence lines for moments or torques applied at these points.

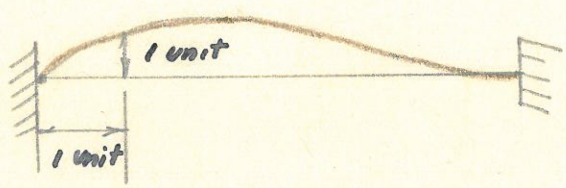
"That is any stress function whatever in any structure, whether simple or complex, will draw to some scale, its own influence lines, this being the deflected load line due to a unit distortion corresponding to this stress function."

First, the influence line for a beam with a fixed end moment and constant cross-section is drawn as follows. At the point on the beam for which the influence line is desired break the beam, put in a hinge, and put a unit rotation at the hinge.

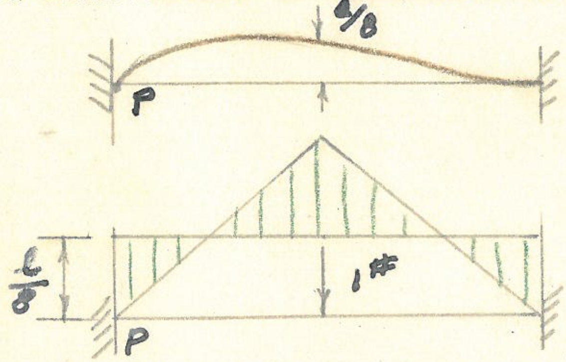
Influence Line of Moments

There are <u>three ways to get the scale</u>. No one way is good all the time, so it may be necessary to combine some or all of the three.

1. Produce a unit angle.

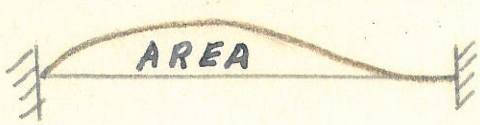


2. Find values of influence ordinates by some indirect method.



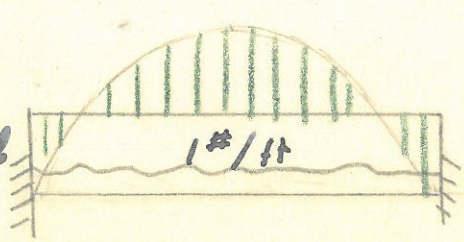
The moment at P due to a 1# load at the center of the span is the influence ordinate at the point the 1# load is applied.

3. Pick out an average value.



$AREA = \frac{1}{12} \cdot l \cdot l$

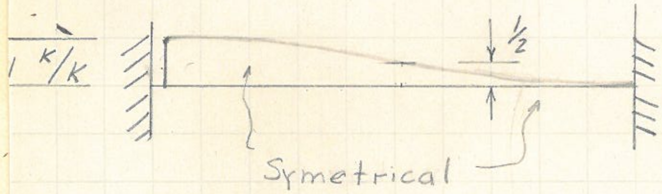
Average Ordinate $= \frac{1}{12} l$



$\frac{1}{12} \cdot l \cdot l$

INFLUENCE LINES

BENDING MOMENT



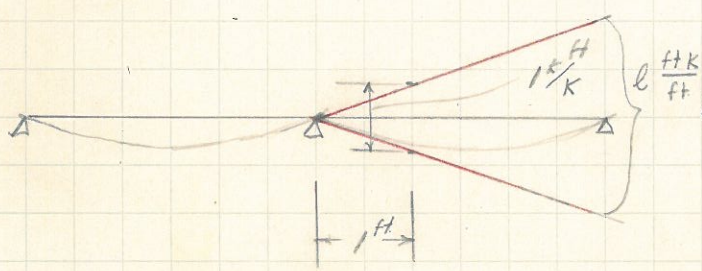
SHEAR

"Let the shear do its stuff through one unit."

REACTION

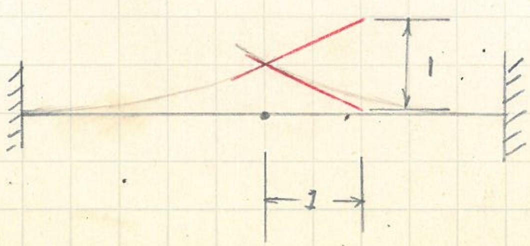
"Let the reaction do its stuff through one unit."

SHEAR



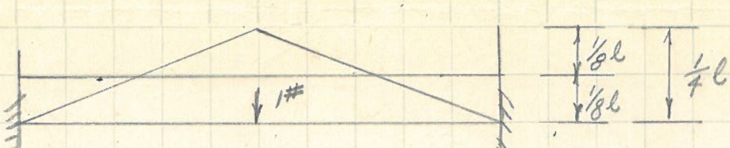
BENDING MOMENT

Rotate a unit angle.

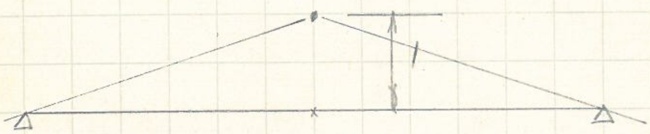


BENDING MOMENT

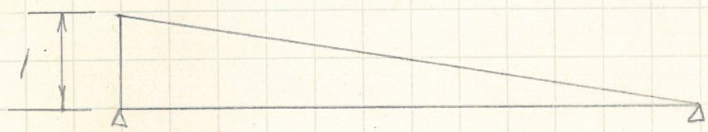
This is messy. Get scale from the following diagram. Place 1# load at point



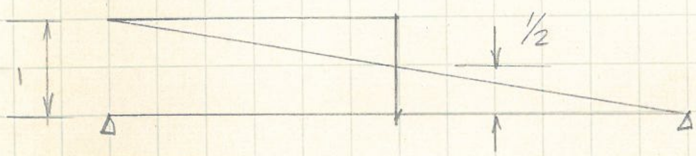
SHEAR AT CENTER



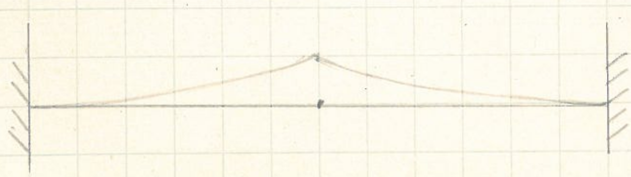
MOMENT



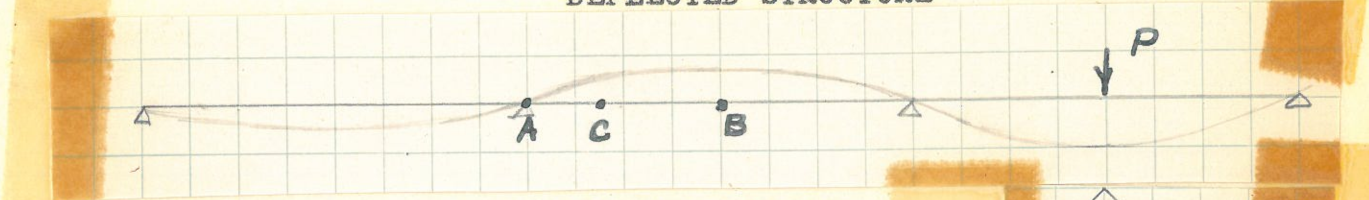
REACTION



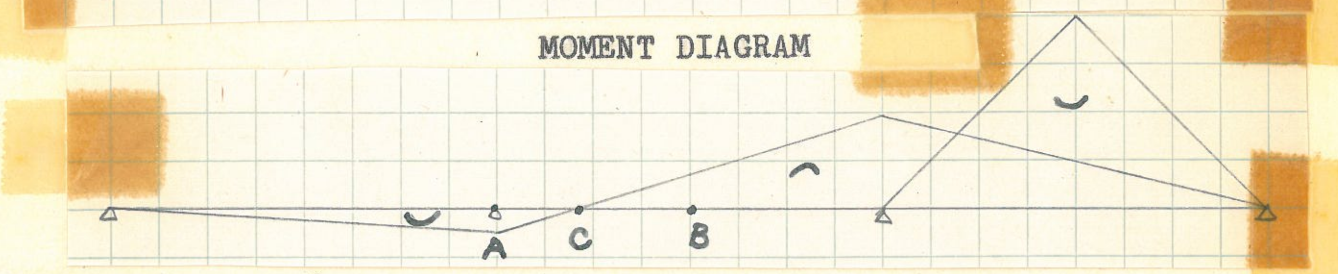
SHEAR



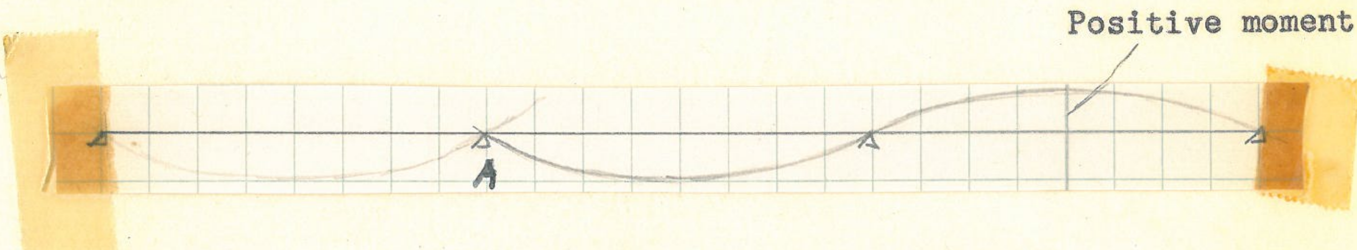
DEFLECTED STRUCTURE



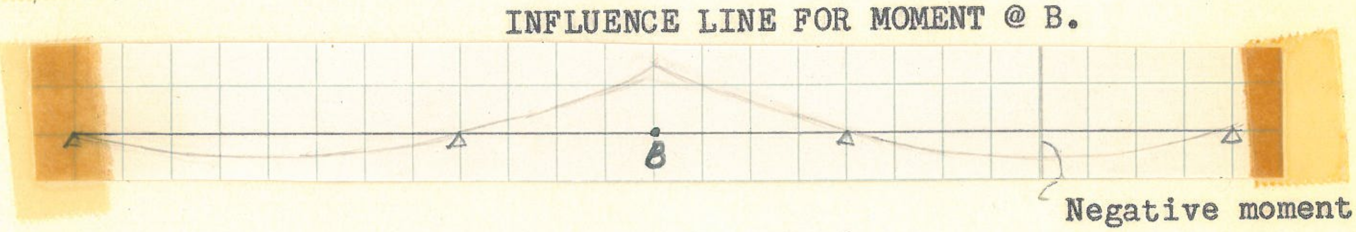
MOMENT DIAGRAM



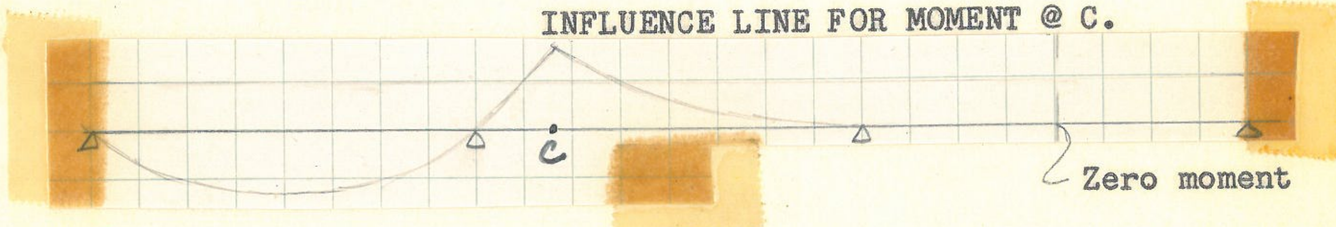
INFLUENCE LINE FOR MOMENT @ A.



INFLUENCE LINE FOR MOMENT @ B.



INFLUENCE LINE FOR MOMENT @ C.



C is the fixed point

To obtain a maximum at the support either load the whole span or none of it. The same thing is true in the middle span but nowhere else.

Although it is a matter of no importance, the fixed point of the span which is about the 2/10 point of the span should be recognized. The "maximum, absolute, theoretical" is slightly greater if the span is not fully loaded.

Even though it is artificial, the concept of influence lines is extremely useful. It will be used a great deal as a short cut. It is, as in most other cases, more easily done by breaking into simple steps. How to load may be seen by drawing either deflected structures or drawing influence lines. It is not too common to compute maximum loads from influence lines.

Later lectures will further instruct how to draw and interpret influence lines.

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Oct. 16, 1952
APP.

NAME
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1. D.L. = 1^k L.L. = 1^k Span length = 30'

(1)		(1)		(1)		
+75	-75					
-75	+37	+38				
+19	-38		+19			
-19	+19	+19	-9	-10		
+10	-10	-4	+10		-5	
-10	+7	+7	-5	-5	+5	
+3	-5	-3	+3	+3	-3	
-3	+4	+4	-3	-3	+3	
0	-61	+61	+15	-15	0	L.L.

		+75	-75			
	-37	-38	+38	+37		
-19		+19	-19		+19	
+19	-10	-9	+9	+10	-19	
-5	+10	+4	-4	-10	-5	
+5	-7	-7	+7	+7	+5	
0	-44	+44	-44	+44	0	L.L.

0	+15	-15	-61	+61	0	L.L.

+75	-75	+75	-75	+75	-75	
-75	0	0	0	0	+75	
	-37			+37		
	+19	+18	-18	-19		
+9		-9	+9		-9	
-9	+4	+5	-5	-4	+9	
+2	-4	-2	+2	+4	-2	
-2	+3	+3	-3	-3	+2	
0	-90	+90	-90	+90	0	D.L.

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2. D.L. = 1^k L.L. = 1^k Span Length = 30'

2		1		2		
+75	-75					
-75	+50	+25				
+25	-37		+12			
-25	+25	+12	-4	-8		
+12	-12	-2	+6		-4	
-12	+9	+5	-2	-4	+4	
+4	-6	-1	+2	+2	-2	
-4	+4	+3	-1	-3	+2	
0	-40	+42	+13	-13	0	L.L.

		+75	-75			
	-50	-25	+25	+50		
-25		+12	-12		+25	
+25	-8	-4	+4	+8	-25	
-4	+12	+2	-2	-12	+4	
+4	-9	-5	+5	+9	-4	
-4	+2	+3	-3	-2	+4	
+4	-3	-2	+2	+3	-4	
0	-56	+56	+56	-56	0	L.L.

0	+13	-13	-40	+40	0	L.L.

+75	-75	+75	-75	+75	-75	
-75					+75	
	-37			+37		
	+25	+12	-12	-25		
+12		-6	+6		-12	
-12	+4	+2	-2	-4	+12	
+2	-6	-1	+1	+6	-2	
-2	+5	+2	-2	-5	+2	
0	-84	+84	-84	+84	0	D.L.

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3. D.L. = 1^k L.L = 1^k SPAN LENGTH = 30'

① ② ①

+75	-75					
-75	+25	+50				
+12	-37		+25			
-12	+12	+25	-12	-9		
+6	-6	-6	+12		-4	
-6	+4	+8	-8	-4	+4	
+2	-3	-4	+4	+2	-2	
-2	+3	+4	-4	-2	+2	
0	-77	+77	+17	-13	0	L.L.

		+75	-75			
	-25	-50	+50	+25		
-12		+25	-25		+12	
+12	-9	-16	+16	+9	-12	
-4	+6	+8	-8	-6	+4	
+4	-5	-9	+9	+5	-4	
-2	+2	+4	-4	-2	+2	
+2	-2	-4	+4	+2	-2	
0	-33	+33	-33	+33	0	L.L.

0	+15	-15	-77	+77	0	L.L.

+75	-75	+75	-75	+75	-75	
-75	0	0	0	0	+75	
	-37			+37		
	+12	+25	-25	-12		
+6		-12	+12		-6	
-6	+4	+8	-8	-4	+6	
+2	-3	-4	+4	+3	-2	
-2	+3	+4	-4	-3	+2	
0	-96	+96	-96	+96	0	D.L.

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4. D.L. = 1^k L.L. = 1^k Span Length = 30'

	9		1		9	
+75	-75					
-75	+67	+8				
+34	-37		+4			
-34	+33	+4	0	-4		
+16	-17		+2		-2	
-16	+15	+2	0	-2	+2	
+7	-8		+1	+1	-1	
-7	+7	+1	0	+2	+1	
0	-15	+15	+7	-3	0	L.L.

		+75	-75			
	-67	-8	+8	+67		
-34		+4	-4		+34	
+34	-4	0	0	+4	-34	
-2	+17			-17	+2	
+2	-15	-2	+2	+15	-2	
-8	+1	+1	-1	-1	+8	
+8	-2			+2	-8	
0	-70	+70	-70	+70	0	L.L.

0	+3	-7	-15	+15	0	L.L.

+75	-75	+75	-75	+75	-75	
-75	0	0	0	0	+75	
	-37			+37		
	+33	+4	-4	-33		
+17		-2	+2		-17	
-17	+2			-2	+17	
+1	-8			+8	-1	
-1	+8			-8	+1	
0	-77	+77	-77	+77	0	D.L.

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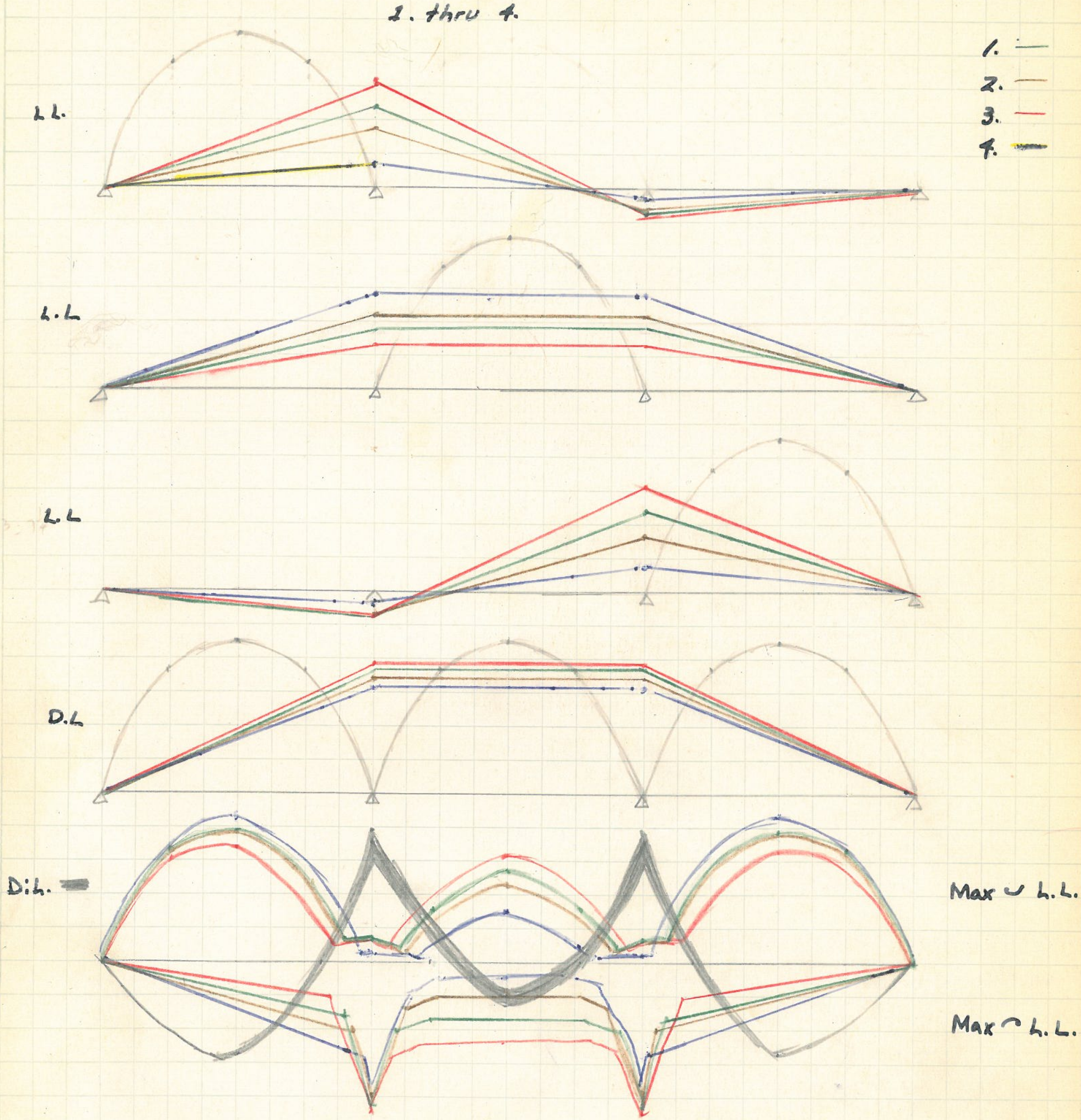
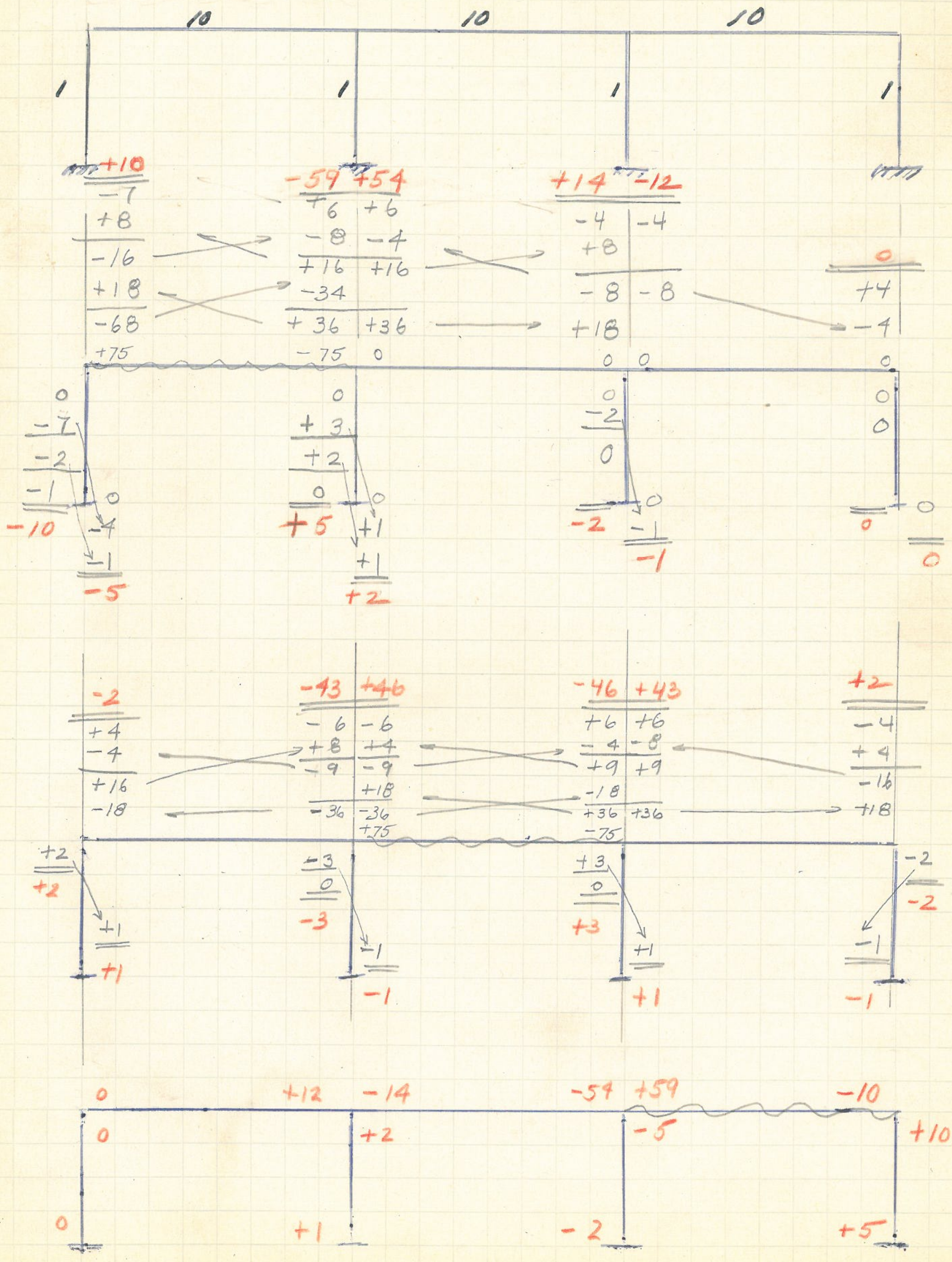
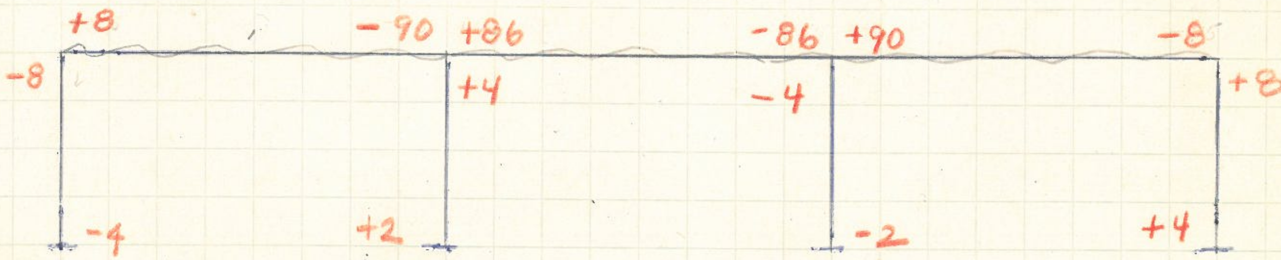


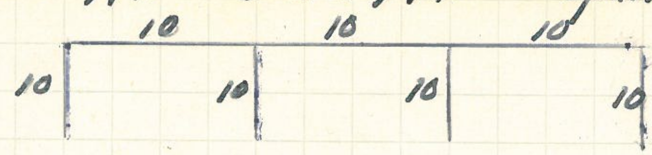
5. L.L. = 1^k/ft. D.L. = 1^k/ft. Span Length = 30'

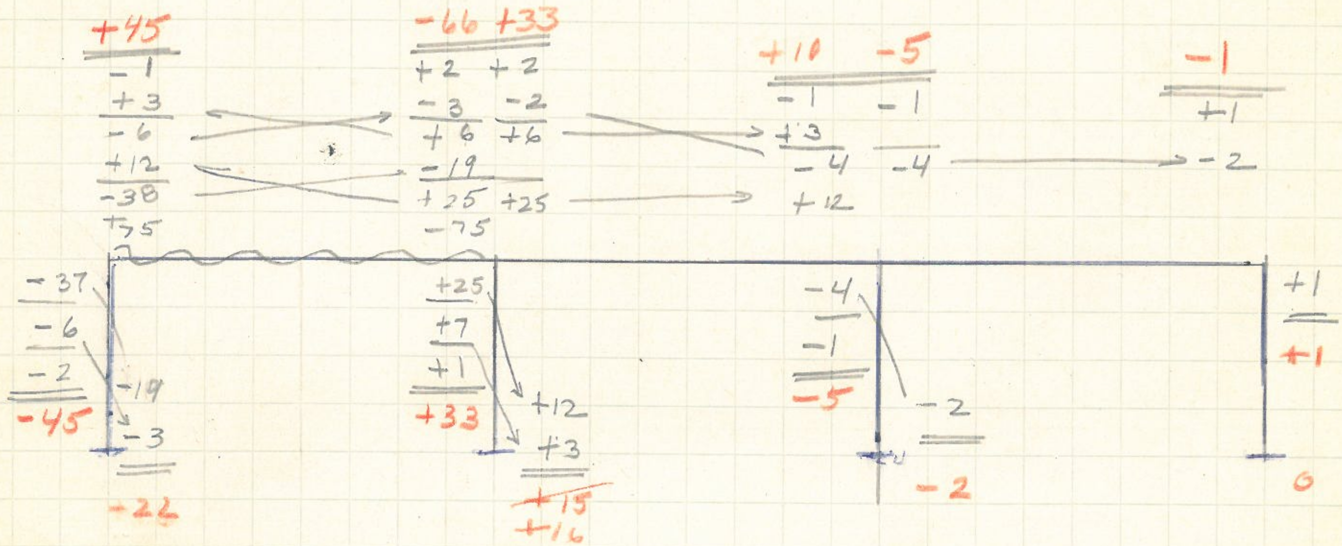


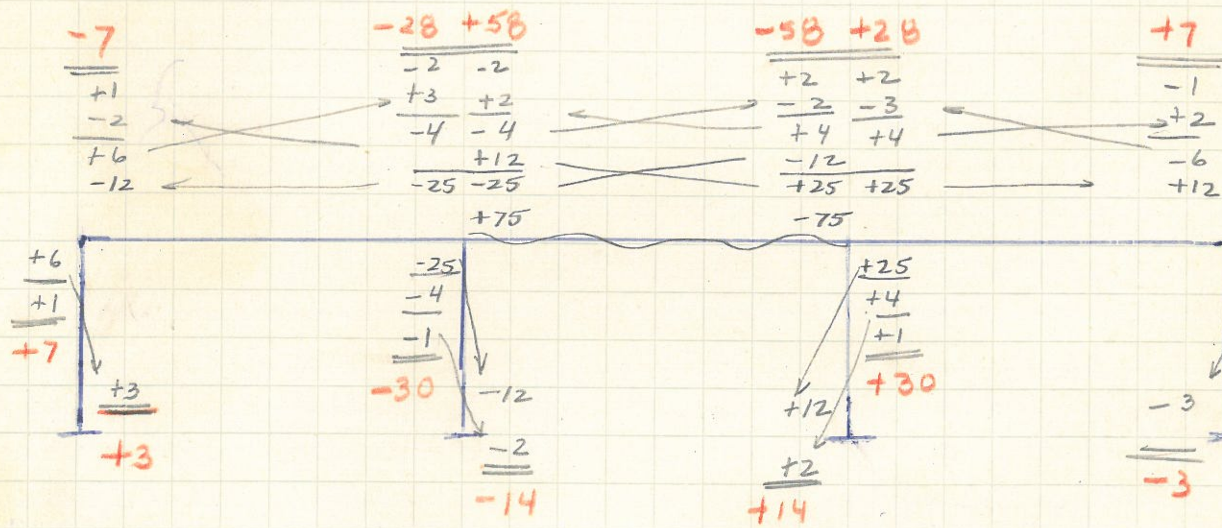
5. (continued)

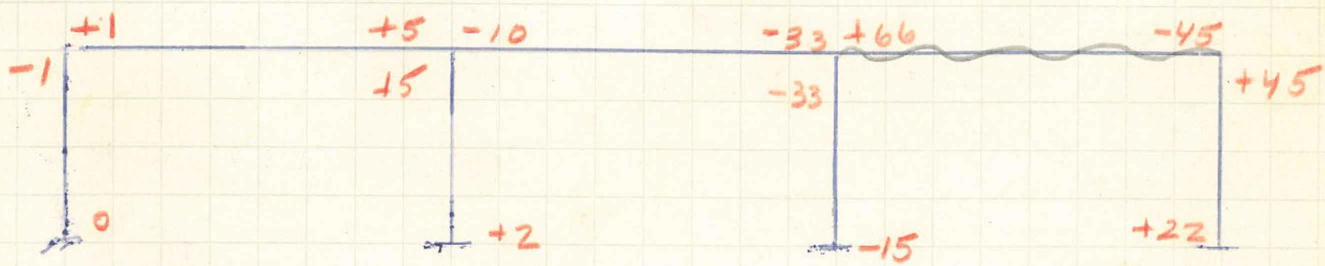


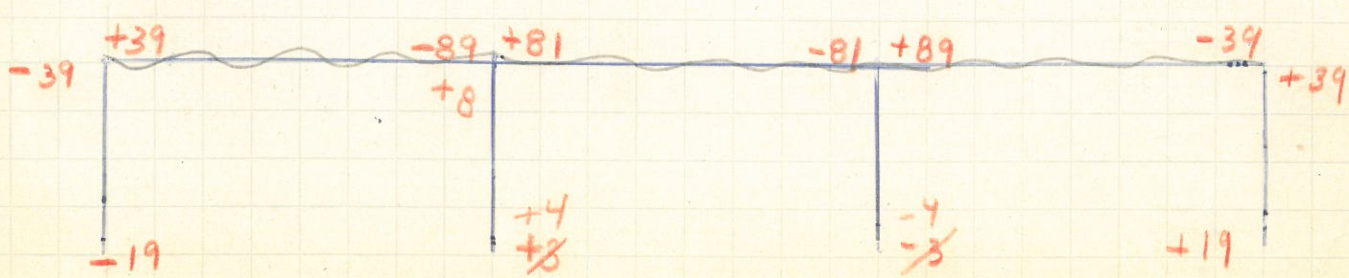
6 L.L. = 1^k/ft. D.L. = 1^k/ft. Span Length = 30 ft.











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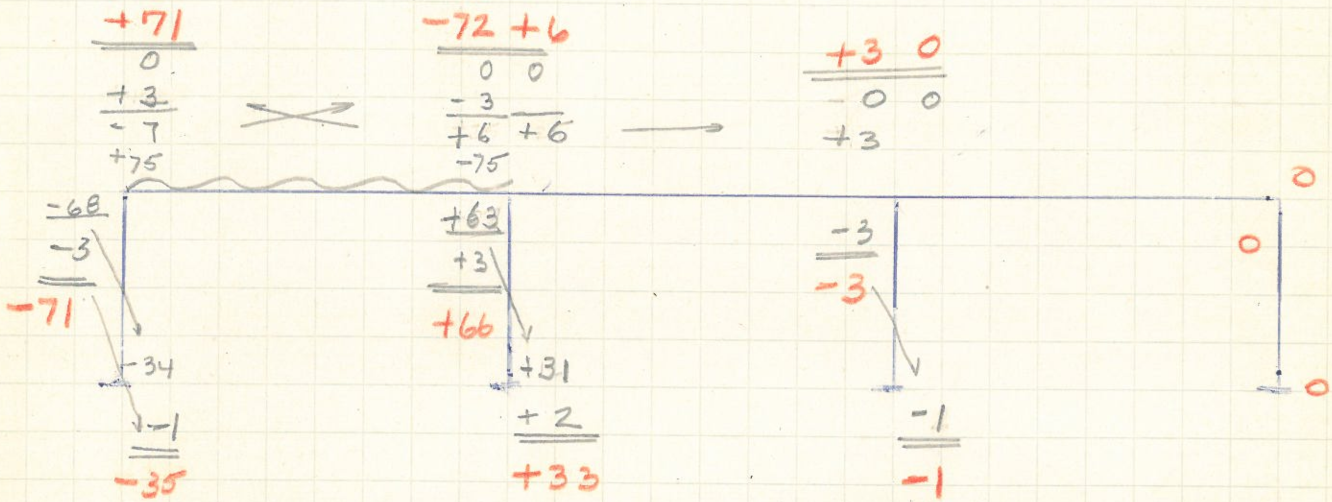
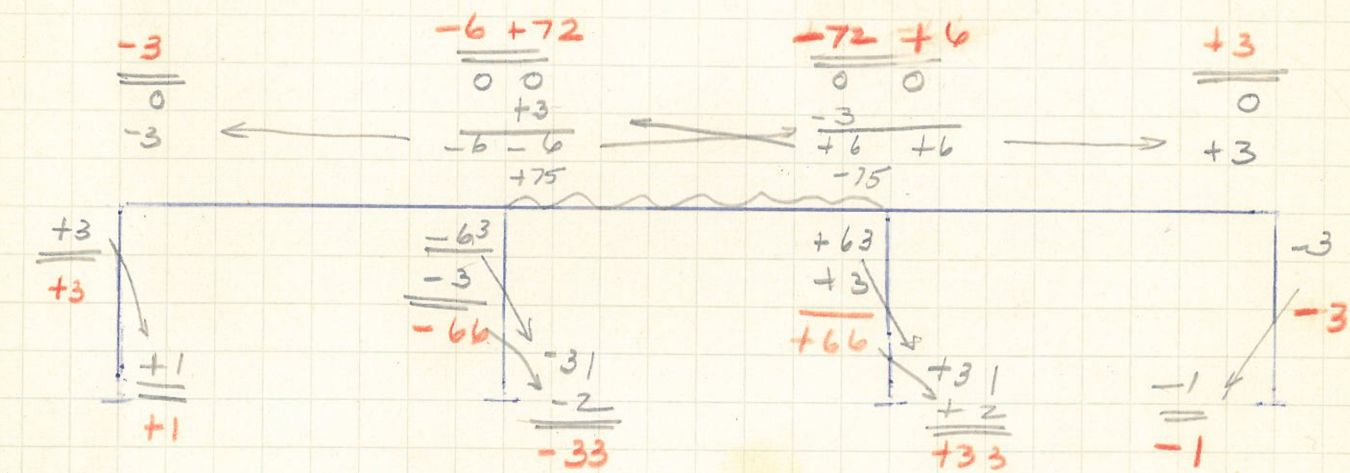
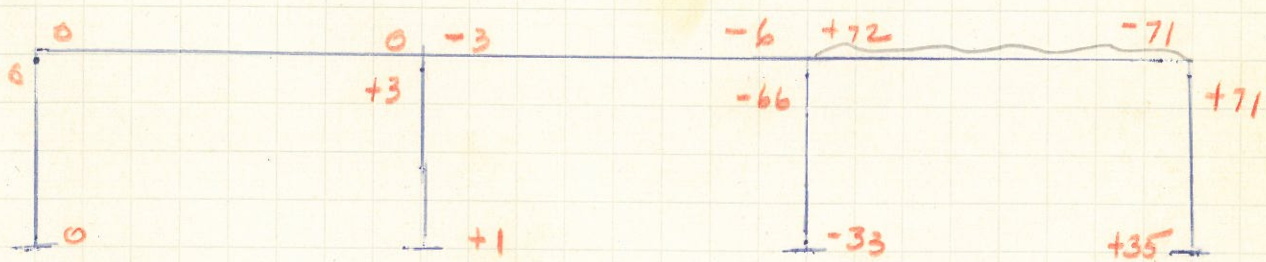
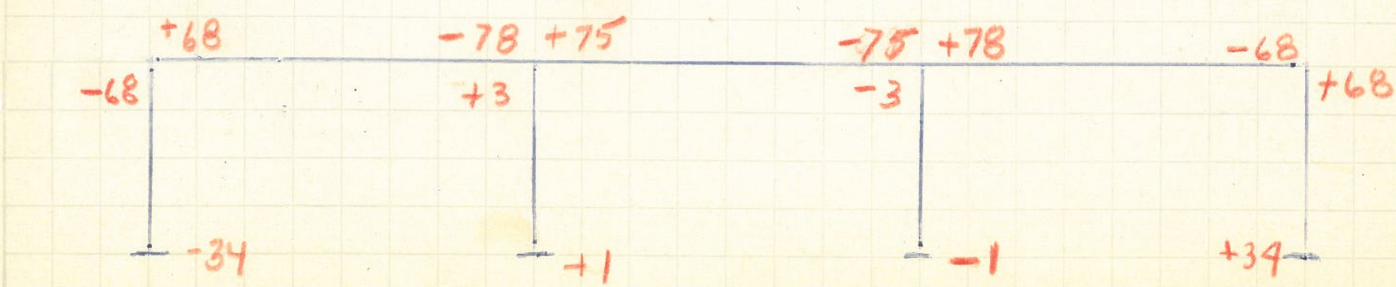
7.

Same span
& loading as
1 through 6









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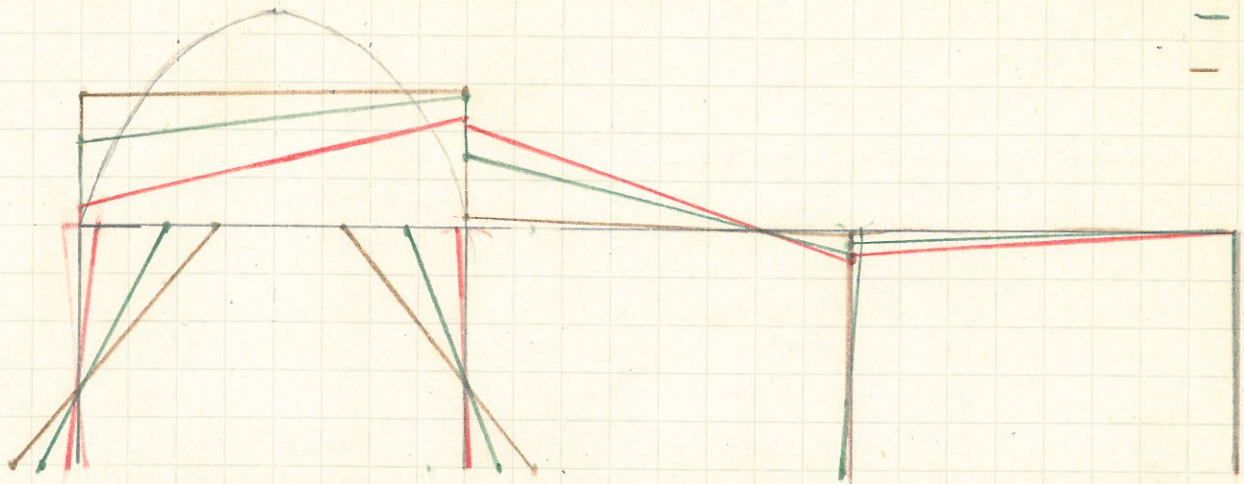
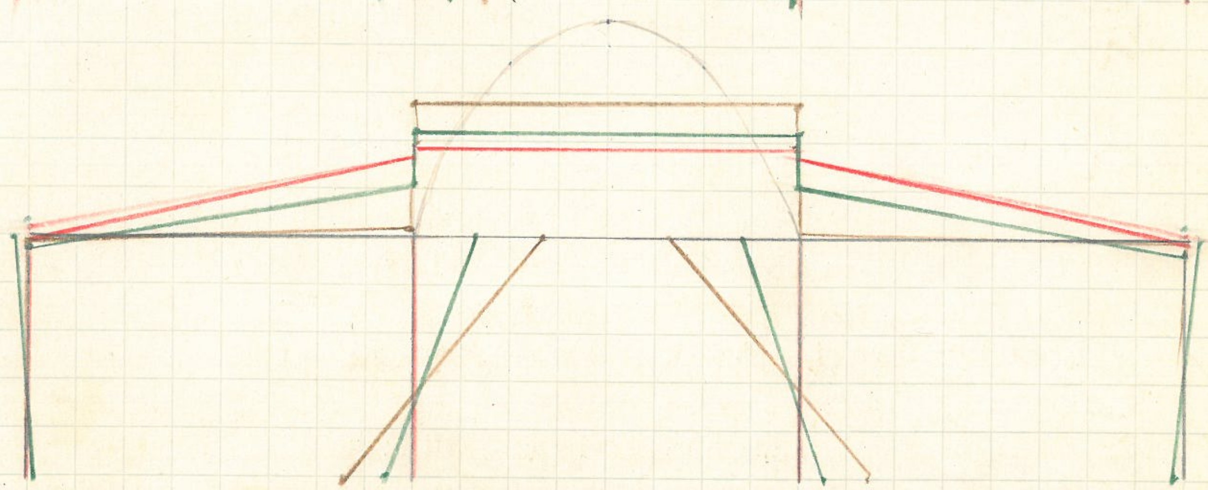
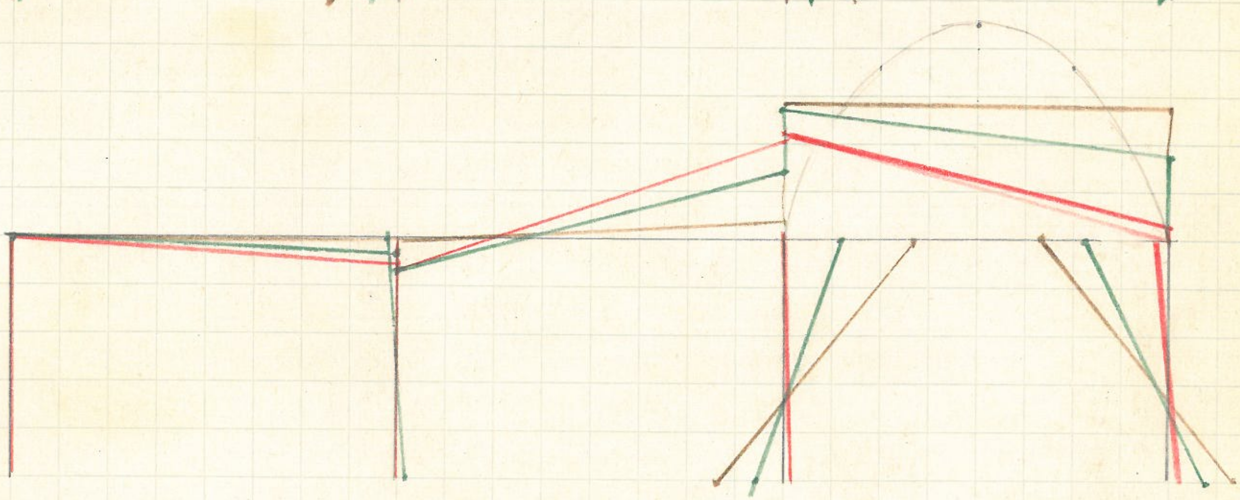
5
6
7



L.L.



L.L.



L.L.

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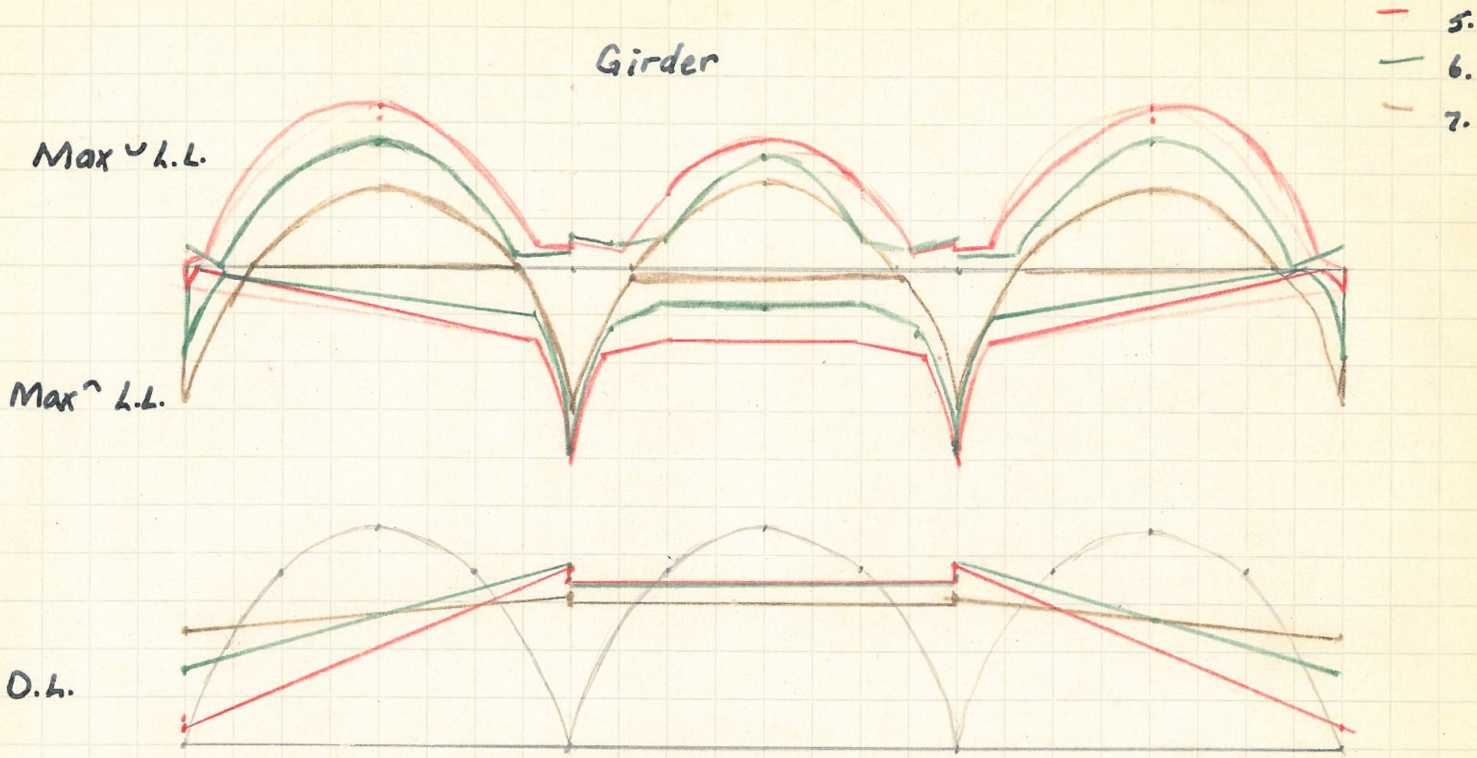
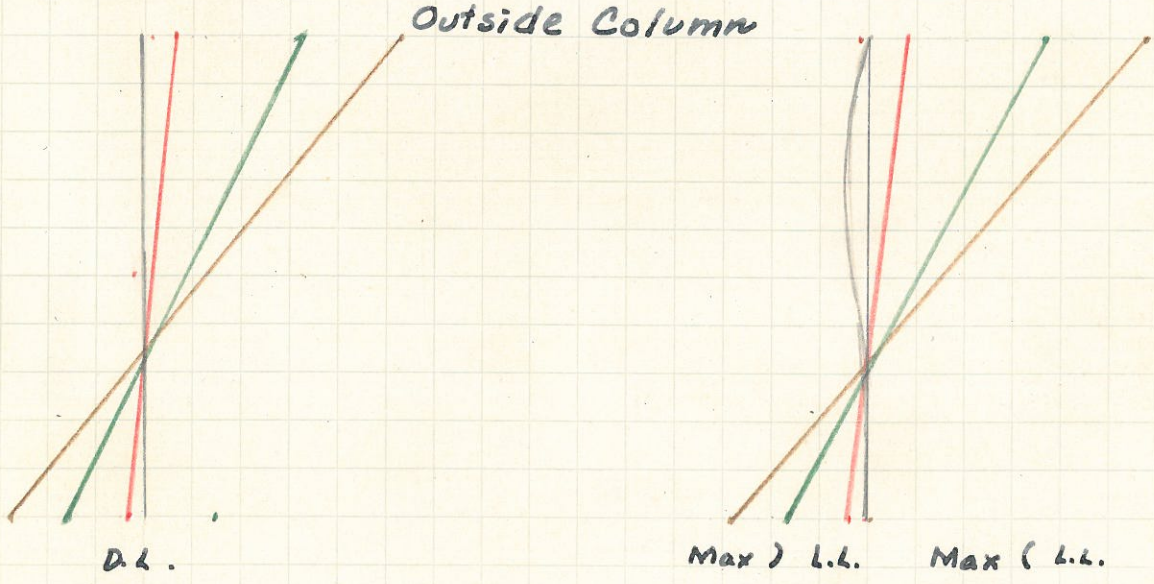


Observing these diagrams, the effect of stiffness on the maximum moments of the girder is small, whereas, the effect of the stiffness on the maximum moments of the outside column, as illustrated below, is quite pronounced. The interior column is also small.

Outside Column



The preceding lecture was concerned with influence lines. However, the discussion was premature because much of it should have been introduced in a lecture with Prof. Looney first.

"Research" is highly recommended for us. We should do intense experimenting with many structures trying to draw deflected structures and check by moment distribution. When this is first undertaken, comparisons may not be close, but after some practice the quick method will give the same results as moment distribution. This is not an argument against so-called "exact" analysis but, as has been emphasized often, the scale is what is wanted, and quickly. Once you have the structure it can be analyzed, then, hire a "routine man." He does not have to think since he uses standard assumptions or regulations.

Our job while at Yale is to get to the point where we can form our own judgement. There are a great many things that progress in structural engineering will create, thinner floors, longer spans, etc. This progress will be attained by dreaming up what is wanted, and seeing how to get these things. The main objective of analysis is not primarily economy of material, a little less steel or concrete. Nevertheless, it is a powerful sales argument. The agencies of the various industries, P.C.A., U.S.S., etc. are concerned with selling steel or concrete. A competitive system is excellent, but an engineer should remember he is representing a client who is entitled to impartial judgement.

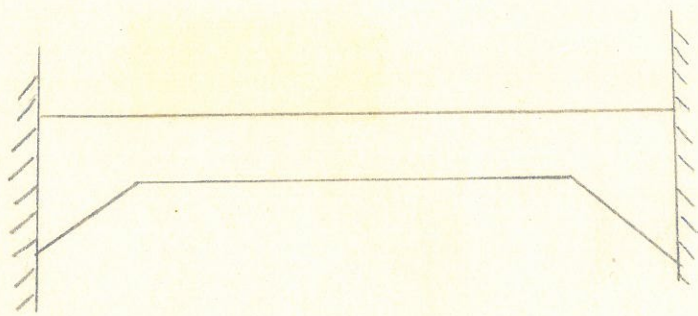
By experimenting, it will be found out quickly what trouble will be gotten into with ordinary concrete frames. The same answer will be attained by drawing either a deflected structure or by any other method because they are based on three assumptions:

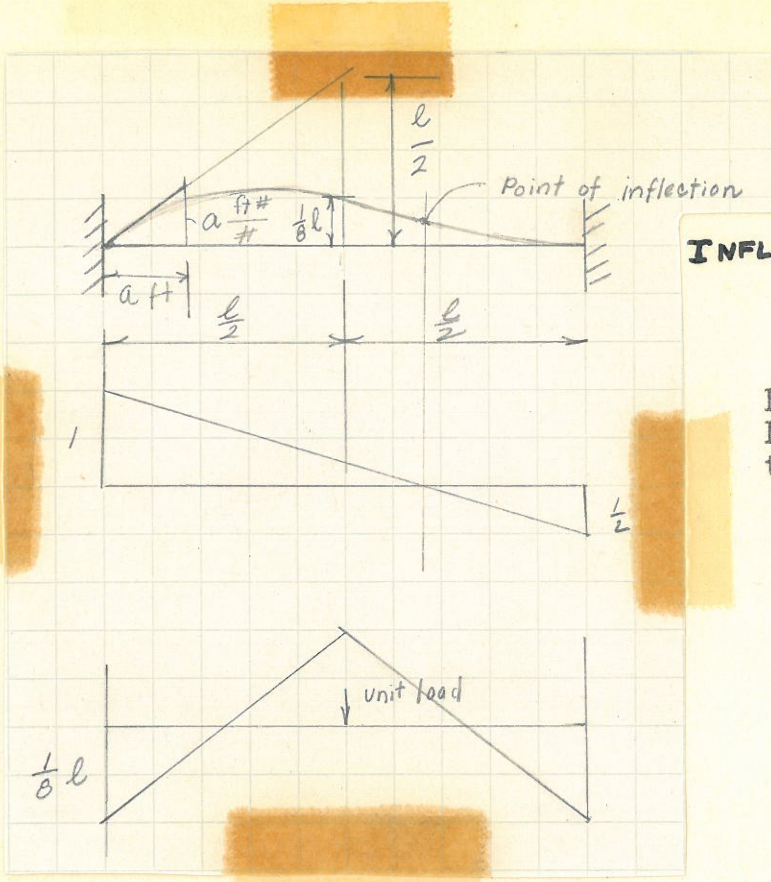
1. The statics must balance.
2. There is continuity so the deformations must be consistent.
3. There is a relationship between the curvature (rate of rotation or 2nd differential) and the moments.

We are not concerned with what rotation is produced by a moment, but what is the variation if there is a change of moments. In the plastic state, the relationship between curvature and moment is not proportional and must be found.

In the preceding lecture are certain constants for loadings in the case where the flexibility is constant throughout the beam. The stiffness is $4EI/L$. Although E is considered constant in most books, it is not at all constant in the structure. The carry-over factor equals 1/2.

This problem has many variations. The depth of haunch relative to the depth of beam, and the length of haunch relative to the length of beam on both sides can be varied. There are a limitless amount of answers possible. There are numerous tabulations of these constants which are largely worthless. Since all the tabulations assume rectangular section anyway, they are not at all precise as they appear to be. Some of these constants can be estimated as accurately by influence lines. However, they are not duplicable.

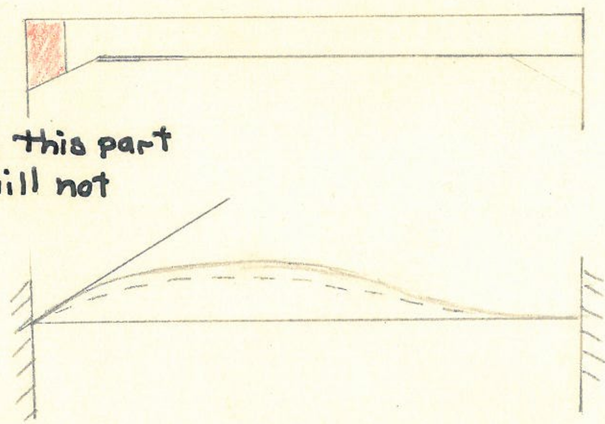




INFLUENCE LINE

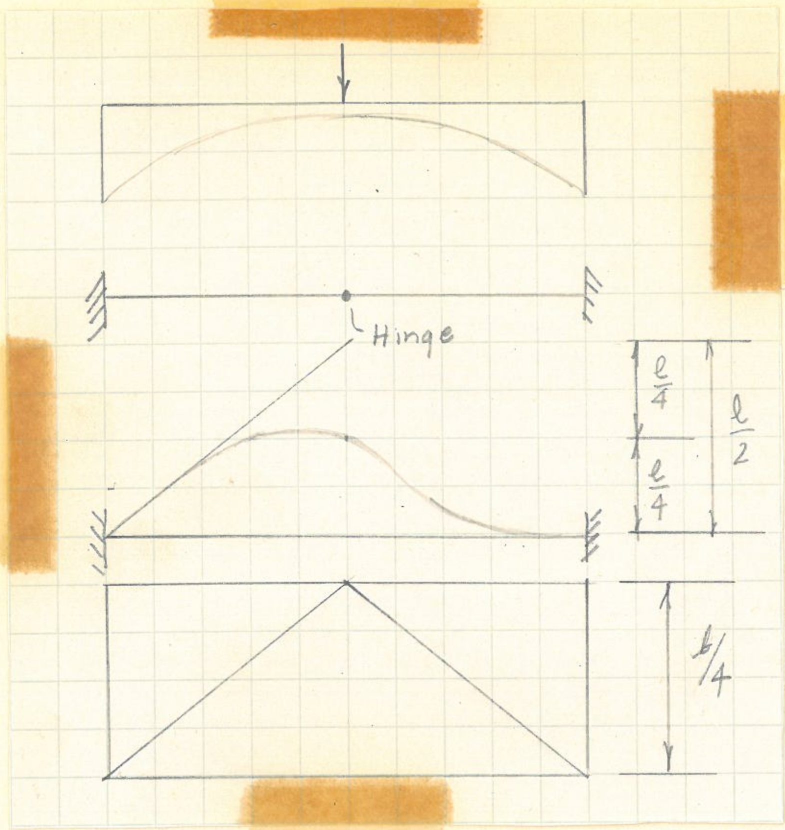
From the influence line, F.E.M. and carry-over Factors can be obtained.

Experimenting in moment distribution, it can be seen that it does not matter if the carry-over factor is not 1/2 but 0.4 or 0.6, but it will make a large difference if the carry-over factor is 0.9.



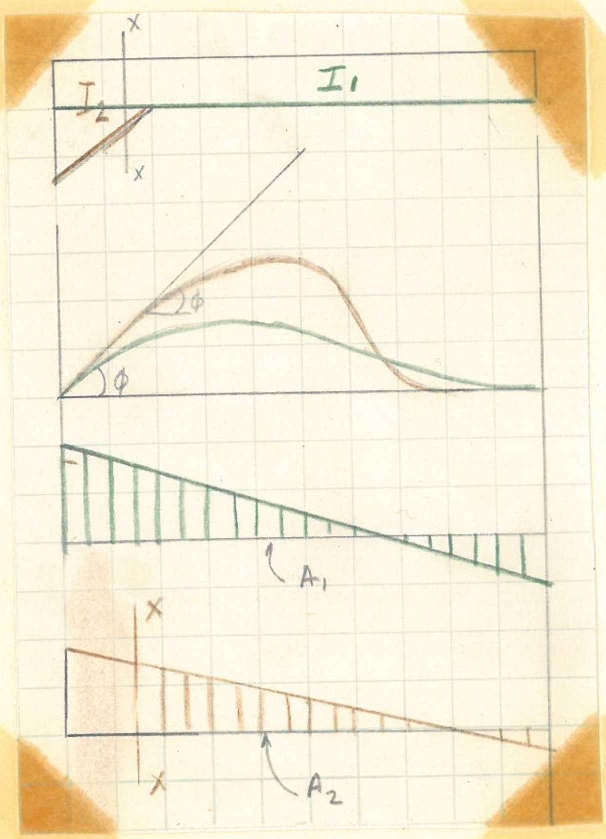
Assume this part of beam will not curve.

The influence ordinates and carry-over factors due to haunching can be compared with those obtained if the beam were of the same cross-section throughout.



Hinge in the Middle.

STIFFNESS

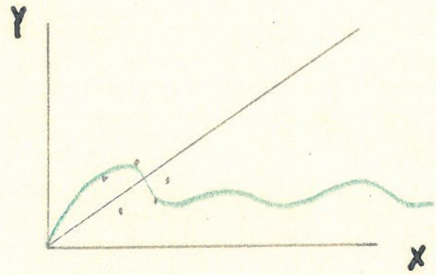


— Uniform Section

— Haunched Beam

Since the angle changes are the same the moment areas are the same. However, it takes a greater moment to produce the angle change in the haunched beam. If the moments are the same, the moment area of the haunched beam is smaller than the moment area of the uniform beam. Stiffness $\propto \frac{M}{\phi}$. The rotation and stiffness are greater in a uniform beam than in a haunched beam.

By the method described a good estimate of relative stiffness can be had. Research will indicate it does not make a difference how stiff thecolumns are to the girder, but it makes a large difference to the column how stiff the girder is. Much published research is poor because experimenters are in the habit of assuming a straight line where there is none.



There is a danger of extrapolating data beyond the range of the experiment.

In summary, make some estimate of the relative stiffness of beams and columns by drawing deflected structures, or by the concept of influence lines.

The next question to be answered is how much steel is going to be needed and where should it be put. Almost all concrete books emphasize that the stresses in concrete are found. It is not necessary to find the stresses in concrete. We want to know if the stresses are too high, and if so what to do about it, or, if too low, can advantage be made of it. It does not matter one bit should the stress be too low if no advantage can be made of it. Later, to satisfy the "old lady" (boss, building commissioner, etc.) figures must be put down.

There are three questions to be answered:

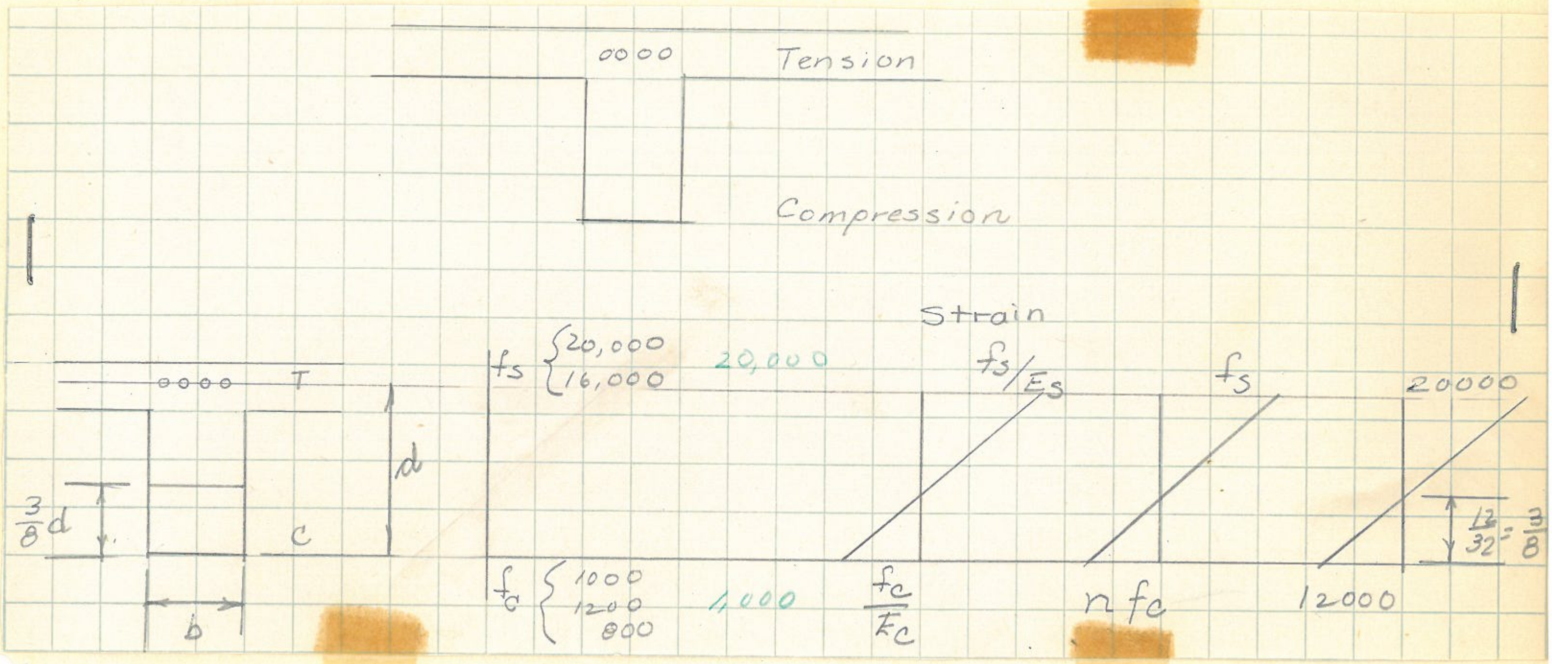
A. Compression in concrete -- fc

B. Tendency of bar to slip in concrete (Bond Stress). -- u

C. Maximum shear -- fv

COMPRESSION IN CONCRETE

Invariably, the question of compression in concrete is only of interest when there are negative moments.



Strain $= \dfrac{fs}{Es}$ by definition

E_s = 30 million psi approximately

Strain $= \dfrac{fc}{E_c}$ The ratio of stress to strain of concrete is not known. It will vary with the intensity of stress, with the duration of stress, and with the age of the concrete. Figure the strains as though they were linear across the section.

Multiply both strains by E_s

$$\frac{fs}{E_s} \times E_s = fs$$

$$\frac{fc}{E_c} \times E_s == fc \; \frac{Es}{E_c} = fc\ n$$

Some sources say fcn = 12000 psi.

T in steel = fs As

C in concrete = 1/2·fc·bkd = (in the above case) 1/2·1000·b· 3/8 d

If T = 150 & C = 100 Overstressed   If T = 150 & C = 200 Understressed

There is little advantage to balancing C & T. If steel were inexpensive and concrete expensive, it would be a great saving to use steel and conserve on concrete. So, relative cost is a great factor. A criterian can be set up for under and over stress.

When
$$T = fs\ As = C = 1/2\ K\ Acfc,$$
we will have balanced reinforcement.

Therefore,

$$\frac{As}{Ac} = 1/2\ K\ \frac{fc}{fs}$$ fc and fs are given, K is known from simple proportioning

Let Pc be the critical ratio which will be called a critical percentage.

Then
$$Pc = \frac{As}{Ac} \quad \& \quad Pc = 1/2\ K\ \frac{fc}{fs}$$

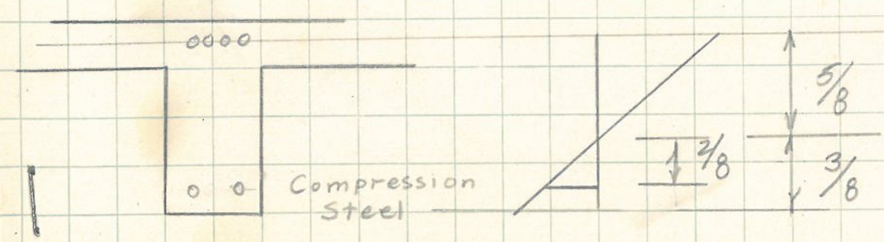
Pc will be about 1-1/4 or 1-1/2 or maybe 1-3/4. If the percentage is higher than the concrete is overstressed.

WHAT TO DO IF CONCRETE IS OVERSTRESSED

1. Make the beam wider.
   Since the bars have already been properly spaced, and the cover is just enough so that the bars will not corrode (cover depends on conditions of exposure), it does not appear proper to make the beam wider.

2. Make the beam deeper.
   Although less steel would be required, since the depth of beam may have been selected for reasons of architecture, it may not be advisable to alter the depth of beam.

3. Put bars in compression.



Stress in compression steel = $\frac{2/8}{5/8}$ = 2/5 of stress in tensile steel

Most regulations are inconsistent and forget about proportioning and just pick a value for compression steel.

The compression in concrete plus the compression in compressive steel equals the tension in tensile steel, or:

$$C_{conc} = T_{steel} - C_{steel}$$

$$T - C_s = C_c$$

If $C_s = 10{,}000$ then $\frac{C_s}{T} = 1/2$

and $\frac{A_s - 1/2\, A_s'}{A_c} \leq P_c$ $\quad A_s$ = area tension steel
A_s' = area compression steel

This method is very good, but, since there are bars top and bottom, it is not necessary to add new bars, just to extend the bars so that they run through and anchor on the other side. Then, the question is resolved to whether running the bars through is enough. It usually is.

BOND

This was first written up at the turn of the century by Talbert in the Western Society of Engineers Journal.

The bond formula, one of our "sacred cows":

$$u = \frac{V}{\Sigma\, ojd}$$

Let n = number of bars

$$V = \Sigma\, ojd \cdot u$$

$$V_{max} = \pi\, D \cdot n \cdot j \cdot d \cdot u$$

$$M \text{ capacity} = As\, fs\, jd = \frac{\pi\, D \times \frac{D}{4} \times n \cdot fs\, jd}{\pi \cdot D \cdot n \cdot j \cdot d \cdot u}$$

$$\frac{M \text{ capacity}}{V \text{ max.}} \geq \frac{fs}{4u} \cdot D \qquad ; \frac{fs}{4u} \text{ is about 25 or 30.}$$

With equal spans and uniform load

$$M = 1/12 \cdot WL \quad \& \quad V = 1/2 \cdot W$$

$$\text{so that } \frac{M}{V} = 1/6\, l$$

$$\text{and } 30\,D = 1/6\, l \quad \text{or } D < \frac{1/6\ l}{30}$$

$$< \frac{l}{180}$$

With 1" bars a span of 15 feet or less would be necessary for bond to be critical. Generally, it is not necessary to worry about bond in other than very short spans. Just look at structure and make a mental computation to see if there is trouble. If there is use smaller bars and more of them.

However, we do not have an allowable value for bond. About all that can be done is to make some beams and find out when the bars slip. Then, set up some criterian of danger. Put loads on the beam and place values in a formula. Suppose there are two formulas, A, and B.

A.

$$u = \frac{V}{bjd}$$

Assume u is found by formula A to be 200 psi

In our real structure we know V and compute by formula A. u to be 100 psi

B.

$$u = \frac{2V}{bjd}$$

Assume u is found by formula B to be 400 psi

In our real structure we know V and compute by formula B.u to be 200 psi

It is obvious that it does not make any difference if the wrong formula is employed since the same material will be used regardless.

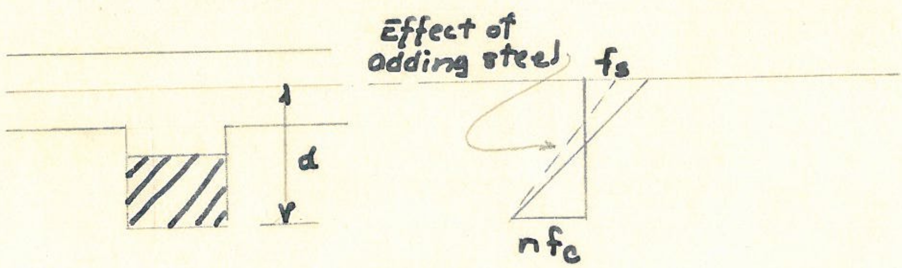
Today the discussion of proportioning of concrete will be continued. We will use only orthodox methods that are presented in texts and used by industry today. The orthodox method is a little bit absurd, but it does not matter as will be seen later.

If we worked out the formula for bond stress and found it to be, $U = \frac{V}{\Sigma ojd}$, it would not be worth five cents unless an engineer can determine how much bond stress is allowable. Bond cannot be measured. Therefore, a criterion must be selected that will show when the limiting value of shear is reached. Then, substitute in formula $u = \frac{V}{\Sigma ojd}$. Obtain u and divide by a factor of safety. Imagine the answer to be 200. Then, assuming formula $u = \frac{V}{\Sigma ojd}$ is not right, but $u = \frac{2V}{\Sigma ojd}$ is right, make the same test and substitute into the new formula and divide by the same factor of safety. Then, u will be 400 if it were 200 the first time.

Since the same formula as was used to compute the allowable stress will be used in the real structure, the amount of material used will be the same regardless if the right or the wrong formula is used.

In computing concrete stress we will stick exactly to textbooks or the existing theories of computing stress. This is a repetition of what was said in the last lecture. Knowing allowable steel stress, knowing allowable concrete stress, and knowing the ratio of steel to concrete, a critical percentage can be set up for steel.

The following diagram shows the effect of adding steel:

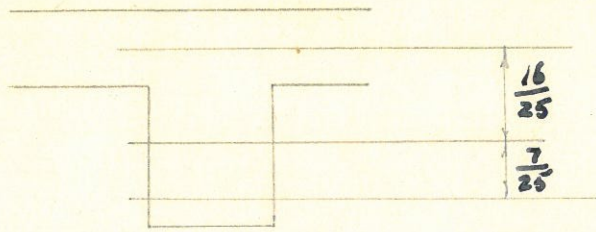


To revise a building code costs a great deal of money. It involves having a large commission to revise the code, advertising in newspapers, public hearings and the. That is why the New Haven code has not been revised for thirty or more years. Under this code fs = 16,000 psi, n = 15 and fc = 600. Therefore, nfc = 9000

$$\text{and } T = 16000\,A_s$$
$$C = \frac{600}{2} \times \frac{9}{25}\,bd$$
$$\frac{A_s}{A_c} = \frac{1}{160}$$

The critical percentage is very low.

A new ratio must be found if compressive reinforcement is used. Old codes do their figuring according to the theory of proportionality:



Stress in compressive steel $= \frac{7}{16} \times 16000 = 7000$. In later codes a fixed stress is specified, of 12,000, or 14,000.

$$\frac{A_s^{\text{tension}} - \frac{7}{16} A_s^{\text{compression}}}{A_c} < Pc$$

With $A_c = bd$
& $Pc = 0.0075$

BOND

n = number of bars $\qquad A_s = n \frac{\pi D^2}{4}$

$$\frac{M = fs \cdot A_s \cdot jd}{V = u \sum o \cdot jd}$$

$$\frac{M}{V} = \frac{f_s \cdot n \cdot \frac{\pi D^2}{4} \cdot jd}{u \cdot n \cdot \pi \cdot D \cdot jd} = \frac{fs}{4u} \cdot D$$

$\frac{fs}{4u} \cdot D$ must be greater than M/V to be in trouble in bond.

Taking a simple case

$M = 1/12\ W\ell$ & $V = 1/12\ W$

$\frac{M}{V} = \ell$ so that $\frac{M}{V}$ is primarily a variation in span length.

There may be trouble with bond where the ratio of bar size to length gets large, i.e., only in short spans. There will be trouble if the moment is small and the shear large. Therefore, if the beam is short use more and smaller bars. With a uniform load on a cantilever beam and l the length of the beam, then

$$M = 1/2\, \ell\, W \quad \& \quad V = W$$

$$\frac{M}{V} = 1/2\, \ell$$

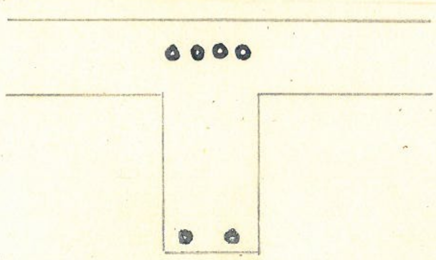
There is, therefore, more likelihood of failure with a cantilever beam. Bond is of importance also in spread footings where it is not a good idea to use many small bars. The alternative is to anchor the bars.

COMPRESSIVE STRENGTH IN CONCRETE

Let n = number of bars in a row.

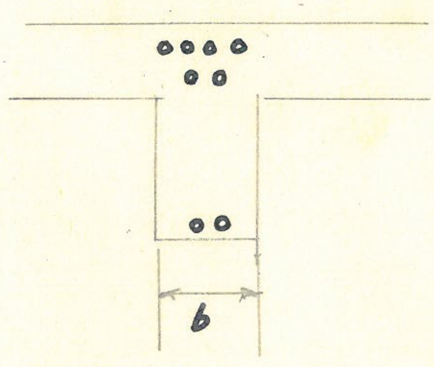
N = number of rows.

N' = number of rows not "balanced" by compressive reinforcement.



$n = 4$
$N = 1$
$N' = 3/4$
$nN' = 3$

Assume 20,000 in tension and 10,000 in compression so that 1 bar on the bottom equals 1/2 bars on top.



$n = 4$
$N = 1\ 1/2$
$N' = 1\ 1/4$
$nN' = 5$

$$A_s' = n \frac{\pi D^2}{4} N' \quad \& \quad A_c = \pi n D d$$

$$\frac{A_s'}{bd} = \frac{nN' \frac{\pi D^2}{4}}{\pi n D d} = \frac{N' D}{4d} = Pc$$

$b > \pi n D$

$\frac{A_s'}{bd} < Po$

Trouble can be expected when the ratio of the depth of beam to the diameter of the bars is small, unless $b > \pi n D$. Also, there will be trouble if there is a large N'.

SUMMARY

1. Look for trouble in bond where the shear is high compared to the bending moment. This usually means short beams or cantilevers.

2. Look for trouble in compression in concrete on short spans with shallow beams.

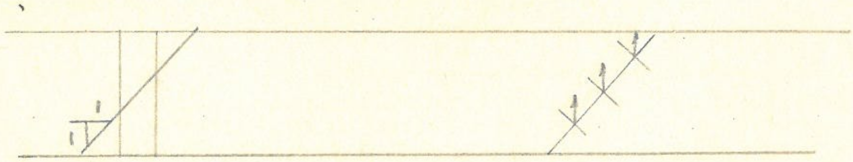
We should learn how to proportion a structure and foresee what trouble is expected and know how to get out of trouble. Later we can put down calculations so as to satisfy "granma". Before analyzing a structure, it must be obtained first. The span, load, reinforcement, and width must be decided upon first by someone before the compressive strength is computed. Also, someone had to select an allowable compressive stress, too.

SHEAR OR DIAGONAL TENSION

Sometimes specifications say in a long-winded way what can be said very quickly. Both ways will arrive at the same answer, but since the long-winded way has been used for many years it would cause too much confusionif the wording were changed.

In most American codes shear and stirrups are handled in a roundabout way. $\frac{V_{max}}{jbd}$ shall be limited. If 1/3 the maximum shear that the concrete can resist is not exceeded, stirrups are not required. If this limit is exceeded but the maximum is not, i.e., it lies between the two limits, stirrups are required, but the amount of shear the concrete can resist is deducted and the stirrups resists the remainder. However, the allowable stress in the stirrups is reduced.

In Britain, more sensibly, if the shear the concrete can resist as above is exceeded, stirrups must be used, but if they are used they must resist all the shear. However, the same stress as in the main reinforcement is used in the stirrups.



Every 45 degree crack must have enough stirrups to take residual stress. In inclined bars, stirrups must take vertical components.

$$M = f_s \cdot A_s \cdot jd$$

a = any distance along beam

V' = shear not carried by concrete.

$$V' a = f_v \cdot A_v jd.$$

Prof. Cross
Oct. 28, 1952

The "grandmas" in engineering are the many organizations and individuals which establish many rules and customs for engineers to follow. These include the following and many more:

- Local building codes
- Specifications and recommendations of the Bureau of Public Roads
- of AREA
- of AASHO
- of Joint Committee
  - ASCE
  - AIA
  - ACI
  - ASTM & Others
- Employer
- Architect

To make up something like the ACI building code, several men, representatives of the different phases of the reinforced concrete industry, meet and contribute towards the making of a code. These men include contractors who are not theorists but know what can be built, manufacturers of reinforcing bars, gravel and stone interests, and manufacturers of cement. Should one of these introduce a new clause it may very well seriously affect another. The chances are it will be successfully opposed but it may slip by if someone sleeps and so put some group out of business. These codes cannot be analyzed on a blackboard.

Engineers build things even if they know nothing about it. Occasionally something falls down and a law suit follows. This is usually when rules get changed.

Back before the First World War they were building reinforced concrete without knowing what they were doing. Steel tonage varied 100 percent. Engineers in competitive bidding can say they can build a project with half as much steel. We could do it too and invent stuff to justify it. Fortunately, in our country the engineer and the contractor are separate which is not so in European countries; to counteract this dangerous competition the Joint Committee met about 1906.

During the First World War someone got the idea of building ships out of concrete. In Norway where steel was difficult to obtain these boats were successful, and they were not an utter failure here. Because of this, a high-powered government sponsored research lab was set up at Lehigh. If the current regulations ~~were~~ had been used, the boat would probably sink. So, much higher allowable stresses were permitted. After the war a second Joint Committee met and issued a relatively poor report. About the same time Frank McMullen, who has been associated with Turner Construction Co., University of Minnesota, and the Portland Cement Association, made studies of concrete and recognized that concrete shrinks as it sets. He noticed that if a load is applied to concrete and the deformation at the end of one year is one inch, then at the end of two years it may be three inches. This is a time-yield effect. Also, he saw that if a one pound load causes a deformation of one inch it did not follow that a two pound load results in two inch deformation. In concrete, therefore, there is yield due to plasticity, a time-yield effect, and, to further complicate the matter, shrinkage.

$$\frac{E_s}{E_c} = n$$

While the modulus of elasticity of steel, E_s, can be found with a reasonable amount of certainty, it is complete nonsense to believe there is an E_c and, therefore, an n. In the old specifications columns were handled in this manner.

$$P = f_c A_c + n f_c A_s$$

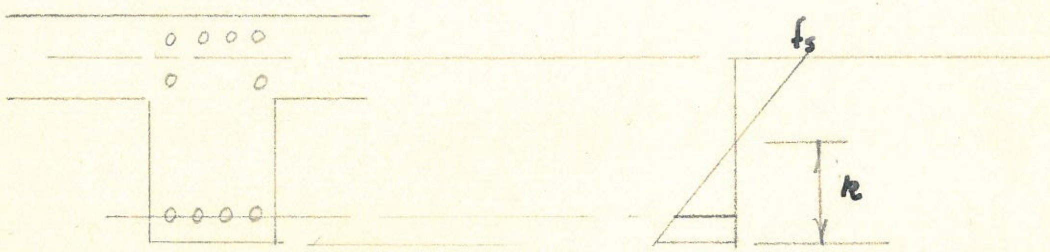
but if $n = 15$ and $f_c = 600$

then $nf_c = 9000$

However, the new specifications say:

$$P = f_c A_c + f_s A_s$$

For beams, as has been stated earlier, it is handled this way:



Returning to the first Joint Committee, we see they did a remarkable job with what little they knew. No one at that time could figure a continuous concrete beam. However, they came up with a useful specification. In ordinary cases, in which the span lengths are nearly alike and the ratio of the dead load to the live load is not greater than 3 to 1 or 1 to 3, the negative moment at the interior support is $\frac{WL}{12}$, at the end support is $\frac{WL}{10}$, and the positive moment at the center of the span is $\frac{WL}{12}$. The L here is measured center to center. In ordinary buildings the ratio of dead load to live load is usually only 2 to 1 or 1 to 2. In these specifications the stiffness of the columns were ignored. However, some accounting was made for the effect of haunches. When there were haunches the clear span for which the structure would be designed would be measured from a point of the haunch where the thickness of the haunch was 1-1/3 times deeper than the depth of the beam. However, from that point towards the center of the span the concrete would be designed as if it all were of uniform thickness.

The second Joint Committee, consisting of college professors, began to vary the column stiffness. Of some humor, is that the first committee said if the structure were not ordinary to make special investigations. At that time an investigation would take years.

The third Joint Committee began to treat structures as rigid frames.

The existing ACI code says that if the structure is to be treated as a rigid frame, ie., bending in the column, you can choose any relative stiffness you like provided you figure the stress on the basis of the same stiffness.

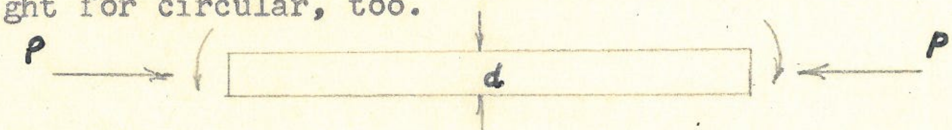
A column in a rigid frame structure is partly a column and partly a beam. It was attempted to work out a specification that would have a rule for proportioning one such structure that would cover the range from all beam, no column to no beam, all column. This was not entirely successful. The n was left in the beam and left out of the column. An n was assumed somewhere between the n in the beam and the equivalent n in the column.

$$P = f_c A + f_s A_s$$

Let $f_c = 600$ and $f_s = 18{,}000$

Therefore, the equivalent $n = 30$

Here is an excellent way to handle beam-columns, although it is not exactly what codes say. Although for rectangular columns, it is all right for circular, too.



a means axial

b means bending

$$\frac{P}{f_c^a} = A^a \qquad \frac{\frac{6M}{d}}{f_c^b} = A^b$$

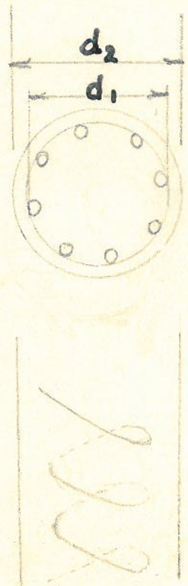
$$A^a + A^b = A^t = A_c + nA_s$$

where n is the adjusted value.

$$n = \frac{n^a A^a + n^b A^b}{A^t}$$

SPIRAL COLUMNS

Our "grandma" has had certain prejudices. One of them has been the spiral column. Seeing that the high strength spiral steel keeps the column that is beginning to crush from spreading out, she has permitted higher allowable stress for spiral columns. There is a tendency, much more so in spiral columns, for the concrete outside the reinforcing bars to come off. The question is then whether to use the whole area or just the area of the concrete within the spiral.



d_1	d_2	Error
10"	12"	40%
30"	32"	neglegible

REVIEW

Bond is of special concern in short spans. If there is trouble, put in a greater number of bars. Compression in concrete is important if the beam is relatively shallow, ie., relative to the diameter of the bar. If there is trouble in compression, put in more compression bars.

NAME

DATE

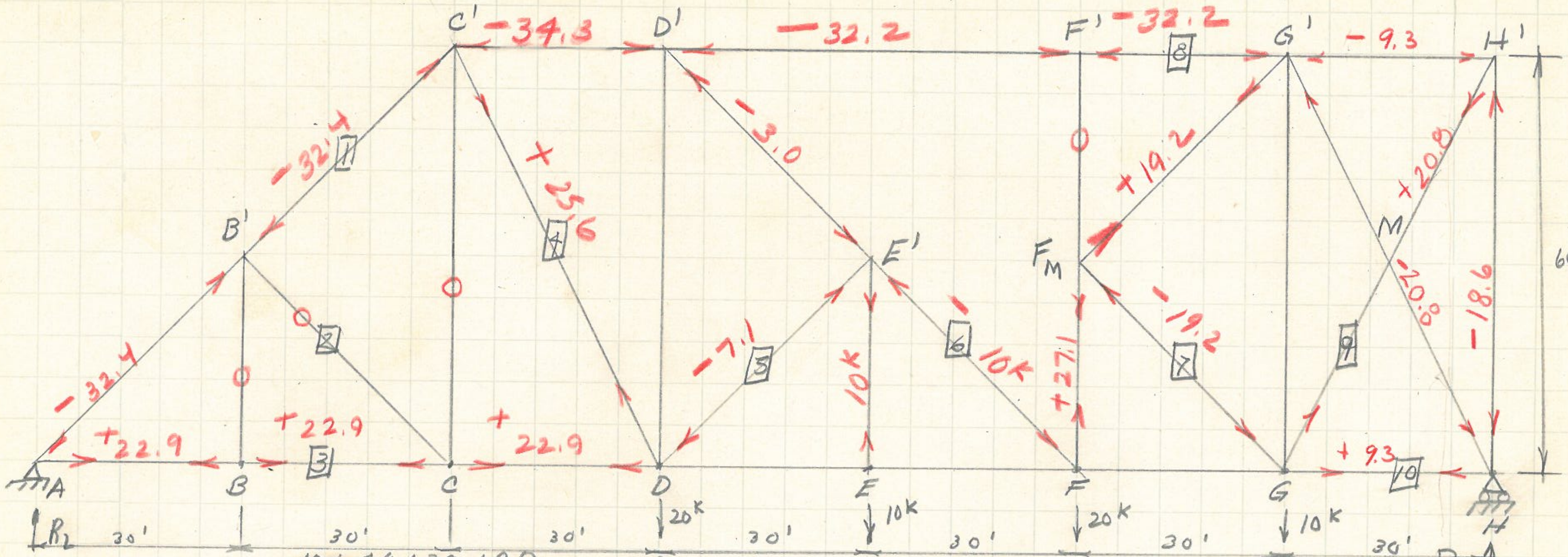
EXAM - PROF. CROSS
Oct 30, 1952
Robert A. Heller

Prof. Cross - Exam

NAME Robert Heller

DATE Oct. 30, 1952

Prob 1



$R_L = \dfrac{10+40+30+80}{7} = \dfrac{160}{7} = 22.9 \qquad R_R = 37.1$

$AB' = \sqrt{2} \cdot 22.9 = 32.4$

$B'C' = 32.4$

$AB = 22.9$

$BC = 22.9$

$CD = 22.9$

$B'C'\text{ vert} = 22.9 = C'D\text{ vert}$

$C'D = 22.9 \dfrac{\sqrt{5}}{2} = 25.6$

$C'D' = \frac{1}{2} \cdot 22.9 + 22.9 = 34.3$

$DE' = 10/\sqrt{2} = 7.1$

$D'F'$

$= \dfrac{22.9 \times 5 - 20 \times 2 - 10 \cdot 1}{2} = \dfrac{64.5}{2} = 32.2$

$D'E'\text{ hor.} = 1.1 \quad D'E' = 1.1\sqrt{2} = 3.0$

$E'F = \dfrac{10}{\sqrt{2}} + 3.0 = 10^k \qquad 30 - 22.9 = 7.1\text{ vert comp.}$

check $E'F = 10^k$

$FF_M = 20 + 7.1 = 27.1$

$F_MG_{vert} = \dfrac{27.1}{2} = 13.6$

$F_MG = 13.6 \times \sqrt{2} = 19.2$

$F_MG' = 19.2$

$F'G' = 32.2$

Assume

$GH' = G'H$

then $GH'_{vert} + G'H\text{ vert} = 37.1$

$\dfrac{37.1}{2}\sqrt{5}\Big/2 = GH'_{vert} = G'H = 20.8$

$GH = G'H_{hor} = \frac{1}{2}\, 18.6 = 9.3$

$\boxed{1}$	32.4 C
$\boxed{2}$	0
$\boxed{3}$	22.9 T
$\boxed{4}$	25.6 T
$\boxed{5}$	7.1 C
$\boxed{6}$	10.0 C
$\boxed{7}$	19.2 C
$\boxed{8}$	32.2 C
$\boxed{9}$	20.8 C
$\boxed{10}$	9.3 T

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Prob 2

II Draw deflected structures [no scale] and curves of moment [with scale]. [State whether approximate or exact]

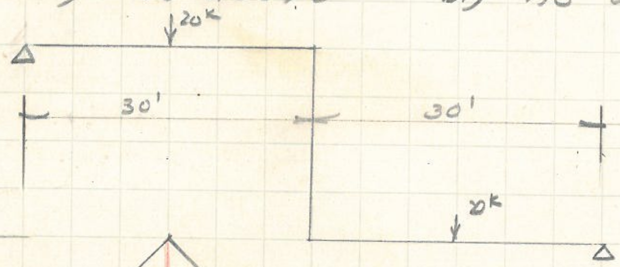
A. Assuming B can move freely. (on rollers)

a) Assuming constant stiffness

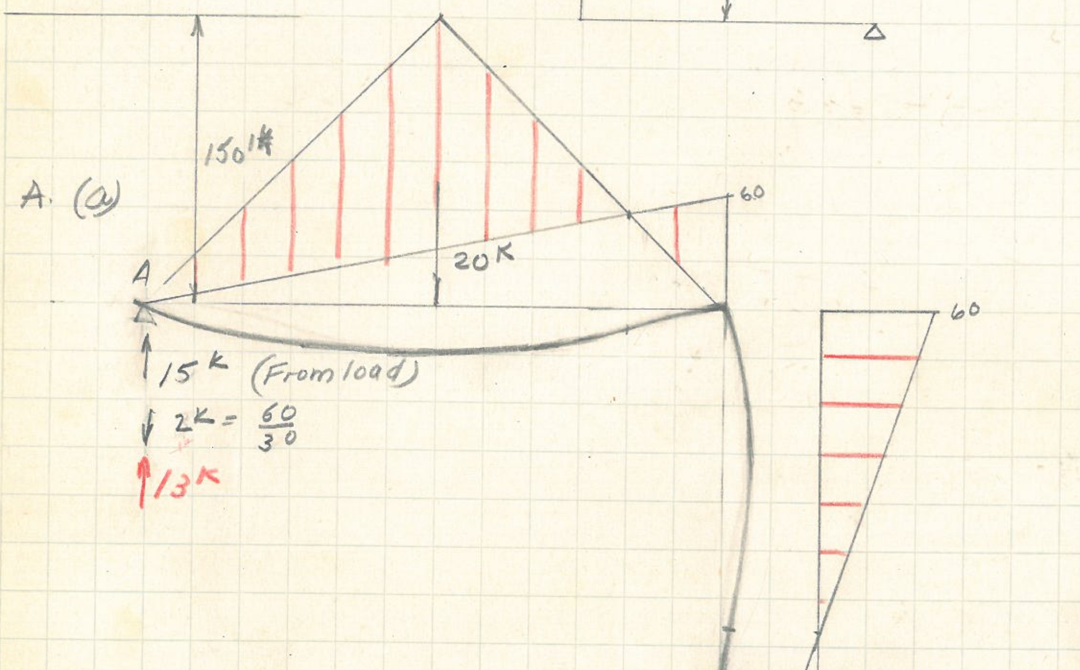
b) " vertical member infinitely stiff.

B Assuming B cannot move at all. (pin connected)

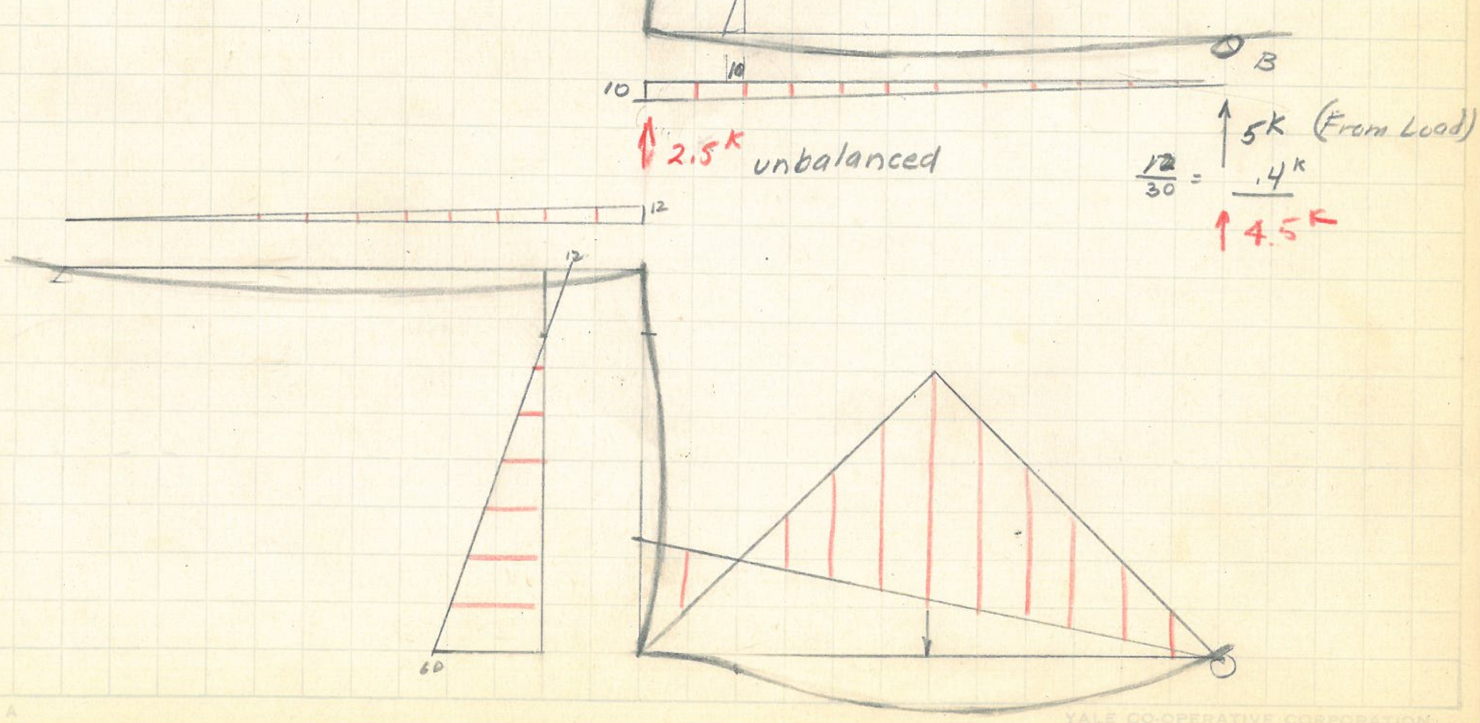
a.) As above b.) As above.



A. (a)



If B is on rollers and only one of the 20k loads were applied the structure is unstable. The moments at the columns' ends must be the same



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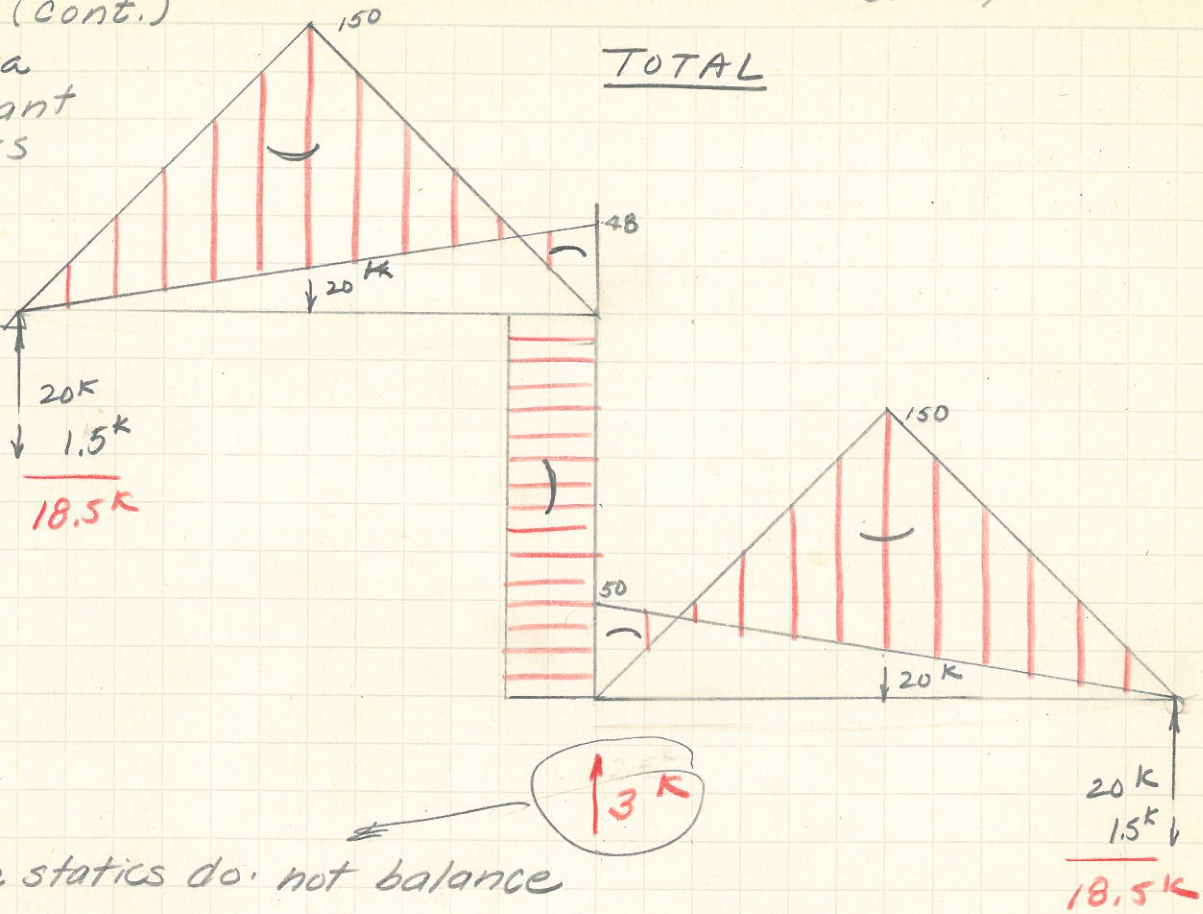
Prob. 2 (cont.)

II. Aa
constant
stiffness

TOTAL



The statics do not balance

II Ba & II Bb

Pinning B will not affect the moments.

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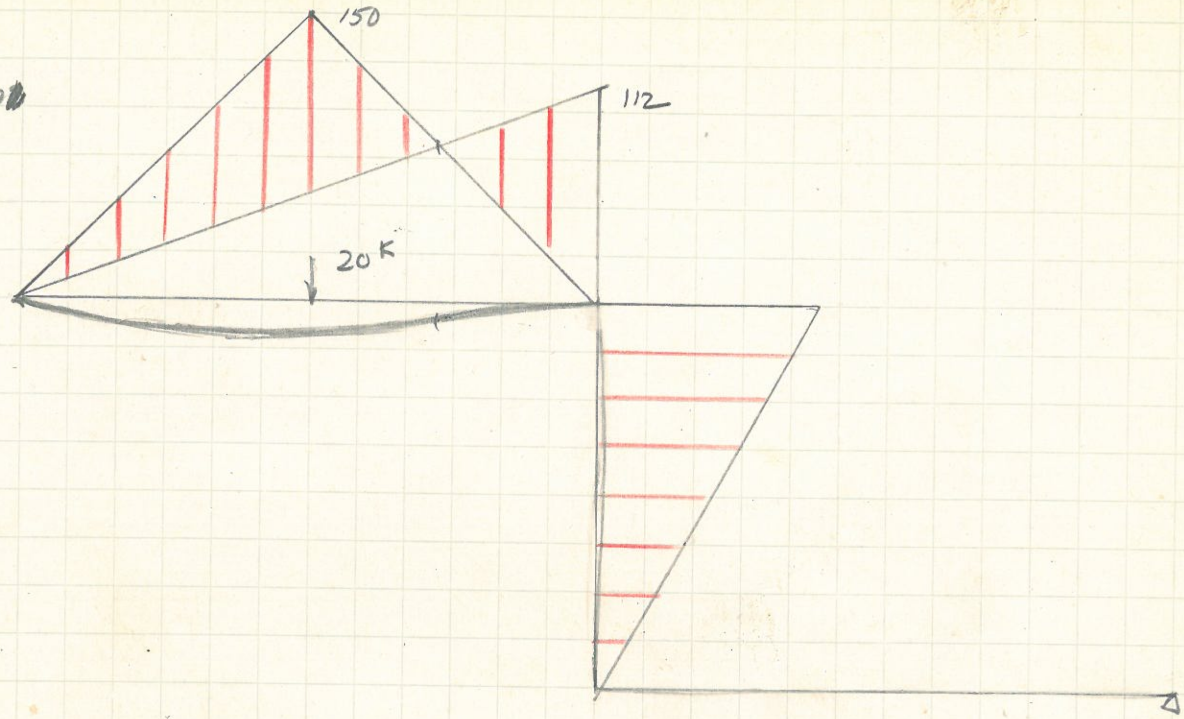
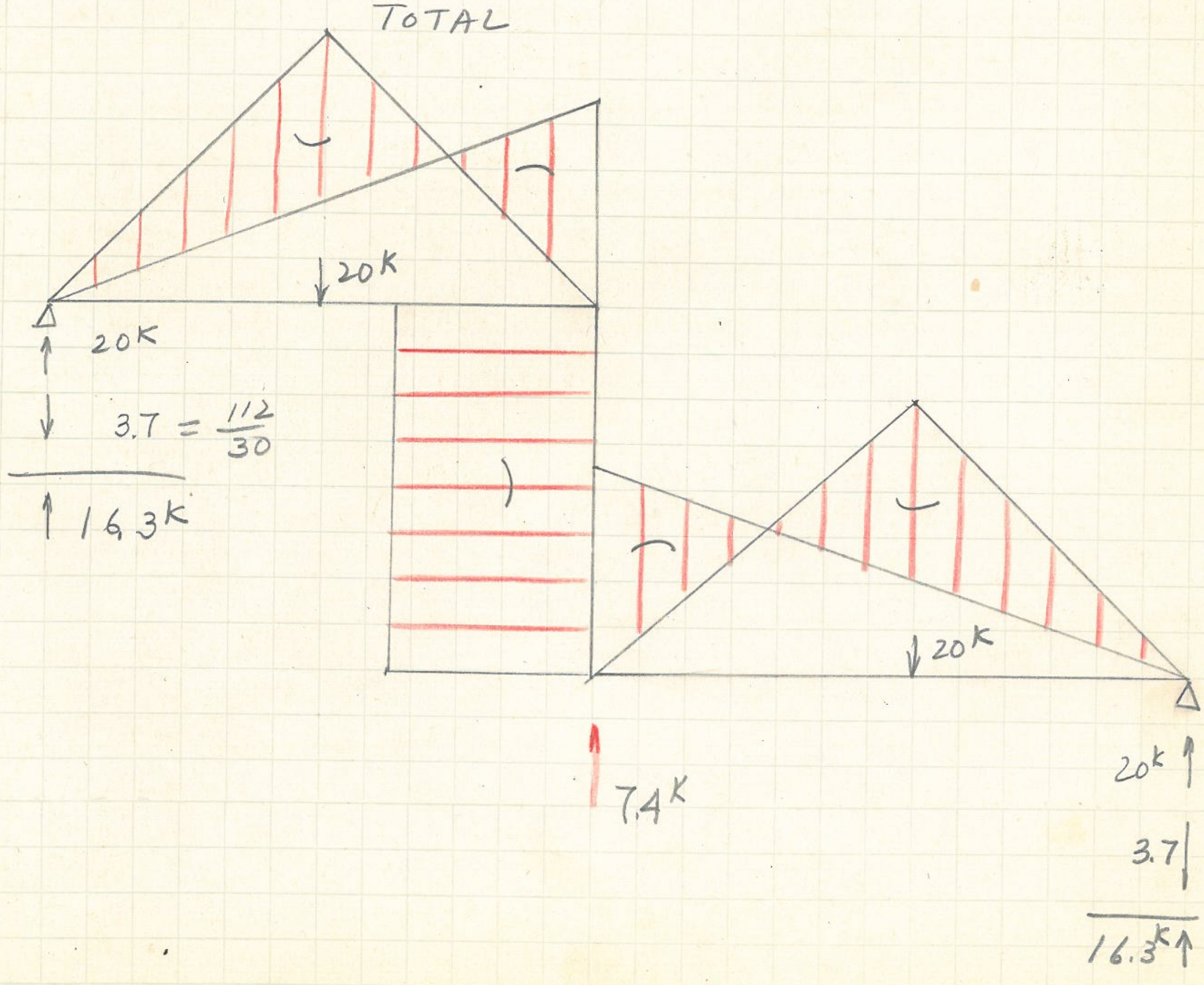
Prob. 2 (cont.)

Ab.
Column stiff.



TOTAL



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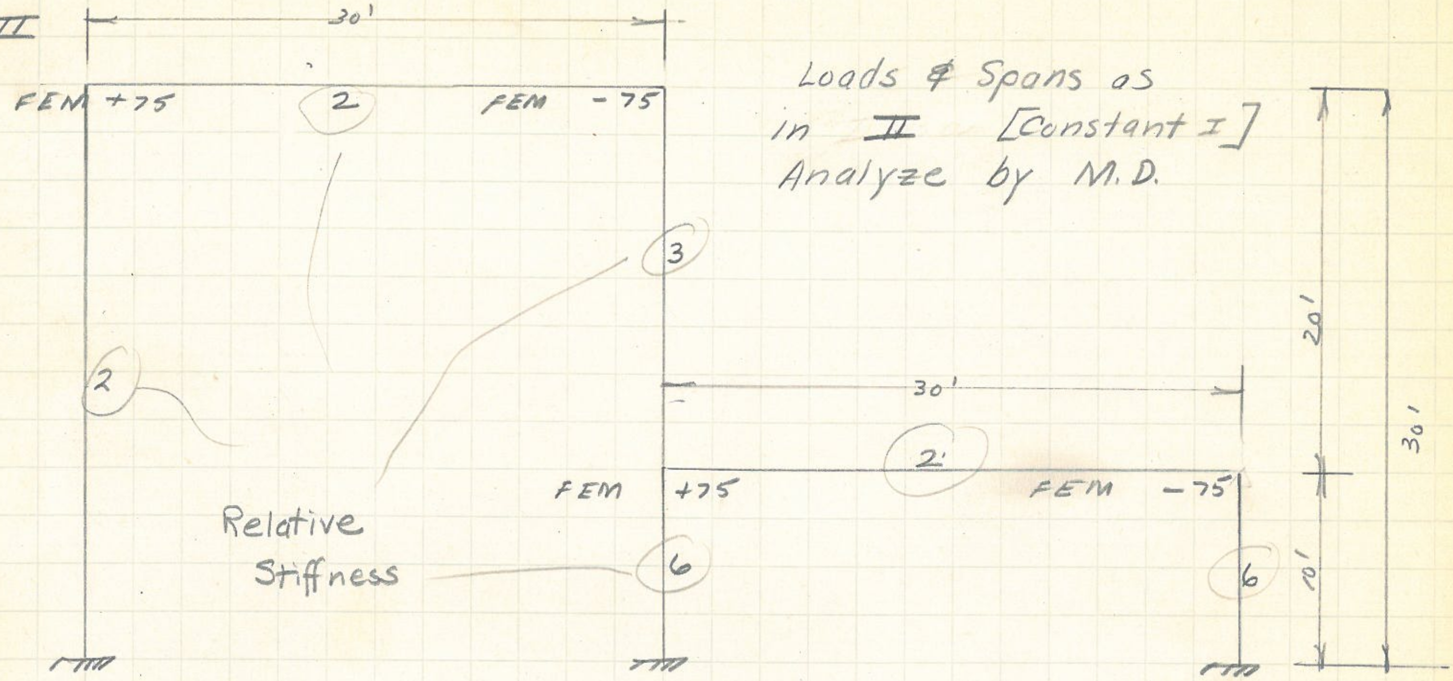
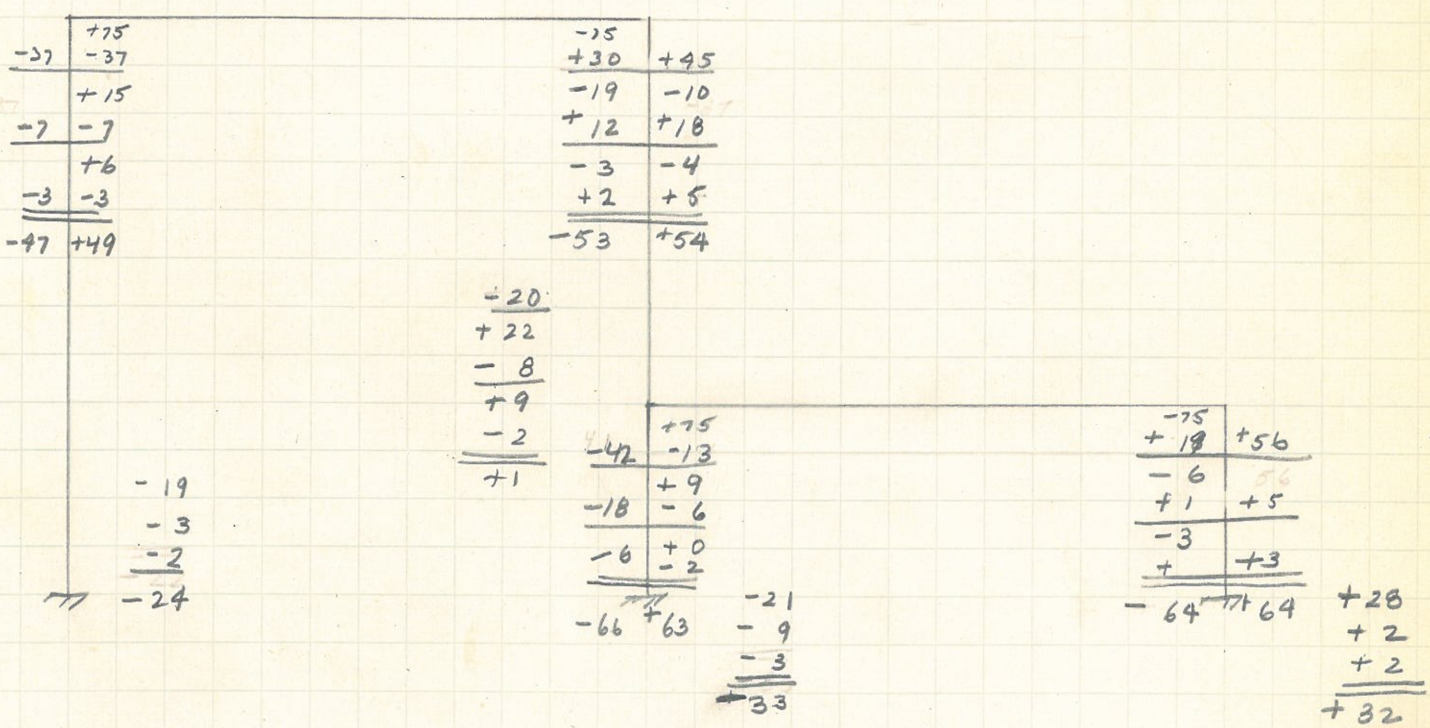
Prob 3

III

Loads & Spans as in II [Constant I]
Analyze by M.D.





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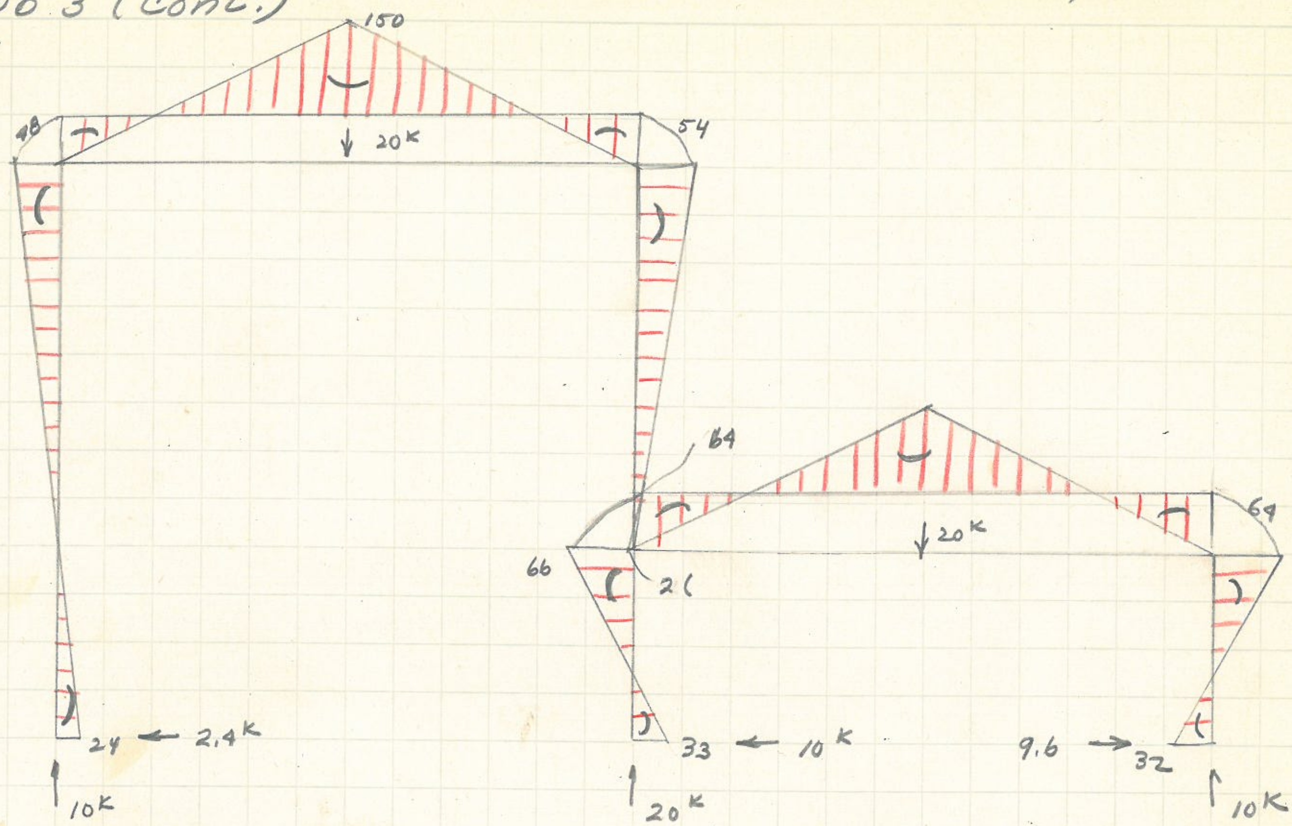
Prob 3 (cont.)

III



Unbalanced reaction 2.8K - neglected.

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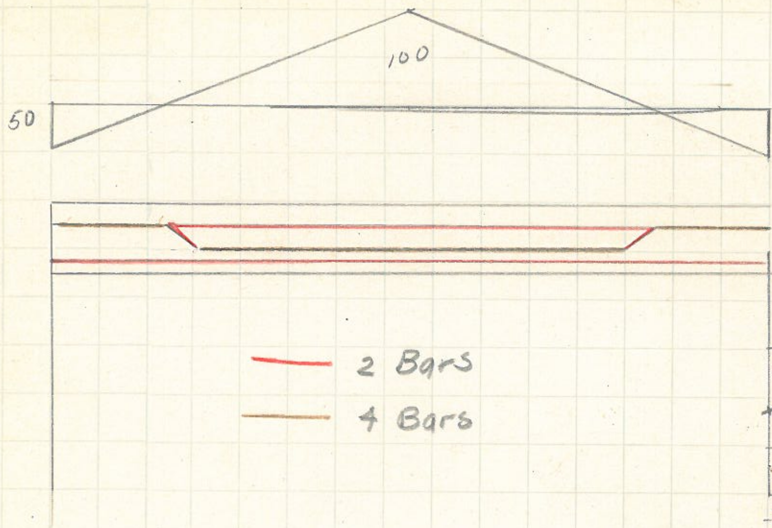
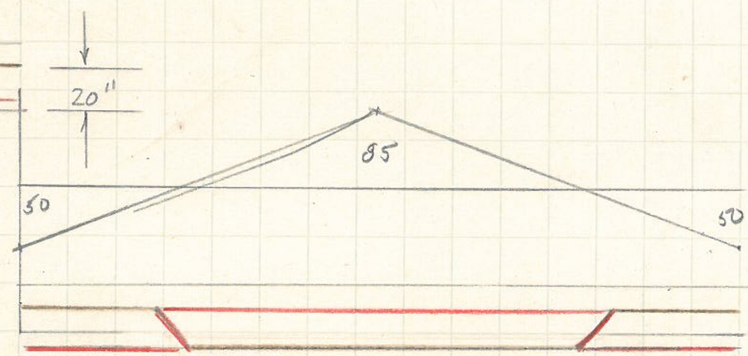
Prob 4.

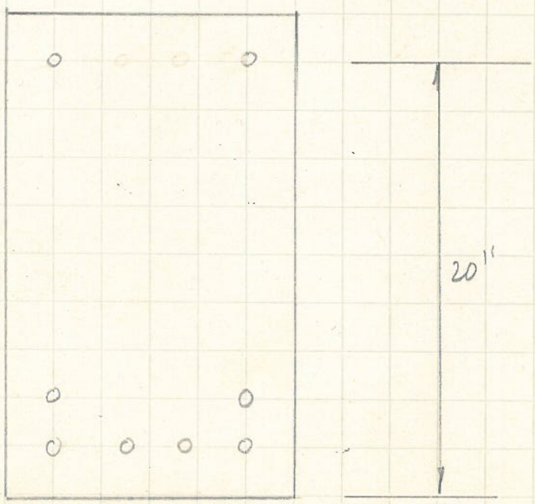
IV Same as III

Choose depths and show reinforcement in

III. Choose depth and bar size







No of Bars $= \frac{M}{jdAfs}$

$\frac{100 \times 1000 \times 12}{.9 \times 20 \times .79 \times 20000} \sim 6$ bars

Columns require only nominal steel, a bar at each corner, to tie it together. I would hesitate, however, to build the thirty foot column of the same size. Instead, I would make its thickness $\frac{1}{10}$ its unsupported length. This equals 3 feet.

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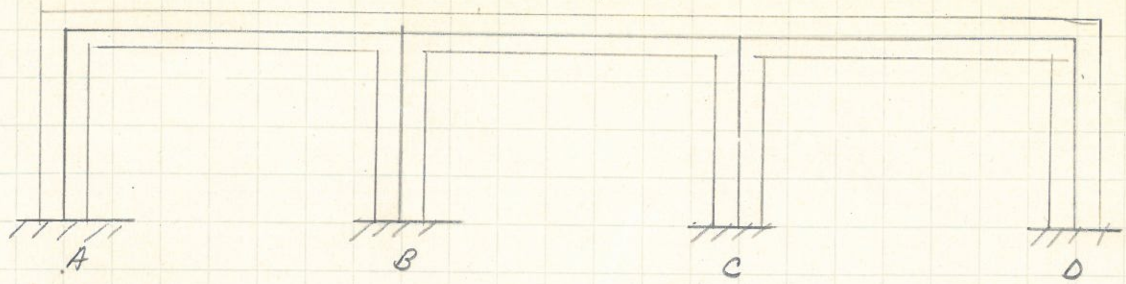
Prob 5

DATE Oct. 30, 1952

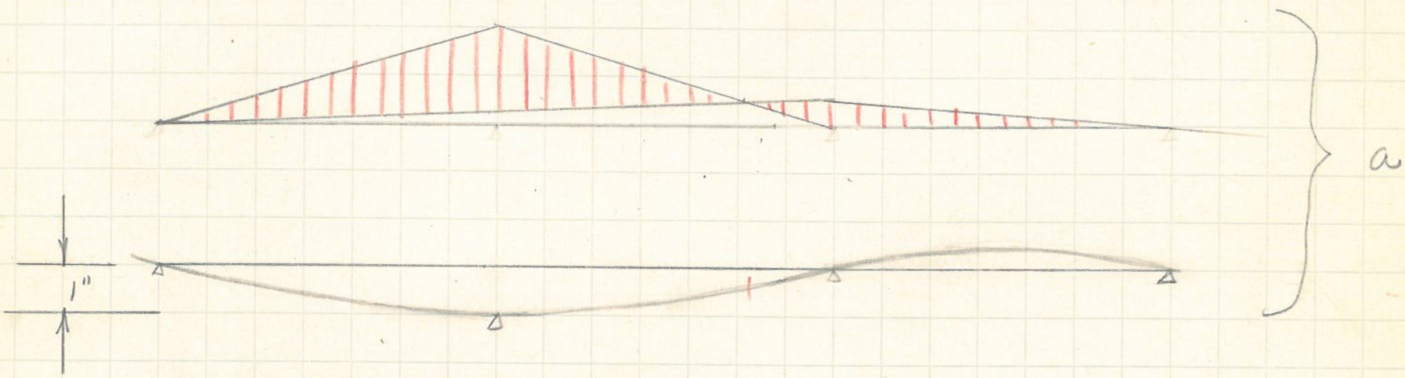
V

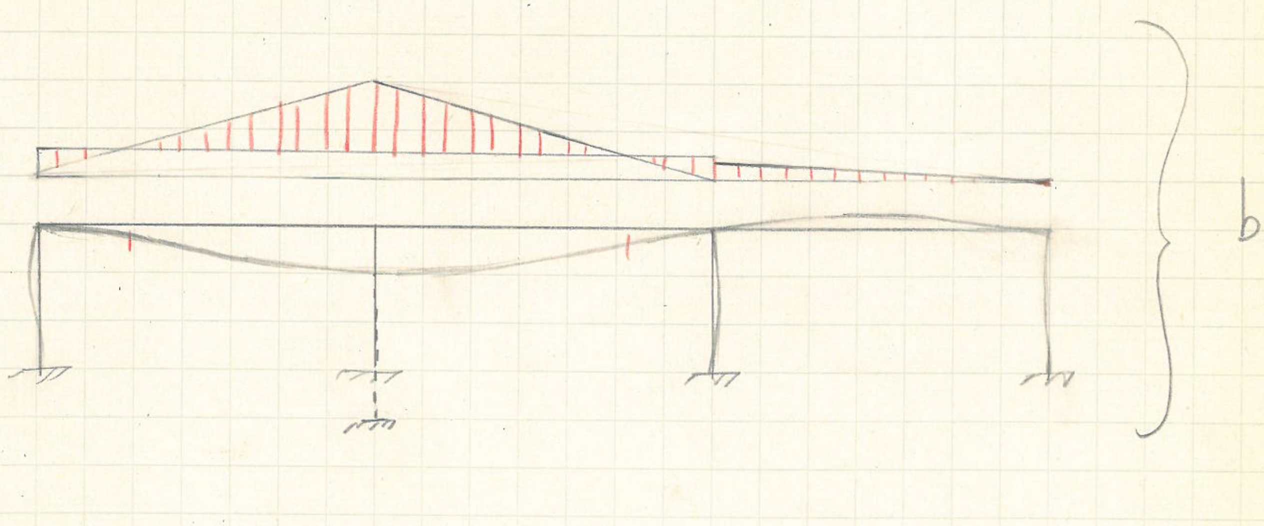


A. Point B settles 1 inch (with reference to other supports) Draw curves of deflections (with scale) and curve of moments (without scale)

a) Neglecting column restraint

b) Guessing at column restraint





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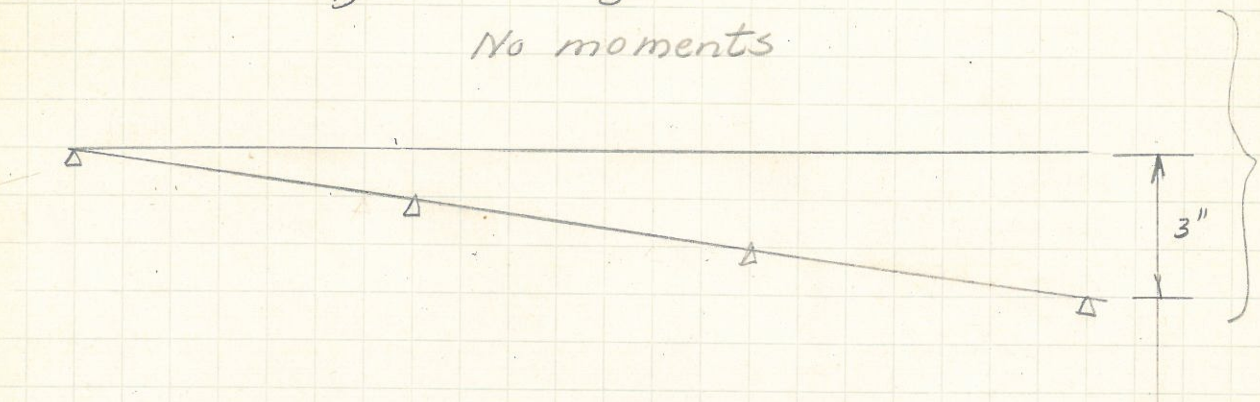
Prob. 5 (Continued)

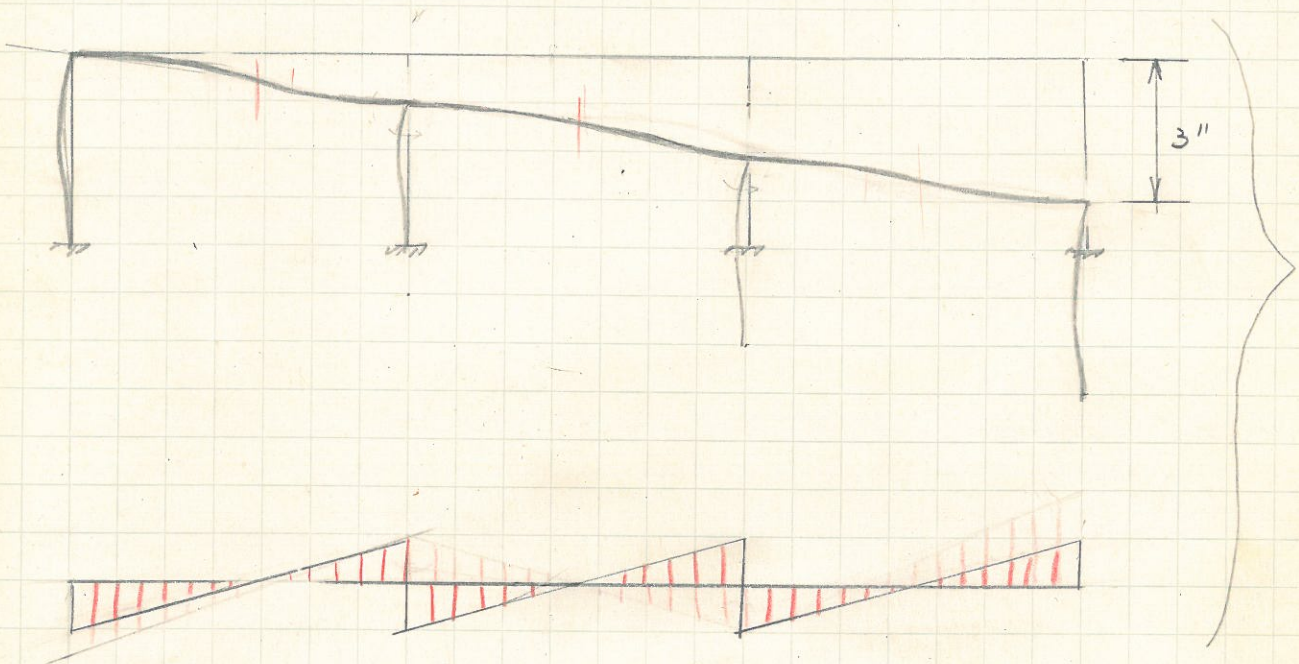
DATE Oct 30, 1952

B. Point A settles 0"
B 1"
C 2"
D 3"

No moments



a.



b

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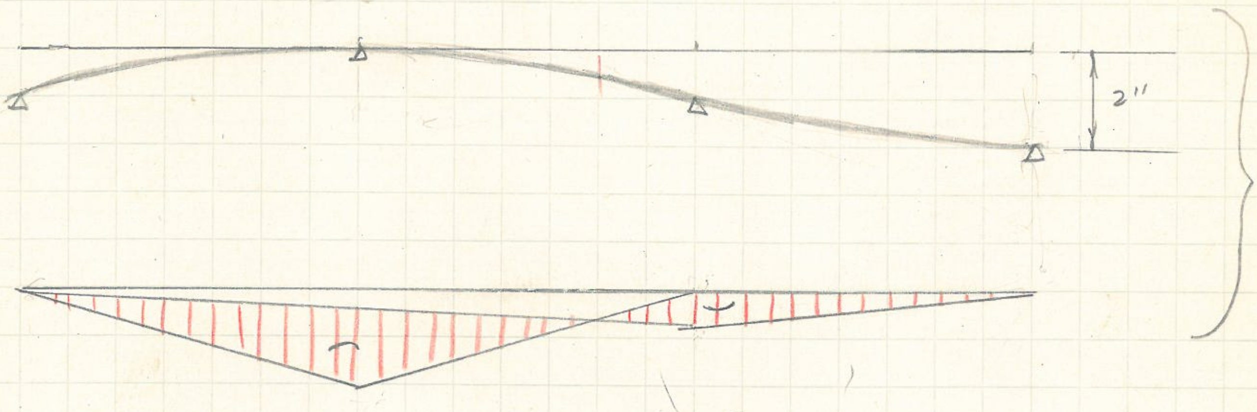
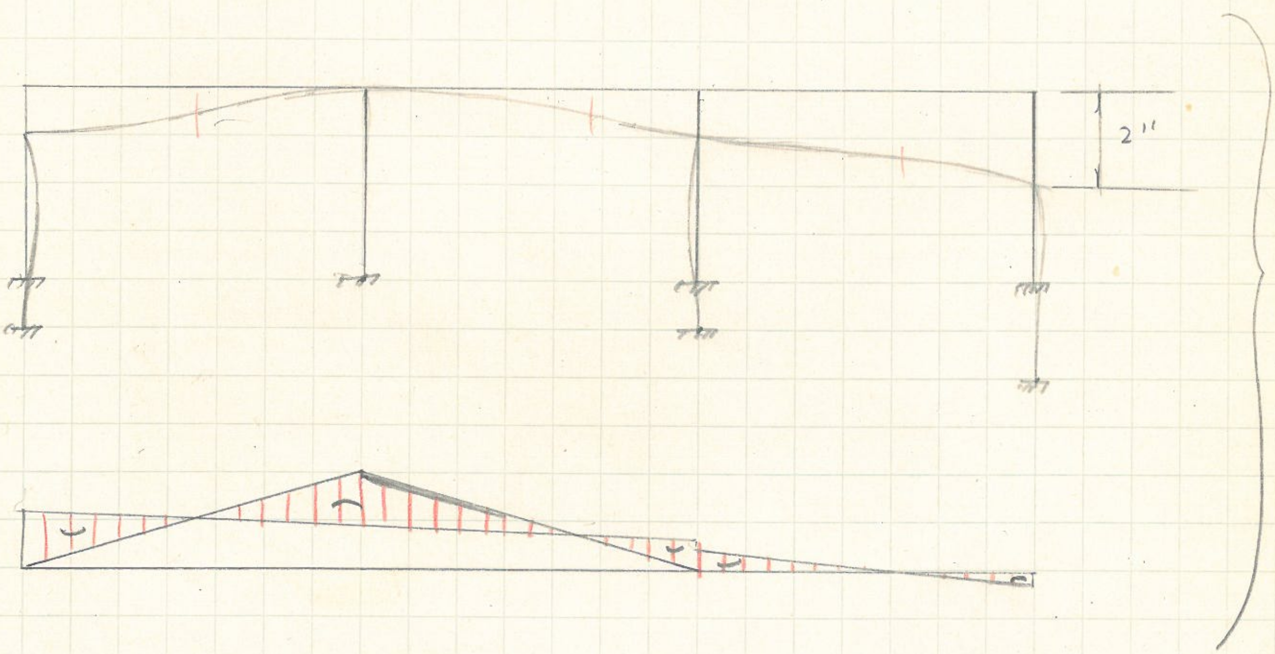
Prob. 5 (continued)

c. Point A settles 1''
B 0''
C 1''
D 2''



a



b

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Prob 6 DATE Oct. 30, 1952

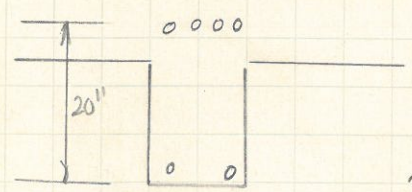
VI A. Establish critical % [State in words not letters)

If a beam is brought to failure and the concrete yields in compression before the steel yields in tension the beam is over-reinforced. It is under-reinforced if the reverse occurs. At some point they both give together. Then, we have balanced reinforcement. At this point the ratio of the area of the steel in tension to the concrete in compression is a critical ratio, or as it is called, critical percentage, p_c. If the critical percentage is exceeded the concrete will be overstressed. The usual solution is to add compression steel. Balanced reinforcement is not necessarily the most ecomomical design even if it is the most efficient design.

If $f_c = 700$ psi & $f_s = 16{,}000$ psi
& assuming $n=15$
then $nf_c = 10{,}500$ & $k = .4$
the critical percentage $= \frac{1}{2}\frac{f_c}{f_s} \cdot k = p_c$
therefore
p_c will equal .009 or .9%

see pg 6 Lecture 9
Oct. 21, 1952 my note
pg 3 Lecture 10
Oct. 23, 1952

1 comp. bar on the bottom equals 1/2 bars on top



$n = 4$
$N = 1$
$N' = 3/4$
$mN' = 3$

$$p_c = \frac{N'D}{4d}$$ assume 1" bars

$$= \frac{3/4 \cdot 1.0}{4 \times 20} = .0094 \text{ or } 9.4\%$$

The critical percentage is exceeded. Solution, put more compression bars in.

Prof Cross Exam

NAME Robert Heller

Prob. 6 (continued)

DATE Oct 30, 1952

VI B. State some criterion for bond limit.

Assume $f_s = 16{,}000$ psi

$u = 100$ psi

" uniformly loaded fixed end beam

$M = \frac{1}{12} WL$ & $V = \frac{1}{2} W$

& $\frac{M}{V} = \frac{1}{6} l$

$\frac{M}{V} \geqq \frac{f_s}{4u} D$ $\qquad \frac{f_s}{4u} = \frac{16{,}000}{400} = 40$

$\frac{M}{V} \geqq 40D$

$\frac{1}{6} l \geqq 40D$

see pg. 9 Lecture 9
Oct. 21, 1952

$D < \frac{l}{240}$

If D is 1" than the span must be greater than 20 ft.

FORM A

YALE CO-OPERATIVE CORPORATION

The next several lectures will be spent on what the codes and specifications have dictated in the past and what is said today, and on what evidence this is based.

Codes for the dimensioning of reinforced concrete are made up of three parts:

1. Loadings: This is not what will be discussed in this and following lectures. Codes specify live load, impact, wind, temperature change and earthquake effects. (See what has been done on earthquakes.) Codes specify for highway bridges the weight of trucks, and for railway bridges the weight of trains. These loadings have changed constantly over the last seventy years. The sources of authority or information are for the railway bridges, the AREA, for the highway bridges, the AASHO, for buildings, the various building codes.

2. Assumptions used in analysis: This part of the code specifies how the stresses should be computed. If a different, so-called better, analysis is used, different working stresses must be used.

3. What stresses are allowed.

4. Limitations on construction: This part specifies methods of placing bars, etc.

A seemingly logical way to find out something about something is to test that something in a laboratory. However, a concrete bridge cannot be brought into a laboratory. Another way is to go into the field and test. This is not satisfactory because proper controls cannot be had in the field. However, many structures fail in the field. This is fortunate for everyone but the designer or builder. In recent years there have not been many failures. Where failures have occurred in concrete structures everyone has been at fault, poor contracting, poor engineering, and poor control. When resulting lawsuits occur they may be for many hundreds of thousand dollars.

In referring to specifications and codes in an earlier lecture, three types of documents were mentioned:

1. Recommendations: This is a general statement of general principles. It is used most advantageously in Europe where the engineer and the contractor are one and the same. It is dangerous to interpret European type recommendations for American use.

2. Specifications: Take as an example the AREA specifications. The railroads are not under obligation to follow the specifications but usually do unless there is good reason not to.

3. Building Codes: These are laws. Theoretically, then, work has to be just as it states. However, in practice it would be fantastic to follow some things that are in these codes.

It is advisable to look at the following codes:

1. First Joint Committee, 1916: This is an unusually valuable work. Note the type or tone of document. It is really

recommendations and was written primarily with buildings in mind. Even though written so long ago modern codes have never gotten away from it, but have merely modified it.

2. Second Joint Committee, 1924: This work is not very important.

3. Third Joint Committee, 1941: While the First Joint Committee was a recommendation, this attempts to be a specification.

4. Continuing work of the building code committee, ACI, 1951.

5. Building Codes: While some of these codes follow other codes, in many big cities they are quite different.

When looking at codes it is not necessary to take notes. It is much better to go into codes with a question than to try to read through them.

1. Look at the composition of the committee and see who each member represents. Engineering is a business. Men working on committees are representing some industry and are looking out for that industry's own interests. As a result the code is a compromise document.

2. Compare what the codes say about the methods of finding ~~dining~~ moments and shears in rigid frames.

L-12 (p. 4)
Prof. Cross
Dec. 2, 1952

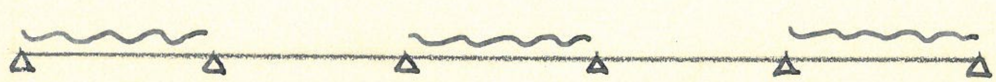
	1 Span		2 Spans			3 Spans or More				
	Center	Support	Center	Center Support	End Support	End Support	End Center	Interior 1st Support	Interior Center	Interior Support
1916			1/10	1/10	1/16	1/16	1/10	1/10	1/12	1/12
					1/12	1/12				
					(small beams, heavy columns)					
1924 into beams girders	1/8	>1/16	1/10	1/8	>1/16	>1/16	1/10	1/10	1/12	1/12
into columns walls			$I/\ell < 2\sum I/h$			1/12	1/12	1/12	1/16	1/12
			$I/\ell > 2\sum I/h$			1/16	1/10	1/10		
1951			1/14	1/9			1/14	1/10	1/16	1/11
									$\frac{\text{column stiffness}}{\text{beam stiffness}} > 8$	1/12

The First Joint Committee simplified most things very much. Some phases were, perhaps, oversimplified. For one thing they lumped together live load and dead load. The coefficients, 1/10, 1/12 are not comparable to rigid frame "exact" analysis. While saying that the negative moment at the support is the same as the positive moment at the center, they neglected the negative moment at the center. They were talking exclusively about equal spans. If the spans were not equal they suggested that a special analysis should be made. This was meaningless at that time because no one knew how to make such an analysis.

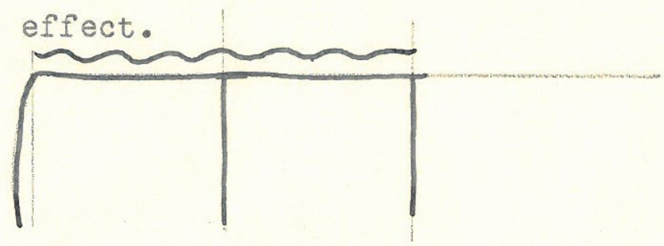
The First Joint Committee accounted for the effect of haunching by measuring the clear span from a point on haunch where the depth is 1.33 times the depth of the girder. Ten percent loss in length means twenty percent loss in moment.

The ACI building code specifies that the negative moment of the first interior support be equal to $\frac{wl^2}{10}$, and the negative moment at the other faces of the interior support be equal to $\frac{wl^2}{11}$. If a three span beam be loaded fully the moment at the first interior supports is $\frac{wl^2}{10}$. However, if only two adjacent spans are loaded the moment will be $\frac{wl^2}{11}$.

The following diagram shows the loading for maximum moment when there are many spans.



The column stiffness would have a considerable effect. If adjacent spans were loaded the stiffness of the end column would have a great effect.



The First Joint Committee did not consider the effect of bending in a column. With them, the column was only a column with its sole function being the support of the load. The Third Joint Committee took the effect of the column into account and treated it like a flexural member. The ACI compromises between the two.

The First Joint Committee said that if the spans were equal or near equal and if the L.L. and D.L ratio was anywhere from 1:3 or 3:1 the analysis could be made as prescribed. However, in the case of unequal spans a special analysis would have to be made. That at that time was meaningless. The Third Joint Committee became overexcited about equal spans and so tried to pin down what assumptions to make in analysis. They, for example, specified how to compute the moment of inertia of beams and columns. However, the ACI sidesteps this and simplifies it by suggesting that the designer go ahead and assume something reasonable for stiffness.

Since the first buildings built of reinforced concrete were industrial buildings like warehouses, the spans were equal. However, schools and apartment houses were soon being built of concrete. These had a narrow passageway running through them. As a result that middle span had little positive moment but an appreciable negative moment. Since the First Joint Committee said nothing about negative moment, no steel was put in the top and much cracking resulted.

It is not the intention of this series of lectures to promote memorization of the various codes or even mastering of them. However, it is to our advantage to familiarize ourselves with them. The New Haven building code and others still use 650 psi working stress for concrete. This dates back to the First Joint Committee. Our purpose is to look somewhat critically into the codes and have a basis for a comparison, so as to find out what is in one code and not in others. In the last secture a comparison was made on the basis of computation of stress. This lecture a comparison will be made on the basis of allowable stresses, i.e., the stresses permissible after the stresses have been computed.

Therefore, the following comparison will be made:

1. Compare (between codes) the length which will be permissible if the bond stress is not to be exceeded. What limitations does the bond have on the length of beam.

$$\frac{M}{V} > \frac{f_s}{4\mu} D$$

Assume 7/8" bars and 3000 lb. concrete.

Sample calculation:

1st Joint Committee - End Span

$$\frac{M}{V} = \frac{\frac{1}{10} wl^2}{\frac{1}{2} wl} = \frac{1}{5} l$$

$$\frac{f_s}{4\mu} D = \frac{16000}{4 \times 150} \times \frac{7}{8} = 23.4 \text{ ins.}$$

$$l = 117 \text{ inches or } 9.7 \text{ feet}$$

L-13 (p. 2)
Prof. Cross
Dec. 4, 1952

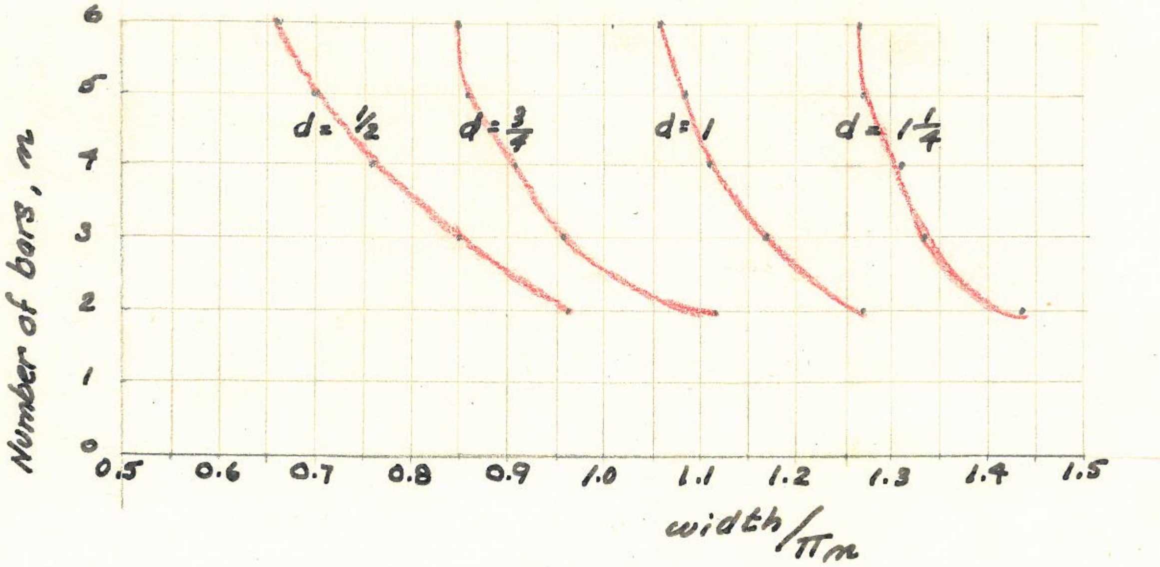
Table

	f_s	f'_c	f_c	u	v	MOMENT COEFF. wl^2 SPAN End	Int.	Cant.	LENGTH ≥ feet SPAN End.	Int.	Cant.
1ST J.C.	16,000	3000	$.375f'_c$ 1125	$.05f'_c$ 150	$.045f'_c$ 135	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{2}$	9.7	11.7	3.9
2ND J.C.	18,000	3000	$.3f'_c$ 900	$.05f'_c$ 150	$.06f'_c$ 180	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{2}$	10.9	13.1	4.3
3RD J.C.	20,000	3000	$.375f'_c$ 1125	$.05f'_c$ 150	$.06f'_c$ 180	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{2}$	10.9	12	4.8
A C I.	20,000	3000	$.45f'_c$ 1350	$0.1f'_c$ 300	$.12f'_c$ 360	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{2}$	6	6.7	2.4

2a. Compare the size and number of bars to the width of beam. Refer to "Theory of Reinforced Concrete" by Dunham.

The minimum width of a beam is $\pi_m D$. It is apparent from the graph below that in practice beams are made wider. The graph based on Table 8 of the Appendix in reference indicated above is for 3/4 in. aggregate.

Graph



2b. Determine the minimum depth without exceeding the allowable compressive stress.

$$p_c = \frac{A_s}{A_c} = \frac{\frac{n\pi D^2}{4}}{n\pi D d}$$

so that $d = \frac{DN'}{4p_c}$ where N' is the number of uncompensated rows of bars.

For balanced design

$$\tfrac{1}{2} f_c k b d = A_s f_s$$

$$f_c = 1350 \text{ psi} \qquad f_s = 20000 \text{ psi}$$

$$\frac{A_s}{bd} = p_c = \frac{\frac{1}{2} f_c k}{f_s} = \frac{1}{2}\,\frac{1350}{20000}\,0.4 = 1.35\%$$

Sample Calculation: $N' = 2 \quad D = \frac{1}{2}''$

$$d_{min} = \frac{DN'}{4p_c} = \frac{\frac{1}{2}\cdot 2}{4\cdot 0.0135} = 18\tfrac{1}{2} \text{ inches}$$

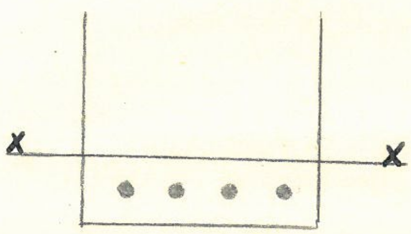
Table

Bar Diameter	N'	d_{min} (inches)
$\frac{1}{2}''\phi$	2	18 1/2
	1 1/2	14
	1	9 1/2
	1/2	5
$\frac{3}{4}''\phi$	2	28
	1 1/2	21
	1	14
	1/2	7
$1''\phi$	2	37
	1 1/2	28
	1	18 1/2
	1/2	9 1/2
$1\frac{1}{4}''\phi$	2	46
	1 1/2	34 1/2
	1	23
	1/2	11 1/2

3. Relate the bond stress, u, to the shearing stress, v.

The difference in stress anywhere along a beam is represented by the bond. Since the periphery of all the bars, Σo, is $n\pi D$ and u is the bond stress, then the bond equals $n\pi D u$.

Bond on the bar must pass through the section, as shown, as shear. If the approximate width of the beam equals $n\pi D$, and since the unit shearing stress equals $\frac{V}{bjd}$ and the unit bond stress equals $\frac{V}{\Sigma o jd}$ where Σo equals $\pi n D$, then solving for V in both cases



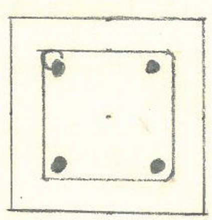
$$V = v(n\pi D)jd$$

and

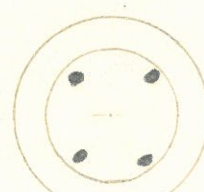
$$V = u(n\pi D)jd$$

or $v = u$ for one layer of bars. For two layers of bars, $v = 2u$.

4. Compare the carrying capacity of the two columns pictured below for axial load for the First Joint Committee, the Third Joint Committee, and the ACI code.



TIED
12" x 12"



SPIRALED
12" ϕ

No general conclusions can be drawn from these two columns since they are fairly small columns. Attention will be given to the difference between figuring the column as an elastic column and as an inelastic column, whether to consider the steel or not, and, if a spiraled column is used, whether the shell can be counted.

All bars will be 7/8" diameter and the concrete cover will be 1-1/2 inches.

First Joint Committee

The effective area will be the core of the concrete within the hoop. According to this committee, under compressive stress the steel and concrete are stressed in proportion to their moduli of elasticity.

$$\delta_c = \delta_s$$

$$\delta_c = f_c/E_c \qquad \delta_s = f_s/E_s$$

$$f_c/E_c = f_s/E_s$$

$$\frac{f_c}{f_s} = \frac{E_c}{E_s} = \frac{1}{n}$$

$$f_s = n f_c$$

$$A_{total} = A_c + nA_s = A_g - A_s + nA_s = A_g + (n-1)A_s$$

$$P = f_c\left[A_g + (n-1)A_s\right]$$

TIED	SPIRAL
$A_g = A_s + A_c = 2.4 + 81 = 83.4^{\square''}$	$A_g = 2.4 + \frac{\pi 81}{4} = 66^{\square''}$
$f_c = 0.225 f_c' = 675$ psi	$f_c = 1.55 f_{c_{TIED}} = 1048$ psi
$P = 675[83.4 + (9)(2.4)] = 70.8^k$	$P = 1048[66 + (9)(2.4)] = 91.8^k$

Third Joint Committee

In the preceding analysis, the stress in the steel does not exceed nfc. However, results of test by the ACI performed at Lehigh and the University of Illinois showed that steel reinforcement was capable of withstanding higher stresses without the bond between the steel and concrete being destroyed. The strength of a column is the sum of the strength of the concrete and the strength of the reinforcement regardless of the ratio of their moduli of elasticity. Included in the calculations is the full gross area of the concrete. For the spirally reinforced column the code specifies the safe axial load to be:

$$P = 0.225 f_c' A_g + f_s A_s$$

$$P = (0.225)(3000)(113.2) + (20000)(2.4)$$

$$P = 76,400 + 48,000 = 124,400^{\#}$$

Columns without spirals develop lower stresses in both concrete and steel so the Third Joint Committee specifies the safe axial load for the tied column to be 0.8 that of the spirally reinforced columns or

$$P = 0.18 f_c' A_g + 0.8 A_s f_s$$

$$P = (0.18)(3000)(144) + (0.8)(2.4)(20,000)$$

$$P = 77,800 + 38,400 = 116,200^{\#}$$

Code	P-Spiral Column	P-Tied Column
1ST JC	91.8 k	70.8 k
3RD JC	124.4 k	116.2 k
ACI	124.4 k	116.2 k

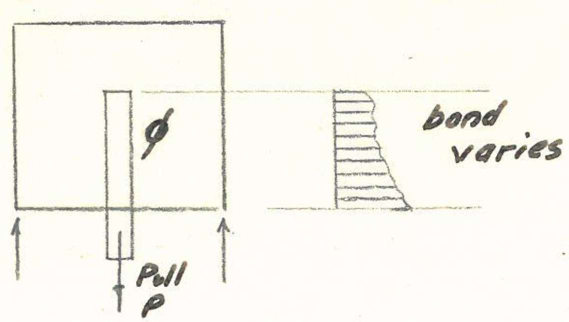
Much information upon which the codes are based was obtained from testing. Testing began very early, probably in France in the 1890s by Considere. Germany continued investigations past the twenties. In this country Talbot of the University of Illinois, and Teaurneure of the University of Wisconsin and later MIT contributed a great amount of information much of which is the basis of working stresses today. Much of the early Illinois work is under criticism today because the tests were made by college seniors who were not trained observers and the concrete was not a "modern concrete."

Investigation of Bond Stress

Referring to Lecture 10, page 1, it can be seen it does not matter what formula is used since a wrong formula will yield to same amount of material.

$$u = \frac{V}{\Sigma o \, jd}$$

How are we to determine the ultimate bond stress as used in concrete beams? A bar can be buried in a concrete block and pulled until something happens.

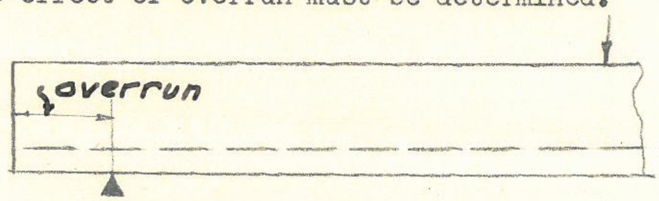


This something that happens can be a movement 1/1000 of an inch or 1/6000 of an inch or even 1/10000 of an inch. The peripheral area of the bar in the concrete and the pull when something happens can be recorded, but it should be remembered that bond cannot be measured. The bond varies but just how cannot be determined. It is the average bond that is determined.

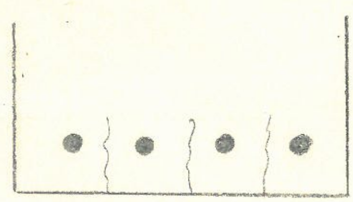
In order to more nearly approach reality a beam can be made and since the loading, P, is known and

$$u = \frac{V}{\Sigma o\, j d}$$

it is possible to determine the bond. However, there are further difficulties. The effect of overrun must be determined.



If the bars are too close there will be splitting and the results of the tests will be invalid.



Further, the beams act differently after cracks open up.

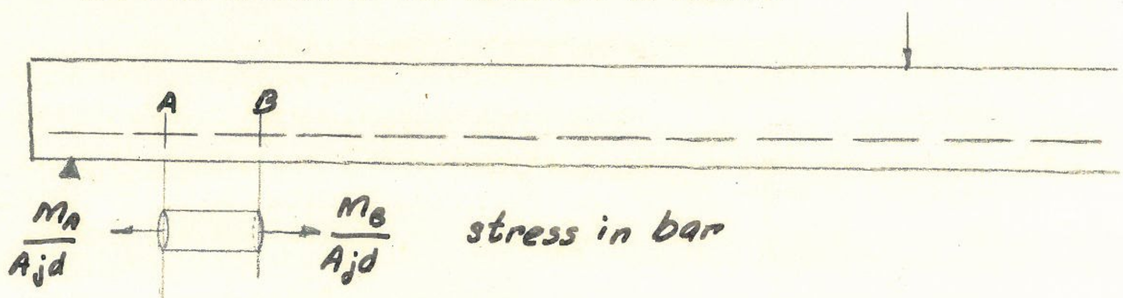
The trouble is that beams are not built simply supported but are continuous. In a continuous beam, where the movement is a maximum the shear is a maximum so that the cracks open up because of moment and spread by shear.

In this and most tests we are on the outside looking in. One way of overcoming this is to hollow out the bars and put in strain gages to measure deformation. In so doing this you get inside. Duff A. Abrams at Illinois did this when testing bond between concrete and steel. He was able to trace out the different stages.

1. Concrete uncracked
2. Concrete begins to crack
3. Concrete cracks so much the bars give in bond
4. Steel yields

He observed that the sliding resistance reached a maximum value for plain bars at a slip of 0.01 inch and that this figure was consistant for a wide range of mixes, ages, sizes of bars, and condition of storage.

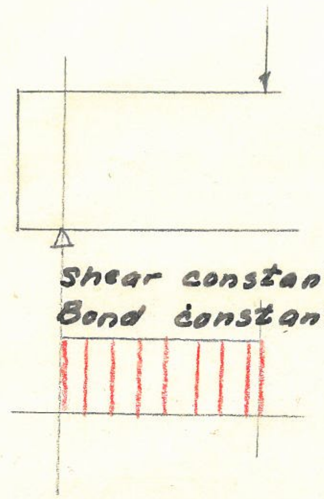
The bond formula is not difficult to derive.

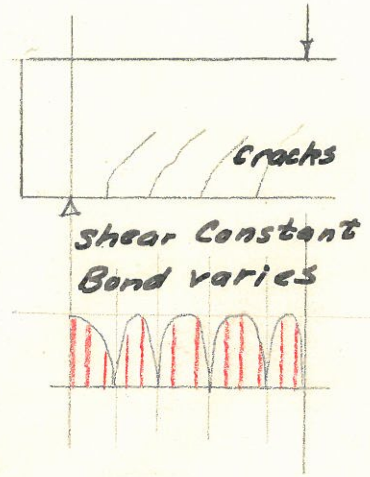


$$\frac{M_A}{A_j d} - \frac{M_B}{A_j d} = \text{Bond from } A \text{ to } B$$

The rate of change of moment per inch is V. The bond stress then is $\frac{V}{\Sigma o\, jd}$.

However, can there be bond where there is a crack in the concrete? This was investigated by Millray at Illinois.





With a continuous beam there is a tendency for the beam to crack earlier than with a simple beam. What is measured is average bond. There are many variations. This matter is not as simple as it appears to the novice.

This matter has been pointed out so that we will not be naive. It is important sometimes to be able to recognize differences. It is true that concrete structures are built perfectly well. There is a danger in graduate work in that the student can argue anything but can not decide anything.

Another factor which is important to note is that concrete when poured will settle around bars when the bars are at the bottom but if the beam were upside down, i.e., the bars on top, there may be water under the bars. With deep beams, because of the shrinkage, the concrete goes down while the steel does not which results in no concrete under the bars. The seriousness of this depends on concrete shrinka ge, moisture, and depth of beam. It is difficult to get uniformity in tests.

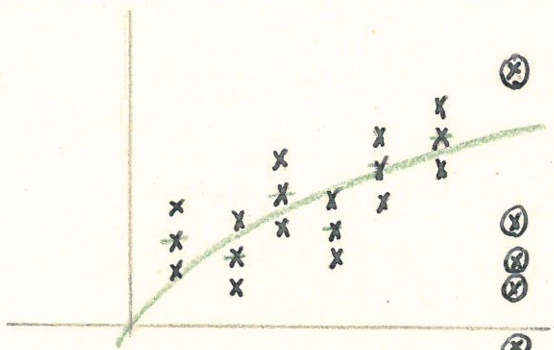
One must be a skilled observor to see cracks in concrete. They can be detected by water and reflected light. The distribution of bond depends on the distribution of the cracks. If the cracks are very close together the beam will fall away.

Generally there will not be a bond failure as we have found out. Nevertheless, there are certain limits put on the "idiots" to be limited by, but there is no cure for idiots who will, anyway, misinterpret the limits if they are left there.

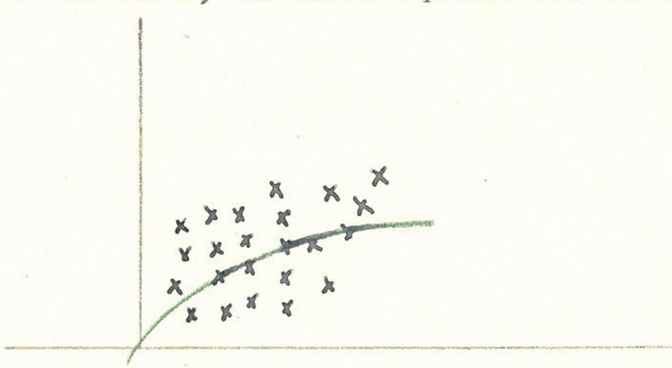
It is almost impossible to get failure in compression in a beam. Concrete beams just do not fail in compression.

L-14 (p.1)
Prof. Cross
Dec. 9, 1952

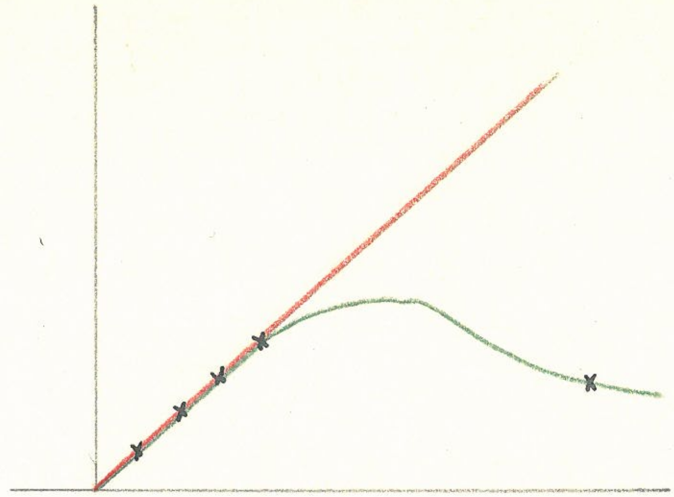
As has been said before, it is not necessary or advisable to plow through the vast literature of testing in reinforced concrete. Although the literature is very voluminous, some of it should be looked over so as to become familiar with it. When trying to find out something be careful not to accept conclusions of bulletins without verifying it by seeing that the evidence is there. Often data is recorded in bulletins by plotting on two axes. The line drawn to represent the many points is usually the author's conclusion. The following three diagrams illustrate mistakes often made in drawing conclusions.



In the above, the circled points are thrown out but may be right.



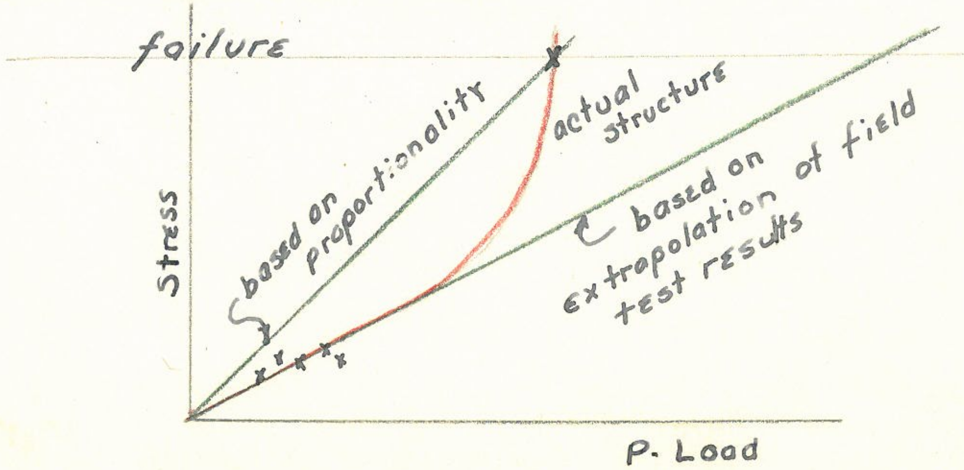
It is really impossible to draw a line through as many points as above. Yet, the psychological effect of the line leads the reader to accept the line as correct.



There is always a tendency to draw simple curves rather than real curves. So, in the above diagram some authors would choose to disregard the last point and to draw a straight line through the other points.

FLAT SLABS

At one time flat slabs were patented. If tests were made on them, through necessity they were made in the field. However, because they were tested in the field there was poor control and they could not be tested to failure. Therefore, the tests were made at relatively low loads. The diagram below shows how the results of the tests were interpreted. From the theory of proportionality one conclusion could be drawn. By extrapolating the short range of field test results another conclusion could be made. However, the structure behaved in a manner as indicated in the diagram.



BEAMS

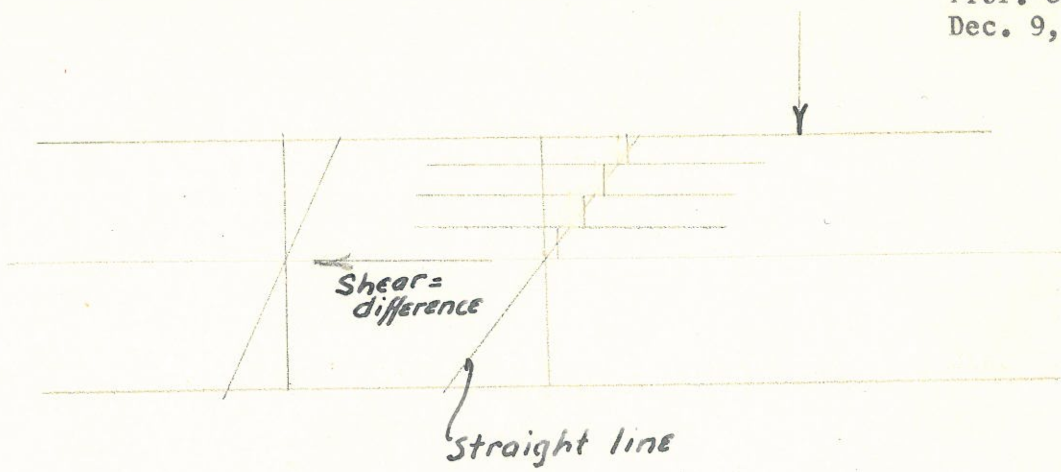
Over several years many men made tests on beams. Bulletins by Richart are based on these tests.

It can be seen that formulas indicate that at every 45 degree crack in the beam there must be enough steel crossing that section to resist shear. However, the cracks do not cross at 45 degrees which can be seen by observing the photos in the bulletins. Further, they are rarely straight. The web steel does not prevent cracks but prevents the cracks from opening up.

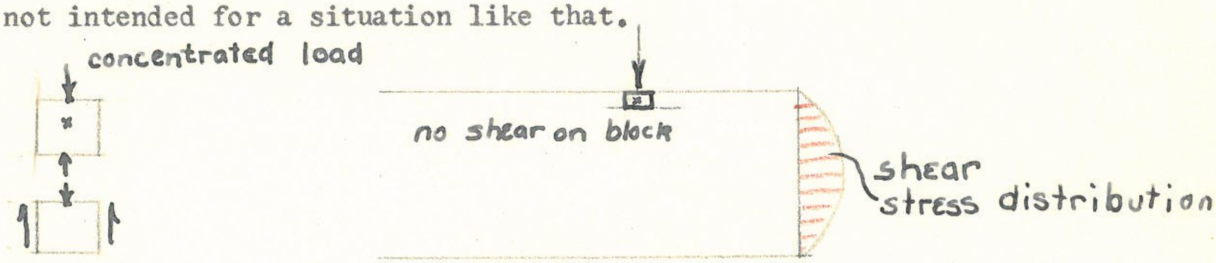
It is more economical to run the steel vertically. If the steel is running diagonally more bars will cross the crack, but there will be a smaller vertical component to resist the shear. Therefore, its effect would be the same as the vertical but more steel would be required.

The theory of flexure, $f = \frac{Mc}{I}$, is based on a homogenious rectangular beam which will follow the theory of proportionality. If the deflections are due solely to the longitudinal deformations, the line of deflection, as can be seen by the deformed structure, is a straight line. Therefore, stress is proportional to strain, and the beam formula, $\frac{Mc}{I}$ holds. The difference of stress by statics is the longitudinal shear.

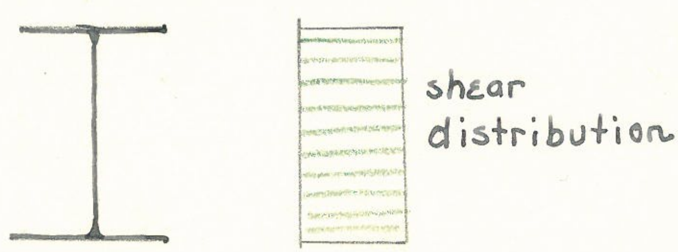
The curvature must be the same throughout the depth of beam or the beam would tend to pull apart.



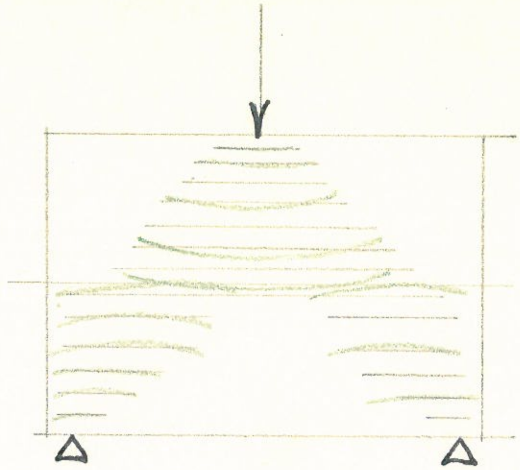
The following diagram indicates that the theory of flexure was not intended for a situation like that.



The flexure theory applies only to timber. Timber is rectangular but not isotropic. Instead of using the formula in the form usually seen, $\frac{Mc}{I}$, it is more likely to be used in the form, $\frac{6M}{Ad}$. Although concrete beams are rectangular, they are never homogeneous. Steel, on the other hand, is never rectangular and, therefore, the shear distribution is almost rectangular.

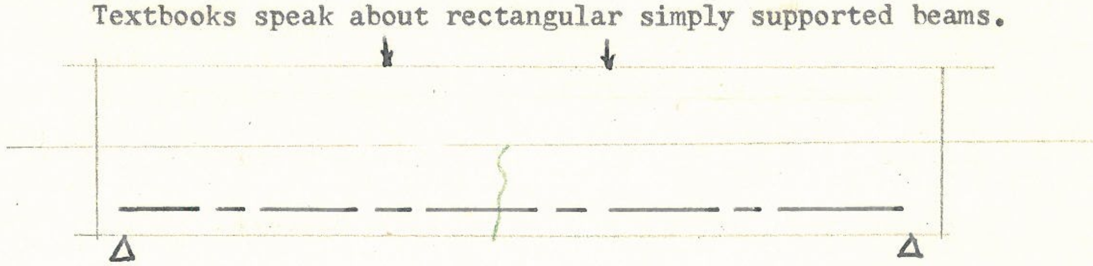


The formula is usually used for steel in the form, $\frac{M}{Z}$, in which Z is tabulated in many manuals.



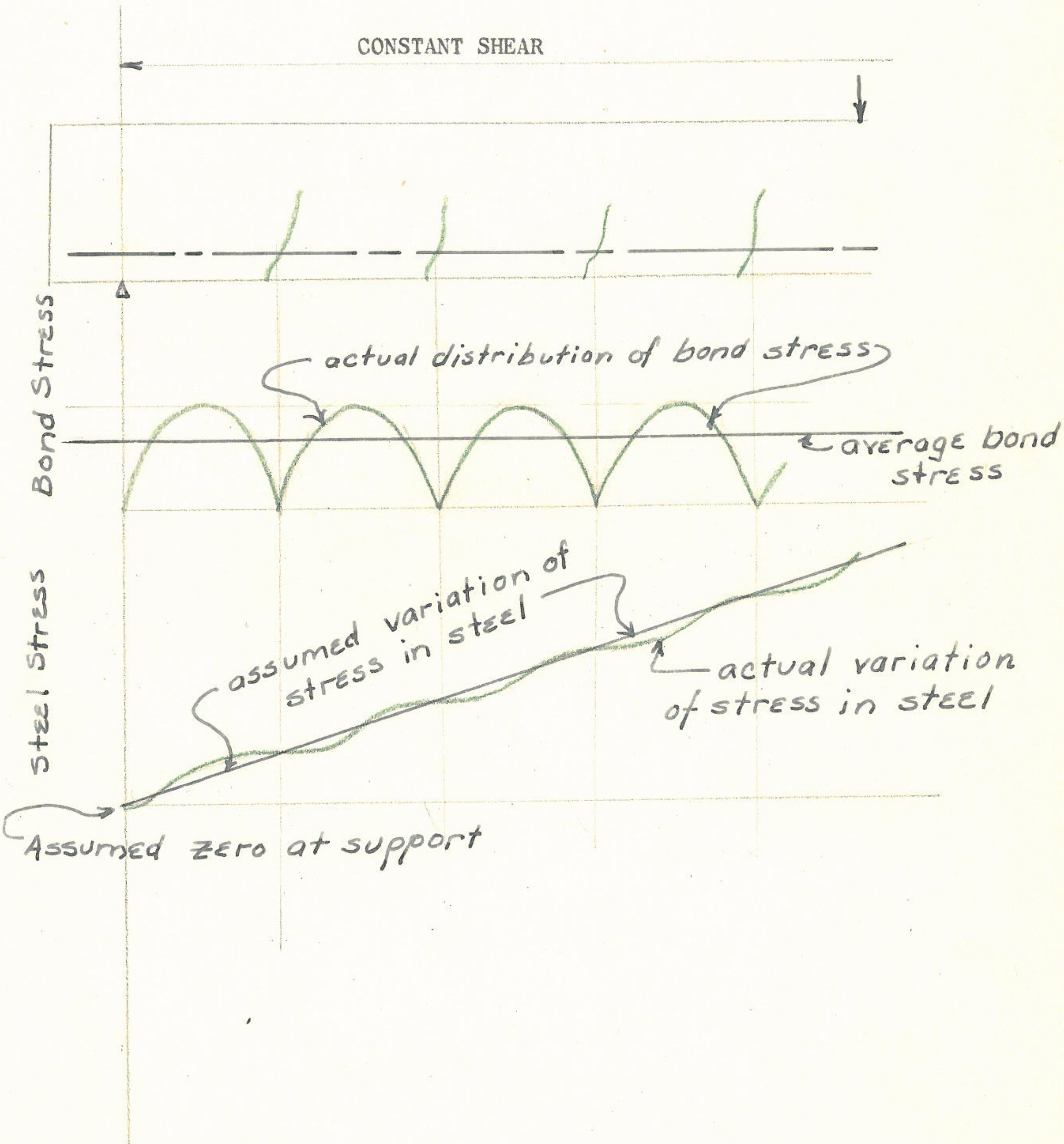
The above diagram illustrates an unusual beam. Here it is possible to be mislead by the beam formula. The fibers under the concentrated load are being squeezed together. The intensity of the stress diminishes and becomes more uniform away from the point of application of the load. This action results in contraction and lateral elongation so that bending is distorted. For one thing, plane sections will not remain plane sections. The curvature is not the same for all the layers throughout the depth. In the ordinary situation the usual starting point was longitudinal shear. In this case it is necessary to work from vertical stress to vertical shear to longitudinal shear. Remember not to push theories too far where they have no application.

Textbooks speak about rectangular simply supported beams.

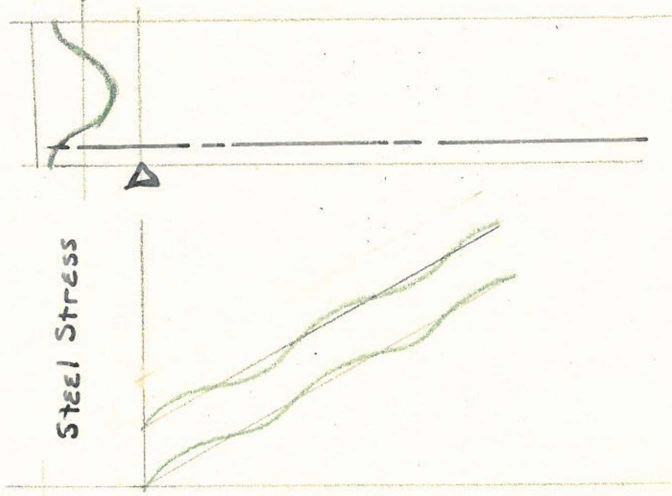


They say that there will be cracks below the neutral axis. It is not that they say there is no resistance to tension in concrete but that the concrete cannot be counted on to resist tension. Computation will verify that below a certain percentage of steel in concrete when the concrete cracks, the steel will flash above the yield. That is why the codes specify a minimum percentage of steel.

The following diagram shows what occurs when a beam is cracked.



When using the theory of flexure for concrete beams in flexure the computations are very hazy. Take, for example, the overhang where there is shear, some bond, but no moment.



What is the shearing strength of concrete? The First Joint Committee made reference to punching shear. However, it has dropped out. There is some doubt if you can get a shear failure in concrete. It seems to break in tension. You will not get a compression failure if the material has no place to go. Although reference is made to compressive strength all the time in concrete, actually the real thing seems to be tension. In a so-called "compressive failure" the pieces break loose like a tensile failure.

FOOTINGS

This lecture will discuss two bulletins, one by Arthur N. Talbot, and the other by Frank E. Richart.

"Reinforced Concrete Wall Footings and Column Footings"
by Arthur N. Talbot

University of Illinois Bulletin, No. 67, March 31, 1913.

The history of this bulletin illustrates what experimenters can drift into in testing. Richart became interested in thick slabs of steel put under columns to distribute the loads. Talbot, while a young man, made tests on concrete. Later he transformed these to steel plates.

Wall footings fail in bond. The concrete strips away from the bar. The bond formula, however, does not express this clearly. From the end towards the center, the bar begins to pick up the stress quite rapidly.

The purpose of a spread footing is to distribute the column load over the soil. Talbot used square footings of roughly the same dimensions but with some variation of depth. He was interested in three things:

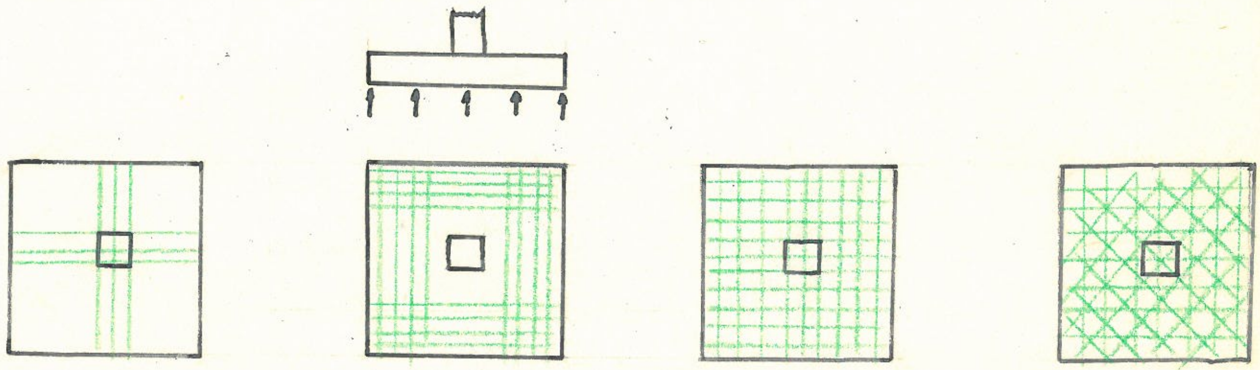
1. Failure due to moment.

2. Failure due to shear (diagonal tension).

3. Failure due to "punching shear."

"Punching shear" is no longer a part of reinforced concrete design but it is being revived for prestressed concrete. Shear is obtained but the crack runs on the <u>tension side</u>.

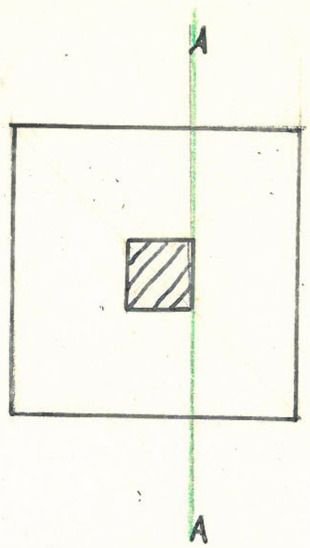
The following diagram shows the conditions of Talbot's tests. The size of column and depth of footing varied. The pressure on thebottom of the footing was kept constant.

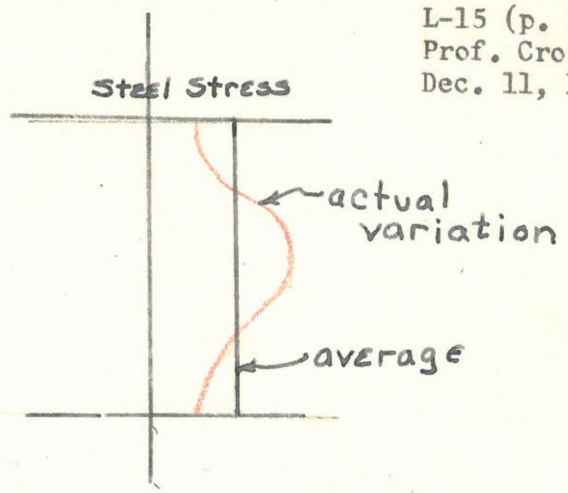


All of the above footings illustrated will work equally well if the same steel is in each.

It was made evident in the tests run that the column tended to punch out a cone in the footing. If a crack started directly under the column face on the underside of the footing it would have no place to run except through the column. If the M to V ratio is small, failure is threatened. Failures were observed by Talbot to be in bending, diagonal tension, and in bond.

Footings are not a very expensive part of construction. A little less steel or a little less concrete does not make much difference in the cost of the footing.

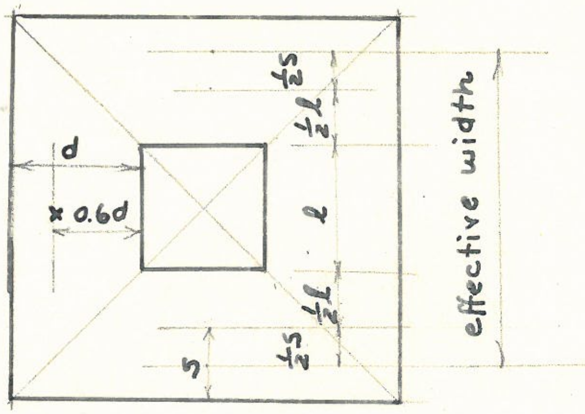




The bending moment is the product of the pressure and the distance of the center of pressure from the section A-A. The bending moment, then, is the tendency to turning, about a straight axis. This tendency is resisted by a couple, the concrete in compression and the steel in tension. Divide this couple by the distance between, jd, and get the stress in the steel. This is no different from any other case.

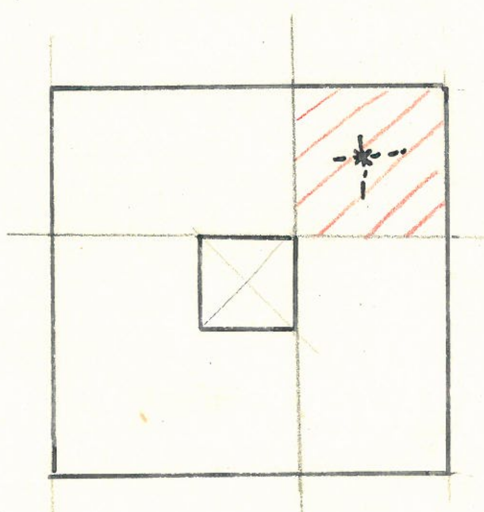
If cracks occur where the moment is maximum, the concrete there becomes less stiff so the moment tends to even out. It does not matter then where the steel is put as long as there is enough steel.

It was deduced by investigators at that time that the bending moment was due to loading on a trapizoid. On a plane, they divided the trapizoid into two triangles and a rectangle. They took the centroid of triangle as 0.6 the distance from the column edge instead of the 2/3 it should have been. Then, equally mysteriously, they took the effective width to resist the bending moment to be the column width, plus, on each side, 1/2 the column width, plus 1/2 the remaining distance.



This idea remained in books and specifications for thirty years. Although the results of tests run verified calculations by this method if the dimensions were changed the results would bedifferent.

If you consider the whole rectangle instead of the trapezoid and the whole width as effective you will get the same results on the dimensions used in testing. What was bothering people was that the load seemed to be taken twice as illustrated below.



Modern specifications scrapped finally the complicated trapezoidal method and used the whole rectangle. However, the ACI specifies that only 85% of the bending moment on that section be accounted for. On flat slabs the ACI code says the moment to be resisted should be $0.09\,wl^2$ or 72% of $\frac{1}{8}wl^2$. 15% of that may be accounted for by the fact that concrete takes a good deal of the tension in a slab as indicated by the cracks in the slab which run zig-zag. The steel strains in the slab are eradict. At the time of Talbot thestrain gage had not been invented.

The tests brought out that since cracks run parallel to the bar be conservative in bond. However, it is not possible to find data in thebulletins to support this conclusion. This brings us to another point. It is wise to go under the assumption that the man who wrote the bulletin did not know what he was talking about. Then say "Smith states his tests show..." However, if you say "Smith's tests indicate the following..." you had better realize it becomes your responsibility.

Slater pointed out that no concrete beam has long survived the yield of the steel. As soon as the steel yields, the steel draws downand the bond loosens. This may not apply to drawn wire but it does not matter because it is not used in reinforced concrete.

Because of its simplicity reinforced concrete will remain with us for a long time. Prestressing requires expert field technique.

If the slab is not square it does not matter. Figure the moments about the long axis. The analysis for shear is the same as for the square slab. Actually, the shear is not so uniform but it is not necessary to fuss about it.

"Reinforced Concrete Wall and Column Footings"
By Frank E. Richart

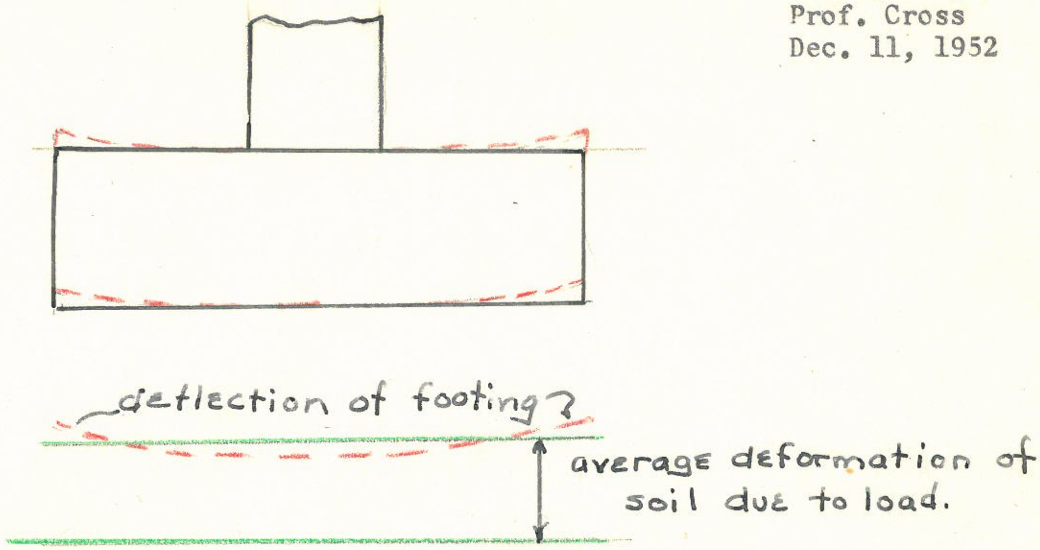
American Iron and Steel Institute
(Reprinted from Journal of ACI, Oct. and Nov., 1948.)

The Iron and Steel Institute is an association of steel manufacturers who were not particularly interested in details of how to construct. It was known that bond was largely a wedging action.

These deformed bars were patented. However, the patents ran out. These people who rolled the bars and the warehouse operators realized it was expensive to maintain stock of all these kind of bars. They thought it would be a good idea to adopt and force the adoption of one or at least fewer bars. There are now three types. They brought pressure on the various committees to force the adoption of these bars by raising the allowable bond. Any other bar was termed a plain bar and did not have any bond at all. Actually, doing this, theoretically, condemned many buildings that were twenty years old. In order to sell these new bars to the profession, the Iron and Steel Institute developed an interest in research. As a result they had many tests made, some good and some bad.

Richart did not like the tests performed by Talbot. Therefore, he ran them again using deformed bars.

L-15 (p. 7)
Prof. Cross
Dec. 11, 1952



The deflection of the footing is insignificant in comparison to the settlement of soil due to a uniform pressure.

In footings of varying size with the same design load, the moment in ft. lb. per ft. width is independent of the area of the footing. Therefore, the percentage of steel will be the same no matter what the size of footing. The shear will be the same, too. How then will the footing fail? If the footing were made very wide, the deflection would no longer be insignificant.

SLABS

The classical algebraic approach to slabs will be presented by Prof. Looney. Here we will becomefamiliar with methods used in the vast literature of this field.

Analysis of the deflected structure starts with a deflected structure. Deflections are represented by the letter Z.

Slope	$\frac{dz}{dx}$	$\frac{dz}{dy}$
Rate of change of slope	$\frac{d^2z}{dx^2}$	$\frac{d^2z}{dy^2}$
Moments that produce curvature	$\frac{d^2z}{dx^2}S$	$\frac{d^2z}{dy^2}S$

S is the stiffness

Shear in X direction	$\frac{d^3z}{dx^3}S$
Shear in X direction due to shearin Y direction	$\frac{\partial^2}{\partial x^2}\frac{\partial}{\partial y}S$
Shear in Y direction	$\frac{d^3z}{dy^3}S$
Shear in Y direction due to shear in X direction	$\frac{\partial^2}{\partial y^2}\cdot\frac{\partial}{\partial x}S$

Rate of change of shear in load per ft.

$$S\left[\frac{\partial^4 z}{\partial x^4} + \frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} + \frac{\partial^4 z}{\partial y^4} + \frac{\partial^2 z}{\partial y^2}\frac{\partial^2 z}{\partial x^2}\right]$$

Lagrange's Equation

$$w = S\left[\frac{\partial^4 z}{\partial x^4} + 2\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} + \frac{\partial^4 z}{\partial y^4}\right]$$

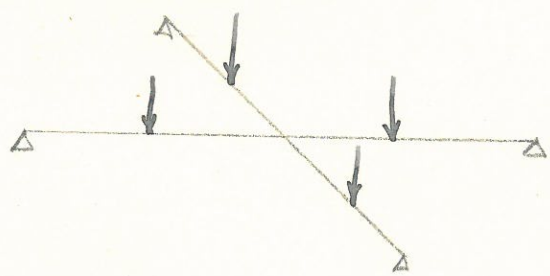
First differential ------ Slope

Second differential ------ Curvature

Third differential ------ Shear

Fourth differential ------ Load

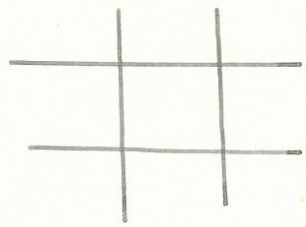
The classical method tries to do in slabs what is almost impossible for beams.



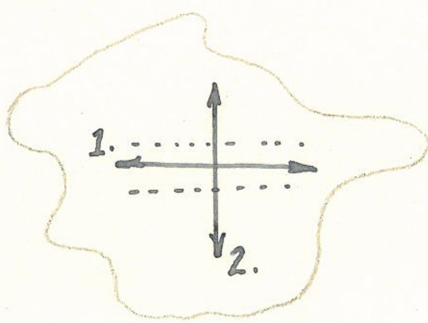
The stiffness which is the ratio of the curvature to the momentmay not be the same in both directions.

In the time ofTalbot's investigations people were thinking in terms of grids not slabs.

GRIDS

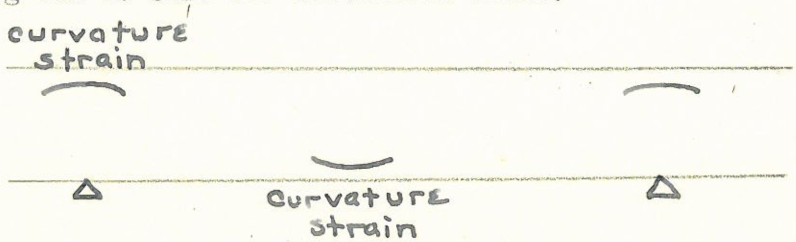


The steel slab acts the same way as a grid but a concrete slab does not because either the concrete cracks or the steel is not working. After the slab cracks the matter may be very different. How does the cracking in one direction effect the moment in theother direction?



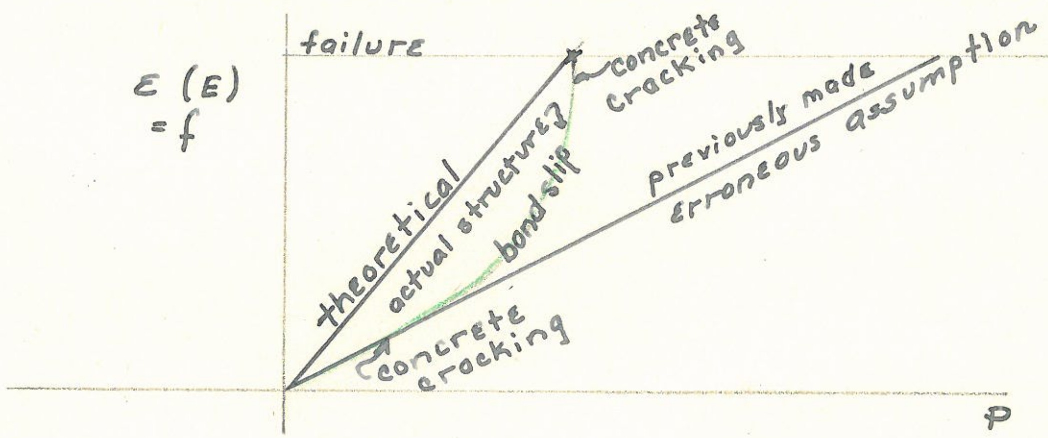
1. Moment, stiffness, and strain in steel 1.
2. Moment, stiffness, and strain in steel 2.

It was necessary for the investigators to swallow that the material is isotropic, i.e., it acts the samein both directions. Therefore, it would need the same amount of steel in both directions. The same thing can be said for continuous beams.



By following this through it can be seen that the same amount of steel is needed in both places.

A slab is probably in pretty bad shape if the cracks are straight. In the early days of testing they tried to find out about the strain in the steel in the field. However, it is hard to make measurements outdoors. Further, and most important, conditions can not be brought about to cause failure because it would endanger people and the rest of the building. Tests run on the Western Newspaper Building by Talbot produced results as indicated below. (See, also, Lecture 14.)



He measured the strain and multiplied by E to get the stress. Making measurements in the field is one way of getting at data. Another way is to build a model slab and test it in the laboratory. Itis true that this method may not be worthwhile, but any test shows something. However, the experimenter has to be smart enough to see what it shows.

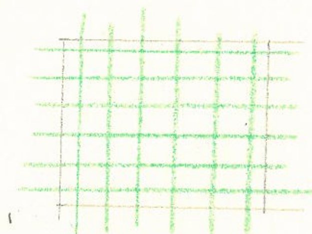
The following are the problems encountered in slabs.

1. One way slabs

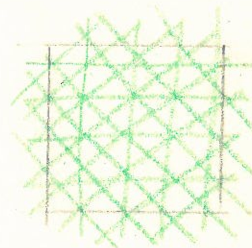
2. Flat slabs

These were figured first and then theorized later. First these flat slabs rested on round columns, later on capitols or drop panels.

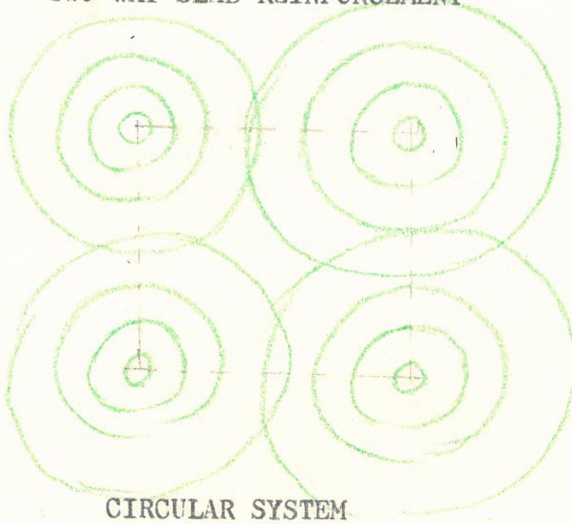
Construction was relatively simple. Further, they were very simple architecturally.



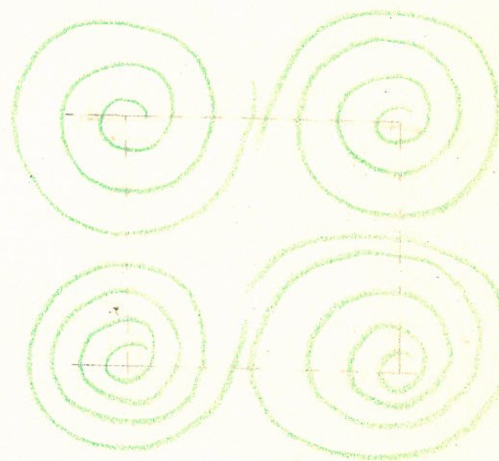
TWO WAY SLAB REINFORCEMENT



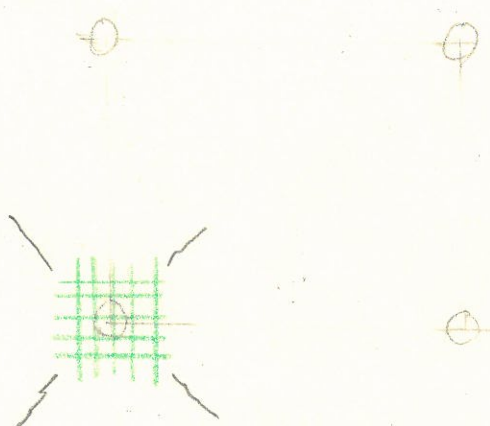
FOUR WAY SLAB REINFORCEMENT



CIRCULAR SYSTEM



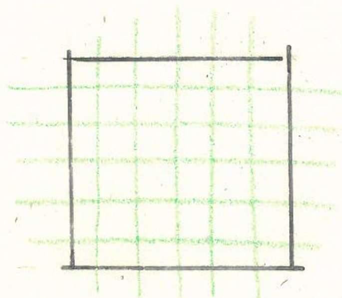
SPIRAL SYSTEM



STEEL GRID

There is a tendency to get circular crack with the circular system. As a result, there was much disagreement as to its satisfactoriness. The steel grid formed cracks from the corners.

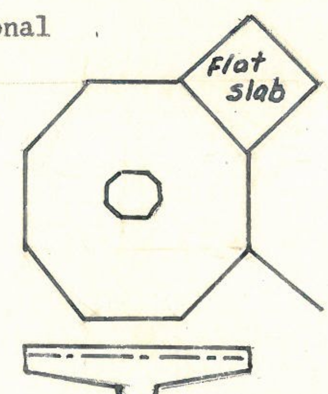
3. Two way slabs



These two way slabs ran up to 30 ft. by 30 ft. They make a very convenient system. However, there are all sorts of complications because the slab does not have to be square, and because it can be simple supported or continuous in one or two directions.

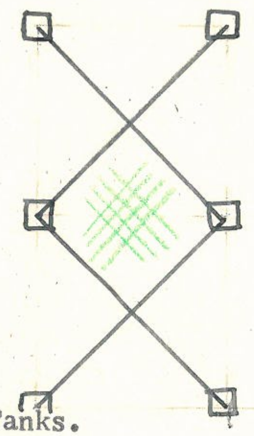
4. Miscellaneous slabs

A. Octagonal



The virtue of this system is that only one easily braced form ~~was~~ is needed. All the steel was straight. However, trouble occurred in the flat slab.

B.



5. Bins, Bunkers, Tanks.

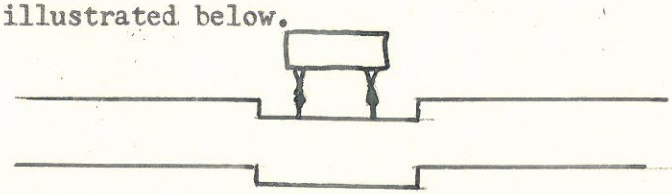
When slabs are used for these purposes, many complications arise. What happens when there is a concentrated load in one way slabs? What beam takes how much?

6. Roadways and Railways.

The problem of skew complicates the matter in slabs of this type.

7. Footings.

Rules and regulations can be written to cover all cases of slabs, but slabs are very irregular. In some cases a hole is required in the slab for an elevator or a stairway, or it may necessarily be constructed as illustrated below.



It is not necessary to become familiar with all the rules and regulations in the codes. In the codes there are limitations put on slabs without regard to analysis.

1. Limitation on depth:
   Example: 1/30th of the span.

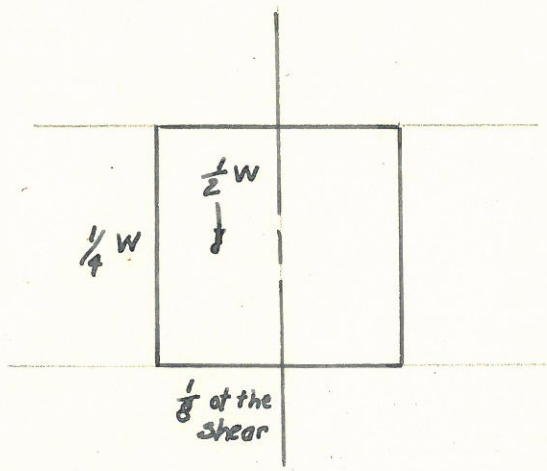
2. Limitation on minimum reinforcement
   Example: 1/2 of 1%

3. Limitation on maximum spacing of bars
   Example: three times the depth of the slab.

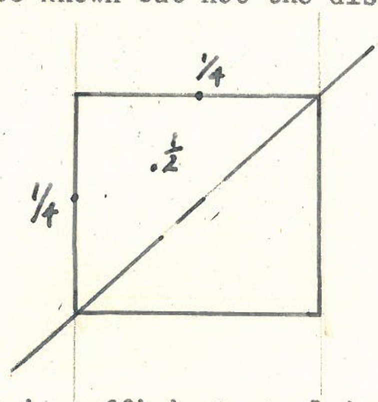
Specifications reflect the fact that flat slab construction was a highly competitive field. The First Joint Committee had many different distributions of moment. For the middle and at the end it said the moment was 0.09 Wl^2 or 72% of 1/8 Wl^2. At about the same time a paper of Nicols said it was 1/8 Wl^2. The tension in the concrete accounts for some of the reduction to 72% but not all. However, it is quite difficult to explain such a thing by pure theory or logic.

In modern specifications the moment is distributed so that the column strip takes about two-thirds of the moment, two-thirds of which is negative, one-third positive, and the middle strip takes about one-third of the moment, one-half of which is negative, one-half positive, the coefficient of the total moment is 0.09. However, the limits on reinforcement, on depth of the slab, and on deflection of the slab may make the question of moment unimportant.

When analyzing two way slabs there are many uncertainties. In the first case, as illustrated below, the distribution of the shear along the lower and upper edge would be uncertain.



The second case as illustrated below : The total moment across the diagonal would be known but not the distribution of the moment.



If there is sufficient steel in the slab itwill take the moment no matter how the moment may distribute. The engineer could find the answer to slab problems by judgement but because of competition it was necessary to lay down rules in codes. However, because in slab construction there is a possibility of most anything, a mess resulted. The two way slab was penalized with regards to the flat slab. Bunkers and tanks are generally neglected in literature. It usually can be easily handled but caution should be exercised if the codes should step in. As soon as rules are set down, things get chaotic.

CIVIL ENGINEERING

Civil Engineering meant originally non-military engineering. Now it is difficult to draw the line. There is no fundamental difference between the architect and the civil engineer, or the architect and the builder. As has been said in earlier lectures, civil engineering means those branches which involve: Planning; Building; Operating (see lectures L-2 and L-3).

Not really engineering, but essential to engineering as are most sciences is the field of research and development, and the field of mechanics which includes Statics, Dynamics, Theory of Elasticity, Strength of Materials, Fluid Mechanics, Soil Mechanics, and Thermodynamics.

Our work is dominated by nature. The earliest cave men tackled the problems of shelter, walks to cave entrances, sanitation, drainage, etc.

The most common way of breaking down the field of civil engineering is as follows:

1. Structural and Architectural

2. Use and Control of Water

3. Transportation

Structures include buildings, bridges, dams, tunnels, structures supporting earth, and containers such as tanks.

A better way of separating the field of civil engineering is as follows:

A. Hydroelectric Power Development
(Steam power and gas power must be considered, too.)

B. Control of Rivers
(This is a specialty of the Army Engineers and includes erosion problems, bank collapse, cut offs, etc. Erosion and drifting concerning harbors is a similar phase. Also in this phase is the building of industrial plants on bottom land of river basins.

C. Sanitary Engineering

1. Obtaining, purifying, and distributing water.

2. Disposing of refuse.

D. Transportation

1. Local Transportation

2. Long distance transportation

a) Railways (Steam, Deisel, Electric)
b) Highways (Trucking industry)
c) Airways
d) Maritime
e) Pipelines

Who wants to get something where and how?

1. Something

a) People
b) Products of the soil (lumber)
c) Products of the ground (coal)

2. Where

a) Port terminal
b) Warehouse
c) Ultimate destination

3. How

a) Equipment
b) Power
c) Ways (bridges, etc.)

There are many organizations dealing with these specialties.

1. ASCE

   Although a source of information, there is still much "poppycock" that should be filtered out.

2. ACI

3. AREA

   Although its publications are written from a railroad view point, they deal with everything including highways, sanitation, bridges, and buildings.

4. ASTM

5. ASME

   Although this may not seem related, in that it deals with heating and ventilating, elevators, and cranes, itis useful to the civil engineer.

6. Others

   Portland Cement Association, PCA.
   American Institute of Steel Construction, AISC
   American Reinforcing Bar Association, AASHO
   Bureau of Public Roads

The literature in our field can be divided into three parts:

1. Newspapers
   (Example: Engineering News Record)

2. University Bulletins

3. Textbooks
   (Textbooks are published for the use of the college and are usually dated.)

Foreign engineers have an influence here. The Italians are excellent engineers. The French are also good, but not aggressive.

The Germans are not so good, but, being aggressive, they have thrust their work on the world and largely have become dominant.

Civil Engineers should become patient with geologists. The accuracy of the geologist is good if within 50 or 100 feet. On the other hand, the accuracy of the engineer is very bad if he is 50 or 100 feet off.

Other phases that are relevant to engineering include:

1. Natural Phenomena

   a) Climate
   b) Weather
   c) Wind
   d) Wave
   e) Earthquake
   f) Fire, rot, and rust

2. Law and Customs

   a) Real Property
   b) Contracts
   c) Agency
   d) Liability

3. Cost and Value

   a) Finance
   b) Economics

4. Human Relations

While we often hear of engineers being criticized for their inability to write, actually they are equally poor at reading. It takes a great deal of experience to be able to glance through a book and pick out the highlights.

Prof. Cross has watched the literature of the field of soil mechanics grow from a wide scattering of information to enormous volumes. The subject is not new but, because it has only recently been coordinated and organized, it seems new. In dealing with soils it should be recognized that there will not be as much uniformity as in concrete or steel. Actually, "soils" is a misnomer because soil to ordinary people means the layer on the top of the ground.

The origin of the earth is worth some study. One should be able to differentiate between igneous rocks, those that have solidified from a molten solution, metamorphic rocks, those changed by heart or pressure, and, finally, sedimentary rock, those which have been carried and deposited by lakes and rivers.

Much of all engineering work is near rivers because of the needs of manufacturing, transportation, and water supply and disposal of waste. Development is also likely in valleys.

The term rock can be misleading. Shale is often called a rock because when dry it appears to be a rock. However, it becomes a clay when weathered or if it be subjected to water under pressure. In river work this has disastrous consequences.

The best place to study earth is from a hole in the ground. At one time personal experience was the only guide to follow in soils. About thirty years ago an Austrian, Karl Terzaghi came to America. He was a geologist but he set up courses in MIT which were the first courses in this new field, soil mechanics. Of course, Terzaghi has received innumerable awards for his outstanding work. The subject was taken up initially by the highway departments, notably the Bureau of Public Roads, and, later, by the Army Engineers.

Soil mechanics may be divided into two catagories:

1. Identification of materials: If a material can be identified to be the same as a material found elsewhere, for which the properties are well established, the properties of this new material can be assumed to be the same.

2. Prediction of what the material will do under certain conditions: Typical questions are as follows: What is the earth pressure against a retaining wall? What is the tendency of soil to flow when unconfined? What are the properties of soils when frozen? Etc.

The following are methods for predicting how a material will act under certain conditions:

1. Experiments: Experiments are not at all conclusive. When making experiments it is important to be able to imagine what may happen and the probability of it happening, but one does not say "this or that will happen."

2. Observation: It is good to form judgement from seeing what happens on actual structures.

3. Analysis of Models: There is not as much tendency to make models in foundation and soils engineering as there is

in structures. This is probably very good.

4. Authority: Soils differ in this phase radically from concrete in that information from old literature is just as good information as that from new literature because soil has not changed through the years. So, although the evidence is there it must be transcribed into modern terminology.

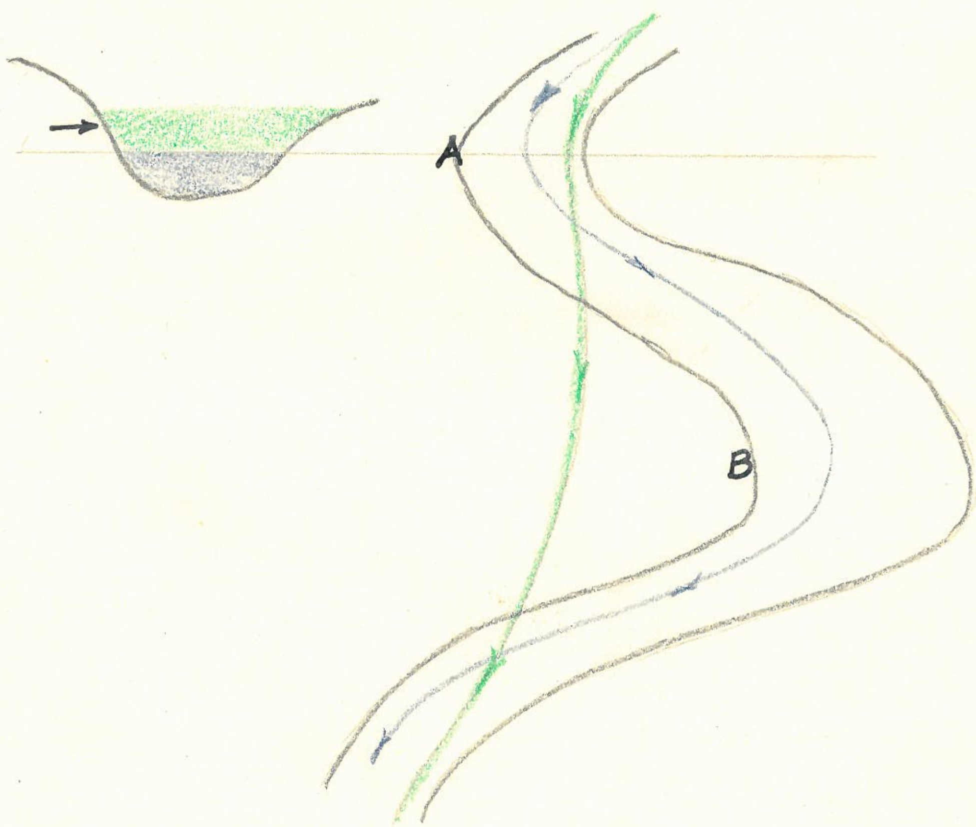
5. Common Sense: Common sense is more important in the foundation field than in any others.

The whole subject of soils is closely affected by the action of water because almost every engineering project is on or near some water source such as a river, lake, or ocean. Water affects foundations in two ways: (1) water action on the properties of the soil, and (2) erosion and constant sliding as a result of water.

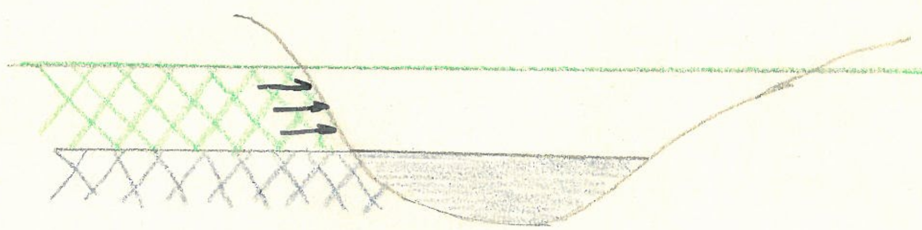
It should follow from the preceding paragraph that it would be wise to become familiar with hydrology, i.e., where water comes from. It is well to develop a scale on hydrology. What constitutes a big river or a little river? It is advisable to get some idea of the river system in the United States and its extent. It seems true that people from the coast regions of the East and West have very little knowledge of the Mississippi Valley and the rivers of the Great Plains. These rivers can wash away a 1000 acre farm overnight and dump it on the other side of the stream. Further, the river may rise 40 or 50 feet.

These rivers have enormous flood valleys for storage. To improve marginal lands these storage areas may have been diked off, rising the flood crest of the river a great amount.

Suppose a bridge is to be built over a river such as the Red River. Books advise rather generally to build the bridge normal to the river. What, however, is the course of the river? At low water it is one way, at high another. Low water is there for a long time and causes the erosion. The sketch below illustrates some of this. The green represents high water, the blue low water.

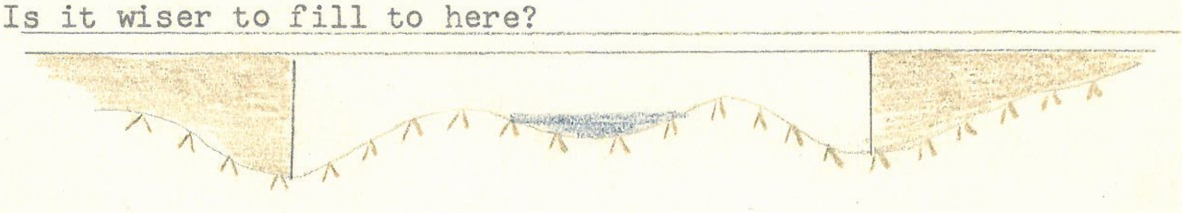


The river picks up at A and deposits at B. There is, therefore, a tendency for the slope at A to slide. This tendency is the greatest at falling water. A hydraulic condition results which pushes the soil out.

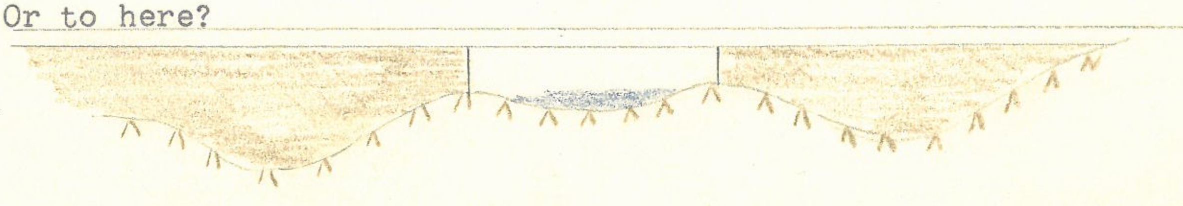


Rivers form natural levees into which they overflow at high water.

Is it wiser to fill to here?



Or to here?

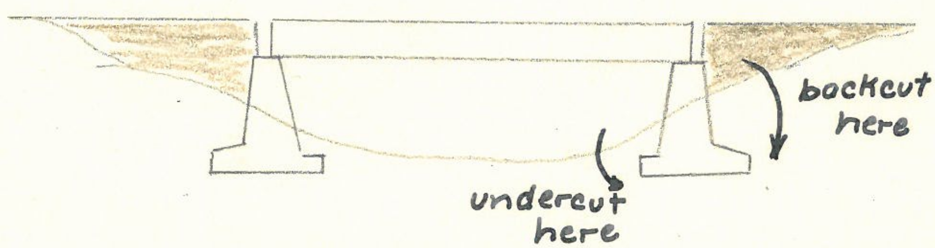


It is traditional in America for the rivers to be run by the Army Engineers.

Schools should have a course in how rivers run down hill. In so doing they would be answering the question of erosion and deposition of soil. Why does soil erode? It is true that the effect on the element is dynamic, i.e., it is pushed along, but, there seems to be something else. When the water flows at high velocity the soil is lifted to the top, spread out, and carried downstream. (For a good description of the building of bars on the inside of curves read "Life on the Mississippi" by Mark Twain.) John B. Eves walked on the bottom of the river and described how the whole bottom of the river is being carried along.

Surprisingly enough a bridge pier is not going to fall down stream but upstream because eddy currents form where the current splits. These eddy currents pick up the soil.

Eighty percent of all bridges are small, i.e., under 80 feet. The following diagram illustrates what might happen to one of these bridges.



When dealing with culverts instead of theorizing on how much water will flow through it, it is more advisable to go down and look at the site. That is the cheapest and best way and it applies equally well for bridges, too. It is the only way to form judgement on the danger of foreign matter piling up. The danger may be ice or drift, such as lumber and wheat shocks, which came down the river during floods. A solution to the problem may be obtained if piles are driven upstream.

Our libraries contain a large amount of material on it, but it is scattered. The material is not at all out of our field. Structurally speaking, the expense of river erosion is greater than the expense of structural inaccuracies. In about 1922 this field became more systematized. Credit goes to Freeman who built dams and similar structures. He did work in Europe.

Soils

It is hard to get information on soils. It is harder yet to evaluate the information. Soil is a material formed by nature and, therefore, it is quite variable. Since it is such a material, there is no control of its manufacture (except a little in special cases of grouting, etc.). There is, however, some control of steel and concrete. It is natural that in the case of soils it is desirable to know something about its manufacture. This is hard to come by. It depends largely on geology. The character of the earth can change, too, by erosion, or, perhaps, in some way a change in the strength properties over a period of time can occur. The earth may have been deposited by a glacier, river, wind (aeolian), or erosion. Salt water and fresh water have different effects on earth, too. It is advantageous to know the limits of uniformity, or non-uniformity of the soils.

The following are different types of soil conditions:

I. Surface Soil

Roots and the like grow in it. It is subject to the effects of climate (wind, water, etc.), and fluctuations of the water table.

2. Deep Soil

Since it is below the water table it is not affected by changes in the water table. Although more dependable than surface soil, it will change when a building is placed on it.

3. Deeper Earth

This strata may be consolidated. Because of excavation the pressure may be removed and swelling result.

4. Rocks

5. Fill

Generally, fill is material placed by man. It has not had an opportunity to become compact.

The following are problems found in soil mechanics:

1. Stability of the earth itself.

This is a problem of motion. Will an earth slope shift or stay where it is? Will a river bank stay put? If a building is placed on an earth mass, will it move?

2. Ability of earth to resist pressures imposed on it.

Can the earth be depended on to resist movement? This may not mean instability if it cannot but unsuitability.

3. Pressure of earth on a structure.

Retaining walls, tunnels, and culverts are problems of this type.

For any material investigations are made to determine the following:

1. Strength

2. Deformation Properties

How much will it move and where will it move when pressure is applied to it?

3. Satisfactoriness

Some materials change characteristics when exposed to moisture or air.

In soils, tests are made for two purposes: (1) to identify the material, and (2) to predict the action of the material under certain conditions. From tests theories can be developed to predict how the soil will act under certain conditions. If you can identify and classify the material, and if you can observe how the material acts for hundreds of structures, then it becomes a matter of bookkeeping to design structures.

It is good to go into literature with a classification system since it assists in selecting what to read.

An outline in brief:

I. Physical Conditions
   A. Type of Structure
   B. Type of Foundation
   C. Location of Structure
   D. Earth and state of site

II. Statics and Geometry
   A. Forces Acting
   B. Movement, Forces Produced, and Causes of Movement
   C. Sources of Evidence of Movement

III. Corrective Conditions
   A. Earth (as a material)
   B. Structure
   C. Site

IV. Construction Method

I. Physical Conditions

A. Type of Structure and

B. Type of Foundation

1. Roadways, Railways, and Runways
The force is applied directly to the earth. A foundation is not added since the structure forms its own foundation.

2. Buildings
The types of foundation used include individual footings, wall or continuous footings, mats, piles and any combination of these types. The structure exerts a force on the earth.

3. Bridges
   Bridges have piers and abutments which in turn have foundations of their own such as mats and piles. The structure exerts a force on the earth.

4. Tunnels, also Culverts, Subways
   In general these form their own foundations. The earth exerts a force on the structure as well as the structure exerting a force on the earth.

5. Retaining Walls, also Bulkheads, Sheeting, Cribbing, and even Tanks and Bunkers
   Although they are really their own foundation, it is often necessary to extend their base.

6. Cuts and Canals
   There is no foundation except the earth below. There is some relationship between a cut and a retaining wall.

7. Earth Dams and Embankments
   These are extensive in river areas. It involves the problem of seepage. Water under pressure can take material with it.

8. Masonry Dams
   The relationship between masonry dams and retaining walls are remote. Masonry dams may have spread footings or piles.

9. Docks, Wharfs, Piers
   Earth and water structures are subjected to forces that other structures are not subjected to.

10. Trestles and Causeways
    Generally they are supported on piles.

11. Marine Structures (breakwaters)
    In general, nothing is known about the forces on marine structures.

12. Fill

C. Location of Structure

Because of the different soil conditions the location of a structure makes a difference on the structure.

1. Land

   a) Upland
      This area is subject to slides

   b) Lowland
      This area is subject to floods

   c) Near Water
      This area has a maintenance problem

2. In Rivers

   It is essential to know:

   a) River type
      Has it a flood plain or a tide?

   b) Location of River

   c) Hydrology of Region

   d) Drift

   e) Erosion

   f) Sedimentation

3. In Lakes

4. On the Coast

   Information is needed on the:

   a) Shoreline

   b) Toral Current

   c) Waves

   d) Tides

   e) Marine Borers

5. In the Ocean

D. Earth and Stratification of Earth on Site

The soil can be classified by:

1. Origin

   a) Transported (and how it is transported)

   b) Weathered

   c) Volcanic

   d) Organic

   e) Marine

   f) Fill

2. Soil Names

   a) Sand

   b) Clay

   c) Gravel

   d) Silt

   e) Adobe

   f) Gumbo

   g) Mud

   This method is deceiving because a name like gumbo means something different elsewhere than it does here.

3. Identification of Properties

   a) Mineral Constituents

   b) Grain Size
   (Gravels and Sands)

   c) Grain Size Distribution

   d) Grain Size Shape
   (Clays and Silts)

   e) Orientation of Grains

   f) Density of Soil

g) Voids Ratio

h) Moisture Content

i) Plastic Limit
(clays)

j) Liquid Limit

k) Permeability

4. Tests

a) Direct Shear

b) Unconfined Compression

c) Triaxial Compression

d) Consolidation

e) Others

Information can be obtained from the tests above. Information should also be obtained on natural modifications by:

1. Erosion

2. Water

3. Frost

4. Drainage
This can be both harmful and good. It depends on whether drainage can be maintained.

5. Chemicals

6. Compression

7. Shearing

II. <u>Statics and Geometry</u>

A. Forces Acting

1. Live Load)
2. Dead Load) Dead load and live load should not be combined since dead load is acting night and day while live load is not. It may be possible to proportion only for dead load.

3. Wind
This is important for buildings, bridges, dams and locks.

4. Water
Water may be either static or dynamic, and it may be uplift.

5. Ice
This may be drift or frost.

6. Drift
This includes trees, ice, wheat, etc.

7. Earth Pressure
This force acts on tunnels, culverts, retaining walls, and many others.

8. Vibration
Vibration may be caused by machinery or by earthquake.

B. Movement Forces Produce....

1. Horizontal

2. Vertical

3. Rotational (in any component of the horizontal and vertical plane).

---Causes of Movement

1. Force

   a) Squeezes out water in earth mass.

   b) compacts soil

   c) Compresses particles

   d) Causes instability which displaces whole mass.

2. Water

   a) Shrinkage (take water out)

   b) Swelling (put water in)

   c) Change in ability of earth mass to resist applied force.

   d) Underground erosion

   e) Frost
   Frost produces swelling when forming, shrinkage when thawing. Ice crystals form on top. These cause a capillary action which draw water to the crystal to form ice lenses. When the ice thaws it melts first at the top. The melted ice is trapped by the ice beneath.

C. Sources of Evidence of Movement

1. Theoretical Analysis

   a) Determination of stresses on various underlying strata. (Boussinesq's equations)

   b) Deformation Theory...
   Theory of Consolidation
   1. amount
   2. time

   c) Sliding (Shear)...
   1. Cuts
   2. Embankments

   d) Pile Theories

   e) Earth Pressure Theories

2. Experiments (full scale)
   Load Test

3. Model Test (small scale)
   This is very poor. Sand particles might act like boulders when used in models.

4. Experience
   This may be the best.

5. Intelligence or Common Sense
   Intangible

6. Study of Displaced Structure

   a) Why
   b) How
   c) Where
   did it move

III. <u>Corrective Conditions</u>

A. Earth (as a material)

1. Drainage
   Wellpoints, etc.

2. Consolidate
   a) Timber piles
   b) Sand Piles

3. Compact

4. Chemical Consolidation
   (Turn sand to weak sandstone, etc.)

5. Electrical Consolidation
   (Used to Stiffen Clays)

B. Structure

1. Use a Rigid Foundation
   This tends to decrease differential settlement.

2. Put Jacks Under Structure
   This method used for Yankee Stadium.

3. Carry Structure to Firm Strata
   This can be done by means of piles or sandpiles.

4. Corrective Conditions to Structure by means of <u>Construction Sequence</u>.

C. Site

1. Preload

2. Make Foundations Deeper

3. Remove Material so as to Reduce Total Load

4. Place Sand Layer on to Distribute Load.

IV. <u>Construction Method</u>

A. How are you going to build it.

1. Equipment

2. Caisson

3. Cofferdam

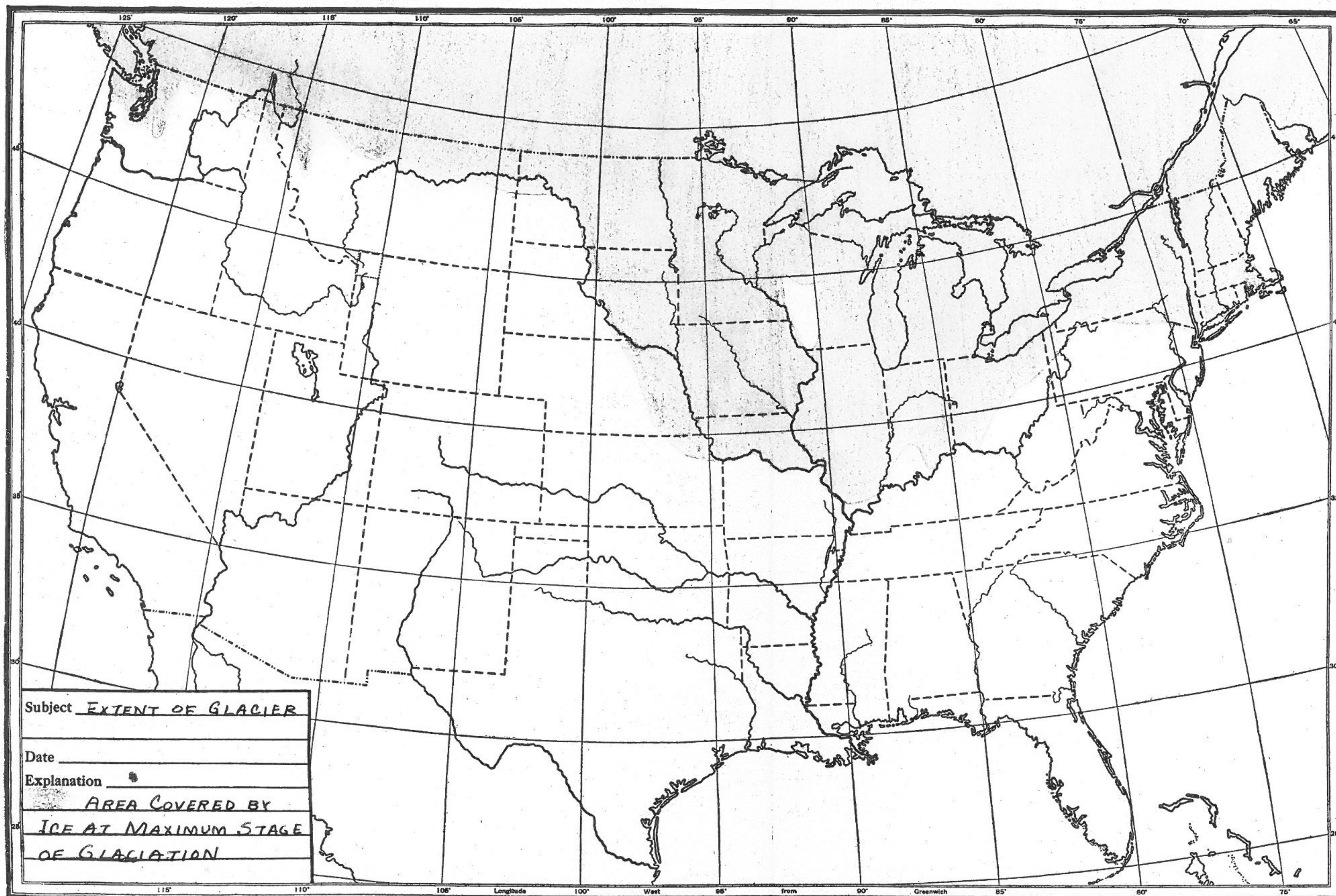
4. Pile Driving

5. Dredging

6. Earth Moving

7. Maintenance

McKINLEY'S DESK OUTLINE MAPS. DOUBLE SIZE. No. 176a. THE UNITED STATES. (STATE BOUNDARIES)



Prof. Brennan
Jan. 13, 1953

Soil mechanics is a very broad subject and one of the most important engineers run into. Here we are interested not so much in theories or tests but whether soil will be satisfactory for use. You cannot talk just soils or site, or, on the other hand, just structures. What happens to the earth may effect the structure which in turn may effect the earth. At the same time we investigate what may happen to the structure or the soil we must consider the possibility of it happening.

Similar to our work in structures, it is advisable to go into the literature of soil mechanics with a question. It will not do to read a book. The following are typical questions:

What soil conditions are suitable for a particular type of structure under suitable conditions?

If the soil is not suitable or conditions are not suitable what will happen to the structure?

What can be done to prevent this from happening?

What information can I get on design?

What information can I get on how to classify conditions?

What theories are available that can help predict what will occur?

What is "soil mechanics?"

There are books on geology, soil mechanics, rivers, and water control. What can we get from geology? We are, of course, not interested in petrology. However, geology will shed much light on the top surface, the top 100 feet. There is good information available in the field of geology on selection of site, what will happen at the site, and on manufacture of the material. There is not much control of soils since they are manufactured naturally. Mostly, expense and time limits control. It is hard to refute anything said in geology. They have been able to get a picture as far as glaciers, erosion, etc. are concerned. We are interested in how the soil got where it is. This can be obtained from geology.

Geology is very broad. A good geologist can determine some things very adeptly. He can select the best site from many. Since he knows something about pre-loading or preconsolidation he can cleverly predict settlement. By noting weathered rock, he can estimate the proximity of rock. By looking at a mountain or hill he can see evidence of sliding which would make the hill undesirable because it may continue to slide, and the valley undesirable because the soil there is unconsolidated. He can look at folds and estimate their extent. The same thing applies to faults. If he finds jointing in the rocks he is aware that water will get through and cause disintegration. Further, he can recognize drumlins and eskers and so predict the soil they contain, clayey till for drumlins, sand and gravel from eskers. He can predict soil and soil properties of valleys that he determines to be old river beds.

Information on colloids can be obtained from ceramic engineers. However, unfortunately, the civil engineer is not often in contact with ceramic engineers.

The following is a partial list of books covering the subjects of geology, soil mechanics, foundations, soil testing, and water.

Geology

Leggett: Since hs is a Canadian, he has Canada in mind. He realizes the application of geology to engineering. In all, it is the best book for our purposes on geology.

Chamberlain and Salisbury: He is primarily interested in historical geology.

Reese and Watson: This book is a conscientious attempt to interpret geology for engineers.

Longwell:

Soil Mechanics

Terzaghi, "Theoretical Soil Mechanics,": Emphasis is placed here on soil mechanics as a means of determining how foundations will act by testing and formulating theories.

Terzaghi and Peck, "Soil Mechanics in Engineering": The emphasis shifts here to a more practical means of determining soil action. Peck was a railroader and through his father gained much practical knowledge. See article by Peck in the Proceedings of the AREA, 1949, 1950.

Krynine, "Soil Mechanics": He explains what assumptions the theories are based on. This book is worth looking at.

Capper and Cassie, "The Mechanics of Soil Engineering": This is a concise, clear job.

Tschebotarioff, "Foundations of Structures": G. Tschebotarioff's background is in erosion and silting in the Nile River. After he came to this country he did extensive work in all phases of foundation and earth work.

Taylor, "Fundamentals of Soil Mechanics": This is an excellent book mostly because he quarrels with theories. The chapters on design are good.

Spangler, "Soil Engineering."

Boston Society of Civil Engineers, "Contributions to Soil Mechanics": This includes articles by many men. A. Casagrande article on flow nets is one of them.

"Proceedings of the Second International Conference of Soil Mechanics": These volumes cover all phases of soil mechanics and foundations.

"Proceedings of the AREA."

"Proceedings of the ASCE."

Highway Research Board: They have published much information on the use of soil for road beds, and on classification, grading, and reworking of soils.

American Road Builders Association.

Foundations

Prentiss and White (Of Spencer, White, and Prentiss), "Underpinning."

Anderson, "Substructure Analysis and Design,": This is a very poor book.

Dunham, "Foundations of Structures": This is a practical book which excels in detailing.

Chellis, "Pile Foundations": This is a good job on a special subject.

Testing

Lambe, T. William, "Test Procedures": This volume gives up-to-date standard soil test procedures.

River Action and Water

Parker, "Control of Water."

Grover and Harrington, "Stream Flow."

"Applied Sedimentation," by a group of authors.

Soil identification is perhaps the most important phase of soil mechanics. If the material is not known it is virtually impossible to know what difficulties may be encountered. The following are methods of identification:

I. Classifying by Origin

A. Transported

1. Glacial
With glacial soils it is possible to find most anything--perhaps material you know about. It is best to be careful. Glacial materials occur as moraines, eskers, drumlins, kames and include till, drift, boulder clay, glacial sands and gravels.

2. Alluvial
This group includes many soils. It is advisable to know where the soil came from. Alluvial soils occur as flood planes, deltas, bars and include sedimentary clays and silts, alluvial sands and gravels.

3. Aeolian
These are wind blown deposits, and, therefore, may be very fine. Aeolian soils include blow sands, dune sands, loess, adobe.

4. Lacustrine and Marine
These are off-shore deposit.

5. Colluvial

These are gravity deposits including cliff debris, talus, avalanches, masses of rock waste.

B. Weathered
It is important with weathered soils to know where they come from and from what rock, and whether they are mature or immature. Weathered soils include wacke, latevite, podzols, residual sands, clays, and gravels.

C. Volcanic
Volcanic deposits include Dakota bentonite, volclay, volcanic ash, lava.

D. Organic
These oganic accumulations include peat, muck, swamp soils, muskeg, humus, bog soils. Classifying by origin is not very good.

II. Classifying by Soil Names

Stones
Gravel
Coarse Sand
Fine Sand (What is fine sand or coarse sand?)

Silt
Clay (What is the difference? What is plastic and non-plastic?)

Loam (Sandy silt)
Adobe (Sandy clay)
Loess

Gumbo (Fine-grained, highly plastic clay)

III. Classifying by the Senses

A. Visual (Color)

B. Odor
Odor is indicative of organic material.

C. Texture (Feel)
Smooth
Sharp
Gritty
Floury

IV. Other Means of Classifying

A. Dilatancy
If water comes to the surface when the soil is shaken and then disappears when squeezed the soil is said to have dilatancy. This is a property of silts.

B. Grain Properties
1. Shape
2. Size
3. Distribution of size and shape of particles.

C. <u>Dry Strength</u>
Pull the specimen apart and see if it has strength or not.

D. <u>Voids Ratio</u>
This must be done in the laboratory.

E. <u>Water Content</u>
This may not mean much because the water content of the soil may change day by day and as a result of excavation.

F. <u>Unit Weight and Dry Weight</u>.

G. <u>Atterberg Limits</u>

H. <u>Permeability</u>

I. <u>Laboratory Tests</u>
1. Direct Shear
2. Unconfined Compression
3. Triaxial Compression
4. Consolidation

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Prof. Brennan
Jan. 13, 1953

CORPS OF ENGINEERS

CLASSIFICATION OF SOILS

MAJOR DIVISIONS (1)	(2)	LETTER (3)	SYMBOL Hatching (4)	SYMBOL Color (5)	NAME (6)	GENERAL IDENTIFICATION Dry Strength (7)	GENERAL IDENTIFICATION Other Pertinent Features (8)	PRINCIPAL CLASSIFICATION TESTS (9)
COARSE GRAINED SOILS	GRAVEL AND GRAVELLY SOILS	GW		RED	Gravel or Sandy Gravel, well-graded	None	Gradation, grain shape	Sieve Analysis
		GP			Gravel or Sandy Gravel, poorly-graded	None	Gradation, grain shape	Sieve Analysis
		GM		YELLOW	Silty Gravel or Silty Sandy Gravel	None to slight	Gradation, grain shape examination of fines	Sieve Analysis LL & PL on "Minus 40"
		GC			Clayey Gravel or Clayey Sandy Gravel	Medium to high	Gradation, grain shape examination of fines	Sieve Analysis LL & PL on "Minus 40"
	SAND AND SANDY SOILS	SW		RED	Sand or Gravelly Sand, well-graded	None	Gradation, grain shape	Sieve Analysis
		SP			Sand or Gravelly Sand, poorly-graded	None	Gradation, grain shape	Sieve Analysis
		SM		YELLOW	Silty Sand or Silty Gravelly Sand	None to slight	Gradation, grain shape examination of fines	Sieve Analysis LL & PL on "Minus 40"
		SC			Clayey Sand or Clayey Gravelly Sand	Medium to high	Gradation, grain shape examination of fines	Sieve Analysis LL & PL on "Minus 40"
FINE GRAINED SOILS	LOW PLASTICITY	ML		GREEN	Silts, Sandy Silts, Gravelly Silts, or Diatomaceous Soils	None to slight	Examination wet (shaking test)	Sieve Analysis LL & PL on "Minus 40"
		CL			Lean Clays, Sandy Clays, or Gravelly Clays	Low to medium	Examination in plastic range	Sieve Analysis, if applicable LL & PL on "Minus 40"
		OL			Organic Silts or Lean Organic Clays	None to slight	Examination in plastic range, color, odor, organic content	LL & PL before and after oven drying
	HIGH PLASTICITY	MH		BLUE	Micaceous Clays or Diatomaceous Soils	None to slight	Examination wet (shaking test)	Sieve Analysis LL & PL on "Minus 40"
		CH			Fat Clays	High	Examination in plastic range	Sieve Analysis, if applicable LL & PL on "Minus 40"
		OH			Fat Organic Clays	Medium to high	Examination in plastic range, color, odor, organic content	LL & PL before and after oven drying
PEAT AND OTHER FIBROUS ORGANIC SOILS		Pt		OR-ANGE	Peat, Humus, and other Organic Swamp Soils	Readily Identified		Consistency, Texture, and Water Content

NOTE: Column 9, "Minus 40" refers to the fraction of a soil that passes the No. 40 mesh sieve.

The Grain Size Test

This is one of the best tests. It gives the range and size of particles in the soil. The primary purpose of the test is to get the percentage of certain size grains in the mass. Secondly, however, it serves to identify the soil.

1. Sieve Analysis

   The grains are put through a series of sieves. A plot is made of the percent of grains smaller than the sieve against the size of the openings in the sieve. However, those grains passing a No. 200 sieve (.003 in.) must be analyzed by a wet analysis.

2. Wet Analysis

   This analysis is suitable to 0.0002 mm. Smaller than that involves colloids and the cation theory.

The hydrometer test is the most common wet analysis made. It is based on an interpretation of Stokes Law which was developed for one sphere falling through a mass.

Stokes Law

$$V = \frac{2}{9} g r^2 \frac{G-1}{\mu}$$

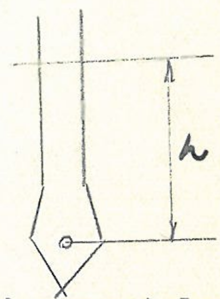
"V" is the velocity of the falling particle. The acceleration of gravity, "g," may be assumed to be 980 cm. per sec. per sec. "r" is the radius of the sphere. The specific gravity, "G," may be assumed to be 2.65. "u" is the viscosity of the liquid. Taking all of this into account:

$$V = 0.545\, D^2 \frac{(G-1)}{\mu}$$

where "D" is the diameter of the sphere. Since $V = \frac{h}{t}$

where "h" is the height of fall and "t" the time,

$$D = \sqrt{\frac{(G-1)}{32.7\mu}}\ \sqrt{\frac{h}{t}}$$



The $\sqrt{\frac{h}{t}}$ is the only variable.

However, the experiment is in error because the particles stick together; there are different specific gravities in soil sample, and the formula is based on a sphere while clay particles are more like slivers. (See Lepper, Lecture 10.)

The grain size classification for which the grain size is determined is good for sands, gravels, silts, and clays but it is not useful for combinations of these. It is customary to use the triangular or grain size distribution scheme. (See Lepper, Lecture 10, for standard soil triangles.)

There are two main types of group classifications, the Casagrande group classification which is employed by the Corp of Engineers and is shown in the table in this lecture, page 8, and the group classification of the Bureau of Public Roads which is listed below.

Group	Textural Class
A-1	Uniformly Graded Granular Coarse to Fine
A-2	Poorly Graded Granular, Coarse, and Fine
A-3	Clean Sand or Gravel
A-4	Silt
A-5	Silt, Very Poorly Graded Soils
A-6	Plastic Clay
A-7	Plastic Clay Loam
A-8	Muck and Peat

Specific Gravity

The specific gravity can be used for classification but this value is used most often in other tests. Although the technique can be obtained from a testing manual, in general, cohesionless soils are tested dried, soils with cohesion are put in suspension.

The dry cohesionless soil after having been accurately weighed, is placed in a calibrated flask and water added. Air is removed mechanically, following which the flask, water, and soil are weighed.

The cohesive soil is put into a suspension by mixing in a mechanical mixer. Then it is placed in a calibrated flask. The weight of the flask and an equal volume of water had been previously determined. Air is removed mechanically. After weighing the flask, water, and soil, the contents of the flask are put into an oven and dried, and the dried soil weighed.

$$\text{Specific Gravity} = \frac{\text{weight of solids}}{\text{weight of water displaced}}$$

$$= \frac{W_s}{W_s - (W_2 - W_1)}$$

W_s = weight of the solid substance, gm.

W_2 = weight of flask with distilled water and with soil, gm.

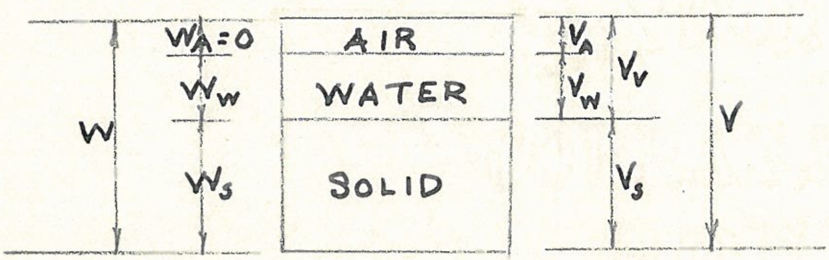
W_1 = weight of flask with distilled water only, gm.

Water Content

This test can be important as a means of measuring the shearing resistance if the water content stays constant.

The water content, "w", is equal to the ratio of the weight of water to the weight of solids in a sample of soil.

Density, Void Ratio, Porosity



$$\gamma = \frac{W}{V} \; ; \; \gamma_D = \frac{W_S}{V}$$

$$e = \frac{V_V}{V_S}$$

$$n = \frac{V_V}{V} = \frac{V_V}{V_S + V_V} = \frac{e}{1+e}$$

V = total volume
V_V = volume of voids
V_S = volume of solids
V_W = volume of water
V_A = volume of air
W_A = weight of air = 0
W_W = weight of water
W_S = weight of solids
W = total weight
γ = density
γ_D = dry density
e = void ratio
n = porosity

Atterberg Limits

Plastic Limit
Liquid Limit
Shrinkage Limit
Plasticity Index
(Refer to Lepper, Lecture 10.)

Permeability

When water flows through a soil material it is for the most part laminar flow and, therefore, Darcy's Law applies, ~~the~~

the quantity of water per unit of time is equal to the product of the cross-section area and the velocity of flow:

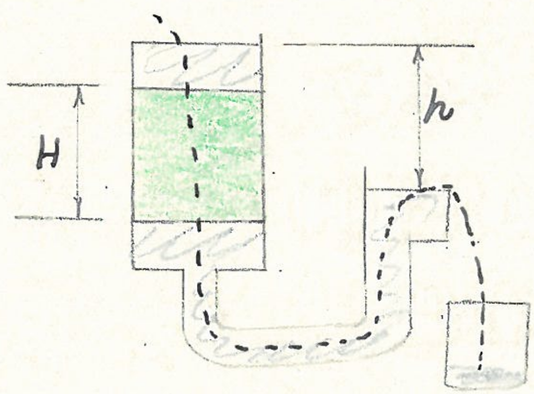
$$Q = AV$$

Let "i" be the hydraulic gradient, i.e., the ratio of the head loss to length of flow, h/ℓ, and let k be the coefficient of permeability. Then, since $V = ki$,

$$Q = Aki \quad \text{and} \quad k = \frac{Q}{Ai} = \frac{Q}{A}\frac{\ell}{h}$$

There are three types of permeameters to measure the coefficient of permeability, the horizontal flow permeameter, the constant head permeameter, and the falling head permeameter.

Constant Head Permeability Test



The volume of water flowing for a specific time, Q, is measured. h and H are constant and known.

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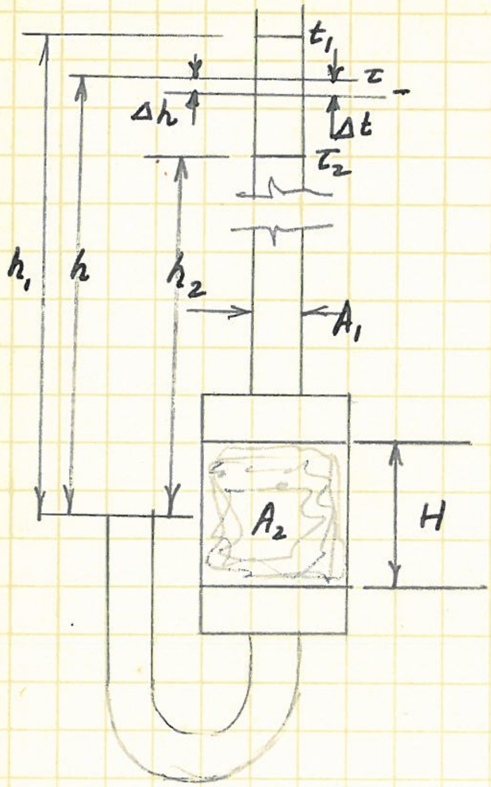
PERMEABILITY TEST (falling head)



In time, Δt, level falls Δh. The volume of amount of water flowing $= A_1 \Delta h$.

The velocity of flow $= \dfrac{\Delta h}{\Delta t}$

The velocity of flow in the specimen, v, equals $\dfrac{\Delta h}{\Delta t} \cdot \dfrac{A_1}{A_2}$

$$i = \frac{h}{H} \qquad v = ki = k\frac{h}{H}$$

$$k\frac{h}{H} = \frac{\Delta h}{\Delta t} \cdot \frac{A_1}{A_2}$$

$$k\frac{h}{H} = -\frac{dh}{dt}\,\frac{A_1}{A_2}$$

minus because as h gets smaller dh gets larger.

$$-\frac{dh}{h} = \frac{k}{H}\,\frac{A_2}{A_1}\,dt$$

$\ln = \log_e$

$\log_{10} = \lg$

$$-\int_{h_1}^{h_2} \frac{dh}{h} = \int_{t_1}^{t_2} \frac{k}{H}\,\frac{A_2}{A_1}\,dt$$

$$-\Big[\ln h\Big]_{h_1}^{h_2} = \frac{k}{H} \cdot \frac{A_2}{A_1}\,(t_2 - t_1)$$

$$\ln \frac{h_1}{h_2} = \frac{k}{H} \cdot \frac{A_2}{A_1}\,(t_2 - t_1)$$

if $t_2 - t_1 = T$

$$k = 2.3 \lg \frac{h_1}{h_2}\,\frac{A_1 H}{A_2 T}$$

$$\text{or } k = 2.3 \lg \frac{h_1}{h_2} \left(\frac{d_1}{d_2}\right)^2 \frac{H}{T}$$

The last lecture was primarily on classification of soil. This is of tremendous importance and is by no means complete. Another source, testing work, concerns properties of soils. It is, of course, undilated by identification. It is important to know what can be obtained from it.

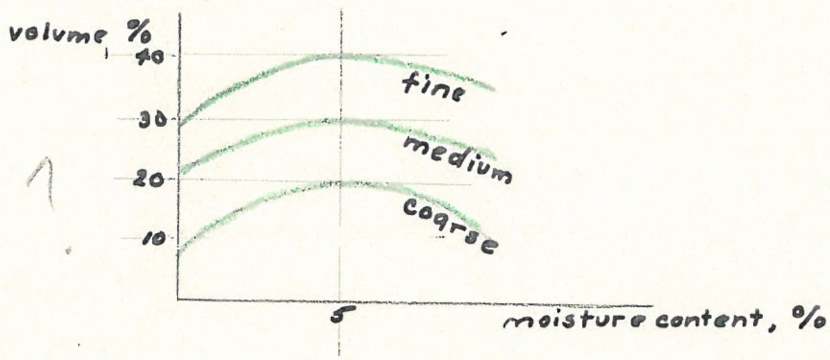
There are two large types or classifications of soils; sands and clays. The coarse-grained, non-cohesive soils are the sands, the fine-grained, cohesive soils the clays.

Coarse Grained Soil

Coarse grained soils vary from boulders to fine sands. They have usually a large quartz content. The water in the soil is either free water or attracted water. Sand is not cohesive and depends solely on friction. Sand when resisting load applied pressure derives their shear strength. The shear strength will be reduced if the load is reduced. Similarly, when water rises, the pressure decreases on the sand and the shear strength goes down.

Loose sand is sensitive to vibration. Vibration may cause detrimental settlement. The permeability rate of loose sands is quite high and there is danger of material being carried away.

Sand is not plastic, only slightly compressible, and when it compresses and settles it does so immediately. The slopes on cuts depends on the water in the sand. Sand is less affected by weathering than are silts and clays. The bearing capacity is higher, too. Dry sand has a certain volume, which, when the moisture content is increased, will increase the volume. As a result, bulking may be serious.



Fine grained soil

Clays which are fine grained soils are cohesive, i.e., they stick together. This depends on the water content. There is little internal friction. Clays are very plastic, and, therefore, may be molded, and, because so much of it is water, clays have a high void ratio. Clays expand when they absorb moisture and soften up. They lose strength as they soften. On the other hand, when clays dry they shrink. Clays have high compressibility but it takes a long time to settle. They also have low permeability because the pore space is small. The bearing value is low. It depends on the shearing strength. Clays are greasy and smooth to the touch and do not exhibit diligency as silts do. The upper grain size limit for clays is 0.005 mm.

When a job begins it is advisable to visit the site and look around. Observation may reveal whether the site is suitable for the proposed work. It may reveal what construction methods will be the best. As far as excavation is concerned, it is important to know if the sides will need supporting, or if the material is difficult to remove. Water problems are common and costly. The water conditions may change after construction perhaps as a result of construction. If the water table is reached during construction it may mean well points, sumps, or even re-routing of the water. A visit to the site may indicate if the soil is suitable, may give information on what the structure may be expected to do, and may help select construction methods and construction material.

<u>Site</u>

1. <u>Geology of Region</u>: Many problems can be avoided by utilizing geology. The limestone problem, i.e., sink holes, may be avoided. There may be evidence of slides or fills resulting from slides, and so on. Most of the United States has been surveyed by the U. S. Geological Survey.

2. <u>Old Maps</u>: An old map can often give valuable information, such as location of old stream beds, etc.

3. <u>History of site</u>: Old buildings may have consolidated part of the site while the remainder may be virtually unconsolidated.

4. Topography: This may reveal evidence of slides, etc.

5. Trees and Vegetation: The roots of trees often get into foundation and sewer lines. Further, they absorb moisture from soils.

6. Streams and Lakes: If there is a stream or lake in the vicinity, information should be gathered on flow, floods, seasonal variation, debris, ice, scour, width and height at high or low state, chemicals in stream, industrial waste, and borers.

7. Adjacent Structures: Much knowledge can be gained from adjacent structures: type of structures applicable to site; loads that are safe; evidence of settlement or deteriation. If the structure is a building the height should be determined. If the structure is a bridge, the location of piers should be determined. Since it is important to build on the same level so as not to cause destruction of the adjacent structure or to your own, the depth of foundation should be found out.

8. Underground Water: The depth of the water table and its variation must be determined but may be difficult to obtain. Information should be obtained also on movement or potential movement of water, its direction and speed.

9. Frost Line.

10. Permeability.

11. Runoff.

12. Vibrations: Vibration may cause compaction. Adjacent buildings, railways, etc. may cause vibrations.

Soil

1. Geology: Information can be obtained about the history of the region and then the soil characteristics determined.

2. Materials: Previous construction jobs will reveal much about the material.

3. Records: The records of federal buildings are usually kept in the building after construction.

The methods used to get soil, discussed extensively in the literature, is briefly as follows:

1. Sounding rod: This is the cheapest method but also the most inadequate. It is supposed to determine rock but it may hit a boulder instead.

2. Hand Auger: This is merely a screw thread turned into the ground.

3. Wash Boring: Two concentric pipes are used;into the interior one water is fitted, through the exterior one the material comes up.

4. Wash Boring and Sampling: At regular intervals the boring is stopped and a sample taken. However, because of the water from the jet, it is not a good indication of the watercondition of the soil.

5. Test Pits: Test pits are very expensive. After four or five feet it may be necessary to shore up excavation. The limit is usually twenty feet.

6. Core Borings: These are the so-called undisturbed samples.

7. Geophysical Means:

A. Seismic

An explosive charge is set off and the time lag is recorded at several pick up points. It requires a special trained crew. It only shows if rock is there although somewhat where the rock is, and it is generally of little use.

B. Electrical Resistivity Method

Four electrodes are set in line. The two outer ones are charged negatively, the two inner ones positively. The resistance of the soil separating them is determined and through comparison it is to a certain extent possible to determine if the soil is silt or clay.

Information as to the suitability of the soil can be obtained through field tests.

1. Permeability: Wells are drilled and pumped. From experience it is possible to tell how water is passing through the soil and where it is coming from.

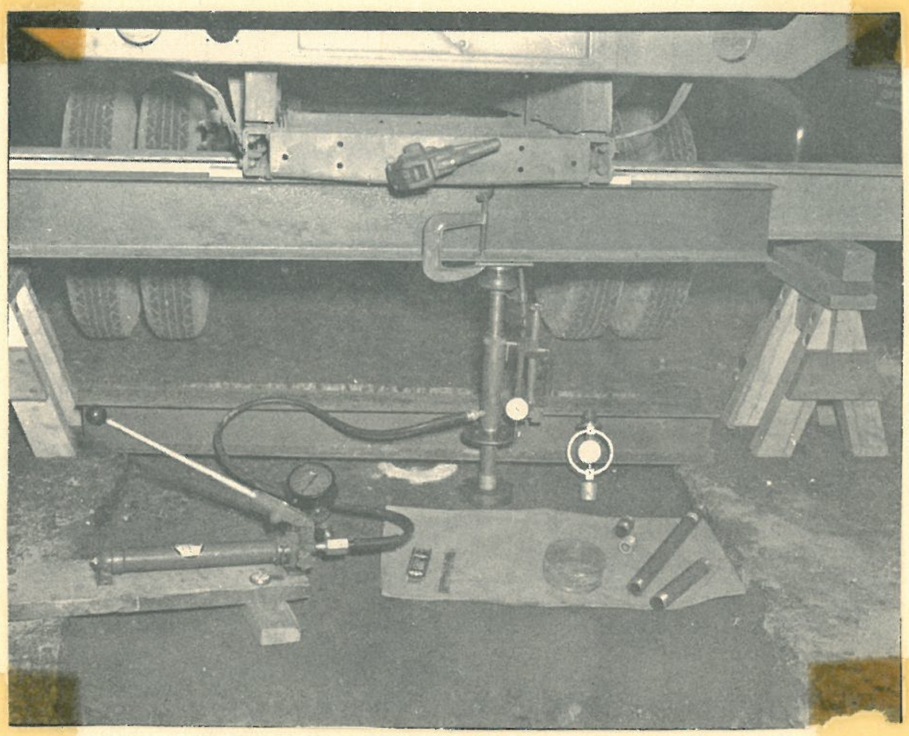
2. Small Scale Load Tests:

A. Proctor penetration needle
A plunger forces a needle into the soil. The load required to advance the needle at a steady rate of 1/2 inch per minute is compared with a standard to get the bearing value of the soil.

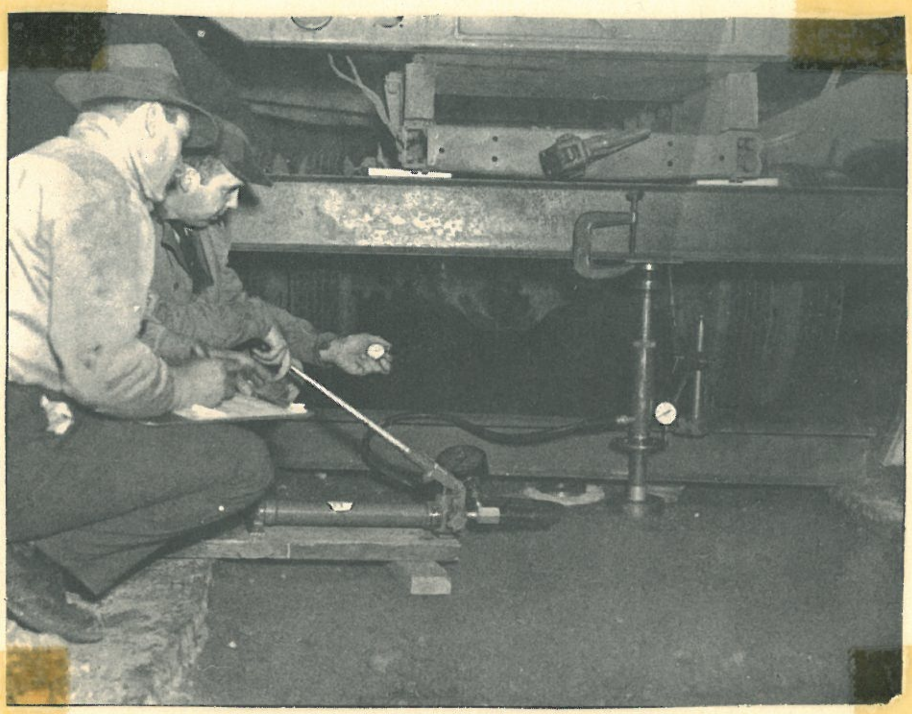
B. California Bearing Ratio
This method is used in flexible material construction and applies to any earth material. The material used in the subgrade is compacted in a mold. A plunger, three square inches in area, is forced in 1/10 inch and the load recorded. The ratio of this load to the load required to push the plunger into crushed rock is the California Bearing Ratio when expressed as a percent. For the purpose of determining the effect on saturated conditions on the soil, tests may be made on soaked samples. As a sideline, the soaked sample may be put away for four days before the test is run and the swelling observed. If the swelling is less than one inch the soil is considered excellent, etc.

3. Full Scale Load Tests: With tests of this sort, it is of utmost importance to have full knowledge of what is below the test area. Generally, all a load test consists of is the loading of an area at increments and observing the settlements for each increment. However, interpretation of the results is difficult. This test is not very good on clays because clays take a long time to consolidate. However, it does give some information on sand and gravel.

The various building codes dictate methods of making a load test and the interpretation of the test. It is generally required to excavate to the depth of foundation and there to build a platform on an area of about 1 ft. x 1 ft. to about 3 ft. x 3 ft., depending on the code. The load is applied directly by gravity or by means of a jack. This load is applied in increments, i.e., when settlement has not taken place for twenty-four hours the load is increased. Settlements must be



Field CBR - Complete assembly of apparatus showing extensions, extra surcharge weights, proving ring for lower loads, and other required equipment.



Field CBR - Field California Bearing Ratio Test in operation.

measured from a reference point but care must be taken so that the reference point does not settle, too. The contact pressure and settlement are recorded and a curve drawn. Some codes, such as the Boston Code, say that the correct bearing pressure is the one where the curve starts to break. However, with most load tests the curve seems to break all along. Therefore, it does not seem to be a good interpretation. Another method would be to put on a load which is somewhat greater than the design load. Then, if the curve does not break, the design load can be used.

<u>N. Y. City Code</u> (Condensed)

Apply sufficient load uniformly on platform to produce a center load of four times the proposed design load per square foot. Read settlement every 24 hours until no settlement occurs in 24 hours. Add 50% more load and read settlement every 24 hours until no settlement occurs in 24 hours. Settlement under proposed load should not show more than 3/4 in., or increment of settlement under 50% overload should not exceed 60% of settlement under proposed load. If the above limitations are not met, repeat test with reduced load.

NAME

DATE

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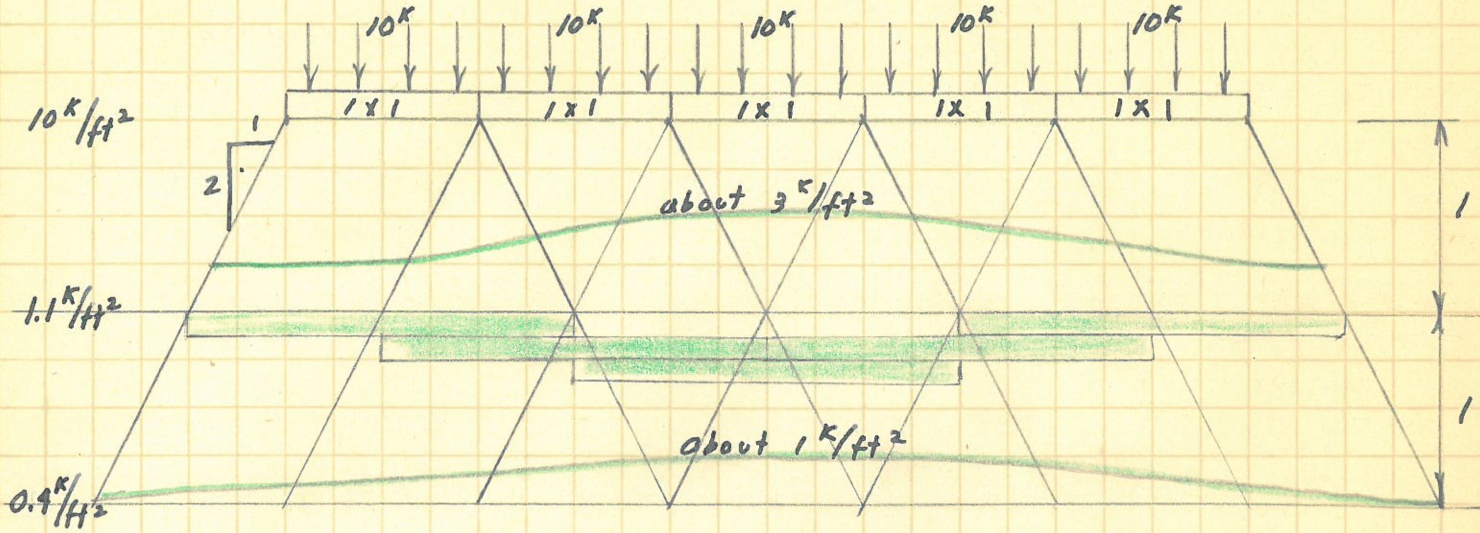
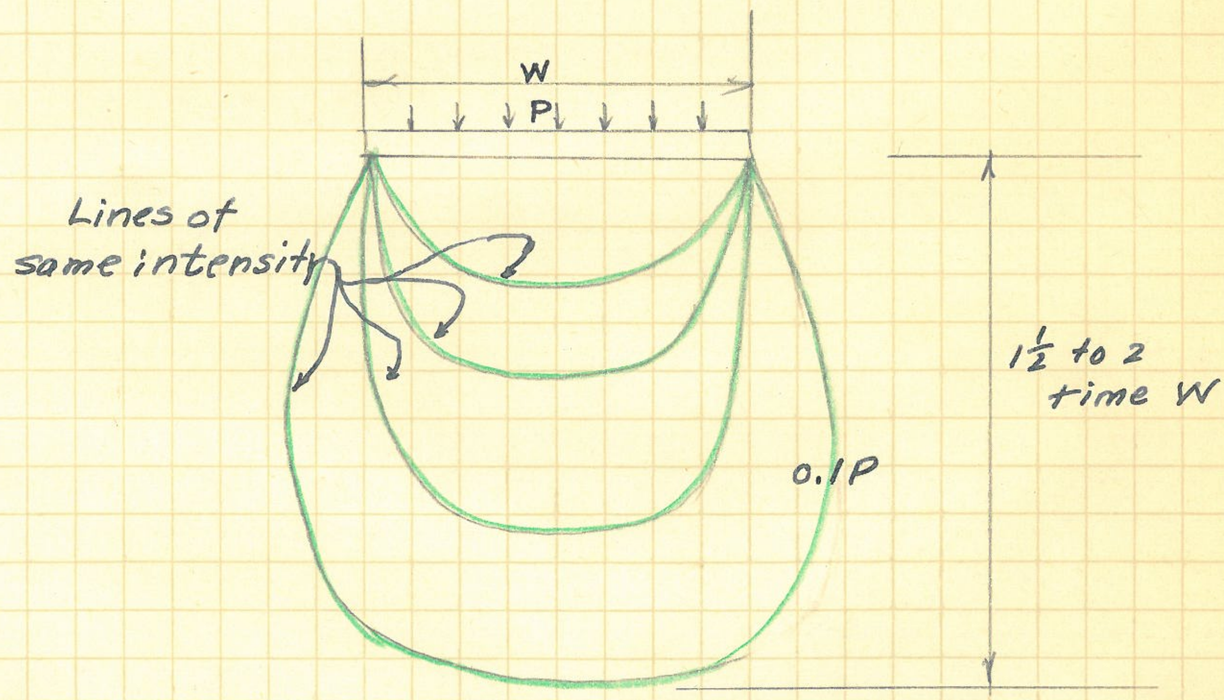
Prof. Brennan
Jan. 15, 1953

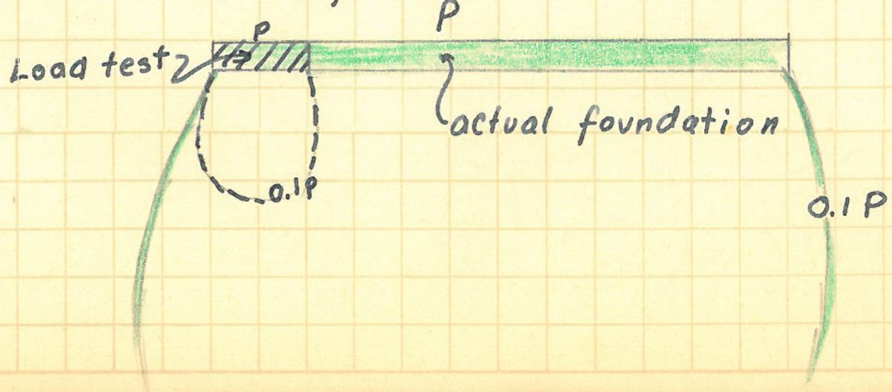
BULB OF PRESSURE

Simplified - based on Boussinesq





The limit is arbitrarily called 1/10 P. Borings are brought to that depth.



This illustration shows why load tests are often misleading.

In the preceding lectures, the basic principles of soil tests and the standard routine of these tests were discussed. It is important to get information on the structure, the design of the structure, or potential displacements. What forces are acting? How much movement can be expected? What movement will there be? These and similar questions need to be answered.

One approach to this problem is to draw pictures on how the structure can move. These can be called displaced structures so as to relate it to deflected structures of earlier lectures. The first thing needed is an idea of how the soil is located under the structure. This may be a soil profile which will indicate the presence of various soil strata. Then, since information will be needed on the material in each soil layer, the soil will need to be classified for identification purposes. Also, information must be obtained on water conditions. With the information a decision can be reached as to what structure and what foundation for the structure to use.

The following questions, when answered, will dictate design and construction:

1. How can the structure act on this particular stratum? Will it settle, slip, slide sideways, settle differentially? A structure can move horizontally, vertically, and can rotate in any plane.

2. How will the movement effect the structure itself?

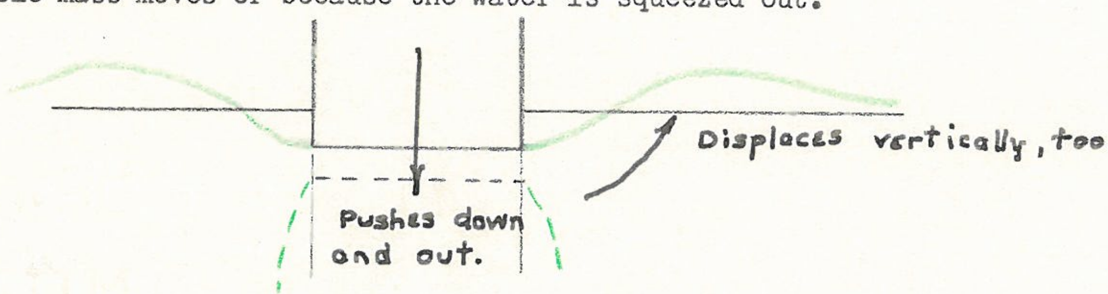
3. What caused the movement to take place?

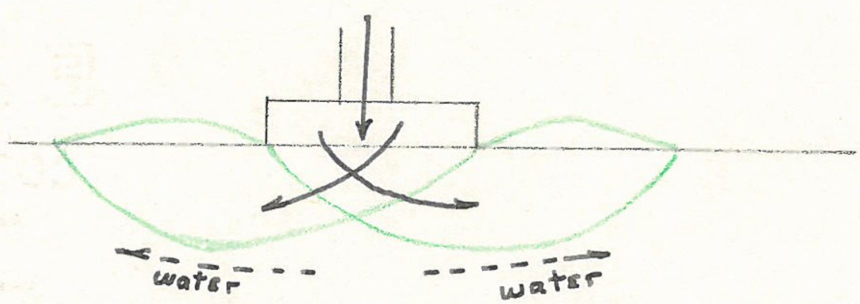
4. How can the structure be designed to resist the effects of movement?

5. What can be done to the earth?

When studying movements usually only a single isolated footing is considered. However, it is advisable to keep in mind that footings are seldom isolated.

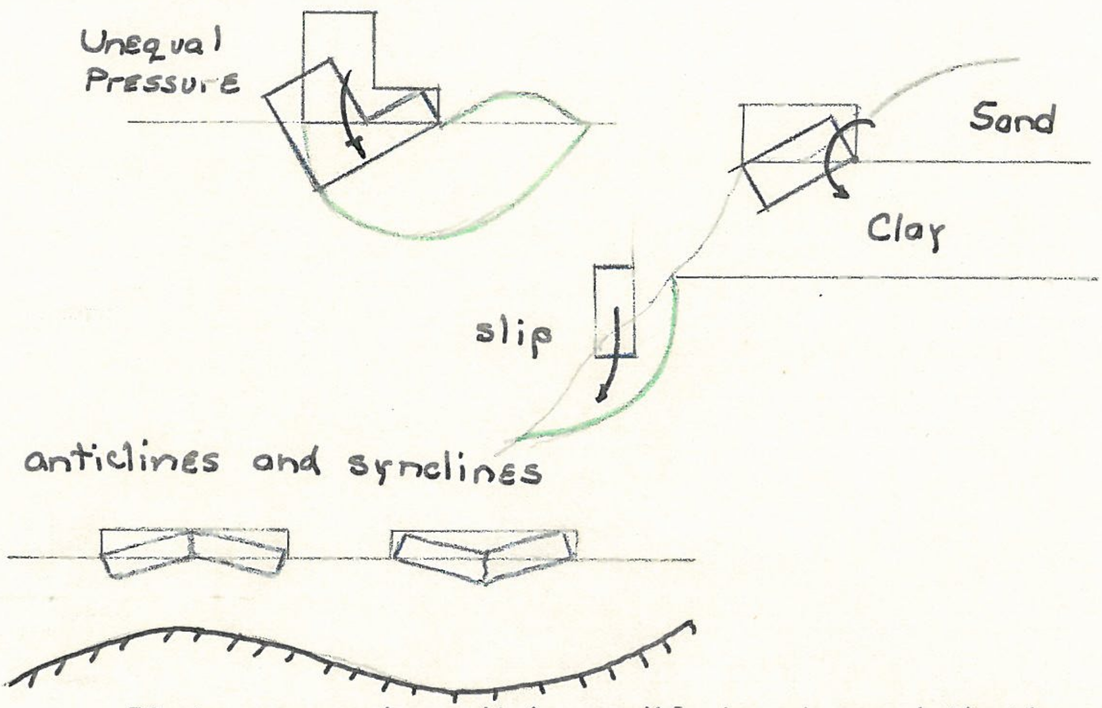
If the layer in question is sand appreciable movement may occur only if the entire mass moves. However, very slight movement may occur if the mass is compacted or if the sand grains are compressed. This small settlement takes place instantaneously. If, on the other hand, the layer is clay appreciable movement may occur either because the whole mass moves or because the water is squeezed out.





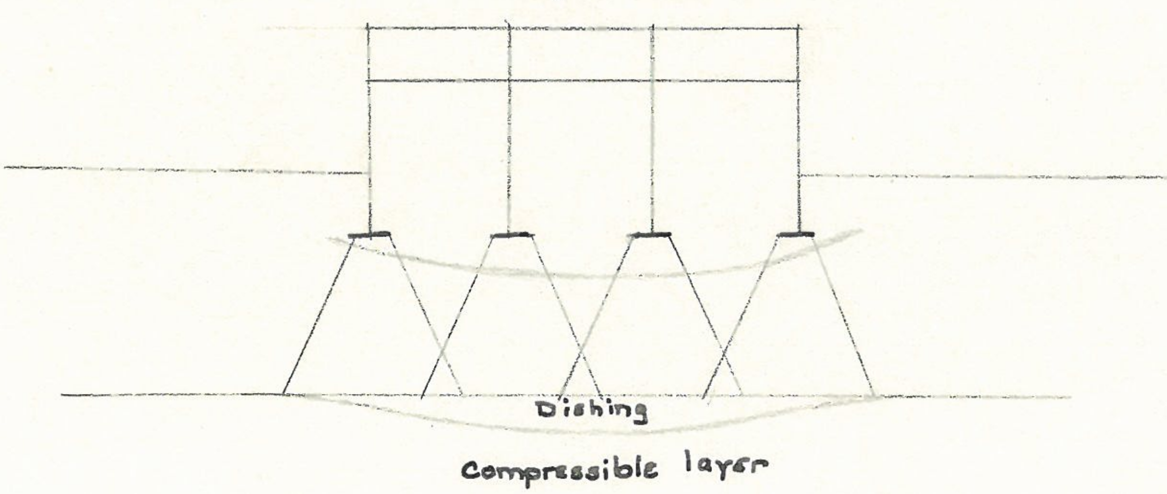
One way of getting rid of differential settlement is to make a rigid structure. On the other hand if the structure is extremely flexible it will not crack under differential settlement. It is apparent then that any degree of stiffness between complete flexibility and complete rigidity is subject to cracking under differential settlement.

Stability, i.e., the ability of the structure to stay put may be more important than plain settlement.



If stresses are known it is possible to get some indication of strain even in soils. If the structure is not continuous the stresses will be greater towards the middle. This will cause a dishing of the compressible layer and unequal settlement of the footings.

Simple Structure



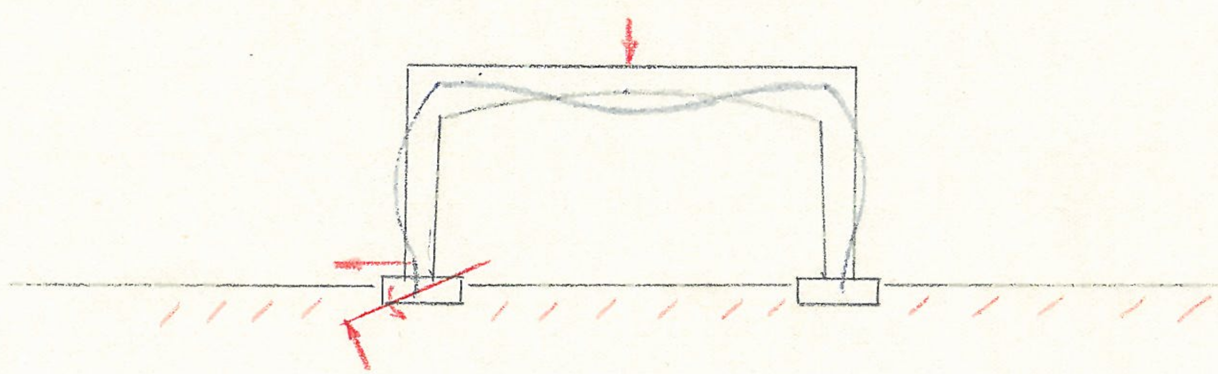
On a continuous structure the interior supports take more load than the end supports. However, as the interior supports settle the reaction on the end support increases. The end supports settle and the load is thrown back on the interior supports. Therefore, the structure may end up with uniform settlement.

In some instances it may be convenient to have the stress the same on each footing. If the footings are proportioned the same, settlement may result. On the other hand, if they are proportioned differently, there may not be settlement.

If cracking takes place the structure will act like a statically determinate structure.

A rigid structure will settle uniformly.

With a rigid frame structure like a bridge founded on a soft compressible material, the abutments will tend to tilt and also to move out. However, as the footing tilts it picks up non-uniform pressure which tends to relieve the tilt. However, the movement out will continue. This movement has a large effect on the moment at the center of the rigid frame.



After having predicted movement it becomes necessary to arrive at the magnitude of the movement. There are four sources of evidence.

1. Load Test

   This was discussed in Lecture 21. It is probably the worst method mostly because it is small scale. A stone would have the effect of a boulder on a full scale structure. Also, the bulb of pressure extends much deeper on a full scale structure than on the small scale load test.

2. Theory of Elasticity

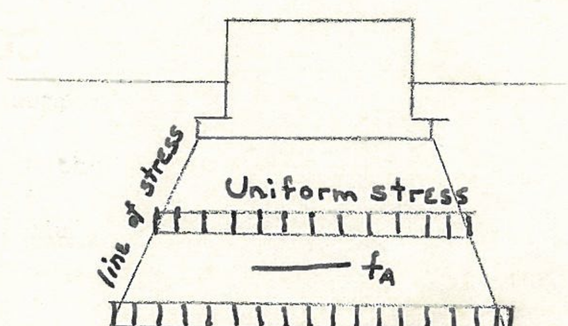
3. Consolidation Test

4. The knowledge of earth material and type of structure so as to build up a file of information from which settlement and its extent may be predicted.

The Theory of Elasticity

It is possible to do this very simply by making certain assumptions. First, assume some line of stress action. Then assume that the stress is uniform. Finally, assume or obtain in the laboratory a modulus of elasticity for the soil.

$$\epsilon = \frac{f}{E} = \frac{f_{A_1} + f_{A_2} + f_{A_3} \text{ etc.}}{E}$$



However, it is common to go farther and use Boussinesq's equations to get variation of stress.

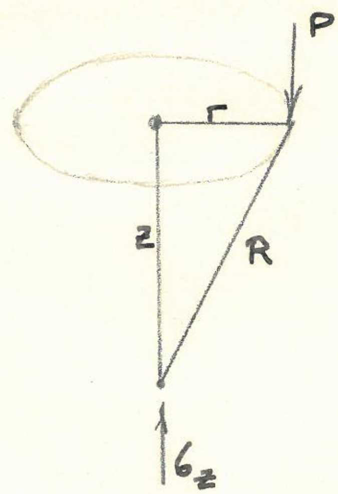
$$\sigma_z = \frac{3}{2}\frac{P}{\pi}\frac{1}{z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

σ_z is the vertical stress at any point due to a load P, z the depth to the point in question, and r the horizontal distance from the point of application of the load to the point in question.

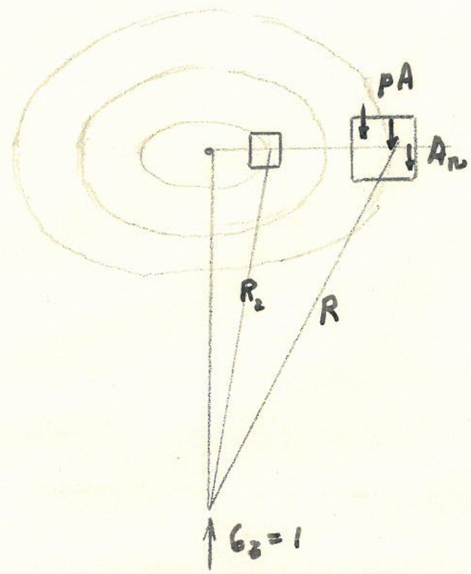
The equation may also be expressed:

$$\sigma_z = \frac{3}{2}\frac{P}{\pi}\frac{z^3}{R^5}$$

where R is the distance from the point of application to the point in question. Derivation of this equation is very long and difficult. It can, however, be found on pages 362-365 of "Theory of Elasticity" by Timoshenko.



Applying a known load to the equation is very simple for one point, but, for many, it is very arduous. Fortunately, it has been simplified by Newmark's influence chart.



$$\sigma_z = \frac{3}{2}\frac{pA_n}{\pi}\frac{z^3}{R^5} \quad \text{or} \quad \sigma_z = \frac{3}{2}\frac{pA_n}{\pi}\frac{1}{z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$A_n = \sigma_z \frac{2}{3}\frac{\pi R^5}{z^3 p}$$

What is wanted is an influence surface which will end up with σ_z being unity.

Integrating, $d\sigma_z = \frac{3}{2}\frac{p}{\pi}\frac{1}{z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} dA$

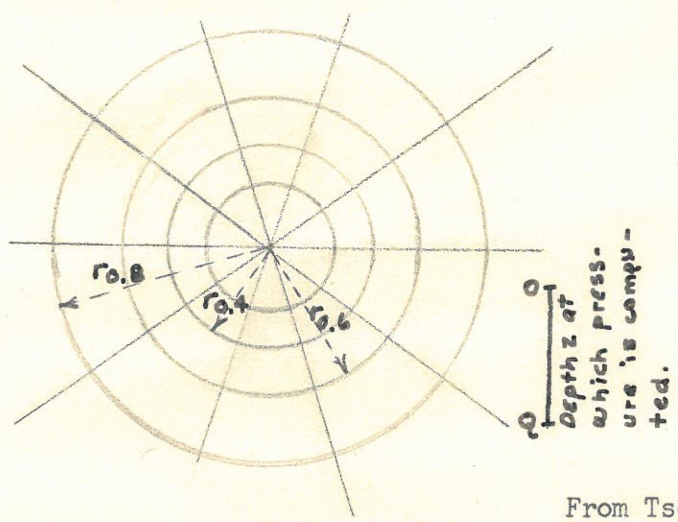
over the loaded area will result in:

$$\sigma_z = p\left[1 - \left(\frac{1}{1+(r/z)^2}\right)^{3/2}\right]$$

When $r = \infty$, $\sigma_z/p = 1.0$. It is possible to determine the ratios r/z for which the ratio $\sigma_z/p = 0.8$. It is 1.387. By selecting some definite scale, like OQ, as shown below, to represent the depth z, we obtain the length of the radius $r_{0.8}$ which corresponds to $\sigma_z/p = 0.8$ by multiplying the distance OQ by 1.387 and drawing a circle with that radius. This procedure is repeated for other values of σ_z/p, $\sigma_z/p = 0.6$ and $\sigma_z/p = 0.4$.

The diagram thus obtained represents an influence chart for a surface unit load of unity $(p = 1.0)$. Thus the vertical stress σ_z

will equal 0.8 if the entire circular area of radius $r_{0.8}$ is loaded by $p = 1.0$. If only a ring between the radii $r_{0.8}$ and $r_{0.6}$ is loaded with $p = 1.0$, then $\sigma_z = 0.8 - 0.6 = 0.2$. Each ring on the chart below is subdivided into 10 equal blocks. Therefore, a load of $p = 1.0$ covering one of these blocks will produce a vertical unit stress of $\sigma_z = 0.1 \times 0.2 = 0.02$. Therefore, the influence value of each loaded block is 0.02.



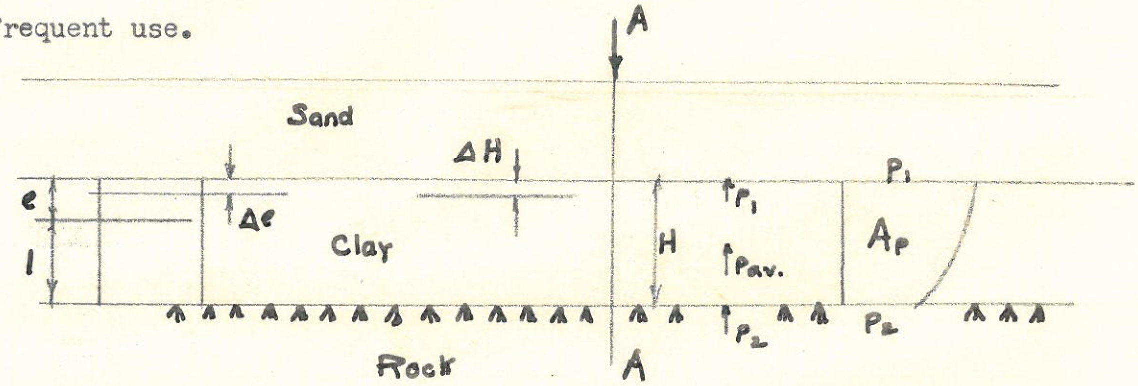
From Tschebotarioff

By taking a plan of the structure, for which the stress caused by it at a depth z is to be found, drawn to the scale for which $z = OQ$, and placing this plan on the chart so that a maximum number of blocks are covered, the stress at a depth z is the number of blocks covered times the influence value of the chart times the value of the pressure per sq. ft. that the structure exerts at the surface of the ground.

There are charts for finding strains directly but Poisson's ratio must be known which means that the soil must be homogeneous and isotropic.

Theory of Consolidation

The theory of consolidation in a simplified form has come into frequent use.



The pressure p_1 and p_2 are found by means of Boussinesq's equations. In this analysis we are interested in settlement along the line A-A. Let $\Delta H = \varepsilon H$ where ε is the unit deformation.

$$\varepsilon = \frac{f}{E}$$

The average stress, f, is A_p/H, which is the additional weight of a structure or fill, so that

$$\Delta H = \frac{A_p}{H} \cdot \frac{H}{E} = \Delta p \, \frac{H}{E}$$

Since $\varepsilon = \frac{\Delta e}{1+e}$, ε can be found in the laboratory.

$$E = \frac{\frac{A_p}{H}}{\frac{\Delta e}{1+e}} = \frac{A_p(1+e)}{H \Delta e}$$

Therefore,

$$\Delta H = \frac{A_p}{H} \cdot \frac{H}{E} = \frac{A_p}{H} \cdot \frac{H}{\frac{A_p(1+e)}{H \Delta e}} = \frac{A_p}{(1+e)} \, \frac{H \Delta e}{A_p}$$

$\frac{H\Delta e}{A_p} = \frac{\Delta e}{\Delta p}$ and is called the compressibility coefficient and is denoted by "a" so that

$$\Delta H = a \frac{A_p}{1+e}$$

"m_v" is known as the "coefficient of volume compressibility" and

$$m_v = \frac{a}{1+e}$$

and
$$\Delta H = A_p m_v = H \Delta p \, m_v$$

To find "a" in the laboratory the void ratio is found when there is zero surcharge and subtracted to get Δe from the void ratio after the sample is subjected to an increment of pressure.

The field consolidation curve for ordinary clays appears in a similogarithm diagram as a straight line represented by the equation:

$$e_{final} = e_{initial} - C_c \log_{10} \frac{p_o + \Delta p}{p_o}$$

(from Terzaghi and Peck)

$$\Delta e = C_c \log_{10} \frac{p_o + \Delta p}{p_o}$$

where C_c (dimensionless) is the compression index and p_o is the weight of the overburden. Then,

$$a = \frac{C_c}{\Delta p} \log_{10} \frac{p_o + \Delta p}{p_o}$$

$$m = \frac{C_c}{\Delta p (1+e)} \log_{10} \frac{p_o + \Delta p}{p_o}$$

$$\Delta H = H \frac{C_c}{1+e} \log_{10} \frac{p_o + \Delta p}{p_o}$$

For a clay of medium or low sensitivity

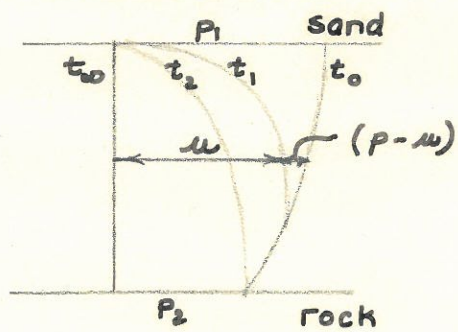
$$C_c \cong 0.009\,(L_w - 10\%)$$

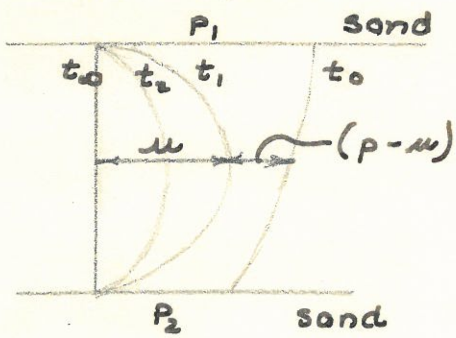
The time it takes to consolidate the soft layer follows from the computation of the total amount.

$$\Delta H = a\,\frac{\Delta p}{1+e}$$

At the instant the load is applied the load is carried entirely by the water in the pores. As the water is forced out of the pores the soil grains come to bear against each other. Consequently, the grains begin to take up the stress. At any time, then, $p = u + f_c$ where "u" is the load carried by the water.

In the following diagrams the condition that exists at any particular time is depicted. Although the time of complete consolidation is different in each of the two cases, the total deformation is the same. Deformation continues until all the water is squeezed out.



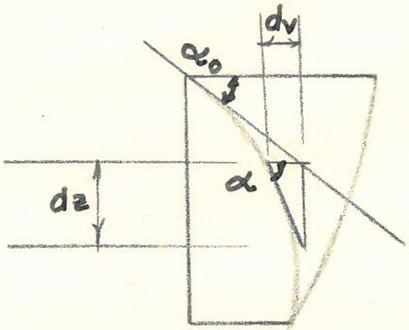


The discharge "Q" during the time "t" through a total cross-sectional area of soil "A" is

$$Q = kiAt$$

where "i" is the hydraulic gradient. The differential discharge "q" will be

$$q = k\,i\,dt$$



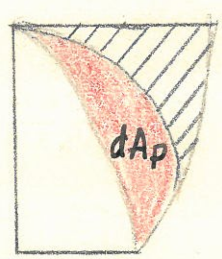
$$\text{Now} \quad i = \frac{\text{head loss}}{L} = \frac{dh}{L}$$

$$dh = \frac{du}{\gamma_w}$$

$$i = \frac{du}{\gamma_w} \Big/ dz = \frac{1}{\gamma_w} \cot \alpha$$

$$i_o = \frac{1}{\gamma_w} \cot \alpha_o$$

so that $q = k \frac{1}{\gamma_w} \cot \alpha_o \, dt$



When the cross-hatched area doubles, the deformation doubles.

$$\Delta H_t = a \frac{(A_p)_t}{1+e}$$

The percentage of consolidation is expressed:

$$U = \frac{(\Delta H)_t}{\Delta H} \times 100$$

and through proportionality

$$U = \frac{p - u}{p} \times 100$$

At any interval of time

$$d(\Delta H)_t = d(A_p)_t \frac{a}{1+e}$$

$$d(\Delta H)_t = q_t$$

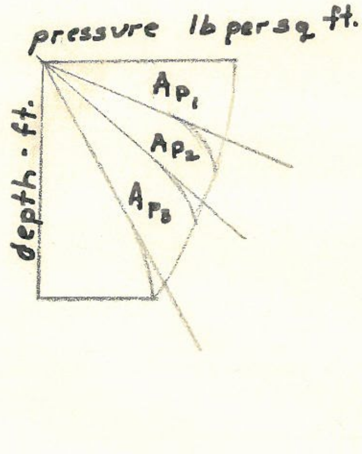
$$\frac{k}{\gamma_w} \cot\alpha_0 \, dt = d(A_p)_t \frac{a}{1+e}$$

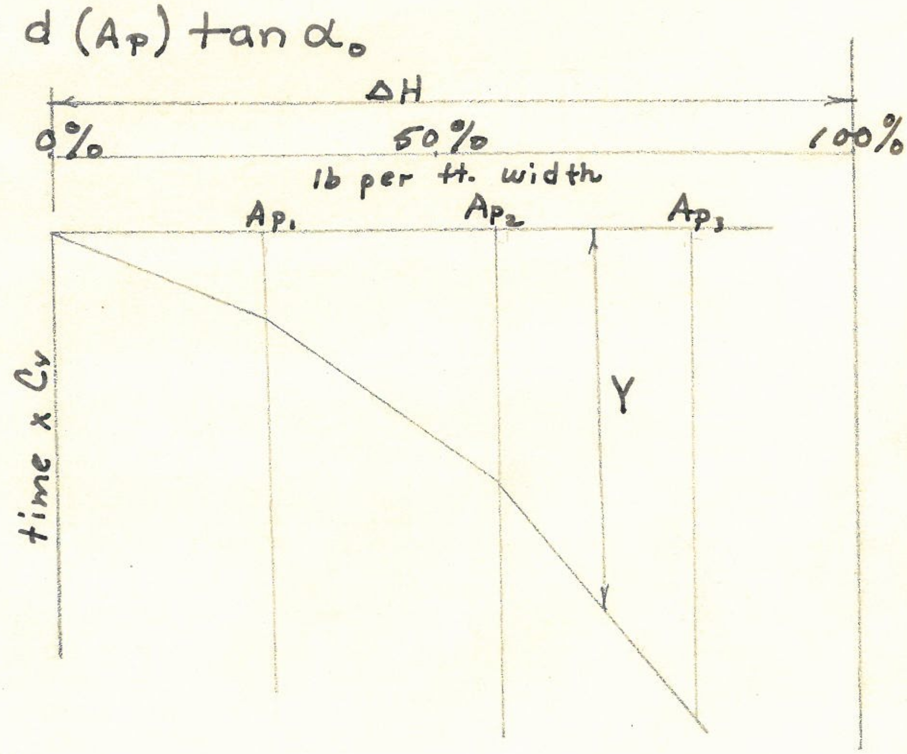
$$dt = \gamma_w \frac{a \; d(A_p)}{1+e} k \tan\alpha_0$$

Introducing a designation C_v, the coefficient of consolidation:

$$C_v = \frac{k(1+e)}{a\gamma_w}$$

$$dt = \frac{1}{C_v} d(A_p) \tan\alpha_0$$





$$t = \frac{1}{C_v} Y$$

$$T = \frac{C_v t}{H^2}$$

T is termed the "time factor" and is dimensionless. When reaching a certain percentage consolidation U, two soil layers of different thickness H_1 and H_2, but made of the same material (same value of C_v), have the same value of the time factor. Designating the time intervals required to reach that percentage consolidation t_1 and t_2.

$$\frac{C_v t_1}{H_1^2} = \frac{C_v t_2}{H_2^2}$$

so that

$$\frac{t_1}{t_2} = \frac{H_1^2}{H_2^2}$$

most important part
not included!

These last lectures have discussed ways of getting an idea on how the structure is going to act and of putting a scale on the amount of movement. Last time settlement was studied. Failure may be by instability also. There are means of getting a scale on instability. The same approach is used for earth dams, cuts, and the like. First, some factors have to be considered:

1. Stratification of bedding planes.

   The situation is very different if the materials are not homogeneous than if they are. Each material has a different tendency to slip.

2. Weight of soil.

3. Water in the soil.

   Water will change the weight, reduce the cohesion.

4. Roots and trees.

   These give strength to the mass.

5. Putting additional load on foundation.

6. Water seepage.

   Water may erode material.

7. Change in material.

8. Excavation.

   This may affect all of the above.

9. Structure.

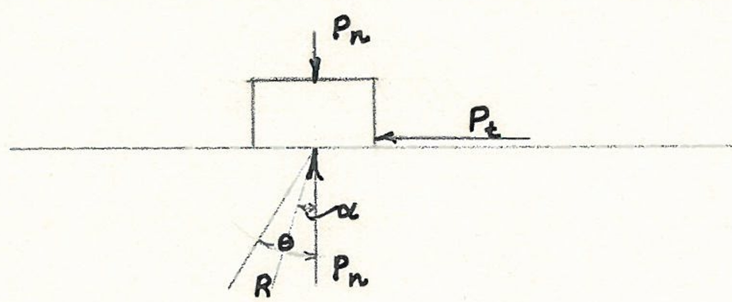
   The structure may block the flow of water.

SHEARING ALONG PLANE OF FAILURE

There are many ways of finding the shear along a plane of failure but all of them assume a plane on which the shear takes place called a critical section. Then, they figure all the forces on that section to see if it will remain in equilibrium. The forces acting include the weight of the material and the structure. The resisting forces must be determined by laboratory analyses.

Two terms are used, cohesion and friction. Cohesion is the ability of the material to resist being pulled apart. Friction is the resistance caused by the material being rubbed together.

The shear resistance of sand depends on normal load. This is not so with clays. Resistance to deformation in sand increases as the voids are reduced. The coefficient of friction of sand varies with the load.

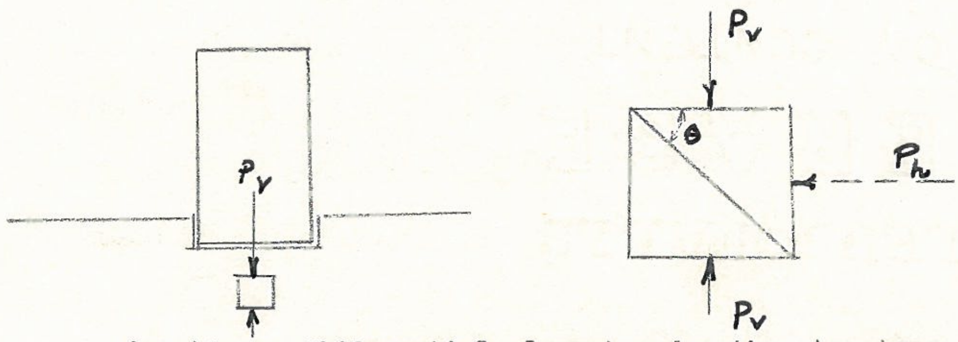


Assume P_n remains constant and P_t increases from zero to the value which will produce sliding. The angle formed by the resultant with the normal to the plane (with the maximum principle stress) is known as the "angle of obliquity" and is denoted by the symbol, α . The body will start sliding along the plane when the force P_t reaches a value which will increase the angle of obliquity to a certain maximum value ϕ

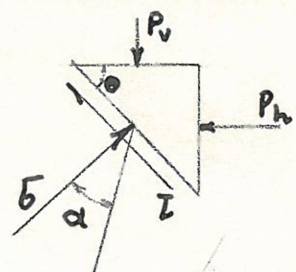
which is termed the angle of friction.

$P_t = P_n \tan\phi$ when sliding occurs, or since s is the frictional stress and σ' the normal,

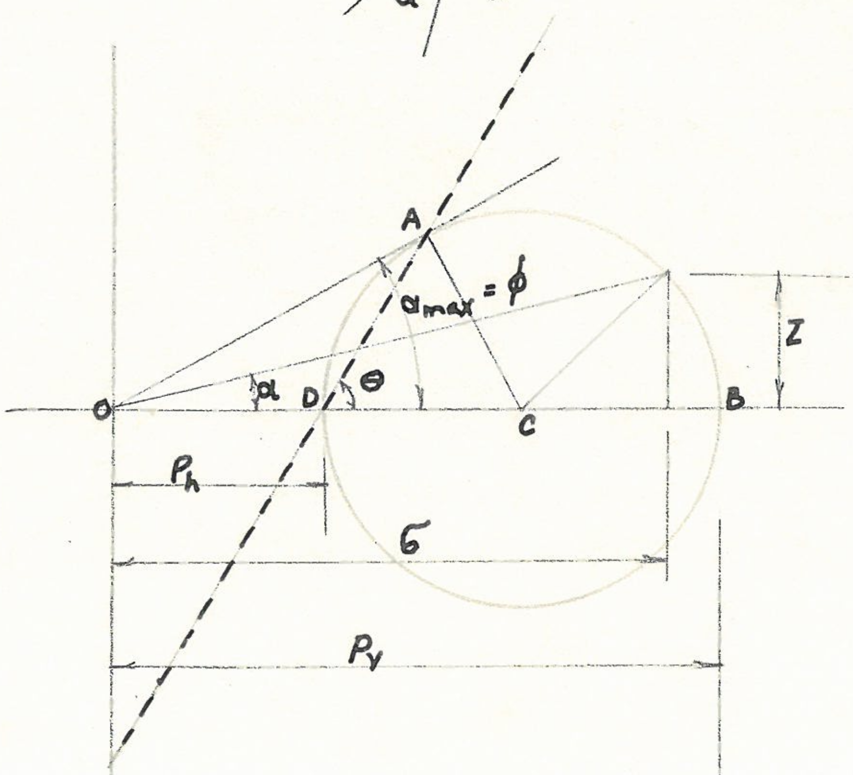
$$sA = \sigma A \tan\phi \quad \text{and} \quad s = \sigma \tan\phi$$



Consider a differential element under the structure shown above and loaded normally. If movement occurs it will set up a force P_h.



$\theta = \alpha$ when $P_h = 0$



Therefore, if P_v is known and ϕ determined as is often the case a Mohr circle can be drawn from a tangent and a point.

$$\frac{P_h}{P_v} = \frac{OC - CD}{OC + CB} \quad \text{and since } CB = CD = CA$$

$$= \frac{OC - CD}{OC + CD}$$

$$= \frac{1 - \frac{CD}{OC}}{1 + \frac{CD}{OC}} \quad \text{and since } \frac{AC}{OC} = \sin\phi$$

$$= \frac{1 - \sin\phi}{1 + \sin\phi} = K_A,$$ the coefficient of active earth pressure.

This applies mostly to sands.

APPROXIMATE RELATION BETWEEN S AND P

$s = p\tan\phi$, valid for cohesionless sands.

$s = c + \tan\phi$, valid for moist or dry cohesive soils located above the water table.

$s = \frac{1}{2}q_u = c$, valid for soft clay ($\phi = 0$)

There are three methods of finding ϕ, the angle of internal friction in the laboratory.

1. Direct Shear Test

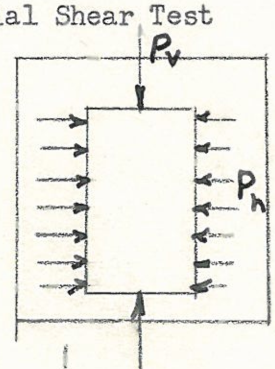
This test is used primarily on cohesionless sands although it may be used for moist or dry cohesive soils located above the water table. Most drainage conditions can be approximated in the test procedure. Basically, the specimen is loaded normally and sheared tangentially. The stress due to the normal load is plotted against the stress due to the tangential pull. The slope of the straight line portion of the curve is the angle of internal friction. Generally the line is extended to intercept the axis of the normal stress and the intercept is assumed to be the cohesion.

2. Unconfined Compression Test

Clay specimens are loaded and permitted to bulge out. The stress at failure is called the unconfined compressive strength, q_u, and if $\phi = 0$

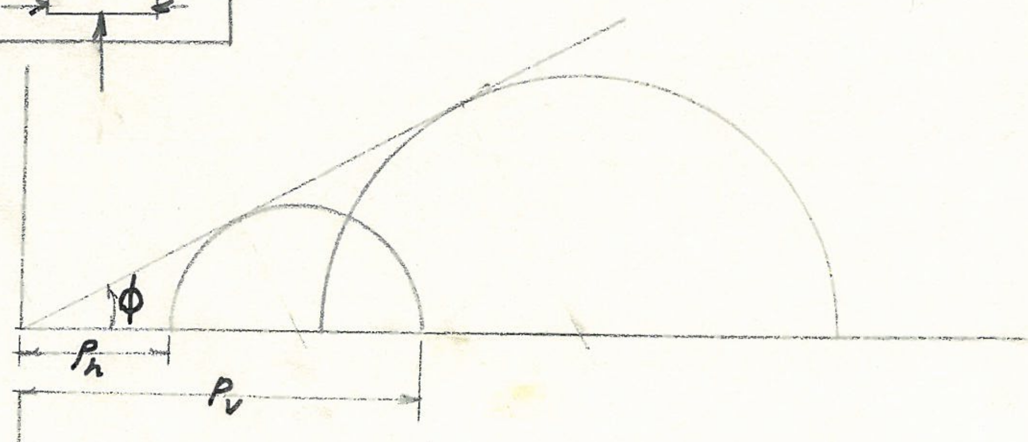
$$c = \frac{1}{2} q_u$$

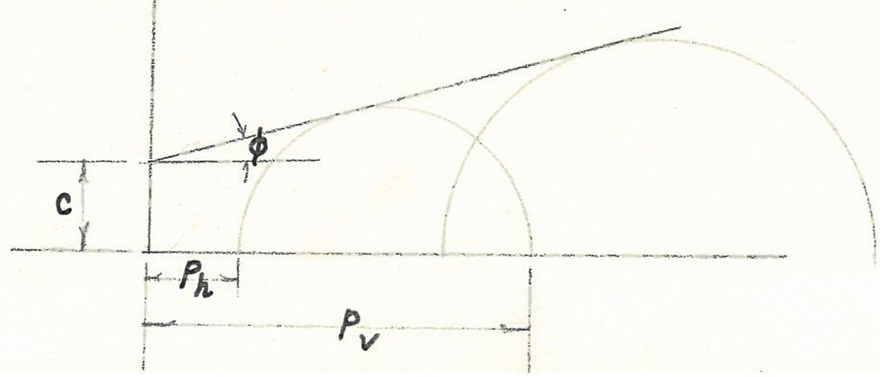
3. Traxial Shear Test



$$S = P_v \tan\phi + C$$

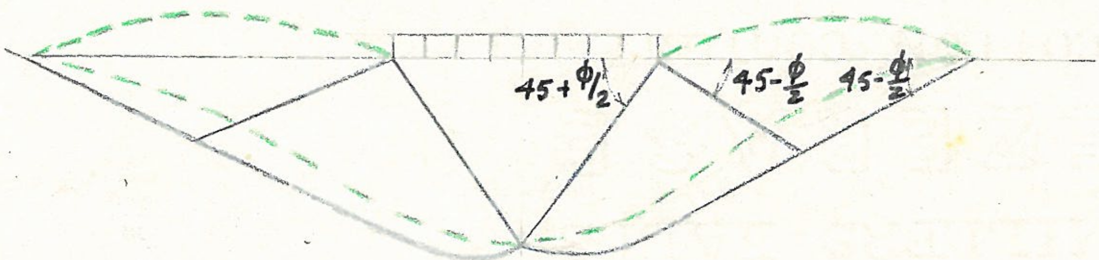
where C is the cohesive force





Prandtl Theory.

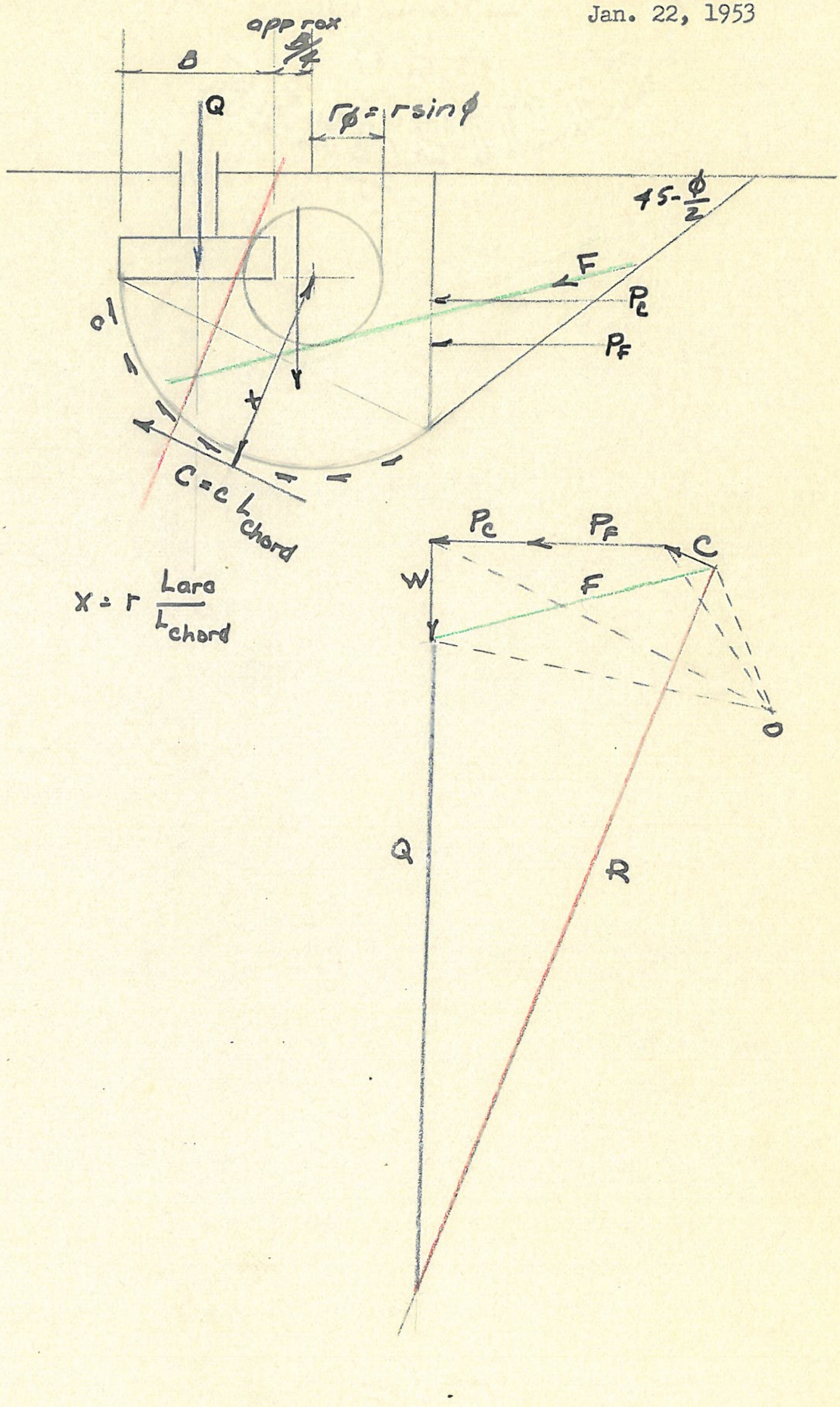
Prandtl investigated the plastic failure of metals. A special case (horizontal surface) of his general solution is applicable to foundations and is illustrated below.



Krey friction circle method

Given γ, ϕ, c

1. Draw footing
2. Assume a center of slip circle
3. Draw circle and wedge as shown
4. Compute C, P_F, P_C, W and lay off force polygon as shown
5. Draw string polygon and position F on diagram. Resultant, R, must intersect F and Q at a mutual point.
6. Draw friction circle with a radius, r_ϕ, equal to r sin ϕ as shown.
7. Draw R tangent to the friction circle and lay it off on force polygon
8. Determine Q
9. Repeat with a different location of center of circle until the minimum Q is found.

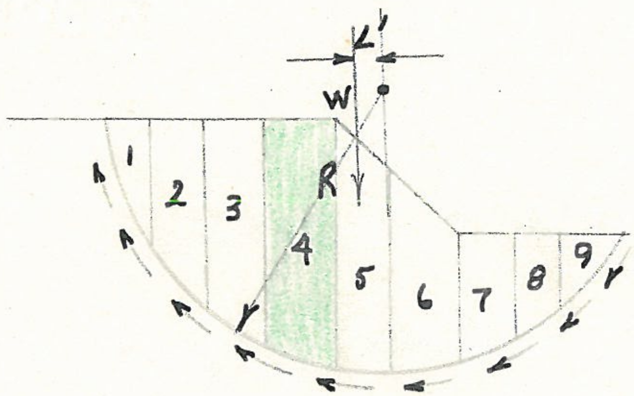


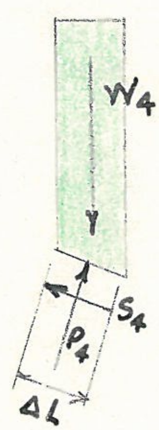
FRICTION CIRCLE

Swedish Circle

This is based on field work on slides on railroad embankments in soft clay sediments along the fjords that line the Swedish coast. This method of predicting failure is based on the observation that failure occurs in a circular arc.

Assume a line of failure. Calculate the factor of safety as indicated below. Repeat until the minimum F. S. is determined.





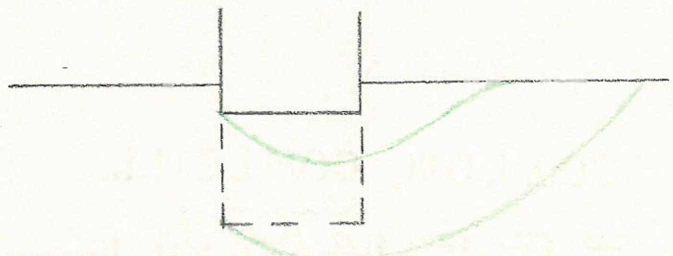
$$M_R = \sum s R \Delta L + cL$$

$$M_F = W L'$$

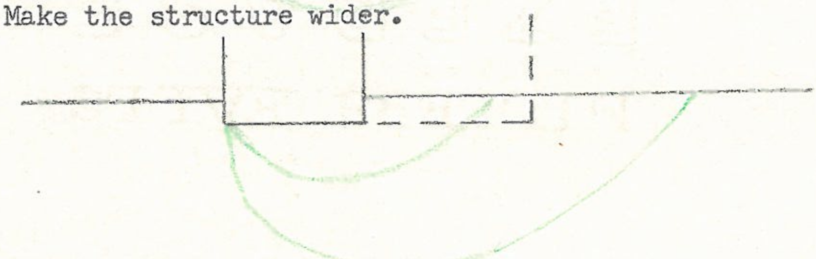
$$F.S. = \frac{M_R}{M_F}$$

The following are methods of avoiding instability.

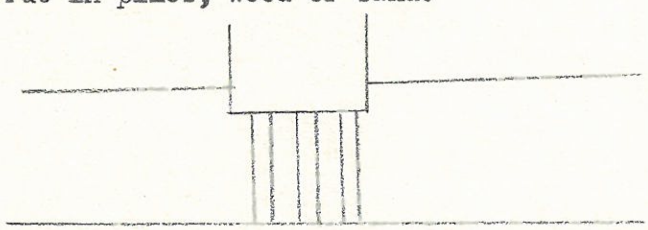
1. Sink the structure deeper.



2. Make the structure wider.



3. Put in piles, wood or sand.



4. Replace the soft compressible layer with sand or gravel.

5. Carry the structure below the soft layer.

6. Consolidate the soft layer

    a. With piles
    b. With sand piles
    c. By preloading

7. Put on a layer of sand

    This spreads out the footing and extends the bulb of pressure.

8. Drain water from soft layer and keep it drained.
    (Most methods are expensive)
    a. Well points (they tend to clog)
    b. Permanent wells
    c. Divert water
    d. Drain tiles
    e. Large conduits
    f. Sheet piling

The following are methods of preventing differential settlement.

1. Build the structure on a rigid foundation.

2. Set the structure on jacks.
   This was done for Yankee Stadium.

3. Chemical consolidation
   This is a very expensive method and it is only used in fine sands. It tends to change the sand to a soft sandstone. Sodium silicate, water glass, is injected into the soft strata.

4. Electrical method
   This method is expensive. It is used only on clays which it tends to solidify. Aluminum is used for the anode and copper for the cathode.

5. Control by construction technique

6. Compaction
   a. Sheep-foot rollers
   b. Double acting hammer

7. Expansion and Contraction Joints

In the preceding lectures and this lecture the following three points were covered:

1. What action takes place.

2. The scale of movements, settlement and instability.

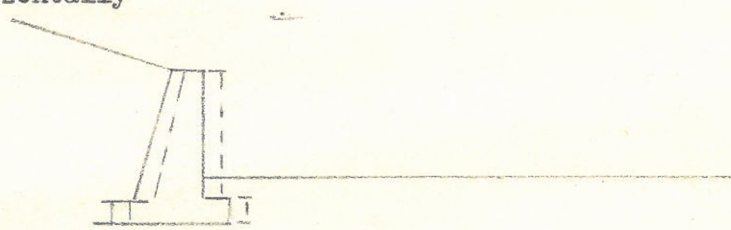
3. Corrective measures .

Prof. Brennan
Jan. 29, 1953

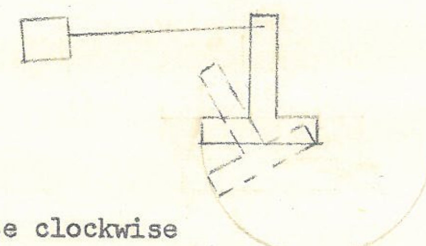
Rankine, Coulamb, and Terzaghi each produced earth pressure theories and amazing as it seems they do not agree. This would seem to mean that we do not know what we are doing. However, factors like the condition of the material itself or the loads on the material effect so greatly the results that variations in theory are perhaps only of academic interest.

When dealing with retaining walls and bulkhead problems two questions need to be answered. 1. How does the earth effect the structure? 2. How do the earth and the structure itself effect the earth?

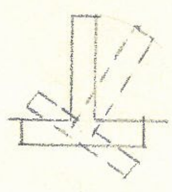
1. Slide horizontally



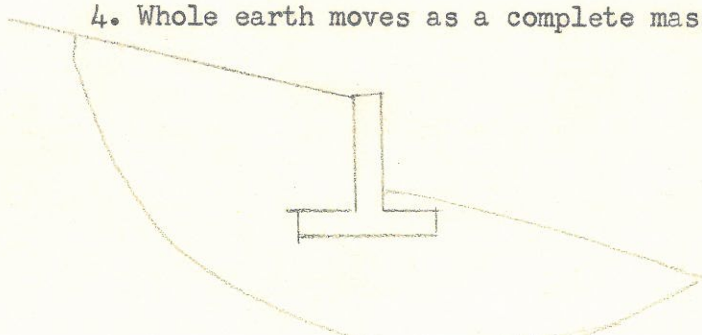
2. Rotate counter-clockwise



3. Rotate clockwise



4. Whole earth moves as a complete mass.

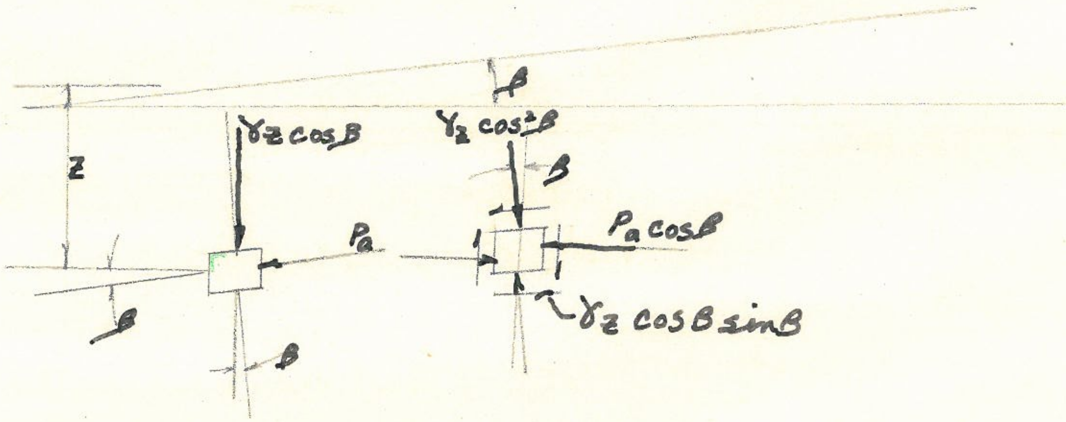


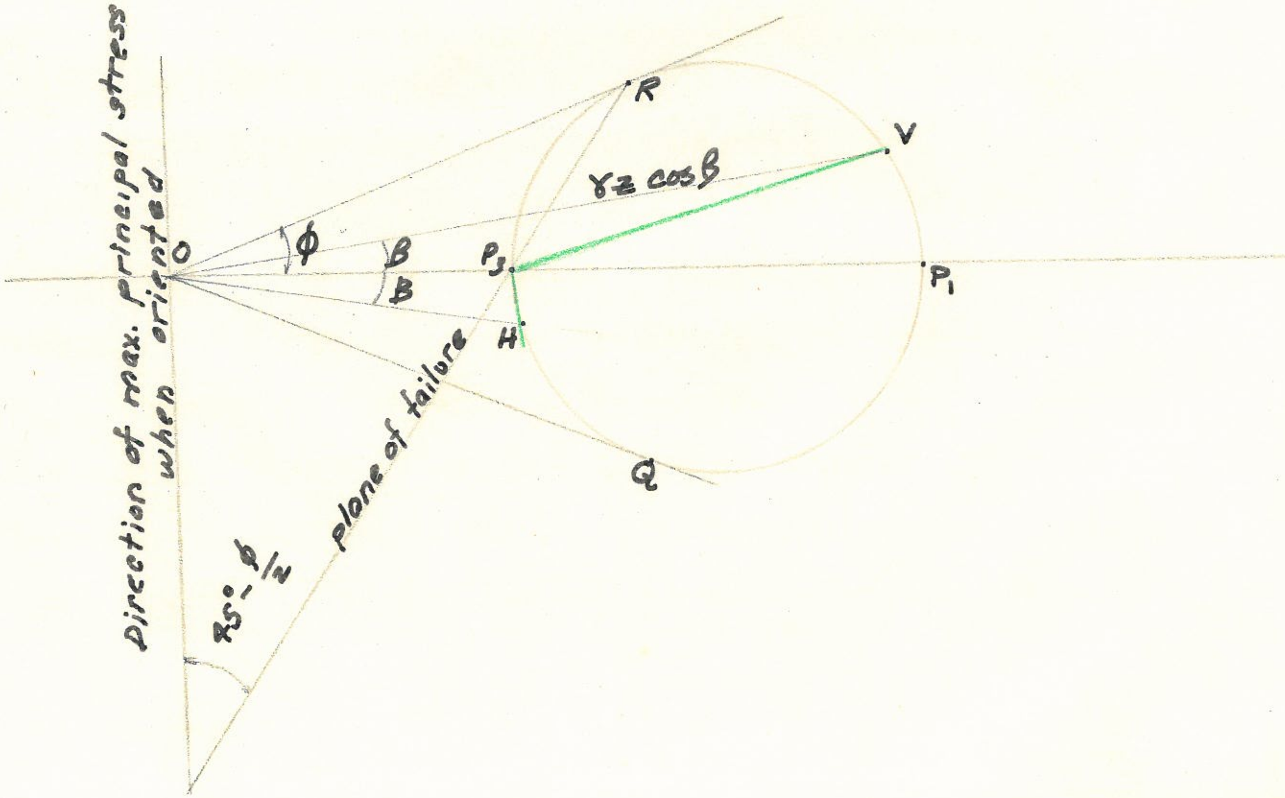
There must be some movement of earth for the retaining wall to move. The magnitude of pressure depends on the movement. Actually there is no such thing as pressure. It is one thing at one time and something else at another time. The answer that is desired is the maximum. Movement in retaining walls are gradual movement. What then is the maximum and how is it found? You must stipulate conditions. The best information on this subject comes from experience. The best experience comes from the railroad.

Rankine's Method when the Surface is inclined to the Horizontal.

Consider the equilibrium of an elemental portion of the material at depth Z. The weight of the soil acts vertically, and the earth pressure considered is the stress which is conjugate to the vertical weight. The earth pressure on a vertical plane thus acts parallel to the surface, an assumption which leads to some inconsistencies and errors when applied to retaining walls.

The direction of the maximum principal stress is not known and the Mohr's Circle diagram cannot be drawn in its correct orientation at the first attempt. For convenience the principal stress axis is drawn horizontally and OV and OH set off at β to the line OP, β being the obliquity of the resultant stresses on the conjugate planes considered. OV represents the known value of the resultant $\gamma z \cos\beta$, and a circle passing through V and tangent to the limiting lines OR and OQ locates the points P_1 and P_3. OH represents the value of the active pressure p_a and the angle VP_3H represents the angle between the planes on which $\gamma z \cos\beta$ and P_a act. When the Mohr's Circle diagram is in its correct orientation, VP_3 should be parallel to the surface of the soil, and P_3H should be vertical. When the circle is orientated correctly the directions of the planes of failure P_3R and P_3Q will be indicated properly. (From Capper and Cassie.)





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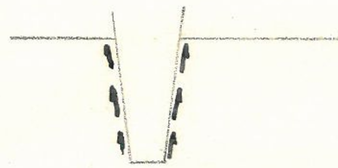
Jan. 29, 1953

PILES

Reasons for driving piles

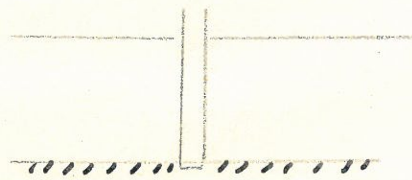
1. Distribute the load

    The Friction Pile



2. Carry load to firm strata

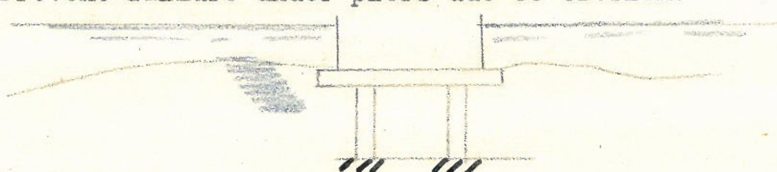
    Bearing Piles



3. Consolidate soil

    Pile driving squeezes particles together which gives greater bearing values for the soil.

4. Prevent failure under piers due to erosion.



5. As fenders

6. As a combination superstructure and bearing pile.

    Trestles

7. As temporary forms

    Falsework

8. Furnish lateral support

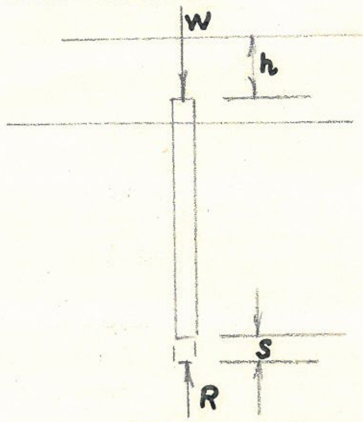
    Batter piles

Driving in clays causes problems of remolding. The effect this has on the carrying capacity of clay has been very controversial. At first it was thought that driving piles was always useful. Then A. Casagrande reported in "Contributions to Soil M Mechanics, Journal of the Boston Society of Civil Engineers, 1932" that driving piles in Boston clays did more harm than good. However, in 1948 A. E. Cummings, G. D. Kerkhoff, and R. B. Peck reported in the Proc. of the ASCE that pile driving in clay not only was not harmful but was even beneficial. Their decision was based on observation of Detroit Blue Clay which showed that with clays of that type the remolding effect was only temporary. Generally, it can be concluded that if the sensitivity of a clay to remolding, which is determined by means of unconfined compression tests, be just slightly sensitive the driving of piles should not cause any concern in respect to a possible weakening of the clay.

Incompletely consolidated and/or remolded clay layers can overload end-bearing piles, owing to the effects of the so-called "negative friction." This term is employed to designate frictional forces which are applied in a downward direction to the pile surface. A dangerous condition may arise when piles are driven through newly placed fill and through underlying soft clay to a deeper lying hard layer. As the clay consolidates the entire weight of the fill may then be transmitted through negative friction to the piles and may crush them. (From Tschebotarioff.)

There are three ways of determining the bearing capacity of piles: by theories, by tests, and by experience. Theories are of two kinds--

static and dynamic. Generally, it can be said of theories that we do not know what we are talking about. Pile driving formulas are a result of these theories. The most popular is the so-called Engineering News Formula.



$$Wh = Rs$$

$$R = \frac{Wh}{s}$$

and so as to prohibit an answer of infinity:

$$R = \frac{Wh}{s+1}$$

However, there is evidence of loss of energy as exemplified by brooming and by heat so that a loss of energy factor, K, must be considered.

$$R = \frac{KWh}{s+1}$$

For the Engineering News Formula $K = 2$ and

$$R = \frac{2Wh}{s+1}$$

This is for a drop hammer. Other hammers are accounted for as is indicated: $R = \dfrac{2Wh}{s+0.1}$ for a single-acting steam hammer.

Although formulas are not to be relied upon for accurate results, they nevertheless serve a useful purpose because they prevent overdriving:

There are many laboratory tests but few of value. The following are the results of a recent field test:

Steel and Timber Pile Tests

West Atchafalaya Floodway--New Orleans, Texas and Mexico Railway as reported in Transactions of the AREA, 1951.

"Single hollow steel piles, as well as a group of nine hollow steel piles, were tested to failure or near failure by placing large concrete blocks on...loading platforms on top of the piles.

"The stresses were determined in the steel piles at 5-ft. intervals of length by placing...[strain] gages on the inside of the piles, and by recording the strains during driving by oscillograph recordings on sensitized photographic paper. The strains were recorded during the static loading of the piles by a portable strain indicator.

"Single timber piles, as well as two groups of nine timber piles each, were tested to shear failure in the same manner as...the steel piles, but no effort was made to record the dynamic strains or the strains along the length of the pile. However, strain gages were used to determine the distribution of load to the various piles of the group.

"...The soils are typical of the sedimentary deposits of the locality and consist principally of clays and silty clays of high plasticity and... would be generally classed as medium and soft clays. ...By the Casagrande classification, practically all the soils are "CH" clays of high plasticity.

"A brief summary...

1. The measured dynamic stresses in the top of the pile became greater as resistance to driving increased. The total stress was about the same at all sections of the pile above the ground line, but decreased below the ground line. Only a relatively small percentage of the dynamic stress was observed at the point of the pile in penetrating the ground, even in piles driven to refusal.

2. The static weight of the pile driving hammer ram was 5000 lbs., but this weight with a 3-ft. stroke developed a measured dynamic force

during driving of over 20 times that amount in the 65-ft. friction pile and over 30 times that amount in the 110-ft. end bearing pile.

3. The static load carried by the steel friction piles driven into clay depended almost entirely upon the shear between the pile and the penetrated material for transfer to the surrounding material. At the termination of the test period the center of bearing resistance for these piles were determined as at about half the penetration of the pile.

4. The long steel pile driven into a layer of sand to refusal, after penetrating very deep clay having a high moisture content, continued to act principally as a friction pile under the short-time loading, which was, however, considerably longer than any duration of a moving line load, with only about 16 percent of the total load carried by end bearing...

5. The shearing forces between the surfaces of the steel piles and the penetrated material became quite large for single piles, attaining a value of 1980 lb.per sq. ft. for the 65-ft. friction pile and 3050 lb. per sq. ft. for the 110-ft. fluted pile. The high shear values occurred for only a relatively short length of the pile and were a maximum just below the ground line.

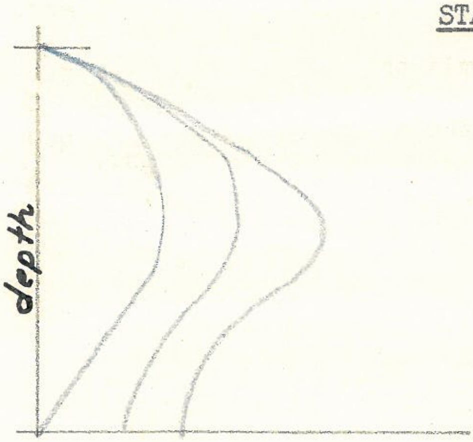
6. The maximum of the shearing forces for the individual piles of the group was considerably below that occurring for the single 65-ft. pile,... The high shearing forces occurred over a much greater length of the pile than that for the single pile and attained a maximum at a much greater depth.

7. (A single tapered steel pile failed at a higherload and shear value than a straight steel pipe pile.)

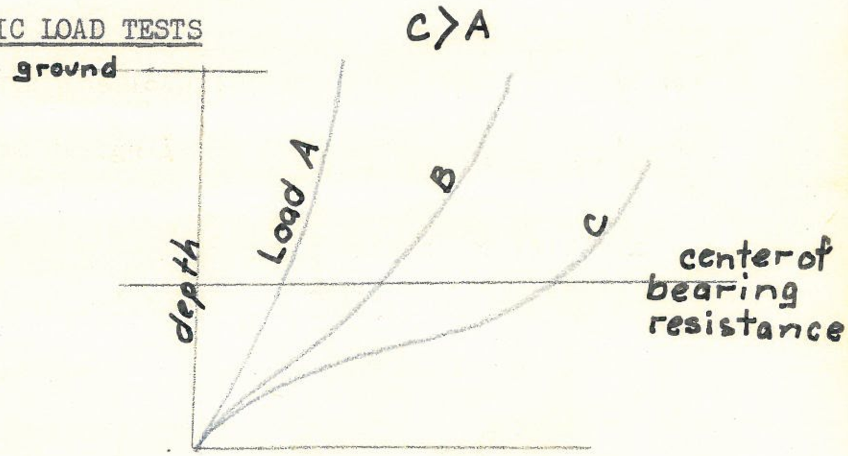
8. There was considerable variation in the magnitude of the load required to produce shear failure in single timber piles driven in the same locality. ... It appears that the number of blows required to drive a single timber friction pile the last foot of penetration is a poor criteria of its ultimate carrying capacity.

9. The distribution of the static load to the various piles of the group both steel and timber, was fairly uniform, with the center pile carrying its approximate share of the load. This uniform distribution was especially true within the usual design loads placed on the piles..."

STATIC LOAD TESTS



Shearing stress, psf
Friction between pile surface and earth.



Pile reactions, Kip at each section

PILE FOUNDATIONS TYPES OF PILES AND AVAILABLE SIZES

Name and description		Maximum length ft	Remarks
Raymond			
	standard	37'-6"	Good friction piles Taper: 1 in 30
	Step	100' +	
Union	fluted steel shell	anything	Heavy steel shell
Western -Button		72' normal 100' + if required	
MacArthur			
	cased or uncased	40'	
Shell-less types		40'	not recommended
Pipe. Open-ended	Hercules or Tuba	anything	Up to 250' piles have been made
Pre-Cast R.C.		150'	40 to 50 ft common 100' exceptional Refer PCA booklet
Composite - wood-concrete		150 ' +	
Timber		100' +	Refer ASCE Manual# 17
Steel H pile	8" to 14"	175' +	Usually in 40' lengths
Sectional pre-cast concrete		any	5' to 10' precast sections in bored hole.
Hollow Concrete Piles		any	150' has been used.
Screwpiles			Special purposes
Drilled-in Caissons	2'-6" d. and up.		"Rotinoff Piling" big diam. pipe pile resembling a pier.

FOUNDATIONS PILE DATA - (From Seelye - Based on the N.Y. Code)

SPACING AND ALLOWABLE LOADS

TYPE	SPACING		LOADS -Tons	
	Min.	Remarks	Usual	Remarks
WOOD	2' -0"	2'-6" usual and preferable	15 for 6" point 20 " 8" "	
COMPOSITE	2' - 6"	"	Same as for wood piles	
CAST-IN-PLACE CONCRETE	2' -6".	Increase spacing 1" per ton for loads above 30 T. Increase to 3'-0" for lengths over 30'-0".	30 T	Increase 2 T for each increase in point diam. over 15", up to 40 T max. and depending on strata.
PRECAST R.C.	2' -6"	Varies with pile design	30	Varies with design
CONCRETE-FILLED PIPE DRIVEN OPEN-ENDED	Rock 2'-0"	o.d. 10"+, 2' min. o.d. 20" 2'-6".	49½-168	
	Hardpan 2'-6". boulders, gravel.		34.7 - 70 30 - 50	Up to 60' lengths. Special for longer piles
	Friction: 2'-6".	Same as c-i-p.	30	
STEEL H-PILES	Rock 2'-0"	Also 2 x max. dimension	40 - 65	Depends on size . 40 T max. in hardpan and when 40' +.
	Friction 2'-6"	Depends also on bearing strata	30	

PENETRATION EXPECTANCY FOR FRICTION PILES

Material	Penetration in feet	Remarks
Clean, compact sand and gravel	slight	piles usually jetted
Other sands and gravels	20	
Sandy clay	30	
Plastic clay	35	
Clay and silt	45	
Silt and mud	50 to 100	
Glacial till		piles cannot be driven

Note : in cohesive soils use $\frac{1}{2}q_u$ for skin friction - ultimate. and compute penetration by surface area x allowable friction. For uplift - use half the allowable bearing load

PRECAST REINFORCED CONCRETE PILES — TYPICAL DIMENSIONS

No	Size in.	Length ft	Main bars No.	Main bars dia. in.	%	Ties Dia in.	Ties Spacing in.	Strength Factor %	% tie steel	% head steel
1	18	65	4	$1\frac{1}{2}$	2.2	$\frac{3}{16}$	9	89	0.054	
2	$16\frac{1}{2}$	60		$1\frac{1}{2}$	2.6	$\frac{5}{16}$	6	113	0.254	1.25
3	16	60		$1\frac{5}{16}$	2.1	$\frac{5}{16}$	7	92	.223	1.30
4	16	50		$1\frac{3}{8}$	2.9	double #6 wire at 5" spacing		153	.217	
5	15	60		$1\frac{5}{16}$	2.4	$\frac{1}{4}$	4	98	.272	1.11
6	15	60		$1\frac{3}{8}$	2.7	double $\frac{5}{16}$ at 5" spacing		100	. 600	
7	15	60		$1\frac{5}{16}$	2.4	$\frac{1}{4}$	$4\frac{1}{2}$	89	.198	0.81
8	15	50		$1\frac{3}{8}$	2.7	$\frac{1}{4}$	6	150	.175	0.70
9	14	60		$1\frac{1}{8}$	2.0	$\frac{1}{4}$	6	76	.183	
10	14	55		$1\frac{3}{8}$	3.0	$\frac{3}{16}$	10	129	.062	
11	14	50		$1\frac{1}{8}$	2.0	$\frac{1}{4}$	6	105	.167	0.50
12	14	50		$1\frac{3}{16}$	2.3	$\frac{1}{4}$	7	121	.157	0.92
13	14	40		$1\frac{1}{4}$	2.5	$\frac{3}{16}$	6	207	.103	0.31
14	14	35		1	1.6	$\frac{1}{4}$	6	180	.183	0.55
15	12	35		1	2.2	$\frac{1}{4}$	7	178	.176	1.03
Averages					2.4%			100	0.2	1.0

$$\text{Strength factor} = \frac{\text{ultimate moment of resistance}}{W\,L/8}$$

assuming concrete 4000 psi and steel 38 000 psi

PRE-CAST REINFORCED-CONCRETE PILES

Maximum lengths for four bars of size given in a square pile of $1\frac{1}{2}$"cover

Dia. of round bar -in.	12" pile	14"	15"	16"
$\frac{3}{4}$	38	36	-	-
$7/8$	44	42	-	-
1	50	47	46	45
$1\frac{1}{2}$	55	52	51	50
$1\frac{1}{4}$		58	56	55
$1\ 3/8$		63	61	60
$1\frac{1}{2}$			66	65

Ties

Size -in	Dia. ties	Spacing of ties
12	$3/16$	$3\frac{1}{2}$
	$\frac{1}{4}$	6
14	$\frac{1}{4}$	6
	$5/16$	9
15	$\frac{1}{4}$	$5\frac{1}{2}$
	$5/16$	$8\frac{1}{2}$
16	$\frac{1}{4}$	5
	$5/16$	8

Long piles in Jetties - Maximum length for each size

Size of pile	Maximum length
12	72
14	71
15	70
16	69

NB
Lift at the one-fifth points.

PRE-CAST REINFORCED-CONCRETE PILES

ALLOWABLE LOAD = Twice the side in inches and not more than 40 Tons.
TONS

Size	Main steel	1st condition	2nd condition	3rd condition
8	4 Φ $\frac{1}{2}$	20	30	45
	8 Φ $^5/_8$	25	38	56
12	4 Φ $\frac{3}{4}$	25	38	56
	$^7/_8$	29	44	65
	1	$33\frac{1}{2}$	51	75
	8 Φ $\frac{3}{4}$	31	47	69
	$^7/_8$	36	54	80
13	4 Φ $^7/_8$	28	42	63
	1	32	48	72
	$1\frac{1}{8}$	30	45	67
	8 Φ $\frac{3}{4}$	$29\frac{1}{2}$	44	66
	$^7/_8$	$34\frac{1}{2}$	52	77
14	4 Φ 1	31	47	69
	$1\frac{1}{8}$	35	53	78
	8 Φ $\frac{3}{4}$	$28\frac{1}{2}$	43	64
	$^7/_8$	33	50	74
	1	38	57	85
15	4 Φ 1	30	45	67
	$1\frac{1}{8}$	34	51	76
	$1\frac{1}{4}$	38	57	85
	8 Φ $\frac{3}{4}$	$27\frac{1}{2}$	41	62

FOUNDATIONS PILE DRIVING HAMMERS (McKiernan - Terry)

Single Acting

No.	Energy per blow ft. lb.	Steam pressure psi	Total weight lb.	Wt. of ram lb	Blows/ min.
S 3	9 000	80	9 000	3 000	65
S 5	16 250	80	12 500	5 000	60
S 8	26 000	80	18 300	8 000	55
S 10	32 500	80	22 400	10 000	55
S 14	37 500 (An exceptional hammer)	100	31 700	14 000	60

Double-Acting

No.	Energy per blow ft. lb.	Steam pressure psi	Total weight lb.	Wt. of ram lb	Blows/ min.
0		100 all sizes	105	5	1 000
1			145	21	500
2			343	48	500
3			675	68	400
5	1 000		1 500	200	300
6	2 500		2 900	400	275
7	4 150		5 000	800	225
9 - B -3	8 750		7 000	1 600	145
10 - B -3	13 100		10 850	3 000	105
11 - B -3	19 150		14 000	5 000	95
Special	55 000		33 000	10 000	

Note. Hammers numbered 0 to 3 are for driving trench sheeting.

Sizes :				
No S 3	is 30	by 34	by 160	inches
S 10	43	43	180	
0	6	4	25	
11 -B -3	26	26	$133\frac{1}{2}$	

Prof. Brennan
Jan. 29, 1953

RIVERS

In studies of rivers information must be gathered on the hydrology of the region. The rainfall, high, average, and low values, runoff, ground water, and evaporation must be determined. The quantity of flow on large rivers like the Mississippi River or the Columbia River is 25 to 60 times greater than the flow on the Hudson River. The sediment carried in suspension or carried along the river bed is estimated on the Mississippi to be 400,000,000 tons a year which is equivalent to 300 feet of delta, and on the Columbia to be 11,000 tons per hour. There is also a relation between velocity of flow and sediment. If the velocity is doubled the amount of sediment will increase about 32 times.

Placing a structure in a river can effect the river's usual process of erosion and deposition. For one thing erosion and sediment can cause the largest expense in river construction. An abutment is designed to hold up the bridge, distribute the load to the earth, and to hold back the earth. At the same time it affects erosion on the whole river. Failure of abutments usually do not concern earth pressure but are failures of sliding, tipping, slipping, erosion, and backcutting as a result of river action. Bridge piers are effected by forces resulting from load from the bridge, uplift by the water, ice and debris, wind, nosing and braking by the locomotive, and vibrations. They can fail by tipping, settling, erosion caused by eddy currents on the upstream side, or by a complete change of river direction.

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Prof. Brennan
Jan. 29, 1953

WAVE AND WAVE ACTION

The problem of wave and wave action is tremendous but unfortunately there is little known. A wave at sea and a wave acting against a structure are much different and cannot be compared. The force of a wave on a structure has been recorded to be as large as 3-1/2 tons per sq. ft. The largest on our coast that has been experienced is approximately 1 ton per sq.ft. Much of the force acts in suction. The force is due to head of water, dynamic action (speed), floatsum (floating objects), and because of a partial vacuum the wave motion sets up. At sea waves are about 60 ft. high but they are not that much at shore. The fetch (the distance to the origin of the wave) which may be 6 or 8 hundred miles is related to the wave height. Waves have a large effect under the surface of the water and often to great depths. For some reason oil floated on the surface of the water quiets the waves.

Breakwaters are built to withstand the force of the waves. Often it is good practice to put the breakwaters in at an angle so as to reduce the force. Unfortunately, the direction of the wave may change and so make the breakwater virtually a failure. Such an event occurred at Northwestern University where a breakwater was put in to stop waves coming from a particular direction. However, as soon as it was built the waves changed direction and the breakwater, still structurally sound, serves no purpose at all.

It is evident that the problem of water is mostly unsolved. It is necessary, therefore, to rely primarily on experience.

It will be attempted here to set up for foundations an analogy to structures. Referring to Lecture 1, three phases of "Building" are considered and summarized by three questions:

1. What do you want?
2. Can you get it?
3. Have you got it?

What do you want in a structure?

Long spans result in heavy concentrated loads. Therefore, the length of the span may depend on the material underneath the structure. Further, whether the structure should have continuous spans or simple supports depends on a consideration of differential settlement. It becomes apparent that it is no longer a question of the structure alone but of the structure, the foundation, and the earth together.

To further this point consider a rigid frame or arch. In computations the assumption is made that the abutments do not move. This depends on the earth and its bearing capacity and the weight of the structure. Next, should the structure be stiff or flexible? If stiff enough there will not be differential settlement. However, if there should be differential settlement cracking may result in a stiff frame. Should the structure be made of steel or concrete? Concrete structures are heavy and continuous. The decision depends on the foundation material.

<u>What do you want in a foundation</u>?

Is a mat foundation advantageous? A mat foundation is heavy, requires much material, and needs much excavation, but it is very stiff. The stiffness depends on the l/d ratio of the mat. Does a pile foundation make heavy loadings feasible? A pile foundation can carry loads to good material but the expense of driving piles may make its use wasteful.

<u>What do you want in the earth itself</u>?

In general, what is wanted is an earth condition which is stable and which does not permit differential settlement.

<u>Can you get it</u>?

The following conditions must be satisfied:

Construction
Appearance
Use
Structural
Economy

1. Construction

Is the contractor available, capable, and willing to do it?

Is the foundation possible? It may not be. Water conditions may cause difficulties which make the foundation virtually impossible.

Is equipment available?

Is the location accessible particularly for large equipment?

Is there a need for temporary structures, sheet-piling, bulkhead, or cofferdam?

Are funds available particularly for contingencies, should there be unfavorable foundation work?

2. Appearance

Appearance is not too important in deep foundations but may be in abutments. These must be tied in with the architecture of the bridge. Retaining walls, dams, and trestles may be important. Also, earthwork may ruin the appearance of a locality. The importance of appearance in foundations above the ground depends entirely on where people will be.

3. Use

Foundations prevent excessive movements and differential movements and help guard against complete failure by instability. With temporary structures like falsework it is not necessary to bother with fancy foundations.

4. Structural

A. Foundation

a. Strength and stability
The forces are known and strains must be computed.

b. Stiffness
This is a problem of differential settlement. It is hard to evaluate but figures can be obtained within 100%.

c. Satisfactoriness
Foundations are subject to cracks and deteriation just like any structure.

B. Earth as a material

a. Strength and Stability
Information must be obtained from geology, from knowledge of the type of material, of stratification, about water and the change in material brought about by water or chemi-

cals, and from classification of the material. The earth may fail completely. Where is the material going to fail? How strong is the material? What are the forces? What is the intensity of the forces?

Sources of evidence include theories; Boussinesq for stress, Swedish circle for stability, pile theories, earth pressure theories, etc. Experiments are tied hand in hand hand with the theories. They involve strength and deformation properties. Experiments include: standard laboratory tests, such as shear tests, unconfined compression tests, triaxial and consolidation tests; small scale model tests; and full scale field tests. Other sources of evidence are experience which is extremely important, commonsense or intelligence, and displaced structures from which the scale can be obtained, probability of it happening, and the consequences if it does happen learned.

It can be seen that "soil mechanics" is a source of evidence. It does not answer everything.

b. Stiffness

This is perhaps the biggest problem in foundation work since E is not known for soils. Even if it were known, it would change constantly because of water conditions. The Theory of Consolidation is actually a matter of stiffness,--how much will it move.

c. Satisfactoriness

In earth satisfactoriness is allied with stiffness. If it does not settle or tip or slide it is satisfactory. Strength, stiffness, and satisfactoriness are all tied to <u>time</u>.

5. Economy

This is difficult to evaluate. A small saving in foundation work may save a lot in the total cost. Borings account for from 1 to 4 percent of the total cost of a job. Bridge piers and abutments amount to 40 or 50 percent of the total cost of the birdge.

<u>Have you got it</u>?

Will it conform with codes or specifications or inspection? Naturally it must conform with local codes which may dictate standard construction procedure and standard foundation materials. There are few specifications on foundations but there are many on testing of materials.

Codes include certain limitations within restricted areas. They may require loading tests or as is often the case put limits on bearing values.

Often unfavorable conditions can be corrected. Corrections may be made to the material, to the structure or foundation, or to the site.

As the lectures deal more intimately with analysis it becomes extremely important not to lose sight of the broad picture. It is all right to specialize if the whole picture is not forgotten. There are three things to do in Civil Engineering, plan, build, and operate (see lectures 1 and 2). Very few men do all three but as has been said before each is closely allied to the other. If you do not know how it is operated you will not know how to plan, etc.

Here, the building aspect will be considered. The elements of building are as follows:

Construction
Appearance
Use
Structural
Economy

Which is the most important depends on whatproject is being considered. The structural element can be subdivided into:

Strength
Stiffness
Satisfactory Performance

Naturally, if the structure is not strong enough and is in danger of failing the design is out. If it is not stiff enough it may be bad but not necessarily. In the past it was true that if a structure were sufficiently strong it would be sufficiently stiff. However, it is not necessarily true now because of many large span structures. The "philosophy" of bracing will be discussed in later lectures but strength will be considered now.

The usual case is to choose the dimensions that are wanted and then to see if the structure is strong enough. There is nothing to do if the structure is strong enough. If it is not strong enough, then the question "what are we going to do about it?" must be asked and answered. The answer may be to make it deeper, shallower, continuous, discontinuous, etc. Pick the simplest way out. What should be done about it depends on what is causing the stress or strain or movement. The kinds of stresses are as follows (see Lecture 2):

1. Primary

   This is the stress which carries the load.

2. Deformation

   This stress occurs because of the action of one member on another member to which it is tied.

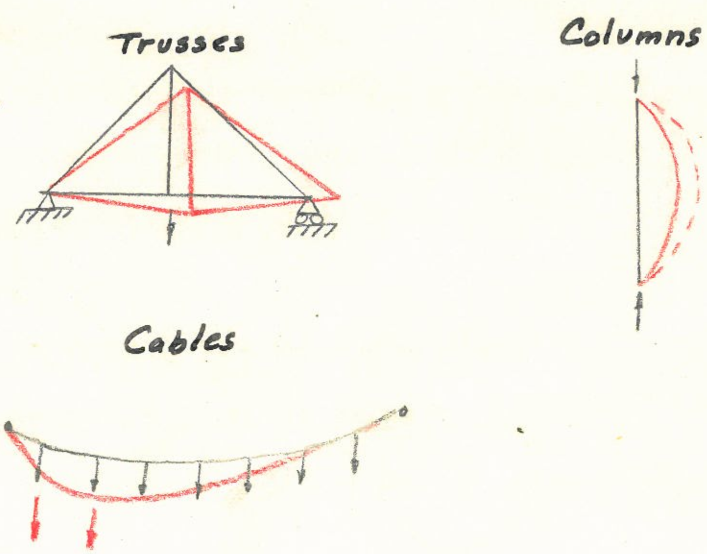
3. Parasitic

   This is a stress caused by movement which resulted from settlement of supports, or changes in temperature, or shrinkage, etc. This stress may or may not be important.

4. Locked

   Due to inequality of internal stresses, it may or may not be intentional. The stress may be locked in because of manufacture. On rolled beams the flange is heavy and the web thin, and, therefore, the web cools faster than the flange causing unequal stresses. On the other hand, prestressing concrete is an example of where stresses are deliberately locked in.

5. Additional



What stresses, then, are you talking about? The primary thing to do is to get some scale on the stresses.

There are three elements in analysis of stress and strain:

Statics --- Proportionality --- Geometry

An interesting thing about the concept of proportionality as used here is that it cannot be said that two loads produce twice the deformation as one load. Failure is elusive. Failure may result from a stress from one cause, but failure may not result from an equal stress from another cause. There is little knowledge about failure. There are five sources of evidence:

1. Analysis (Models)
2. Experiment
3. Experience
4. Common Sense
5. Authority

Much of our information comes from analysis because it can be done quickly, easily, and inexpensively. Most of the literature is devoted to that phase. There are many books on experiments. Although it is not the popular opinion, it takes a better man to make a useful experiment than to make an analysis. The collection and interpretation of experience is even more difficult. The engineer places much value on common sense but little value on authority.

Statics and Geometry

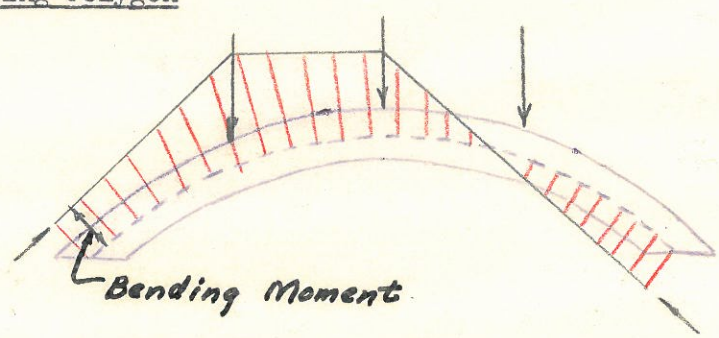
The remaining portion of this lecture will describe certain tricks for manipulating problems in statics and geometry.

Theorem I: Any problem in statics may be made a problem in structural geometry (microgeometry).

Theorem II: Any problem in structural geometry may be converted to a problem in statics.

Problems of relative movements in structures may be solved as problems in statics if (1) rotations are treated as forces along the axis of rotation and (2) linear displacements are treated as couples about the axis of displacement and, further, (3) about any traverse the movements thus considered must be in "geometrical balance."

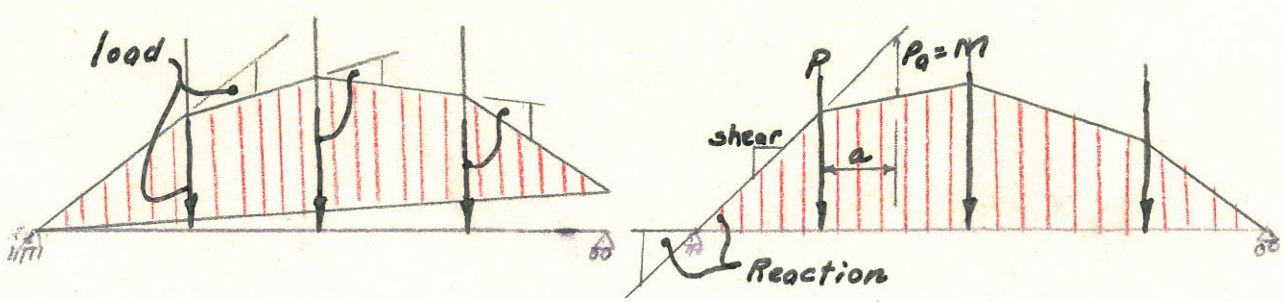
The String Polygon



The slope of the string relative to the closing line is the shear in the beam.

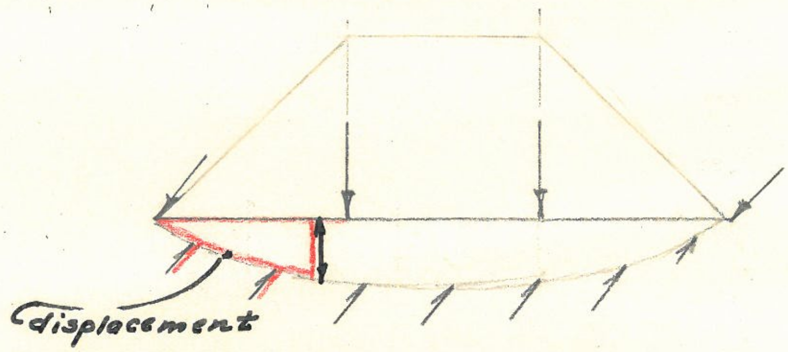
The deflection of the string relative to the closing line is the bending moment in the beam.

The angle formed at the intersections, measured in terms of the offset in so much horizontal distance, represents the intensity of load.



The string polygon can actually be shown by attaching at the reactions of a model structure a string and suspending weights from the string at points of application of load.

As has been said, if the rotations on any structure along a short length are considered as forces along the axis of rotation along any closed circuit the statics must balance. Therefore, a problem in geometry has been turned into a problem in statics. What follows is that a linear movement, a displacement, is represented by a couple.



Greene's Theory and the Column Analogy are a result of these theorems. When Greene's Theory is used, the third dimension for convenience is turned to the plane of the blackboard.

For a string polygon to be correct, it must satisfy statics and continuity. There is only one possible string polygon.

It is well to ask why should stress, strain, movement, and deflection in structural parts and assemblies be computed. In many cases it may be only because the "boss" wants it. On the other hand, it may be desired for general scale, to see if the deflection is 3/4 inch or 1/2 inch or if the stress is 20 ksi or 30 ksi. Stresses must not be just added together. They must be interpreted as they are added. There are five sources of stresses:

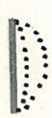
1. Primary Stress
   This is the stress that carries the loads.

2. Deformation Stress
   Also known as secondary stress. It is an unwanted stress caused by movement of other members.

3. Parasitic Stress
   This stress is due to change in temperature, settlement, and the like.

4. Locked Stress
   Welding and differential shrinkage cause stresses to be locked in. When driving rivets a stress is locked in equal to the yield stress of the rivet.

5. Additional Stress

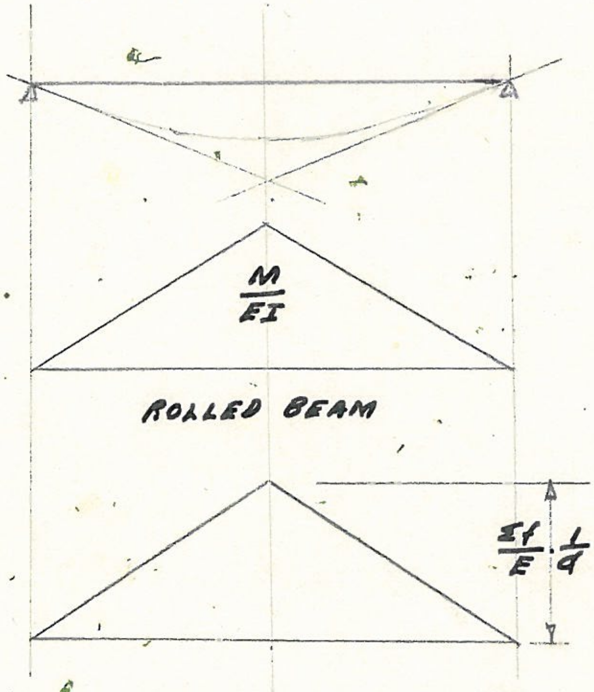


Undergraduates only think of primary stresses. If stresses are too large and the member seems overstressed, the answer would seem to be make the member larger. This may only be true if it were primary stresses that were involved. However, with deformation and parasitic stresses it may make no difference if the member be made larger. These stresses are a result of geometry not statics. It would, therefore, be better to fool with the proportions of the structure rather than the size. It is well to remember that little will be saved if the working stress be doubled--perhaps only 5 percent. This is apparent because spacing bars, and the like carry no stress.

The answer may on the other hand be that the member is not over-stressed. If the minimum section is obtained, why worry? However, "grandma" has something to say about all this, too. As with everything else, figures may have to be put down that can be duplicated. Specifications and codes are not theoretically correct. It is not wise, therefore, to design an entirely different structure by these rules. It is always difficult and usually impossible to find out why a committee does what it does. Do not assume then that a code represents the theoretically correct way of doing a thing. "Grandma" may want it done her way, but it may not be a good way.

Whether stresses are figured loosely or exactly, they are figured through three aspects: statics, geometry, and proportionality of the material. There are two rules: (1) the stactics must balance, and the continuity in the structure as exists must be maintained. Statics can be made into a problem in geometry by what is called graphic statics. Most of us are at home with statics but not with geometry.

The best way to solve a problem in geometry may be by geometry.

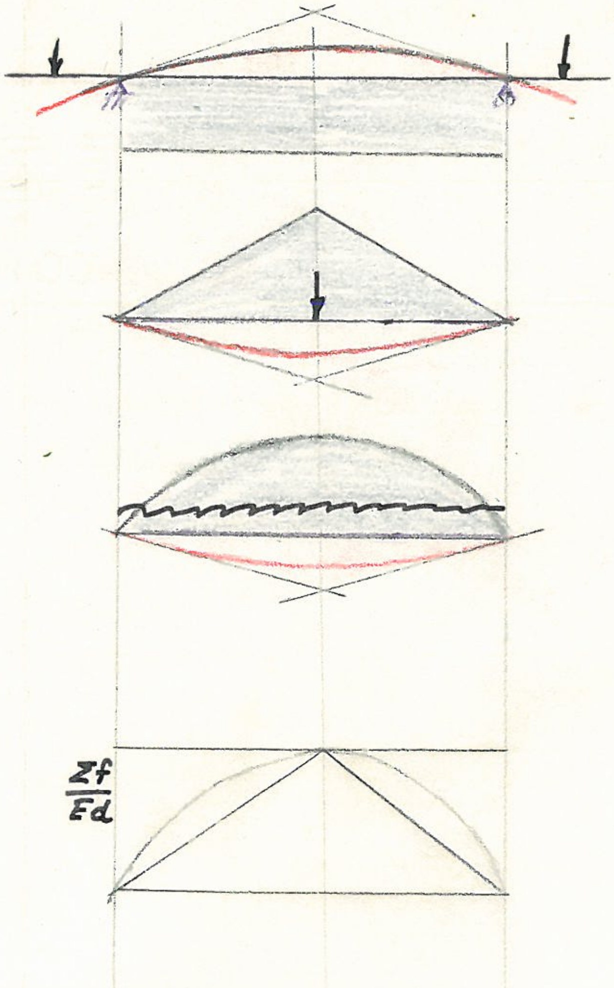


One way of getting deflection is to compute the moments and apply them as loads on a conjugate beam. The moment of these loads will be the deflection. However, usually only the maximum deflection is needed. Then,

$$\frac{M}{EI} = \frac{fI}{c} \cdot \frac{1}{EI} = \frac{f}{E} \cdot \frac{1}{c} = \frac{2f}{E} \cdot \frac{1}{d}.$$

Since the fibre stress is known the computation follows with ease.

Let the curvature, C, equal $\frac{2f}{E} \cdot \frac{1}{d}$ then the end rotation $\phi = K_\phi \frac{2f}{E} \frac{L}{d}$ and if the deflection is Δ then $\frac{\Delta}{L} = K_\Delta \frac{2f}{E} \frac{L}{d}$.

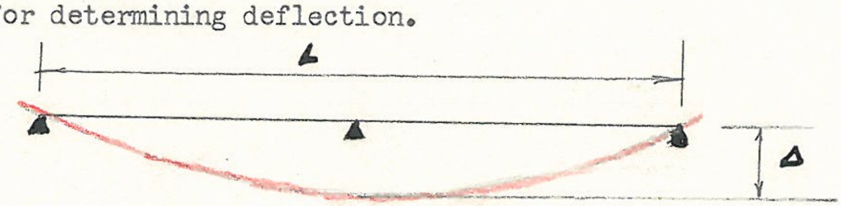


	K_ϕ	K_Δ
RECTANGLE	1/2	1/8
TRIANGLE	1/4	1/12
PARABOLA	1/3	1/9.6
AVERAGE	1/3	1/10

	Σf	E	$\frac{\Sigma f}{E}$	$\frac{L}{d}$	$\frac{\Sigma f}{E}\frac{L}{d}$	
Steel	30,000	30,000,000	$\frac{1}{1000}$	24	$\frac{24}{1000}$	1
Concrete	2,000	3,000,000	$\frac{.7}{1000}$	12	$\frac{8}{1000}$	0.3
Timber	2,000	1,000,000	$\frac{2}{1000}$	12	$\frac{24}{1000}$	1
Aluminum	30,000	10,000,000	$\frac{3}{1000}$	24	$\frac{72}{1000}$	3

Figures have been placed on maximum allowable deflection. For timber the deflection should not exceed 1/360 of the span length. Where this rule came from is not known. It is at least a hundred years old and has been applied for years without any knowledge of what it controls. There are similar rules on deflections for slabs. It is probably based on the dishing of the slab because of time yield.

The effect of settlement can be estimated easily by reversing the procedure for determining deflection.

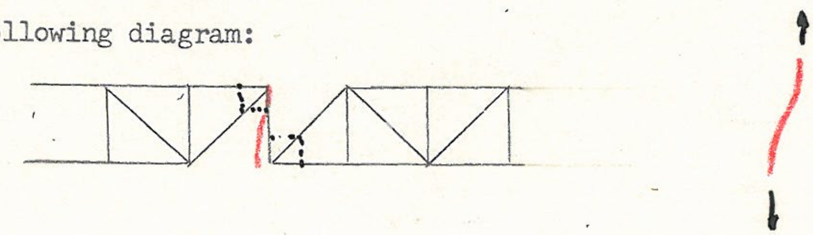


The length, depth, and modulus of elasticity must be known, and the expected settlement estimated so that, since

$$\frac{\Delta}{L} \simeq \frac{1}{10}\,\frac{\Sigma f}{E}\,\frac{L}{d},$$

a figure can be put on the fiber stress resulting from the settlement. For an example take a two span continuous concrete girder with 20 ft. span and with 20" depth. The additional fiber stress due to a 1 inch settlement of the center support would be less than 1500 psi. The argument of many years between Americans (con) and Europeans (pro)as to whether continuous girders are more economical may have been settled if the Americans had put figures on the effects of differential settlement and seen that it was not appreciable.

The designing of a hinge can be very "messy" as is illustrated in the following diagram:

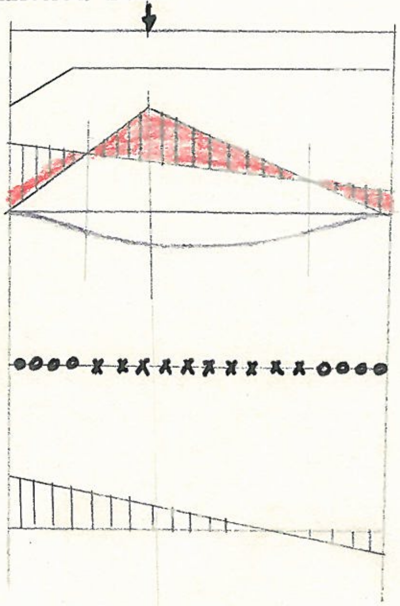


Bending
Stress

A general theorem for relative movement in a structure is if rotations are considered as loads and displacements as couples in a plane along the axis of rotation, the movements about any traverse must balance. Deflection changes the shape of the structure but the error of taking the rotations as though they act on the undeflected structure is very small, perhaps one part in a million.

In order to use moment distribution, the fixed end moment, the stiffness, and the carry-over factor must be obtained.

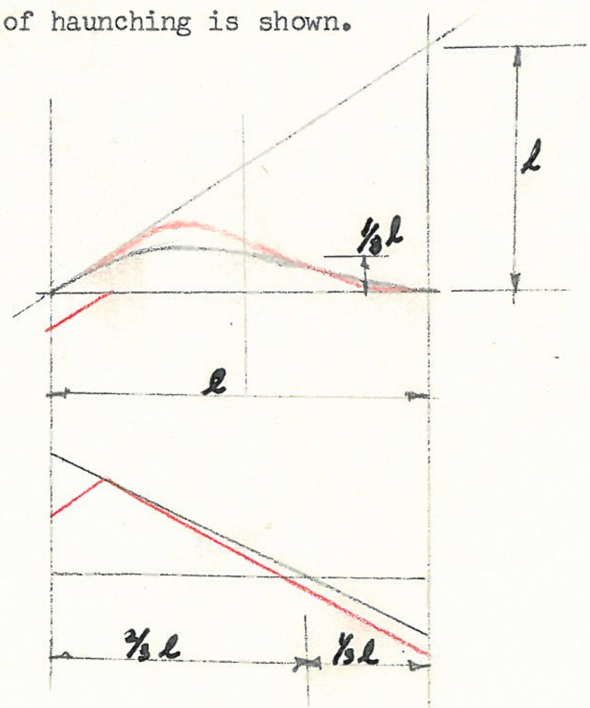
Haunched Beam



$\frac{M}{EI}$

Draw the $\frac{M}{EI}$ diagram as shown by trial. Write two equations, the summation of the $\frac{Mds}{EI}$ forces, and the rotation of these $\frac{Mds}{EI}$ forces. If they do not sum up to zero correct by adding moments as shown.

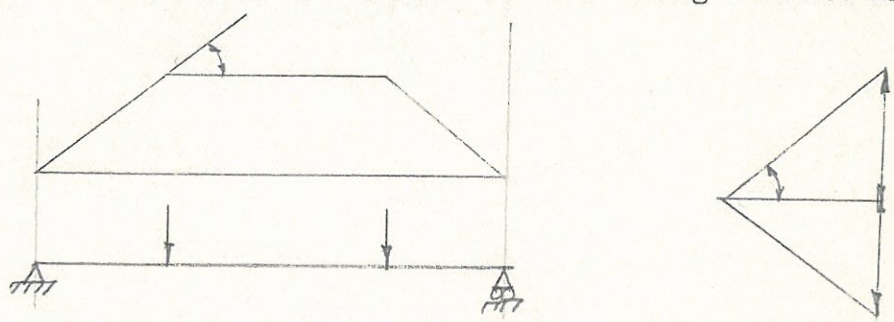
Draw the influence line for the fixed end moment. The rotation must balance about the left hand reaction. It becomes evident that the carry over factor is 1/2. The stiffness is equal to M/ϕ. The effect of haunching is shown.



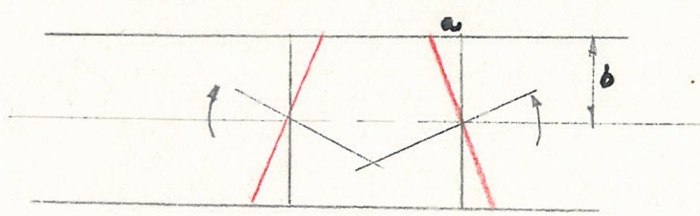
haunch

In this lecture the tools for working problems in geometry and statics will be discussed. Problems in geometry can be worked as problems in geometry. Problems in statics can be worked as problems in statics. The best way, however, may be to work problems in statics as problems in geometry and problems in geometry as problems in statics, or even work them as both.

The string polygon is a useful tool for seeing relative magnitude.



The curvature of a beam is the summation of the changes in length divided by the distance as illustrated.



$$\text{Curvature} = \frac{a}{b} = \frac{d\phi}{ds} = \frac{M}{EI}$$

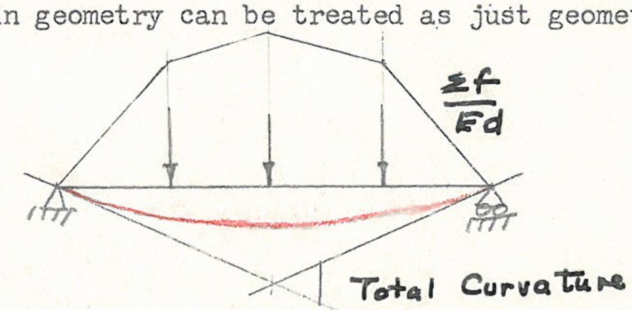
The objection to the expression $\frac{M}{EI}$ is that "I" has no reality, i.e., it has no meaning to the structural engineer. So, another expression is more suitable.

$$\frac{M}{EI} = \frac{\Sigma f}{Ed}$$

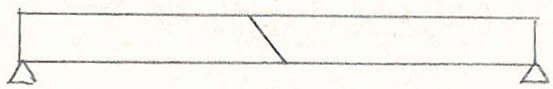
As was shown in the preceding lecture, if the settlement and the depth of the member are known the stress can be figured.

When books say plane sections remain plane what they mean is that strain is planar. Although they intimate that they can it is doubtful if it can be proved in laboratory that strains are not planar mainly because measurements of strains are not very good. Where there is a load or a reaction there is going to be transverse distortion. This is unimportant unless the beams (if it can be called a beam) is as long as it is deep.

Problems in geometry can be treated as just geometry.



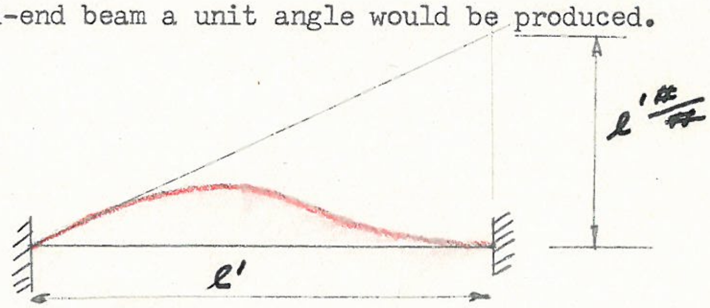
An influence line is an artificial but convenient method. It is important, therefore, that we be familiar with it. An influence line can be drawn for any stress fuction in a beam by means of a model, i.e., if the moment of inertia or "modulus of flexure" varies along the model as it does along the structure. If a model is made and broken so that it will be distorted by the stress with which we are interested, then the displacement of the load line in the direction in which the loads are applied is the influence ordinate for that particular stress. So, if it be a reaction with which we are interested, then by breaking the model at the reaction and letting the "reaction react" then the influence line will draw itself. If this is easy to do and the model easy to make then it is not worth the making of the model, and if it is complicated then there is so much doubt that the model is not worth making. It is very difficult to do this well with a model. When rotating the model to trace the influence line for moments, if the point about which the model is rotated be only slightly off, the results will, nevertheless, be considerably off. When shearing a model the shear plane will not be vertical as illustrated because the beam will wedge together.



By means of Müller-Breslau's Principle a problem in statics may be turned into a problem in geometry. It is not necessary to make and deform a model but merely to sketch a deflected structure.

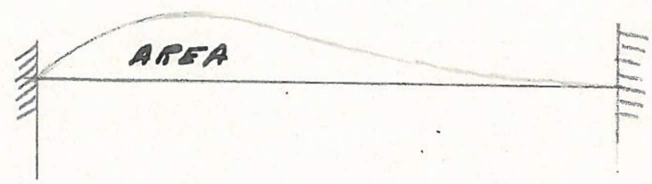
There are three ways of putting a scale on the influence ordinate. No one way is good all the time.

1. Produce a unit deformation. For the influence line for the moment at the end of a fixed-end beam a unit angle would be produced.



For this case this method would not be satisfactory but for the influence line of the center reaction of a two span continuous beam it is very satisfactory.

2. Find the area under the influence line.

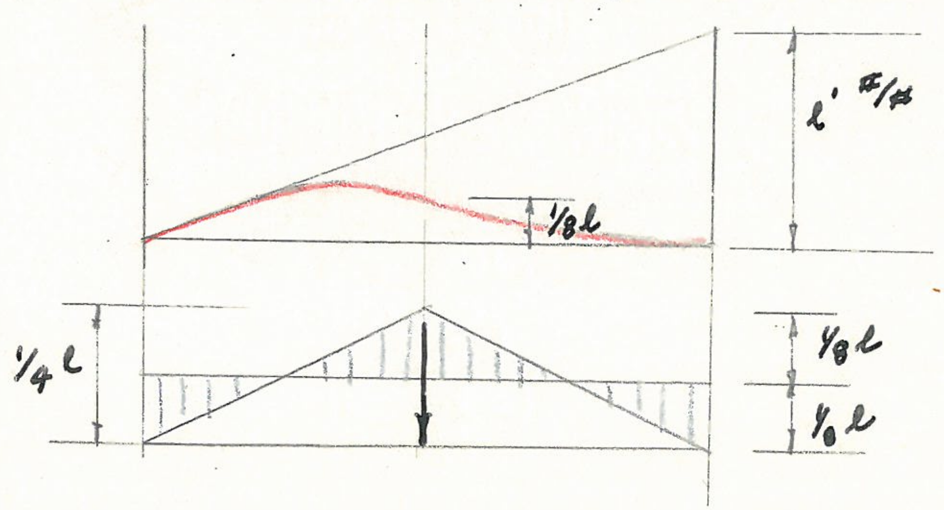


Area = $\frac{1}{12} \cdot l \cdot l$

Average ordinate
= $\frac{1}{12} l$

This method is not too satisfactory for fixed-end beams but it is quite useful in arches because the area under the influence line for an arch is zero.

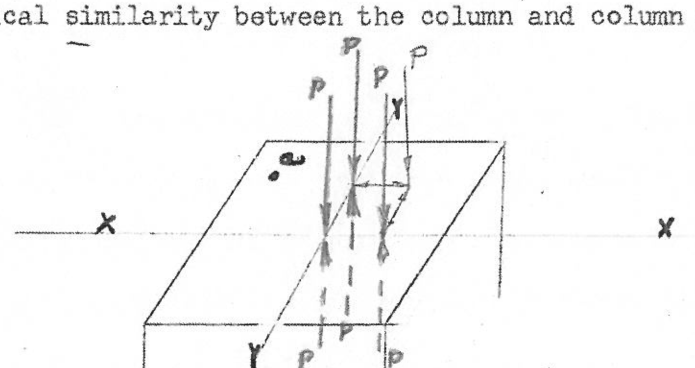
3. Find values of the influence ordinate by some indirect method. The moment at the end of a fixed-end beam due to a 1# load at the center of the span is the influence ordinate at the point the 1# load is applied.



In this case the indirect method is very convenient.

The Column Analogy

The column analogy is essentially a mechanical means of finding indeterminate moments. The indeterminate moments become analogous to fiber stresses in a column. It is of use to the engineer because his thoughts and habits are preserved and no new formulae are introduced even though there is no physical similarity between the column and column analogy.



Take a column cross-section as shown loaded with an eccentric force P, m and n distance away from the centroidal axis. By adding equal and opposite forces, P is converted to axial force P and two couples, so that the stress at any point *a* is

$$S_a = \frac{P}{A} + \frac{P_m X}{I_x} + \frac{P_n Y}{I_y} = \frac{P}{A} + \frac{M_x X}{I_x} + \frac{M_Y Y}{I_Y}$$

It has already been stated that problems of relative movement in structures may be solved as problems in statics if rotations are treated as forces along the axis of rotation and linear displacements treated as couples about the axis of displacement and that about any traverse the movements thus considered must be in "geometrical balance." Therefore, if there is no total rotation

$$\int \frac{m\,ds}{EI} = 0$$

$$\int \frac{m\,ds}{EI} x = 0$$

$$\int \frac{m\,ds}{EI} y = 0$$

where x and y are distances to some axis and $\frac{m\,ds}{EI}$ is the flexural rotation in a short length of beam. If "m_o" were the original moment, i.e., a statically determinate moment or any assumed moment, and "m_c" were the change in moment or the correcting moment, then if there is no total rotation

$$\int \frac{(m_o + m_c)\,ds}{EI} = 0$$

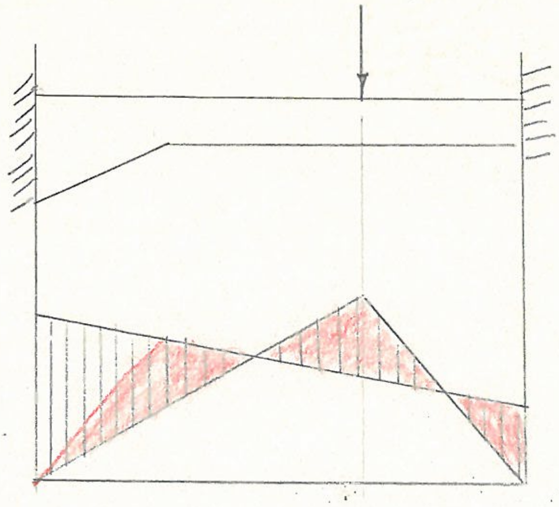
$$\int \frac{(m_o + m_c)\,ds}{EI} x = 0$$

$$\int \frac{(m_o + m_c)\,ds}{EI} y = 0$$

and neglecting signs

$$\int \frac{m_o\,ds}{EI} = \int \frac{m_c\,ds}{EI}$$

If, for an example, a haunched beam be considered, the moments and moment areas would be as shown:



— Bending Moments

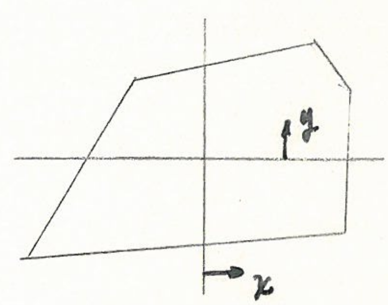
— Moment Areas, $\frac{M}{EI}$

If the moment areas balance, then the drawing is correct. Let P in the column formula equal $\int \frac{m_o\,ds}{EI}$; A, $\int \frac{ds}{EI}$, the total elastic area; M_x, $\int \frac{m_o\,ds}{EI}x$, where x is the distance from the centroidal axis in the X-direction; M_y, $\int \frac{m_o\,ds}{EI}y$, where y is the distance from the centroidal axis in the Y-direction; and I equals the $\int \frac{ds}{EI}x^2$ or $\int \frac{ds}{EI}y^2$. The moment at any point will correspond to the fiber stress.

Then, if the total elastic area $\int \frac{ds}{EI}$ is computed and the centroid of this area found, and P, $\int \frac{m_o\,ds}{EI}$ is computed and its centroid found then the moment at any point can be found by the beam formula.

For situations where there is no symmetry complicated formulas must be written.

$$f = \frac{P}{A} + \frac{M_x x}{I_x} + \frac{M_y y}{I_y}$$



Where I_x is the moment of inertia about the Y-axis in the X-direction and I_y is the moment of inertia about the X-axis in the Y-direction.

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$$f = P/A + \frac{M_x}{I_x - I_{xy}\frac{I_{xy}}{I_y}} x + \frac{M_y}{I_y - I_{xy}\frac{I_{xy}}{I_x}} y$$

This is right if it satisfies the conditions:

$$\int f\,dA + P = 0$$

$$\int f\,dA_x + M_x = 0$$

$$\int f\,dA_y + M_y = 0$$

The kern of a section is the area within which a load must lie if there is to be no tension on the section. Flexural formulas may be written in terms of moments about the kern points.

$$f_a = \frac{P}{A} + \frac{Pec}{I} = \frac{M_k \frac{d}{2}}{I} = \frac{P}{A}\,\frac{e_k}{\frac{\rho^2}{x_a}}$$

where P is the load,

A is the area of the section,

e_k is the eccentricity of the load with reference to the kern point,

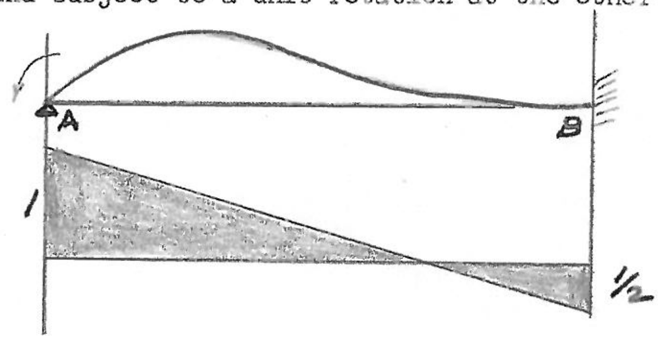
ρ^2/x_a is the distance of the edge of the kern from the centroid, and

M_k is the moment about the kern point.

The kern is sometimes useful where it is known (as is the middle third of a rectangular homogeneous section) or where it is conveniently found.

It is sometimes useful in applying the column analogy. It is never essential.

As a problem useful in the study of moment distribution and as an illustration of the column analogy consider a beam of uniform section fixed at one end B and subject to a unit rotation at the other end.



The rotation or angle change

$$\phi = \frac{1}{2} \times 1 \times \frac{l}{EI} - 1/2 \cdot 1/2 \cdot \frac{l}{EI} = \frac{1}{4} \frac{l}{EI}$$

per unit of moment. That is, since

$$\int \frac{m\,ds}{EI} = -\phi, \quad \phi_{m=1} = \frac{1}{4} \frac{l}{EI}$$

and the stiffness, $\frac{M}{\phi} = \frac{4EI}{l}$

For a rectangular section, a convenient form of the flexure formula is:

$$f = P/A \left(1 \pm \frac{6e}{d}\right)$$

$$P = \phi$$

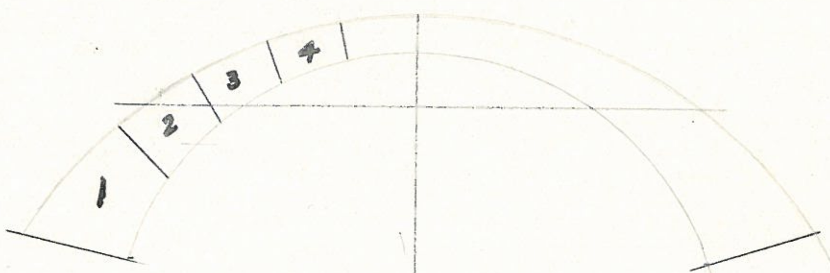
$$A = 1/EI$$

For the analogy,

$$f = M$$
$$p = \phi$$
$$A = L/EI$$
$$e = \frac{L}{2}$$

so $$M/\phi = \frac{4EI}{\ell}$$

Application of the column analogy to the arch.



No.	x	y	$\frac{ds}{EI}$	$\frac{ds}{EI}x$	$\frac{ds}{EI}y$	$\frac{ds}{EI}x^2$	$\frac{ds}{EI}y^2$	m_o	$m_o\frac{ds}{EI}$	$m_o\frac{ds}{EI}x$	$m_o\frac{ds}{EI}y$
1											
2											
3											
4											
.											
.											
.											
.											
.											
Totals	X	Y	A			I_x	I_y		P	M_x	M_y

PROPERTIES OF SECTION											LOADS ON SECTION				
GIVEN PROPERTIES						COMPUTED PROPERTIES					GIVEN			COMPUTED	
						STATICAL MO.		PRODUCTS OF INERTIA			LOAD	COORD. OF LOADS		MOMENTS	
MEMBER	LENGTH L	WIDTH $\frac{1}{I}$	AREA $\frac{L}{I}=a$	X	Y	ax	ay	ax^2+i_x	ay^2+i_y	$axy+i_{xy}$	$P=M_s a$	X_1	Y_1	$M_x=Px_1$	$M_y=Py_1$
TOTALS															
LOCATE CENTROID $\begin{cases}\bar{x}=A\bar{x}/A\\ \bar{y}=A\bar{y}/A\end{cases}$															
CORRECTIONS TO CENTROID															
SUBTRACT CORRECTIONS															
CORRECTIONS FOR DISSYMMETRY															
SUBTRACT CORRECTIONS															

COMPUTE MOMENTS OR FORCES BY FORMULAS

$m_i' =$ $\quad$ $M =$ $\quad$ INTERCEPTS OF INDETERMINATE FORCE +

$v_i' =$ $\quad$ $V =$ $\quad$ $X_1 =$ $\quad$ $Y_1 =$

$h_i' =$ $\quad$ $H =$

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Feb. 10, 1953

References:

1. "The Column Analogy," Hardy Cross.
Bulletin No. 215. University of Illinois Engineering Experiment Station.
2. "Continuous Frames of Reinforced Concrete," Cross and Morgan, pages 46 through 76.
3. "Statically Indeterminate Structures," Hardy Cross, Chapter I.

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Prof. Cross
Feb. 12, 1953

Arches

The tools that we have developed will be used to study arches and then truss problems. Gradually we will get away from concrete.

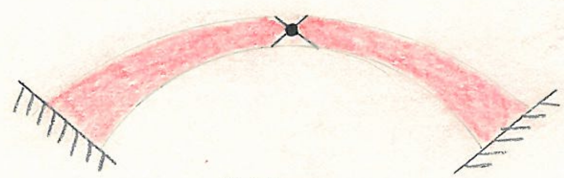
Architecturally speaking an arch is a structural member usually curved for spanning an opening and capable of supporting weight from above. From the mechanics' viewpoint an arch is a structural member that has for vertical loads horizontal reactions as well as vertical reactions. The many books on arches are generally of five types:

1. The analysis of stresses in arches.
2. The architectural aspect of arches in bridges.
3. The architectural aspect of arches in buildings.
4. The construction of arches.
5. The settlement and buckling of arches.

An arch can have anywhere from zero to three hinges:

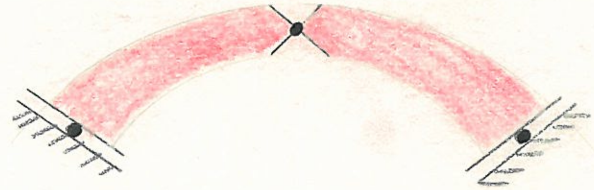
Hingleless

1 Hinge

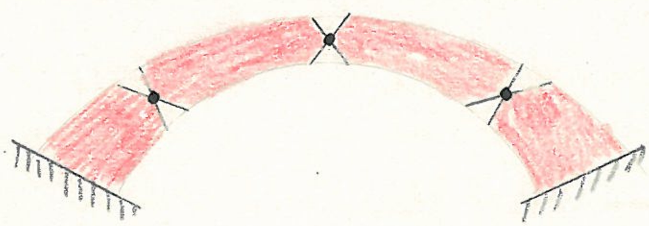


2 Hinge

3 Hinge



Although hinges if used are usually placed as shown above, occasionally they may be placed elsewhere.



As to what is meant by a hinge is questionable. It may be quite a serious matter to detail a hinge. In concrete a hinge may consist of a thinned down section. This aspect will be discussed later.

The hingeless arch will be taken up first.

As a vertical load is applied the arch tends to spread. Therefore, a horizontal thrust must be furnished by the soil or column. This arch differs from the two-hinged arch because there is moment at both ends which tends to rotate the ends. With a change in temperature the arch will tend to move up and out (dashed line). However, the abutments will hold the ends of the arch fixed just as if forces were applied to the dashed line (colored line).

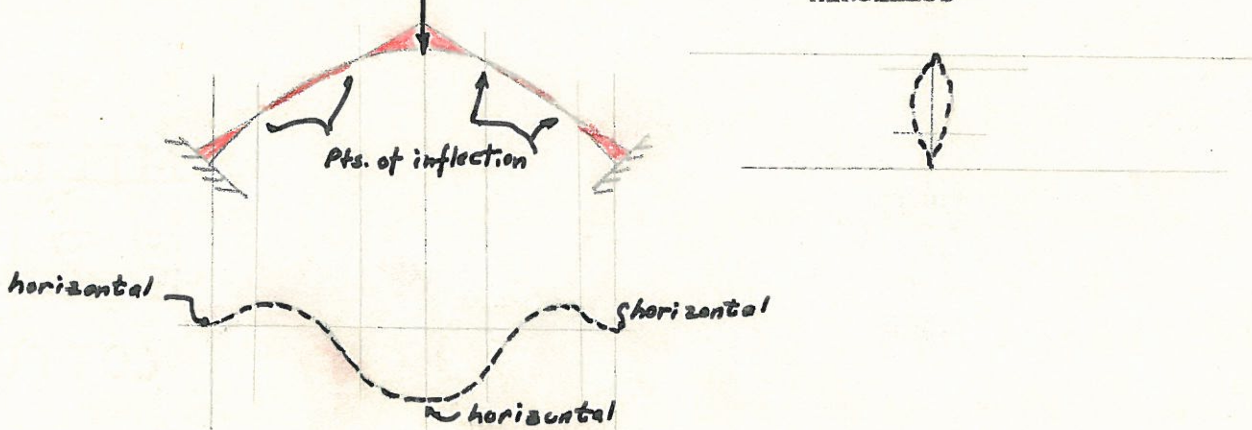
The first tool to apply is the deflected structure. In general a deflected structure cannot be drawn in two dimensions.

To draw a deflected structure with clarity two deflected structures should be drawn, one vertical and one horizontal.

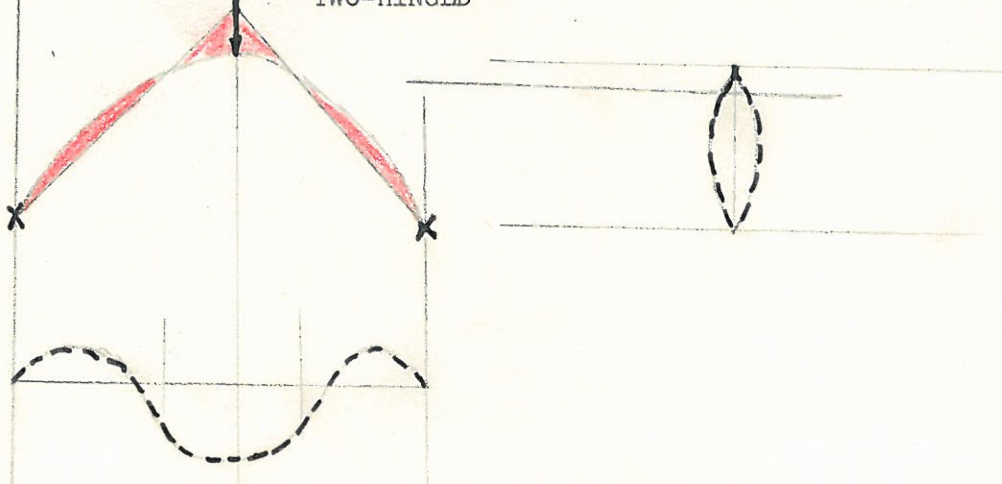
Before proceding, what we want to know should be determined. It usually is only the shape the arch ought to have and the thickness of the arch so that it will not be overstressed. Also, the shrinkage stress and the stress resulting from movement of the abutment may be very important. The live load stresses can be investigated by drawing of influence lines.

In the following diagram the deflected sturcture will be drawn by separating horizontal and vertical movements.

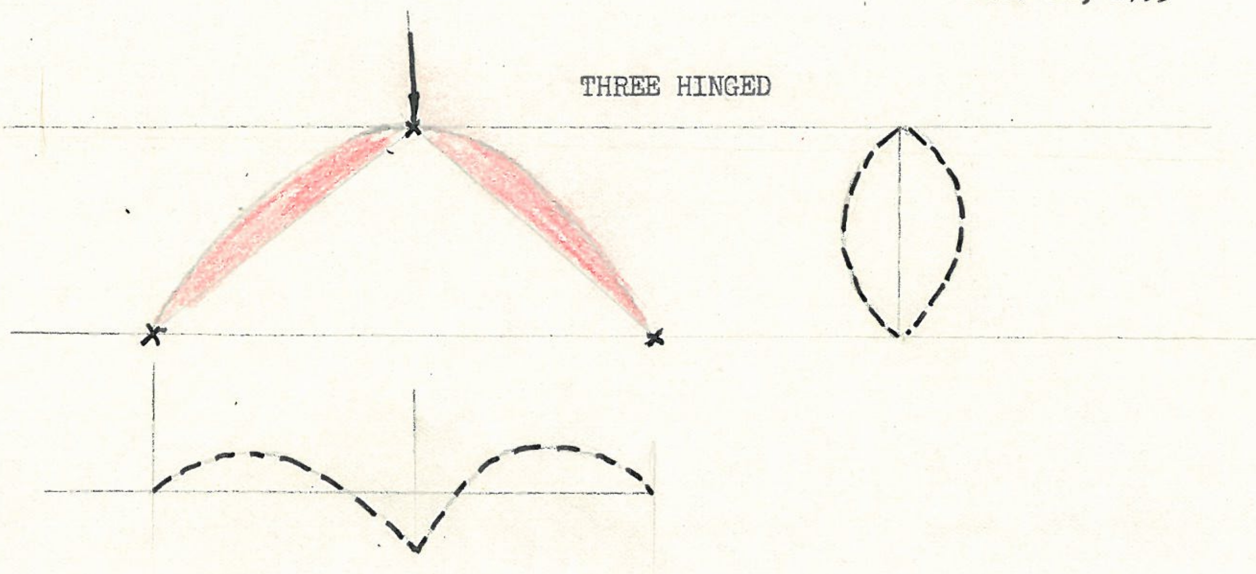
HINGELESS



TWO-HINGED

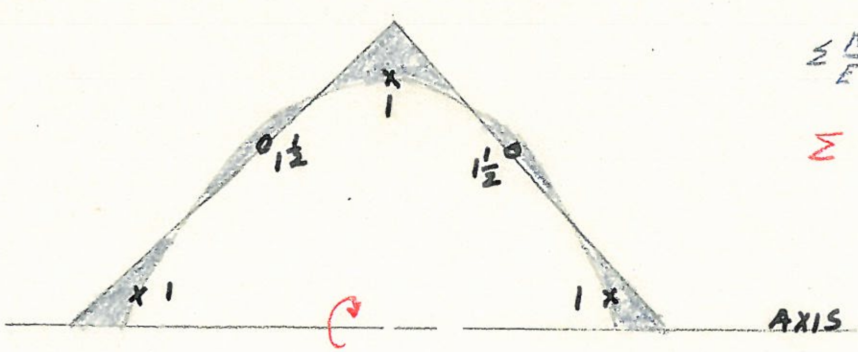


L-29 (p. 4)
Prof. Cross
Feb. 12, 1953



However, drawing of deflected structures breaks down very rapidly when the load or loads are unsymmetrical.

The next tool to utilize is the balancing of rotations. On the hingeless arch the summation of the rotations must equal zero, i.e., $\sum M/EI = 0$ and the moments of the centroids of the moment areas about an axis as shown below:



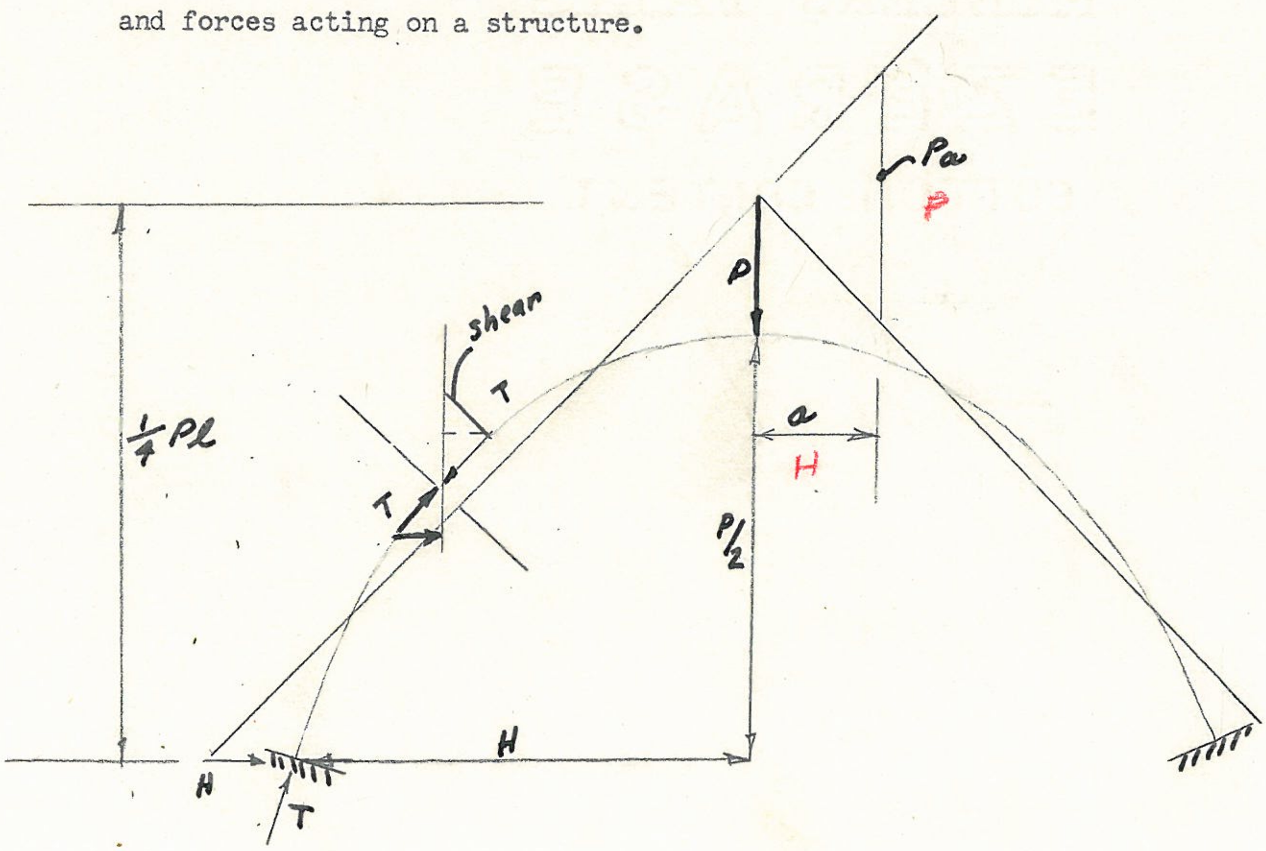
$\sum \frac{M}{EI} = 0$

$\sum \frac{M}{EI} y = 0$

Now the pressure line can be drawn fairly well by sight and trial and error.

The pressure line tends to move toward, i.e., hug the weak section, and to run away from the stiffened section. You can, therefore, within reason dictate to the structure the amount of moment it should have at the crown. Making a structure deeper at the crown may not reduce the fiber stress. In fact, it may increase it very much. Therefore, variation of the thickness of the crown may be of great importance. Structural engineers and authors of books on indeterminate structures think in terms of determinate structures. As a result, they may say that making a structure twice as strong reduces the fiber stress by half. This is not necessarily true as exemplified in arch analysis.

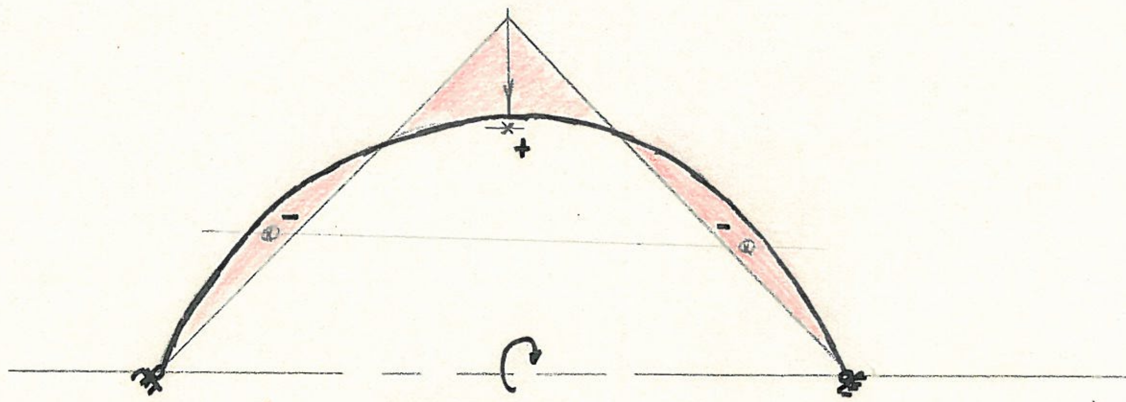
The following diagram illustrates how a scale may be put on moments and forces acting on a structure.



In order not to get confused on moments and shears do not think of them as moments and shears but as forces acting normally, etc.

Two Hinged Arch

With two hinged arches the rotations do not have to balance, i.e., $\frac{M}{EI} \neq 0$ but the rotation of the M/EI areas must balance.



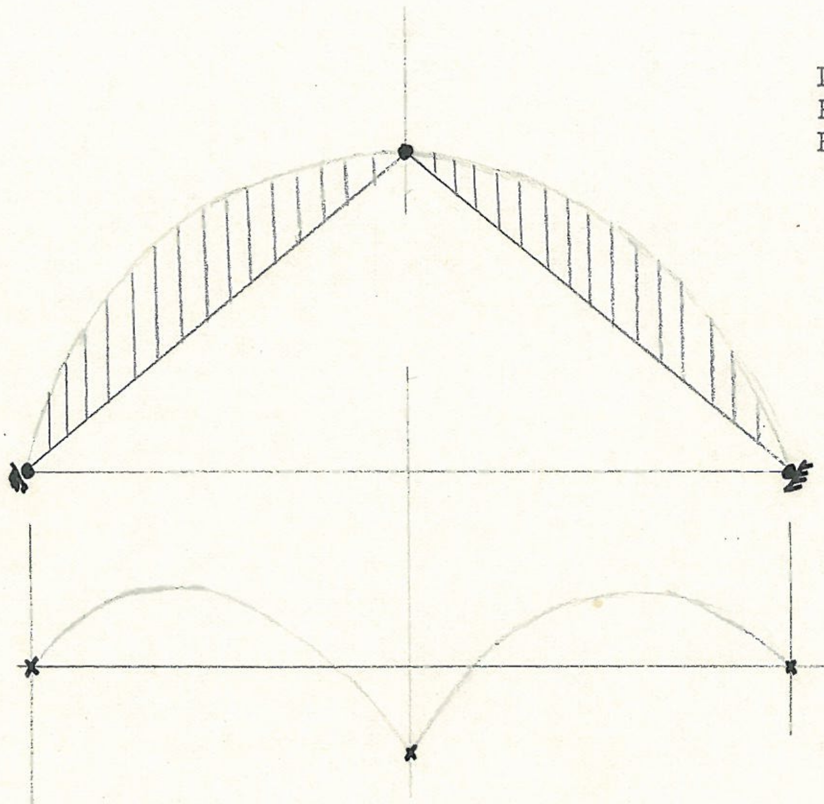
$$\Sigma \, M/EI \; y = 0$$

In this instance the total negative M/EI should be about twice the total positive M/EI.

Three Hinged Arch

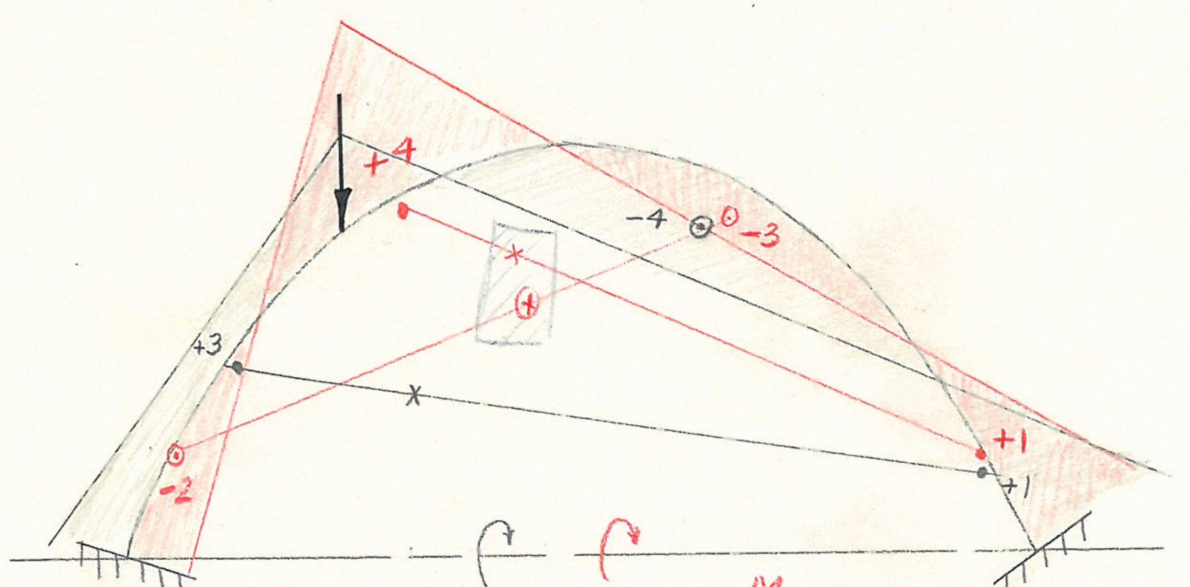
With a three hinged arch neither the rotations nor the rotations around any axis need balance.

L-29 (p. 7)
Prof. Cross
Feb. 12, 1953



Unsymmetrical Loads

By applying the principles just presented unsymmetrical loads on all types of arches do not cause much inconvenience. The rotations on the hingeless arch and the rotations around the closing line in the hingeless and two hinged arches will balance, but it may take patience to get them to balance.



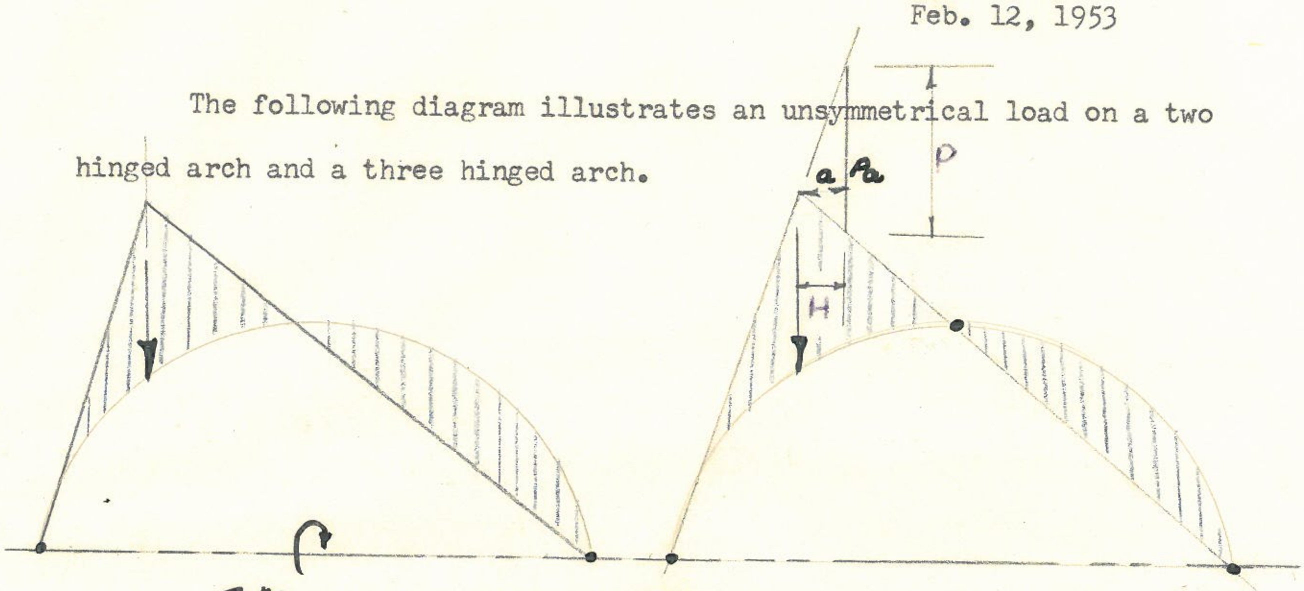
$\Sigma \frac{M}{EI}$ balances

$\Sigma \frac{M}{EI} y$ does not balance

$\Sigma \frac{M}{EI}$ balances

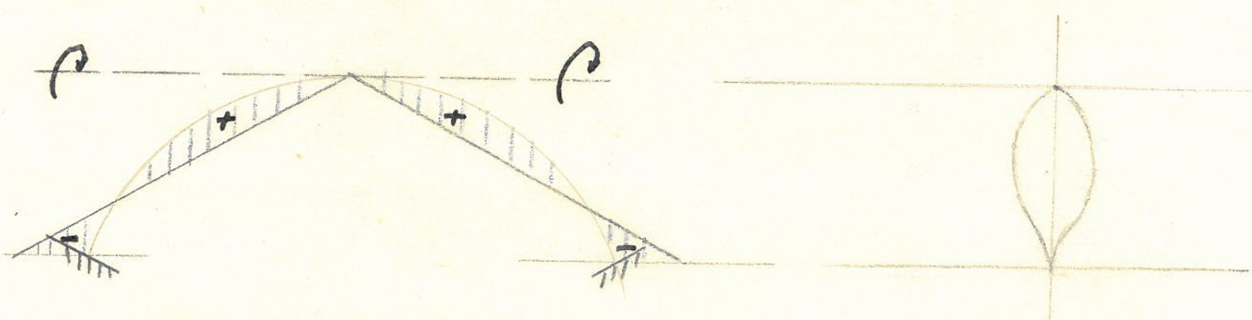
$\Sigma \frac{M}{EI} y$ almost balances

The following diagram illustrates an unsymmetrical load on a two hinged arch and a three hinged arch.

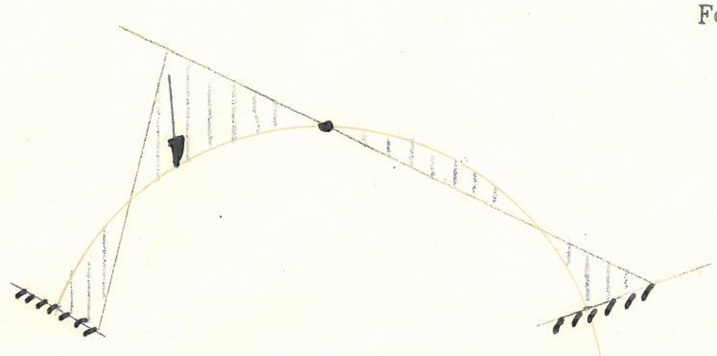


$$\Sigma \frac{M}{EI} y = 0$$

The effect of thinning down the crown can be explored by placing a hinge at the crown. The following diagrams show the effect of a hinge at the crown.



The rotation of the M/EI areas will balance about the horizontal axis shown. As can be seen the positive moment should be about twice the negative moment.



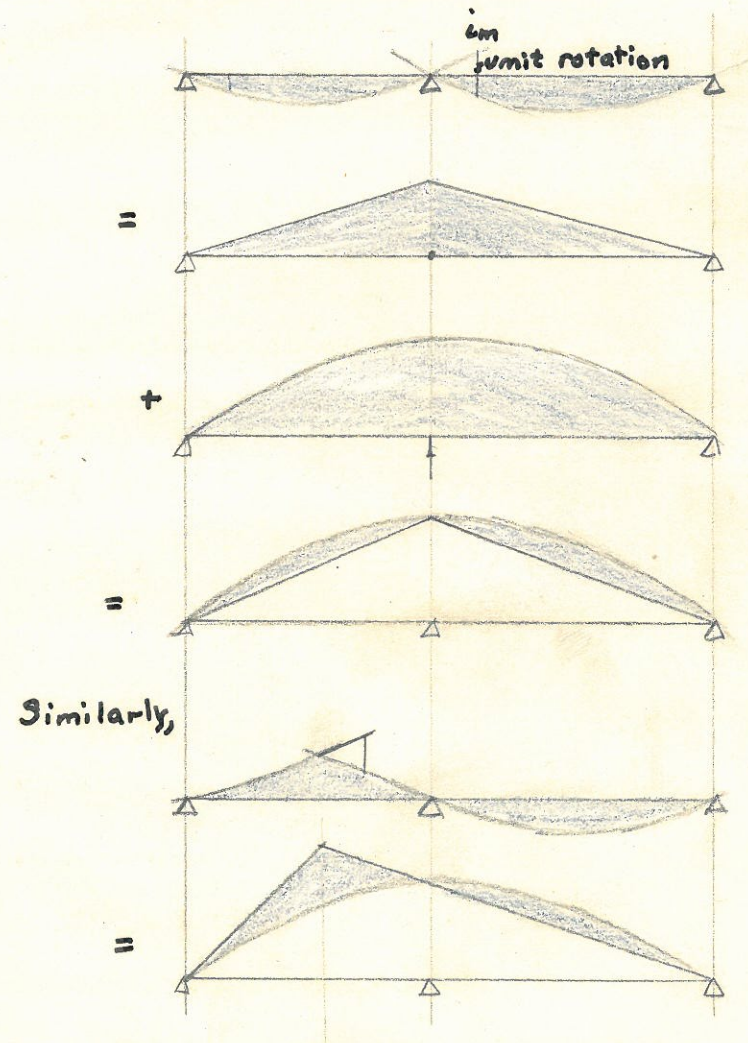
The curve of equilibrium for loads uniformly distributed horizontally is a parabola. A parabolic arch with full uniform load is subject only to compressive stresses if rib-shortening is neglected. The rate of change of slope of curve per horizontal foot is proportional to the intensity of vertical load per foot. An elliptical arch, then, is appropriate for relatively heavy loads near the abutment, a triangular arch for central concentrated load. The geostatic arch of Rankine is of interest in this connection, being the curve of equilibrium for active earth pressure as given by his theory and hence appropriate for tunnel arches. Due to the relatively greater dead load at the springings, the string polygon for dead load will usually fall outside the parabola. (From Cross and Morgan.)

<u>The Pressure Line Theorem</u>

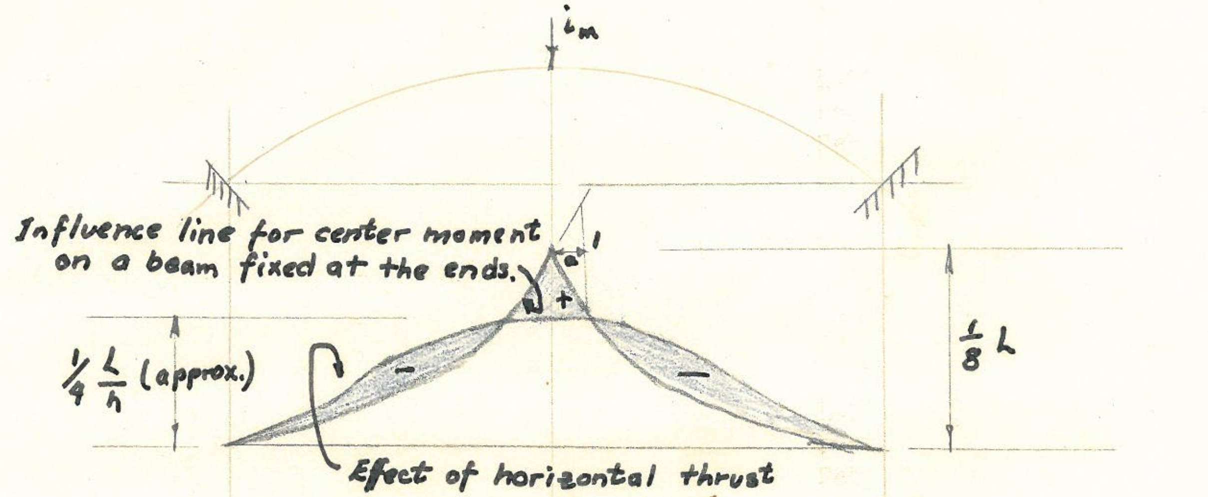
The pressure line in an arch is that string polygon for the loads which most nearly fits the arch axis. The Theorem which was advanced by Winkler in 1879 may be proved by the column analogy. If we draw a string polygon which coincides with the axis, m_o will be zero throughout, which makes m_c also zero, and this polygon is the true pressure line as determined by the elastic theory. If the polygon lies very near the axis, m_o is small and hence m_c is very small, and little if any change is required in the pressure line. P, M_x, and M_y of the column analogy all equal zero.

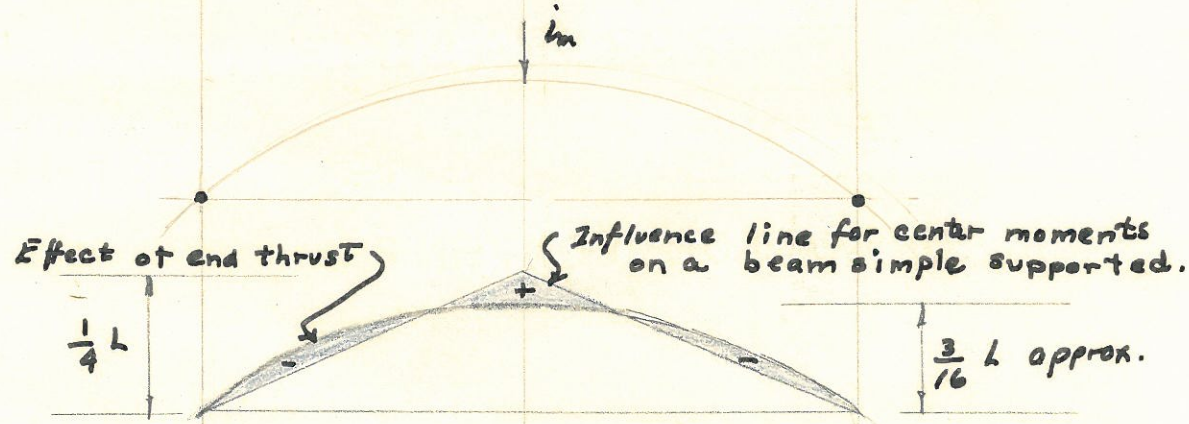
<u>Influence Lines</u> Refer to page 263 Cross & Morgan

Instead of drawing an influence line for a structure directly, it is sometimes more convenient to break the structure down so that simple influence lines can be drawn and combined.



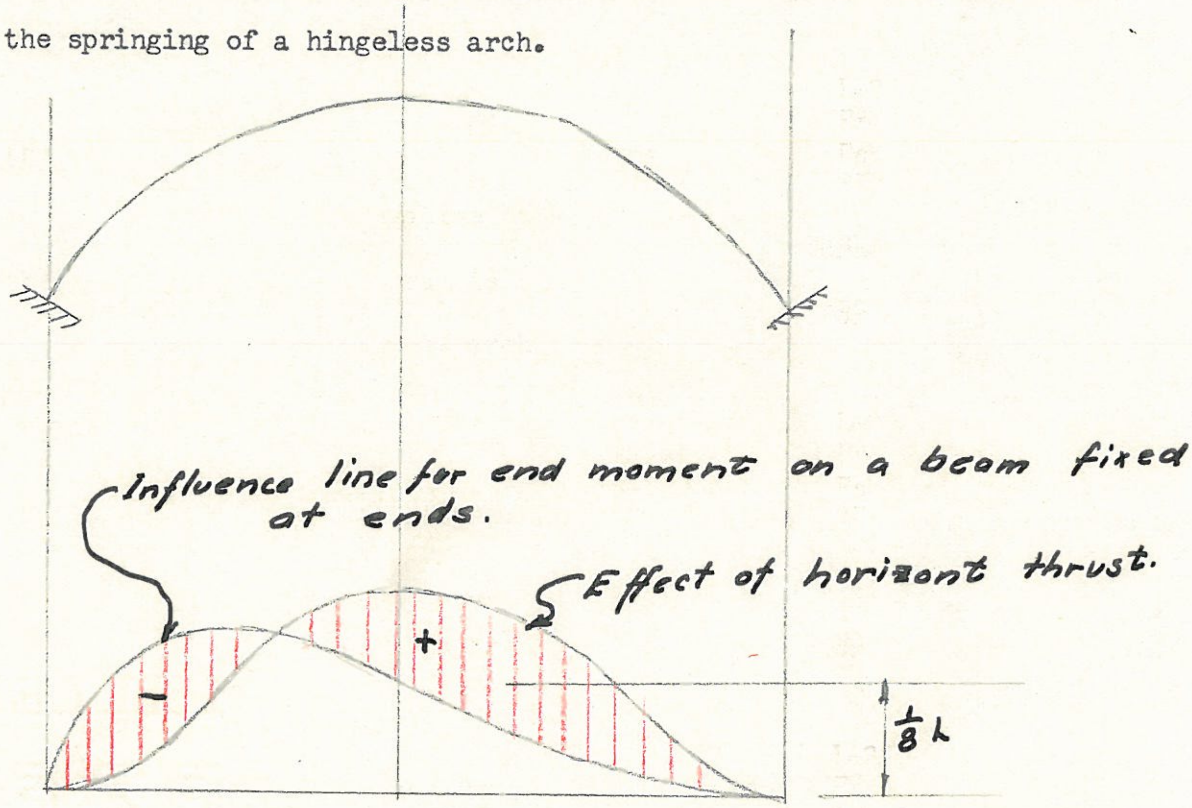
The same procedure can be used for arches but the scale may not be quite as good.



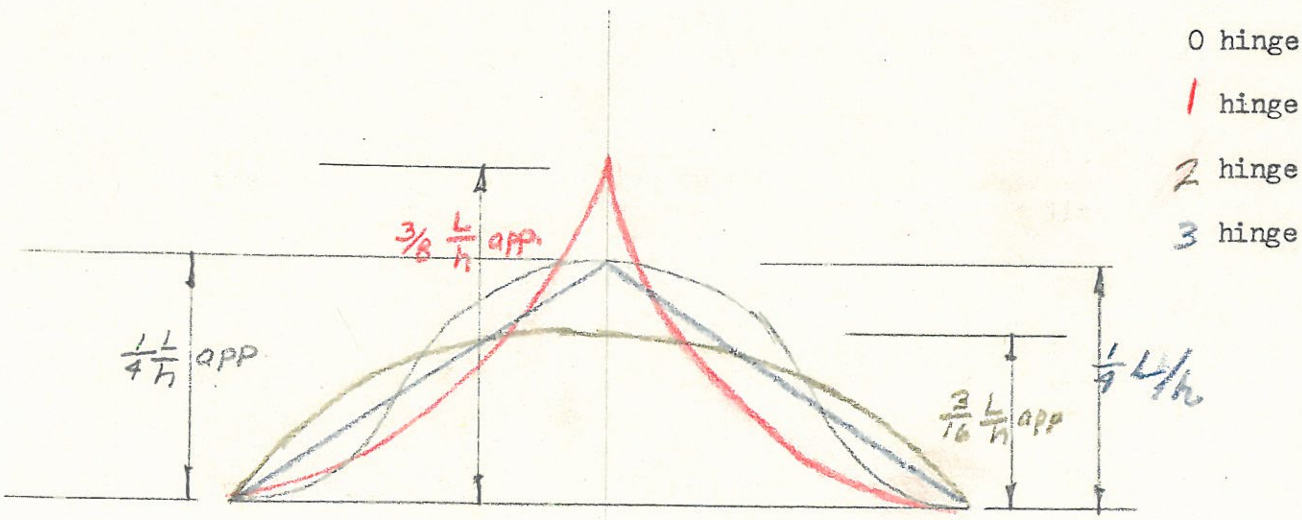


For the three hinged arch the influence line will be zero. If the arch is parabolic the area under the influence line is zero, and if the arch is nearly parabolic the area is nearly zero. Therefore, if the arch is parabolic and uniformly loaded there will be no moment on the arch. An arch will not be very good if the live load is much more predominate as compared with the dead load. It can be seen by these sketches that the maximum positive moment can be obtained by loading the middle quarter of the arch, and maximum negative by loading the end three-eighths.

The following diagram is the influence line for the moments at the springing of a hingeless arch.



The following diagram compares the influence lines for thrust of the different type arches.



The scale shown is based on assumption of the shape of the influence line. There is no bending moment at any section of a parabolic arch if the arch supports a uniformly distributed load over the full span length. The horizontal thrust for this condition of full uniform load is:

$$H = w\,L^2/8h$$

and the area under the influence line is, therefore, equal to:

$$H/w = L^2/8h$$

Thus, if we assume that the influence line for thrust in a two hinged arch is parabolic, the area under the influence line is

$$\frac{2}{3} i_c L = \frac{L^2}{8h}$$

and the middle ordinate of the influence line

$$i_c = \frac{3}{16} \frac{L}{h}$$

Concrete Arches

It is important to be able to distinguish between a short, medium, and long arch. Generally a short arch is one of less than 75 ft. span while a long one has a span greater than 250 feet. The Europeans build long arches. Most arches are short and of single span.

```
                         (Self-imposed
            (Dead Load-  (Super-imposed
            (Live Load
Stresses    (Parasitic
            (Locked
            (Additional
```

The arch is a desirable structure where it has unyielding abutments and carries a relatively small live load. If the abutments yield, it is usually not economical. If the live load is relatively great compared with dead load it is likely to prove uneconomical and probably also subject to serious vibrations.

The amount of concrete in the arch rib is a small factor as compared with the concrete in the deck. As far as relative costs are concerned, the foundations, deck, and ribs share almost equally in the total cost. The cost of foundations and deck are practically independent of the design of the arch rib itself and, subtracting the cost of forms, staging,and equipment which are also largely independent of the design of the arch ribs, only 10 percent to 20 percent of the cost of the structure can possibly be affected by more refined design of the arch ribs. The forces produced by dead load is practically certain. Any economies which may result from great refinements in analysis or design, then, are possible only in connection with stresses produced by rib-shortening, temperature changes, shrinkage and with those produced by flexure due to live load. Only about 5 to 10 percent of the total cost is thus under control.

Temperature change and shrinkage are primarily parasitic stresses. Since the dead load stresses make up about half of the total, the remainder is divided between the live load and parastic stresses. There has been an outbreak of enthusiasm of locking in stresses in the arch. This is done by putting an initial stress in at the crown. This putting in of a thrust serves two purposes. It raises the arch off the forms so that they can be stripped and it places in the structure a beneficial stress.

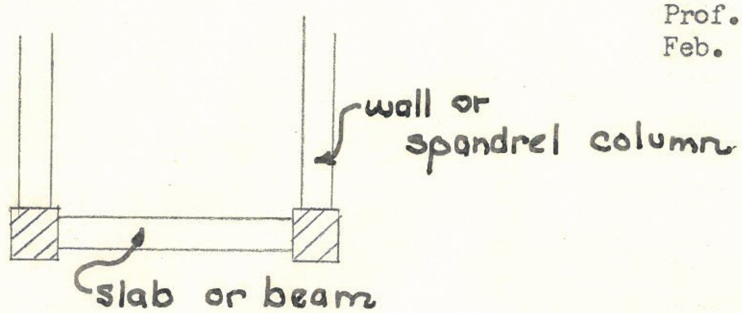
An arch should primarily support dead load. The Roman arches were practically all dead load. However, the tendency in modern construction is to extend the arch concept to where live load is of greater importance. As regards proportioning, the arch should be made of a shape so that there is no bending due to dead load. In order for there to be as little bending as possible, the spandrels of an open spandrel arch should be very close together so that the arch will fit the string polygon.

If there is an arch failure it is usually a foundation failure. It is well to remember that an arch can break in three places and still be good. It would then be a three hinged arch. However, if an arch cracks it usually results in poor appearance.

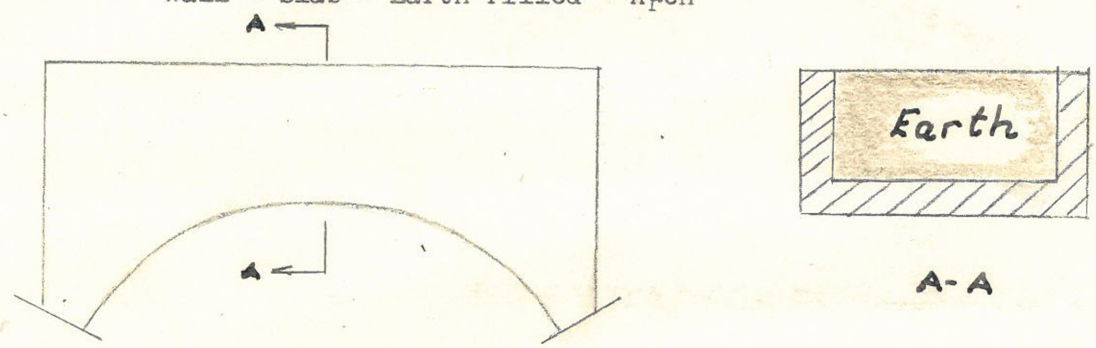
There is much elaboration in the texts of analysis. It is quite proper to know how to put figures on it.

A ribbed arch may have wall or spandrel columns, may be joined by a slab or beam, and if of wall and slab construction may be earth filled.

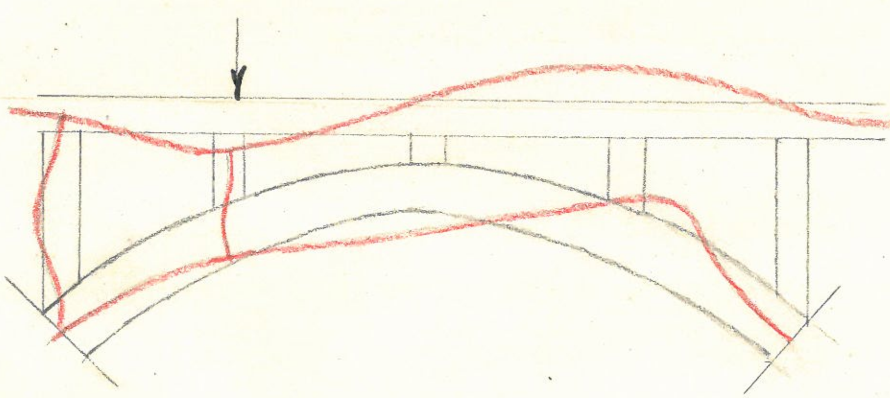
L-30 (p. 3)
Prof. Cross
Feb. 17, 1953



Wall - Slab - Earth Filled Arch

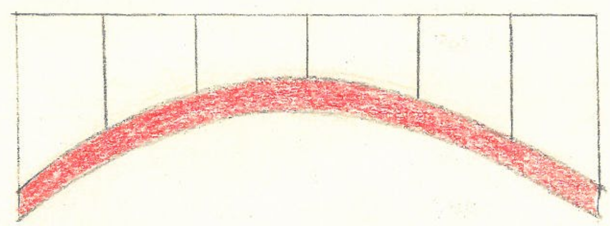


There will be some interaction between the floor girder, columns and ribs as illustrated below:

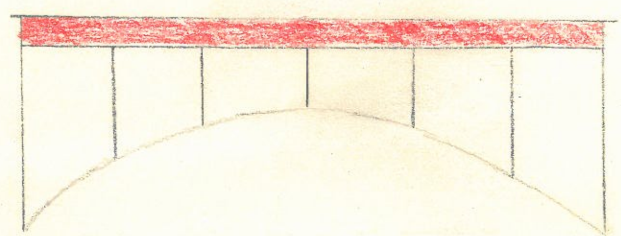


The arch must take the thrust but either the arch or the girder can takethe moment. This can be accomplished with steel construction.

Arch takes moment

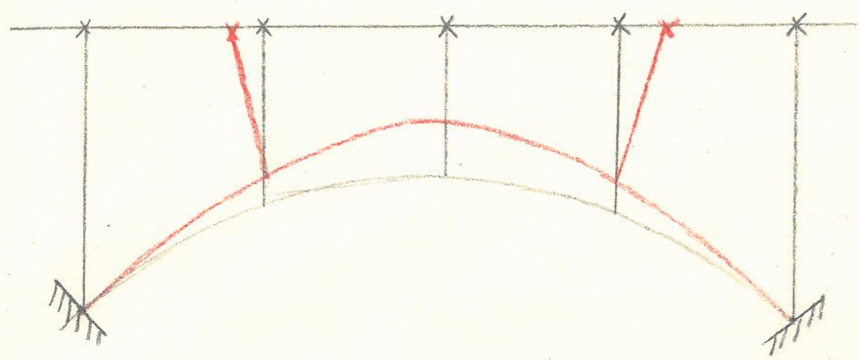


Deck takes moment



In concrete the girders, columns, and arch are all stiff. In short short span arches the interaction of the arch with the column and girder need not be used. However, it may be needed with long arches. There is no interaction with dead load but there is interaction with live load. Theoretically, it could damage the deck and the columns. Practically, however, if advantage cannot be made of it, and it does no damage, then, forget about interaction.

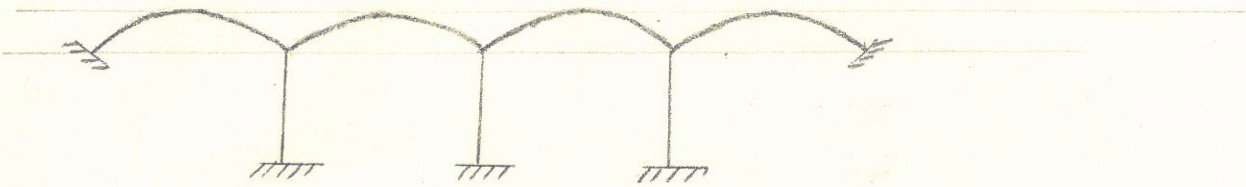
Of the parasitic stresses, temperature can be most serious. The following diagram shows the effect of an expansion due to temperature change on an open spandrel arch.



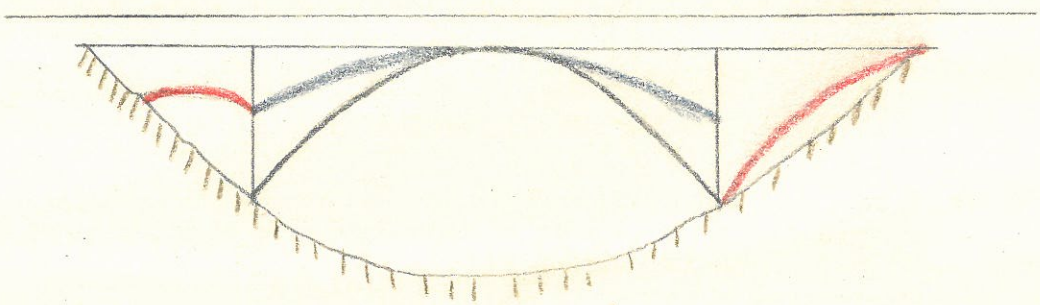
As can be seen, the columns tend to move. In order not to restrain the movement expansion joints can be put in the deck. Closed spandrels are likely to get cracks because of expansion. Expansion joints are ineffective in separating the action of rib and deck. They are probably not needed except at the end of the span. Further, there are objections to their use on the grounds that joints are usually points of disintegration due to weathering, that they produce a slightly objectionable discontinuity in the roadway, and that they produce local movements in the handrail or parapet, which often result in unsightly spalling. The prevailing tendency in practice is against the use of expansion joints except at the ends of the span.

Purposely, exact analysis will be dodged in order to show that there are other more pertinent problems.

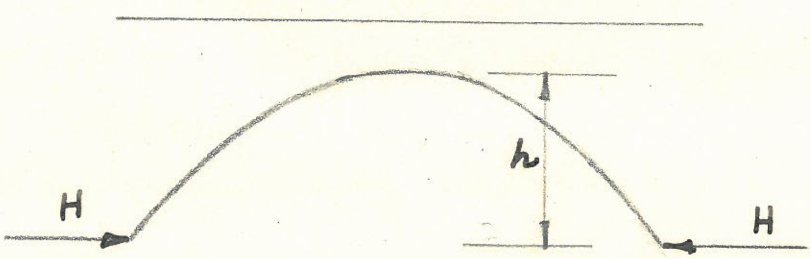
On a continuous arch as shown below the dead load thrust and the temperature effect will be balance. However, live load will not balance and there will be a rotation and spread on the pier tops. Therefore, live loads should not be so big that they have much effect.



The problem now being encountered is that of the continuous arch on elastic piers. The following diagram shows some of the possible arches for a particular situation.



If the shape of the arch axis fits the dead load pressure line then the moment is $\frac{1}{8} w l^2$



and

$$Hh = \frac{1}{8} w l^2$$

$$H = \frac{1}{8} w \frac{l^2}{h} \text{ or } \frac{1}{8} W \frac{l}{h}$$

Therefore, in order for the horizontal thrust to balance at the piers when the span length of the continuous arches are not the same, the rise of the smaller arch must be much shallower.

The ASCE appointed a committee to look into the matter of continuous arches. Reported in the Transactions and in a University of Illinois Bulletin by Wilson, it is worth looking at since all sources of evidence were brought into it. One member of the committee made models while another made analysis by means of the <u>ellipse of elasticity</u>, a cute graphical trick of a nature similar to the dyadic circle. Wilson, himself, did large scale experimentation, building continuous arches 17 feet high and 40 feet long in the laboratory on elastic piers. This report has a large amount of very interesting material. However, although the novice maintains the attitude that if there is a great mass of literature at a certain time about a subject the subject is very important, actually the people who are so concerned do not know what they are talking about. When they do know what it is all about, then nothing is written about it.

Consider now the stress at the crown of the arch rib.

1. <u>Self-imposed dead load stress</u>.

This of course is the dead load stress due to the rib itself. If the arch axis is made the proper shape for dead load the thrust in the rib due to the weight of the rib itself will vary with the area of the rib and the stress in the rib will vary inversely as the area so that the stress in the rib due to the weight of the rib is independent of the area of the rib.

2. Super-imposed dead load stress.

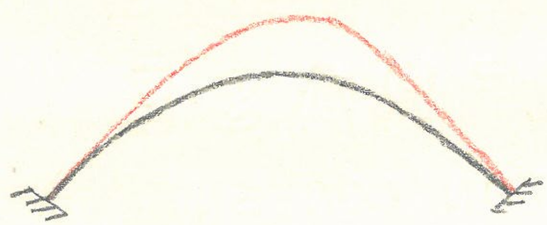
This stress is due to the uniform load of the deck. The stress in the rib caused by the deck will vary inversely as the area of the rib, assuming the arch axis fits the dead load string polygon.

3. Live load stress.

The live load stress is mostly due to a moment. The moment is going to vary as a coefficient times wl^2. This coefficient is very much smaller than that of a fixed end beam. The live load stress will vary inversely as the area of the rib in the same manner as dead load and also it will vary inversely as the square of the depth of the beam or rib.

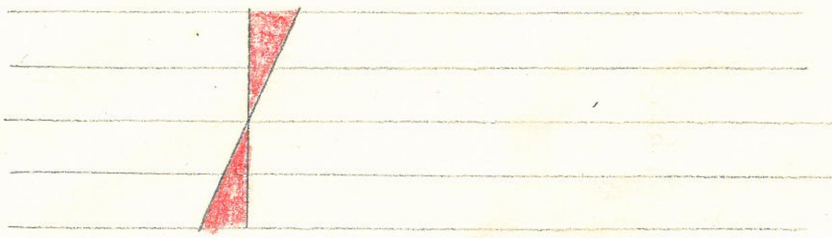
4. Parasitic stresses.

Temperature rise causes the most serious parasitic stress.



We are talking here of geometry. There has got to be a certain amount of movement produced by strains in the upper and lower fiber. It can be anticipated then that the temperature stress is going to vary by some geometric relationship. The easier the rib is made to bend, the less strain will be needed to bend it. The stiffer the rib is, the harder it is to bend. The bigger the thrust, the higher the moment.

Strain $\propto$ Stress



The above diagram shows that a decrease in depth results in a smaller stress. Common sense will show that an arch with a large rise will bend easier than an arch with a shallow rise. So, to reduce parasitic stresses increase the rise and decrease the depth. A three hinged arch will be perfectly free of parasitic stress.

5. <u>Additional stress</u>.

This is a result of the arch changing shape. If the change in shape is insignificant it is not important. With a concrete arch changes in shape are relatively small but in the steel arch it is something to be concerned with. The steel arch buckles. This causes the arch to get out of line and move away from the pressure line. Therefore, the moment increases. Since the moment has increased the arch buckles some more and moves farther away and so on.

Statements have been made about the moment areas balancing. This is not exactly right. The many books go into elaboration on rib shortening, i.e., the change in length of the rib due to the thrust. In a shallow arch the rib shortening becomes more important than the bending.

This leads to the argument, when is an arch not an arch. And to the often meaningless terms, arch action and catenary action. The effect of rib shortening is the same as a <u>drop</u> in temperature. It usually is not figured. Instead 15 or 20 percent is added to the temperature change.

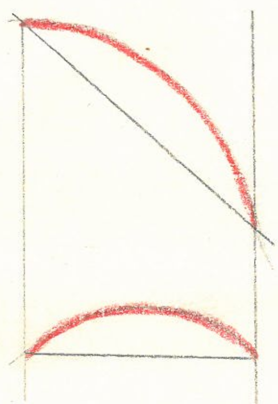
In summary,

Stresses	Dead Load	Self-imposed, independent of size of rib.
		Superimposed, varies as $1/A$
	Live Load, varies as $\frac{1}{Ad^2}$	
	Parasitic, varies as d/h	

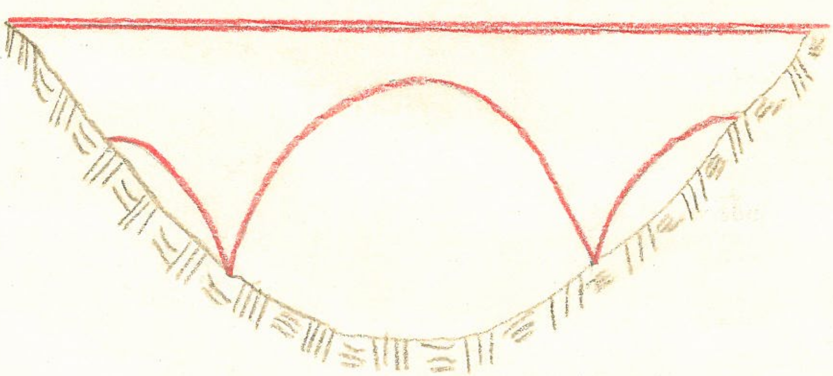
How much change in temperature is there in a concrete arch? It may be from 110 degrees F. above zero to 40 degrees F. below zero. However, the change in temperature takes place very, very slowly. Then maybe there is a time yield of the concrete. It is only "maybe" because it is not known just how much the concrete is able to adjust itself. An allowance of one half the temperature change is enough since the rib can adjust itself because of the time yield. For a time, quick setting cement was in vogue but since it tried to shrink quickly it cracked a great deal. The questions, therefore, still remain. How quickly does the rib respond to temperature change? Is temperature stresses as important as the other stresses?

Interest in interaction was brought about because of temperature rather than live load stresses. It has given some trouble. Some people do not want expansion joints while others try to have too many.

The horizontal thrust of the following two arches are the same:



This makes analysis of a continuous arch as illustrated below somewhat more direct.



Remember that the difficult structure to analyze is the little one. As the structure becomes larger, everything becomes more definite.

"The Effect of Climatic Changes upon a Multiple-Span Reinforced Concrete Arch Bridge"

by

Wilbur M. Wilson

University of Illinois Bulletin, No. 174, 1928

Bridge Tested: Six-span (145 to195 ft.), two -rib highway bridge, Danville Illinois. The bridge is of the open spandrel type except that, for the middle three panels of each span the rib is connected to the deck by means of a spandrel wall. Expansion joints in all but the end spans were adjacent to the piers and at the extremities of the saddle.

Conclusions:

1. Observed range in mean temperature of the arch rib was 83 deg. F., the maximum range about 90 deg. F.

2. The most rapid change in temperature of the rib was a drop of 26 deg. in three days followed by a rise of 35 deg. in the succeeding nine days.

3. The maximum range in the vertical movement was 0.62 inches.

4. The thermal coefficient of the concrete was 0.0000049 for the rib and 0.0000056 for the deck.

5. The change in the temperature caused the piers to rotate but the movement was not sufficient in magnitude to produce appreciable stresses.

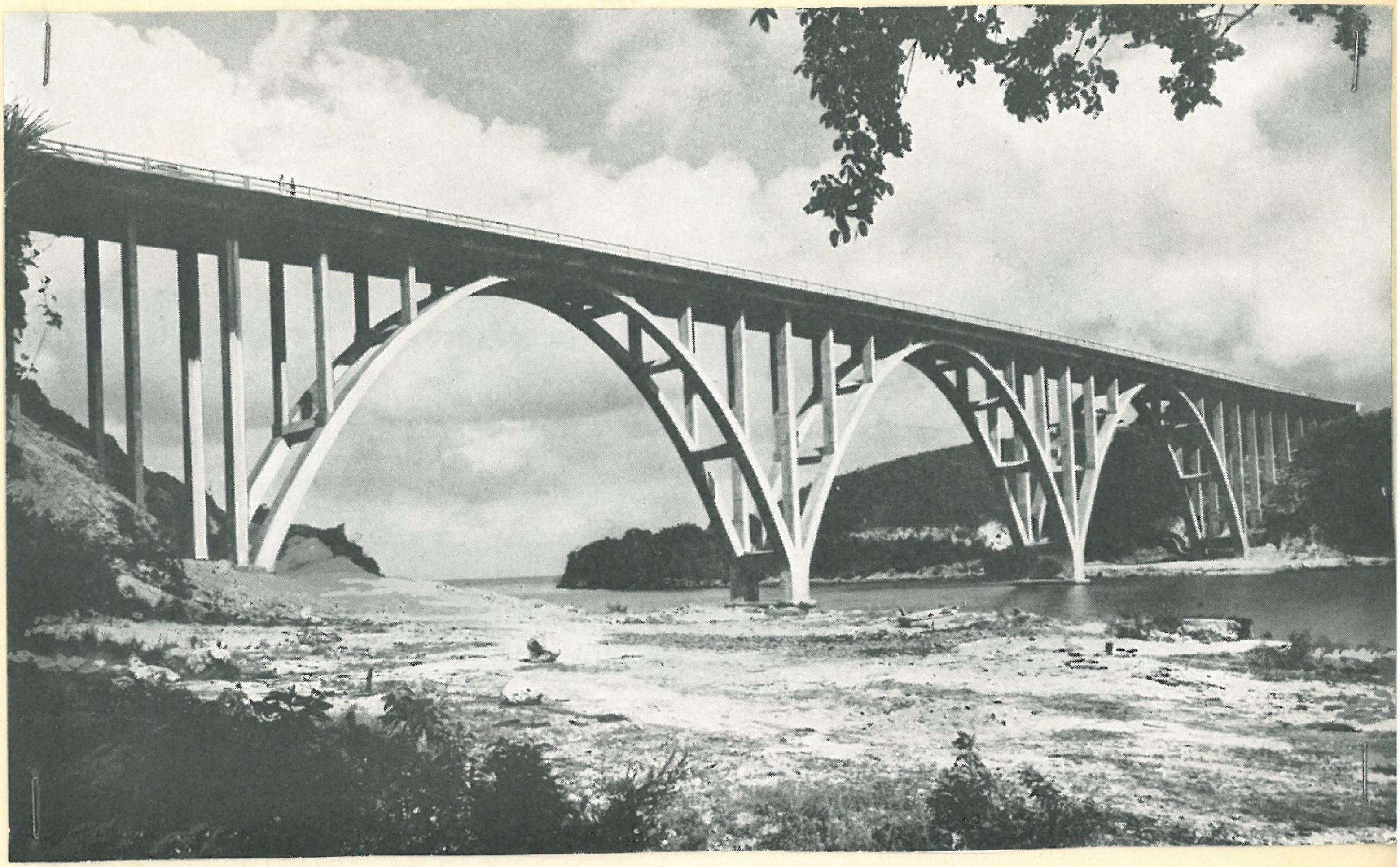
6. There was practically no movement at the expansion

joints adjacent to the saddles.

7. The rib, saddle, and deck functioned as one solid piece. The deck and rib outside of the expansion joints adjacent to the saddle did not function as one piece, but, instead, there was appreciable relative horizontal motion between the two.

8. The expansion joints adjacent to the saddle are not needed to relieve the temperature stress in the deck and they probably do more harm than good.

CANIMAR RIVER BRIDGE



Spanning the Canimar River in Cuba, this structure was designed under the direction of Ing. Jose Menendez of Cuban Comision de Fomento Nacional and built by Raymond Concrete Pile Co. This three span bridge on the Havana-Varadero Highway is 115 feet high and has an overall length of 973 feet.

Robert Heller

Note: This abstract was submitted and a talk based on this article presented by me at WPI in 1952.

ARCH BRIDGE CONSTRUCTION BEATS RAINY SEASON

Abstracted from Engineering News Record

January 31, 1952

A Cuban paradise can be easily reached this season since the Canimar River has been bridged. Cuba's Varadaro area, probably the finest winter resort in this hemisphere, is the terminus for the new coastal highway extending eastward from Havana; and the proposed Canimar River Bridge, designed by the Cuban government, was the missing link. The Raymond Concrete Pile Co., mostly a pile contracting firm in the United States, but proved capable of almost any kind of work outside the country, completed the bridge in nine months.

Everything was ideal for a general contractor. There were good foundations and approaches already in place, and perfect weather for the work - for nine months of the year, between rainy and hurricane seasons.

Nevertheless, the bridge, a three-span parabolic arch with an overall length of 973 feet, and deck-to-water height of 115 feet, was quickly built as a result of full planning, precision built falsework, and production line techniques. Although the river piers supported by deep friction caissons were built in 1948, it was not until December 31, 1950 that the first falsework supporting pile was driven. Seven months later the bridge was open to traffic, and in September the falsework removal was completed.

In planning the construction, three support systems were investigated. The first, steel arch-truss centering was ~~impractical~~ impracticable, since steel could not be delivered promptly. The second, timber arch-truss centering is not economical unless used several times. Since it is imperative to keep balanced horizontal loads on the river piers, it would be necessary to place at least one line of three arch ribs simultaneously. Hence, only one or two uses of arch centering ~~is~~ would be possible. However, the third system, a pile-supported timber falsework, which could be designed quickly, and erected with the same light equipment required for the rest of the work, proved very satisfactory. Furthermore, the timber had high salvage value in Cuba.

The first step in construction was to erect a 1200-foot-span cableway, with a 2" main cable, over the centerline of each row of arch ribs on stationary A-frame towers, 100 feet high, while a jobmade floating piledriver set 206 80-foot piles in 14 feet of water to support the falsework. The bents so formed, capped with 12x12" timber, were located 14 feet apart at panel points of the falsework.

The falsework was placed in four lifts. In the first lift, 26 feet high, each falsework bay weighed 9 tons. The second lift was 32 feet high and the third 18 feet high. This lift had intermediate support points for screw jacks to support cross trusses of the fourth lift, which cantilevered 10 feet ~~both~~ each ~~ways~~ way to support overhang of roadway and sidewalks.

Throughout, falsework consisted of 8x8" columns, 3x8" horizontals, and 3x6" diagonal braces, spliced and bolted with steel ring connectors, totaling 800 Mbfm shortleaf southern pine and imposing an accumulated comparatively light weight of twenty tons per pile.

The falsework was designed to be stable under ordinary wind conditions and to support either arch ribs and spandrel columns, which would be self supporting with the first decentering, or the deck - not both.

The concrete was carried up at the same time in each arch rib in order to balance the longitudinal forces on the falsework and the thrust on the piers. This was completed in less than two months with a peak crew of 237 men. The spandrel columns were then placed, also with consideration to proper balance.

Before tackling the deck the arches and columns were decentered. In each half of an arch there were eighteen jacking levels, and five jacks at each level, for a total of 540 jacks. Five men went downward each way from each of the three crowns at given signals, lowering simultaneously their jacks a half turn. With three passes all the jacks were free. A 2-foot gap was left in the concrete at the crown and four jacks were inserted. By placing 40-ton thrusts at the crown two days before decentering, effects of shrinkage and elastic flow were overcome. The effects of decentering observed were negligible.

Because of the limits imposed by the design of the falsework, the construction of the deck also required balanced concrete pours. Upon completion of the deck, recorded thrust in the crown jacks was 240 tons, showing that much of the load in the deck was already in the arch ribs. After the deck was decentered the crown jacks were pumped up to 325 tons. This load was transferred to short thrust beams and the jacks were removed. Filler pieces were butt welded across the gap, and the gap closed with high early strength concrete just three days before traffic was allowed on the bridge. Handrail construction and bituminous concrete roadway surfacing completed the job well before the first expected storm.

The bridge materials were fed from an efficient contractors' yard. An inventory was kept at all times of 200 to 300 Mfbm of timber. Everypiece was marked with a code number, as were completed panels and forms, so that quick erection could be accomplished. A production line system of cutting, boring, and grooving, working two shifts daily, was necessary to supply a one-shift erection crew. To avoid mistakes, tolerances on wood were the same as steel. Panels were fabricated in the yard and transported by small railroad cars 200 feet to the cables, and there sent by cable together with the bracing to the erection crew on the site. Special framing, such as the 40x40-foot opening left for navigation through the center span, required one special layout and boring crew.

The arch pattern was laid out full size on a coordinate system. Points of the parabolic curve were computed and established on the master layout frame, as were points for traverse beams, columns, and column pedestals. Carpenters made master templets matching the master layout.

The reinforcing steel was stacked upon arrival according to size with their ends lined up, making it possible to cut all the steel in the storage pile.

The concrete plant, consisting of a 100-ton three-compartment batch bin for weighing batches to a 28S mixer, was located near one approach. Material came from various locations; bagged cement from Havana, coarse aggregates from a limestone quarry 50 miles away, silicious sand from Pinar del Rio province 150 miles away, while fresh water was 1/2 mile away. The concrete was transported in one-yard bottom-dump buckets on shuttle cars, and then by cableway to the point of deposit. A chute was attached to the bucket for deck concreting.

Finally, in September 1951, nine months after beginning construction, Arthur Fertell, project manager, C. A. Sutherland, superintendent, and Duff Williamson, chief engineer deserving the credit, the Canimar River Bridge was completed, a beautiful product of efficient construction.

HISTORY OF THE BRIDGE

The Ancient Period

This period includes all that preceded the Roman Era during which most bridges, in Europe, at least, were of the beam type. Arch Bridges were probably built in China prior to the Roman Era and the arch was used by the ancient Egyptians in other constructions. Timber or stone was generally used, but the inhabitants of the "fertile crescent" who lacked timber or stone used brick and clay.

The Roman Period

The Romans gained supremacy of civilization from the Greeks and constructed their roads and bridges from one end to the other end of the Empire. One of their favorite bridge types was the pile structure and another, the extensive use of which they introduced, was the semicircular masonry arch.

The Middle Ages

This period extended from the eleventh to the sixteenth centuries, and was characterized chiefly by the construction of massive, more or less crudely designed and executed arches of masonry (generally inferior to the Roman structures), but including also arches of bold and slender proportions (the flat segmental arch introduced by the Italians). During this period, practically all culture was centered in the religious orders, and, therefore, most of the bridges were built by monks (the Brothers of the

Bridge). During this period finer work was probably being done by the Chinese.

The Renaissance

Occurring during the sixteenth and seventeenth centuries, this period exhibited much greater refinement of both design and construction. It was in this period that the truss was (in a sense) invented by Palladio, an Italian architect, but it was not in this period that the importance of the truss was realized.

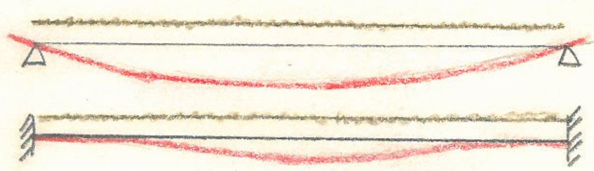
The Eighteenth Century (And the first Quarter of the Nineteenth)

In this period the masonry arch reached its greatest perfection. The truss was used initially as a combination of the truss and arch principles, i.e., trussed arches. The first truss since Palladio was the Town or lattice truss patented in 1820 by Ithiel Town of New Haven.

The Modern Period

This period began with the advent of the railroad about 1830, and is characterized by the utilization of all the basic types. Following the industrial revolution eye bars of wrought iron were produced. There was a transition from the all timber truss to the combination truss of wrought iron and timber (the first Howe and Pratt truss) and to the all wrought iron truss. The bridge builders tended to utilize the tension values of wrought iron as much as possible. Then came steel and the whole picture changed. Later came concrete, an imitation stone, and the trend started back towards compression. Then, still later, came steel reinforcing for concrete to take tension and most recently prestressing which makes the concrete remain in compression. Since timber is becoming scarce and steel is scarce overseas, concrete is a large competitor.

A truss is a girder without a solid web. Material is concentrated in a few members where it is needed. Structural types have certain weak spots. Timber tends to fail in shear for as the member dries it splits along the grain. Concrete has limitations which have been discussed previously, mainly bond since the steel and the concrete do not deform together. The early structures were mainly determinate since the timber was not long enough and stone was used as a simple slab. Continuous structures (or fixed ends) may be used to reduce the maximum moment and to stiffen the structure. If the same material is used in each, i.e., the same beam, there will be less deflection in the fixed end beam than in the simple supported beam.



Now, if the material is reduced in the continuous structure the advantage in stiffness will be lost since

$$\Delta = \frac{\Sigma f \ell^2}{E d}$$

and $\frac{\Sigma f \ell^2}{E}$ will remain the same in both cases.

Bender (the "practical engineer") and Marriman (the "professor") had long arguments, arguing strongly about the economy of the continuous structure--Bender con, Merriman pro. Bender approved of the simplicity of simple discontinuous structures. In 1900 in the Transactions of the ASCE, Frank B. Cilley wrote a paper and said that the indeterminate structure works against itself.

There has been an idea, mainly bunkum, of applying the theory of the one horse shay to statically determinate structures.

> Have you heard of the wonderful one-horse shay
> That was built in such a logical way
> It ran for a hundred years to a day?
> ***
> And then of a sudden ***
> It went to pieces all at once--
> All at once and nothing first,
> Just as bubbles do when they burst.
>
> Oliver Wendell Holmes

However, it cannot be done with a statically determinate structure. Many of the reasons will be discussed. If a rolled steel beam were designed to do anything like that, it would be necessary to shave down the flanges little by little until there would be nothing at the ends. The web must be shaved down until there is nothing at the center. There is an attempt to do something of this sort with cover plates on plate girders. Further, with large plate girders a smaller web can be used at the center. Nevertheless, it would still not come near to the ideal. In a truss the members can be varied to fit the stress but there is a limit on the minimum section that can be used. If a compression member is too thin it will buckle. It should be remembered in this matter of economy of material that if the working stress were doubled in small structures there would be a saving of only 5 percent in material.

In structures, the purpose generally of the material is to carry gravity loads and in tall buildings wind loads. The function of a bridge is to carry you over; of a building to hold you up and keep out the weather.

Selection of material depends on what sections are rolled and what sections are available in the warehouse, and depends on minimum permissible thickness because of corrosion, buckling of compression members, or, in the case of concrete, the minimum thickness, that can be poured.

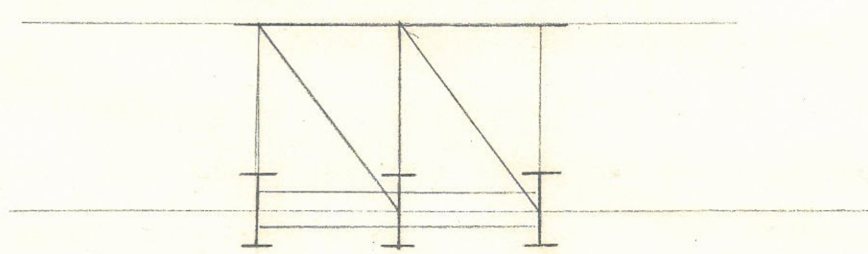
Details such as gusset plates, connecting angles, rivets, and bearing plates add to the gravity load of the structure. Of importance, but varying in importance with the type of material and structure, is bracing. No amount of theory will modify design; it is largely a matter of judgement and experience. A decision to cut bracing in order to save material may be the most dangerous thing to do. Only about 1/2 or less of the material in a structure is there for the primary purpose for which the structure was built while the remainder is for details and bracing.

As has been said, the concept of concentrating material in members is the basis of the truss. There are various types of trusses adopted or modified depending on material to be used or available (wrought iron or steel). There is not much sense to the concrete truss. The truss development was influenced by fabrication or rolling processes, by difficulties arising in construction, and by simplicity of fabrication, punching and cutting. In steel, fabrication is an important factor. Structural types are influenced by several things as pointed out above and will be mentioned later. In the beginning of truss building there was little theory as regards the modern sense but modern theorists sometimes forget these factors.

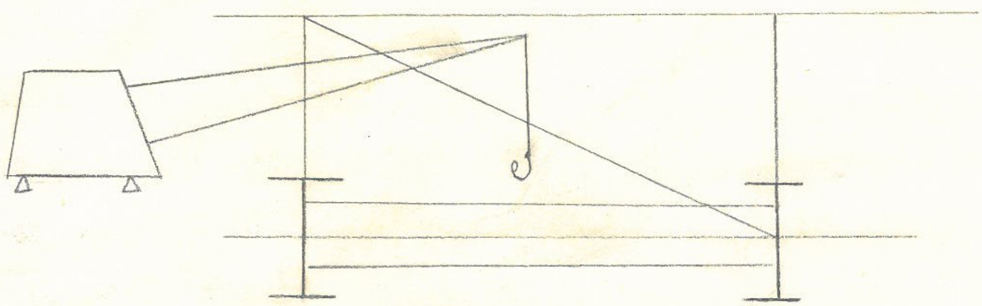
In this country trusses were influenced by the prevalence of wrought iron and frontier conditions. Transportation in the West was very difficult. The old type trusses used timber in compression and eyebars and rolled wrought iron bars in tension.

There are many reasons for the through truss, underclearance or trees and ice floating down the river when the water rises. As to the shape and articulation, it is a matter of fabrication, erection, and economy of material. The length of the bridge depends mainly on location of terminals and level of approaching road.

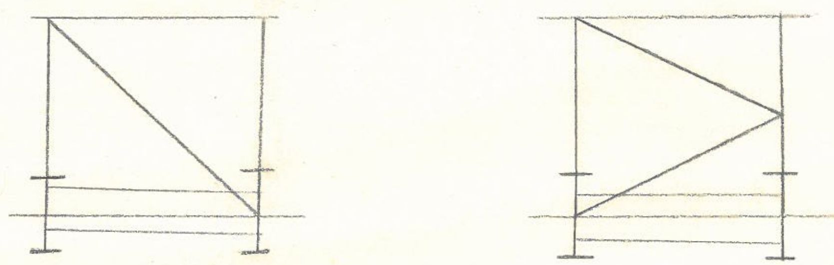
The web of a truss is not solid. The diagonals are designed to take the shear. A slope of 30 to 45 degrees will probably yield the most economical panel length. If the panel length is too short there will be many floor beams and short stringers, a poor layout.



If the panel length is too long there are few floor beams and very long stringers. Further, if the boom were too short it would be impossible to set the stringers.



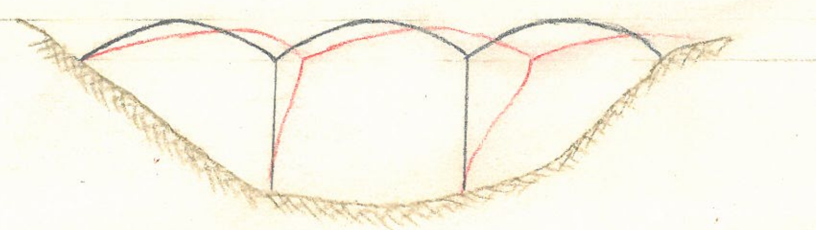
The American Bridge Co. has a boom that can span 100 feet. The following proportions are more satisfactory.



It can be seen now that when selecting the proportions of the truss you become involved with the depth of truss, whether through or deck truss, the shape of truss, and the panel length.

Consider the floor system. Floor beams all cost the same. The cost varies inversely as the panel length. The cost of stringers varies directly as the panel length. It would seem, therefore, that the minimum occurs where the cost of one floor beam equals the cost of one stringer. Actually it makes little difference in the range we are interested in.

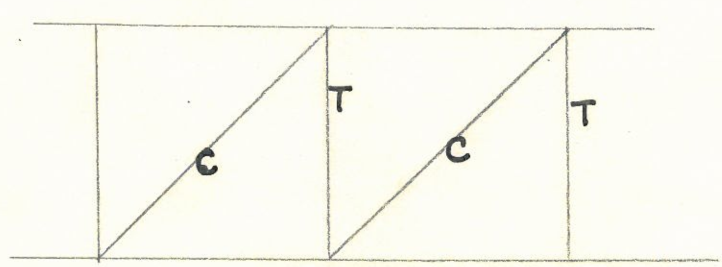
The chords can be made parallel, rectangular, or curved. It is of great importance to compare dead and live load when discussing form and economy. When does the load come on the structure? With a concrete structure, it is not until the structure is completed that the dead load comes on the structure. With a multiple arch structure it is possible to get a pier failure if the forms are not removed (struck) at the same time.



Sometimes the government has much to say as to whether forms which obstruct navigation may be used. Nature too has much to say since spring floods may wash away forms. Steel trusses may have some framework or they may be cantilevered.

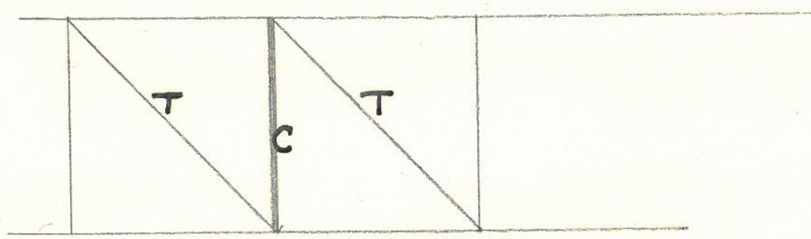
After the depth, form, and panel length of the truss is determined, the system of articulation must be decided upon.

HOWE TRUSS



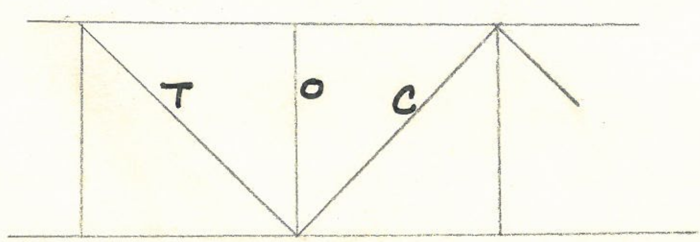
In the Howe Truss there are more compression members than tension members. The railroad builders in the West preferred this system of articulation because the timber takes compression while the eyebars take tension.

PRATT TRUSS



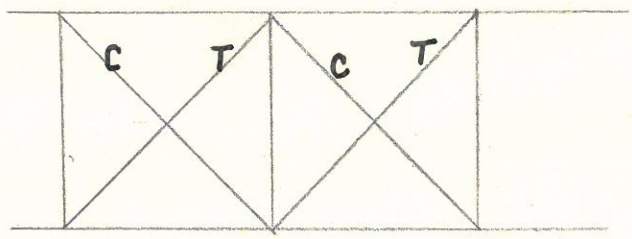
In the Pratt Truss there are more tension members than compression members. An all steel truss favors tension members because of the reduction of area of the tension members or conversely the increase of area in compression members because of buckling.

WARREN TRUSS



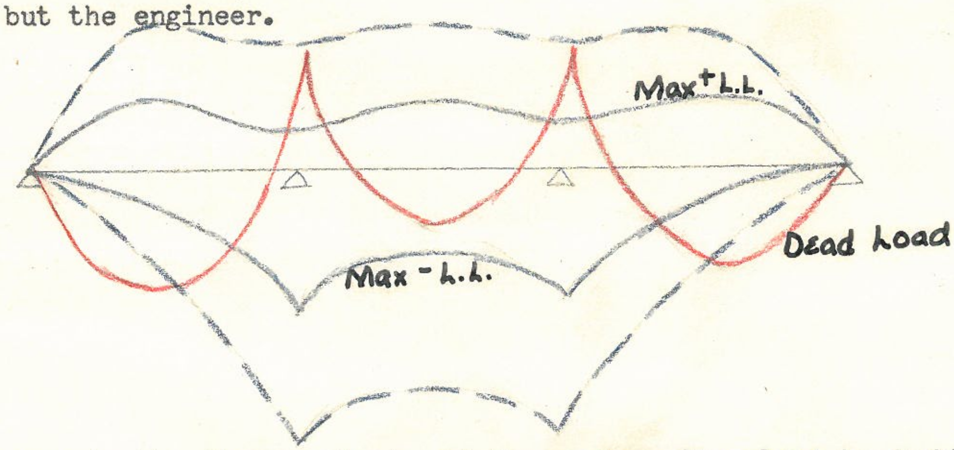
The Warren Truss is a compromise between the two previously mentioned trusses. The vertical member, as shown, takes no stress but it is useful as bracing.

If on a simple truss the live to dead load ratio is large the diagonal members will reverse, i.e., a member in compression will become a tension member. Therefore, every member must take both tension or compression, or counters can be used.



This type of truss is supposed to be determinate.

Next, consider the factor of safety. Everyone knows about factors of safety but the engineer.



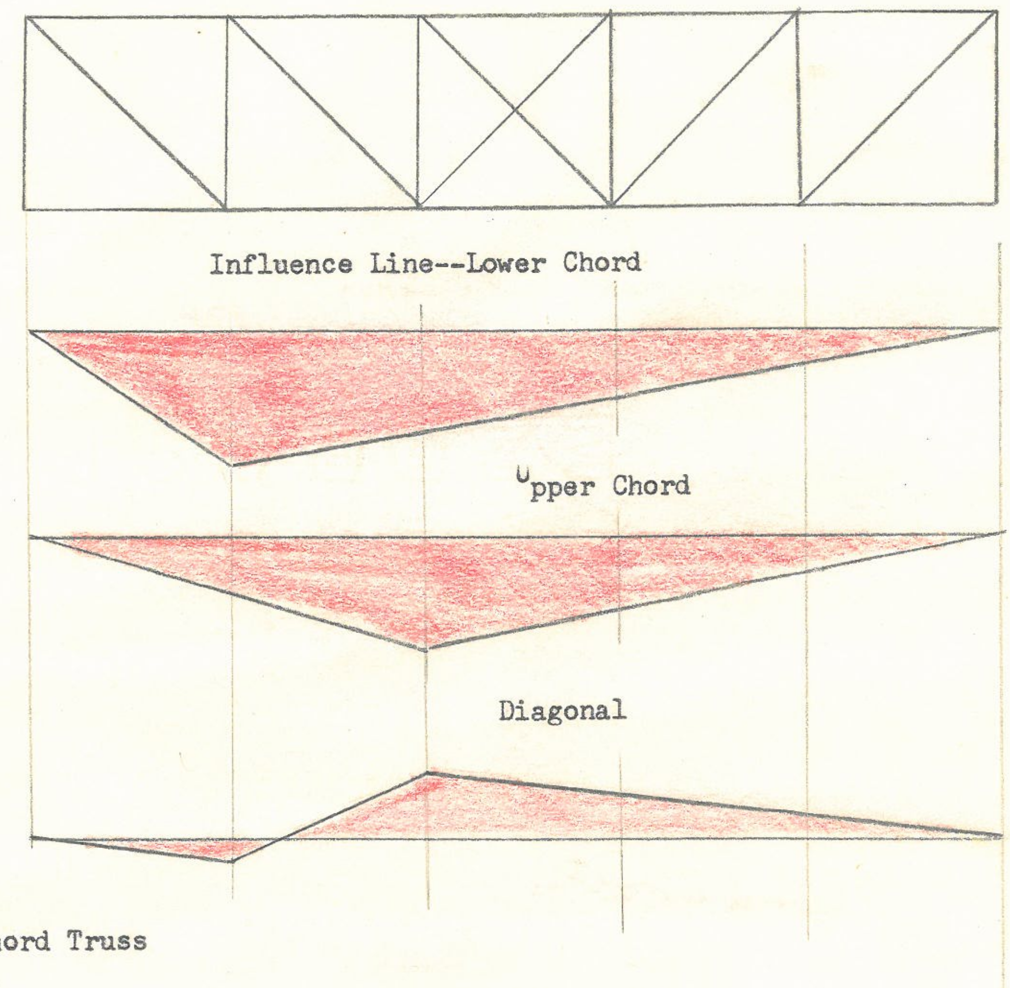
As can be seen in the diagram above, if the maximum live load is doubled, the maximum moment is increased tremendously. In this country we have gone from very light live loads to extremely heavy live loads. With the exception of wartime overloading, the railroad loads because of the diesel have become lighter, whereas the highway loadings have continued to increase. However, before the bridge can give out, the roads leading to the bridge go to pieces. Leon Mossieff thought Russian bridges during World War I would fail because of overloading, but they did not.

Specifications for railroad bridges give allowance for reversal known as Launhardt's Formula. Where reversals occur many times it involves the question of fatigue. Railroad men are not in agreement as to whether it is important or not. As early as 1900 there were extensive symposiums where authorities discussed many such questions among them this matter of fatigue.

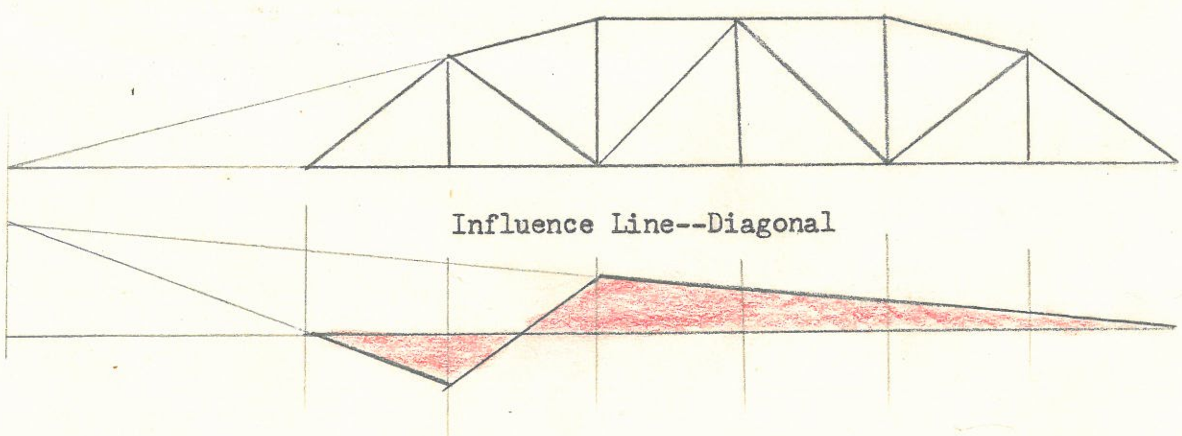
Design as full amount of one kind and one half of the other. If you have stresses of 100T and 100C you must design for 150T and 150C. If you have 100T and 80C you must design for 140T and 130C.

INFLUENCE LINES FOR TRUSSES

Parallel Chord Truss



Curved Chord Truss



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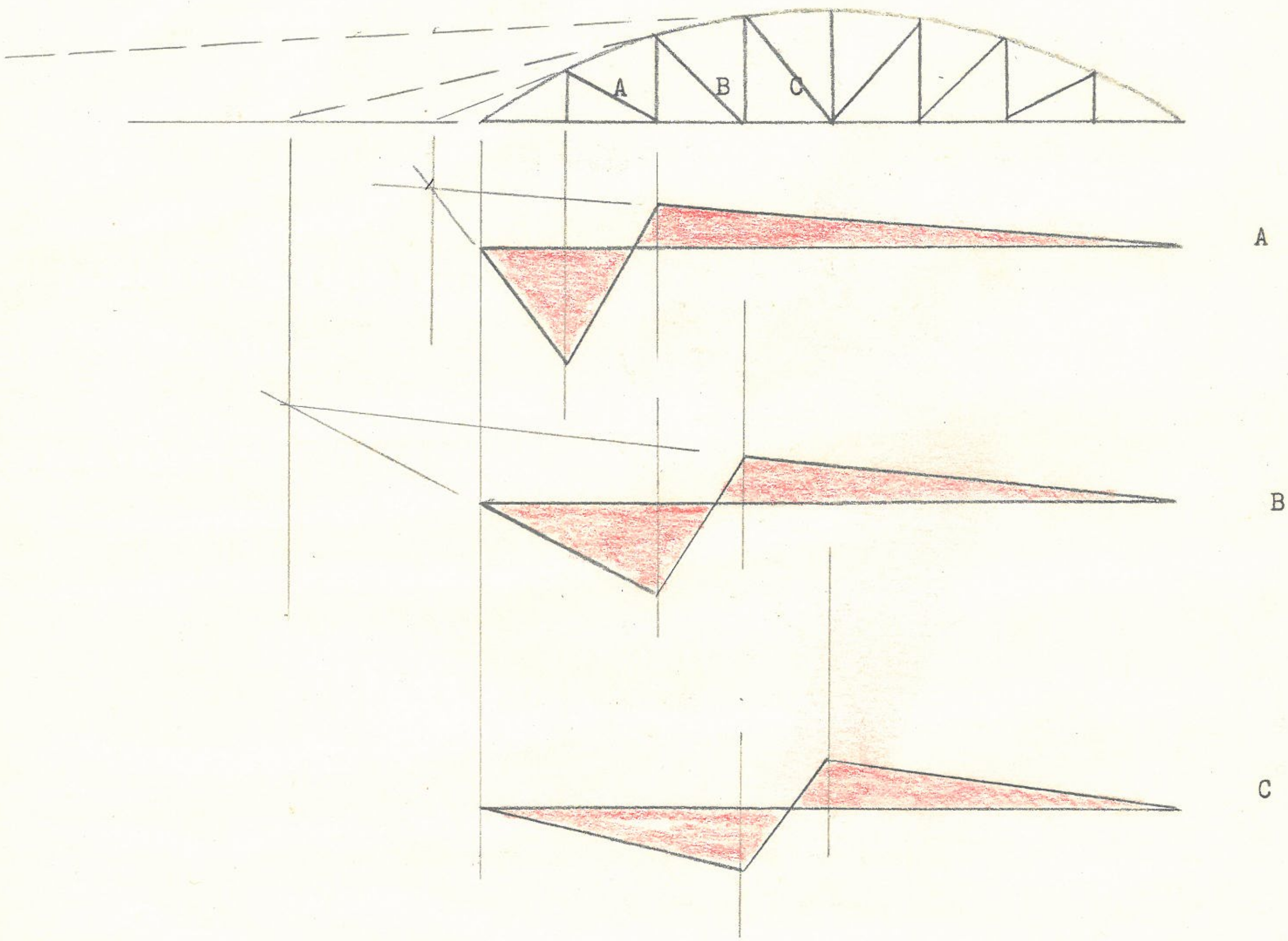
Prof. Cross
Feb. 20, 1953

Parabolic Curved Chord Truss

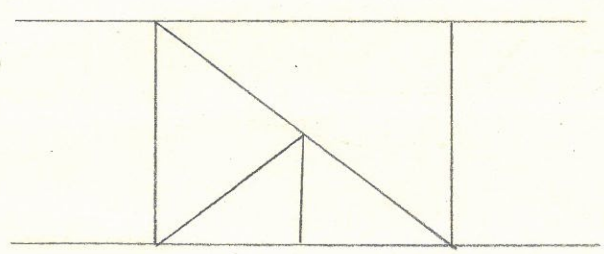


With a parabolic curved chord truss the stress in the diagonals will always equal zero when the truss is loaded uniformly.

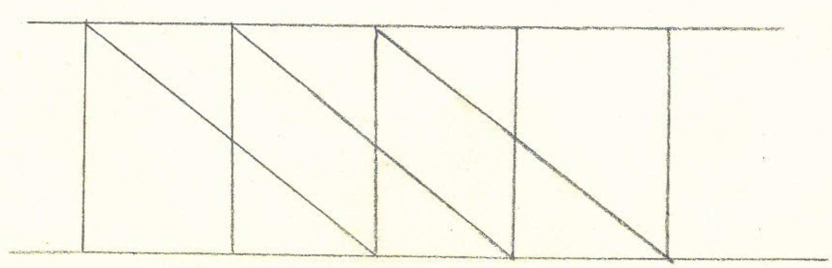
Reversal occurred out towards the center of the span so that was where counters were used. The objection to use of counters is that it is always bad to bring a member suddenly into play. As fabrication procedure improved, it became common to use a member for a double purpose and to design for reversal.

The curved chord truss has an advantage because when under dead load the stress in the diagonal is zero and the verticals serve as hangers. There are, however, disadvantages, too. Each curved chord is different. Further, any argument depends on the ratio of live load to dead load. The tendency is away from the curved chord truss.

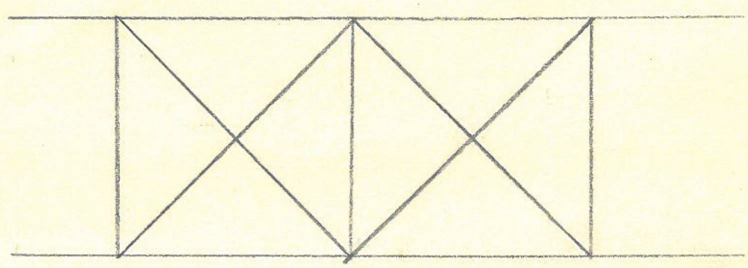
There were many ways of getting around the difficulty of long panel lengths and subsequently long stringers. One way is to sub-divide the truss.



As a result, the stringers are shorter. Another way is the principle of the Whipple Truss.



In truss problems there are two types of indeterminacy, internal and external. The double intersection truss is an example of internal indeterminacy.



Diagonals in trusses are ugly because there is no elementary concept of the thing working the way it does. There has been a strong development to leave out the diagonals, such as the tied arch and the open webbed girder. These are forms of external indeterminacy.

The preceding lecture and this one concerns beams in which the web is articulated, i.e., individual members are put in instead of a solid plate. There is added field work but economy of material.

What is the proper depth of truss? If the truss is made deep there is little chord but much web while if the truss is made shallow there is much chord and little web. In the last lecture, the length of panel and depth of panel were compared. Certain expedients toward correcting the misproportioned truss panel are the Whipple Truss and the subdivided panel. The economy of the truss frame is related to the economy of the deck. How long a panel should be used? There is a tendency to use as long a panel as the stringer can be swung and as long as the longest rolled beam that can be obtained. There has been a great change since the modern heavy rolled beam came into use. The matter can be further complicated by welding.

In general, trusses are statically determinate. Trusses may be made continuous or as an arch. Another aspect of indeterminacy found in literature is called "secondary stresses" and the interaction between or participation of members. Both can be called deformation stresses. If a member is connected to another member the second member will move because the first member moves. This is a deformation stress. The members are connected merely for convenience.

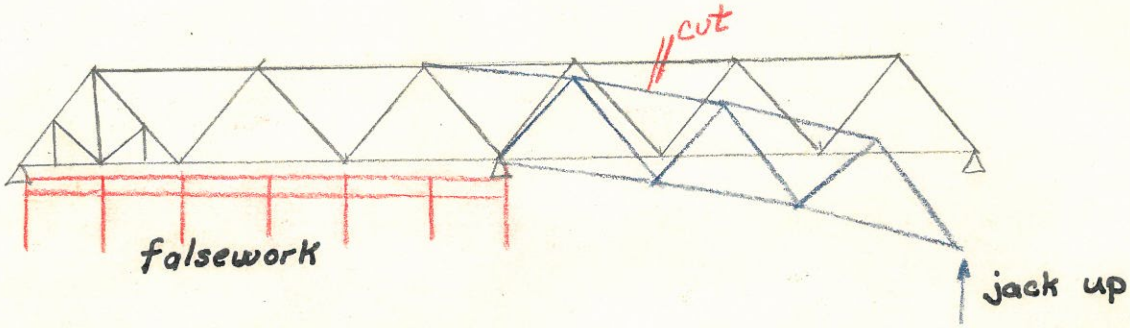
The main problems of continuity in continuous trusses does not differ much except in one important detail. One of the vital questions is the relative value of the weights, self-imposed dead load, superimposed dead load and superimposed live load.

Bender and Merriman argued for many years whether there was or was not an advantage in indeterminacy or determinacy. Cilly stated that indeterminate structures were uneconomical because they worked against themselves. Still later there was a debate between Waddell who favored the statically determinate structure and Lindenthal who supported continuous trusses. Waddell criticized Lindenthal's design of the Sciotoville Bridge. They were talking about big trusses. This bridge was a continuous two span bridge. Waddell later said that there was 15 percent economy in material between simple and continuous structures. However, he should have made a comparison between the cantilever and the continuous because they were talking about a long span.

The novice thinks someone has settled this argument by comparing economy of material. No one can. Suppose one man made both designs. If he had experience in continuous structures but not in cantilever, he naturally will design the continuous structure better. So, comparison should not be made like this.

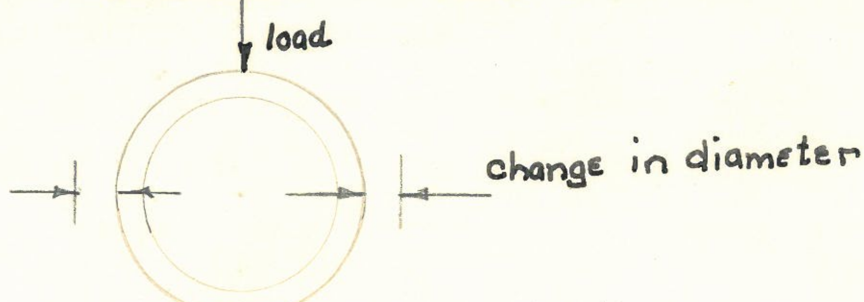
Another important aspect is that of erection. Today, plate girders are built up to 300 ft. In this way a large amount of field work is cut out.

In a two span continuous bridge such as that at Sciotoville, it was necessary to start off with a good deal of falsework. After the first span was built of falsework, the second span was cantilevered out and allowed to deflect. Finally, the end was jacked into place.



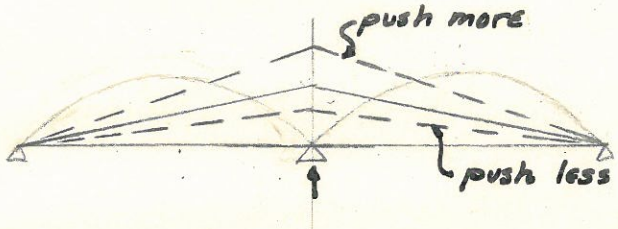
It would be necessary to cut as shown above to make this bridge a cantilever instead of a continuous structure. This would involve an additional field operation. The point is, what is the use in doing anything when you have got a continuous structure already. There is an obvious objection to a two span cantilever in that it would be unsymmetrical. Waddell's objection to the continuous structure was the fear of differential settlement. Figures will be put on this later.

By jacking up the structure as was mentioned above, thestresses will be known since the reactions can be weighed when the end is raised. Weighing can be accomplished by means of a pressure capsule or by a proving ring. The proving ring is very satisfactory. The ring is calibrated so that a change in diameter is a measure of the load.

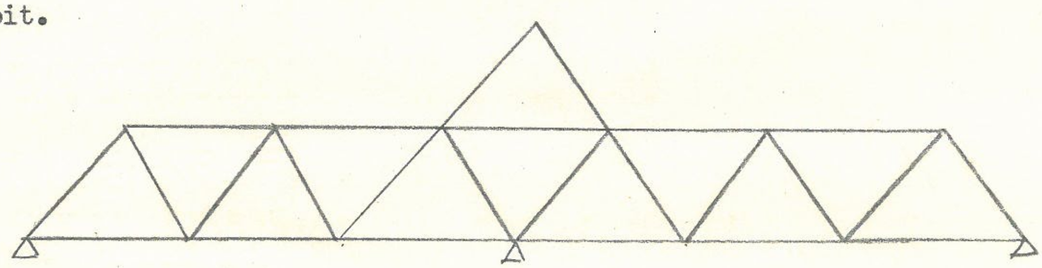


It is a different matter with concrete because there is no dead load at first and so the dead load cannot be bossed around.

By pushing on the center reaction, the dead load moments can be made anything at all. With any additional live load and dead load it will act like any continuous structure.



A truss can be treated for shears and moments just like any beam or girder although in the following case it may be necessary to modify it a bit.



Therefore, analyze a truss as though it were a beam of constant cross-section. Except on certain railroad bridges, no one nowadays bothers to design a truss. After the truss members are layed out, there are not going to be any appreciable changes. The maximum dead load moments and shears in beams of constant cross-section are known. The live load can be obtained from influence lines.

Influence lines for continuous trusses can be drawn in the same manner as influence lines for continuous beams.

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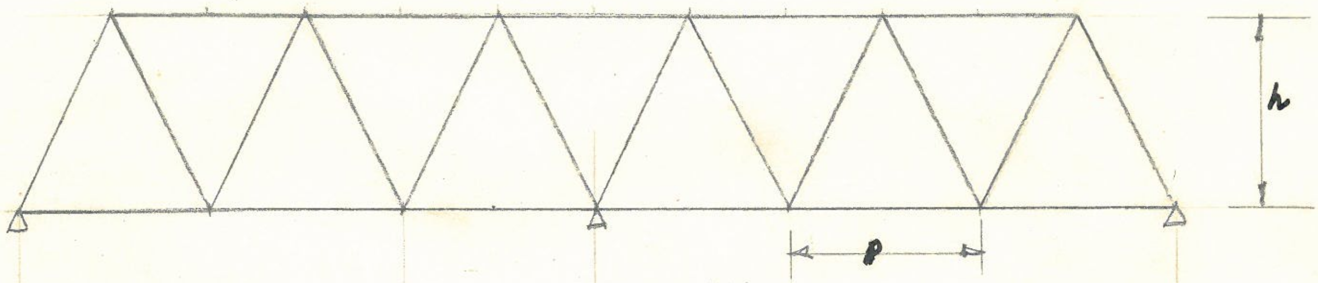
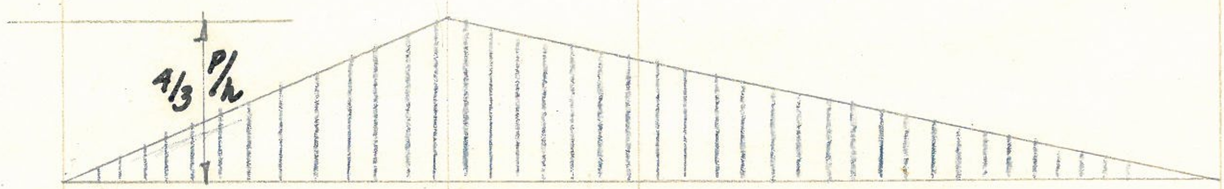
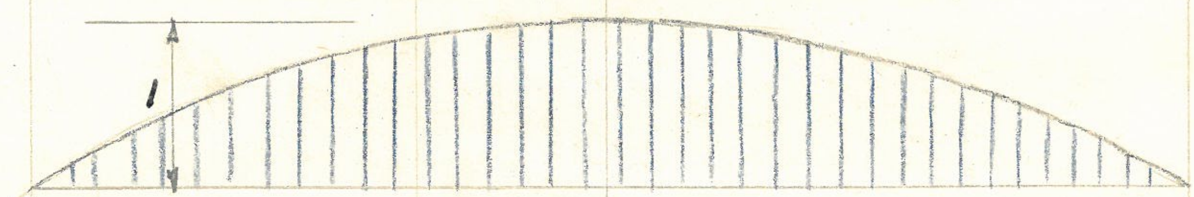
Prof. Cross
Feb. 24, 1953

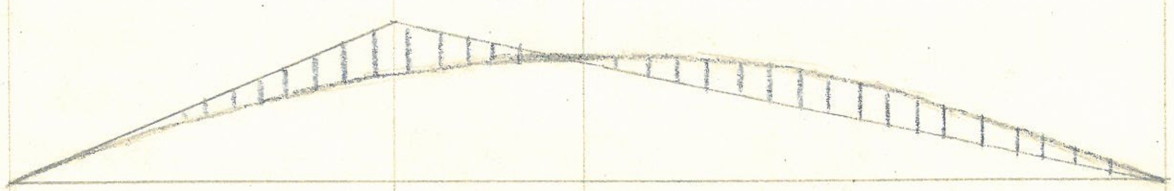




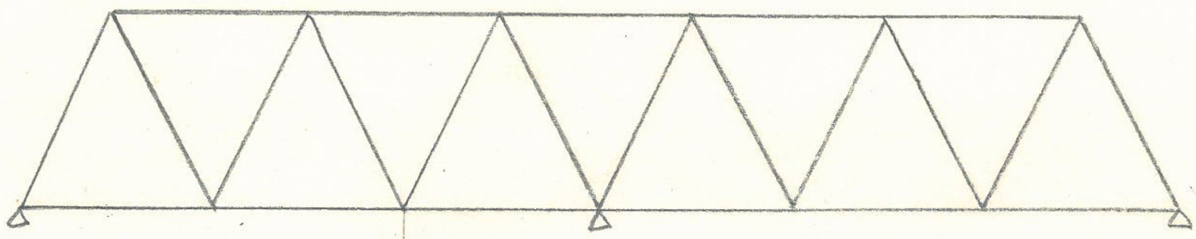
Influence line for simple truss



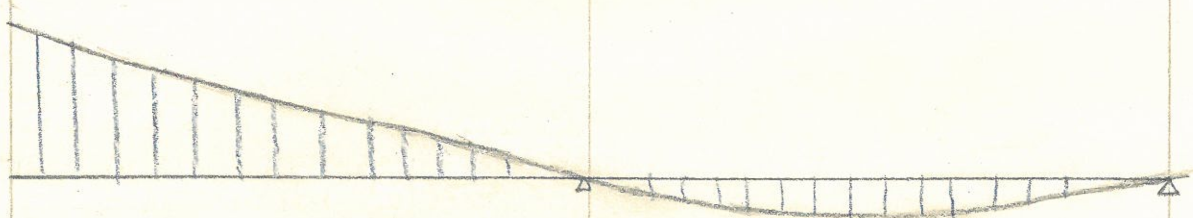
Influence line for center reaction



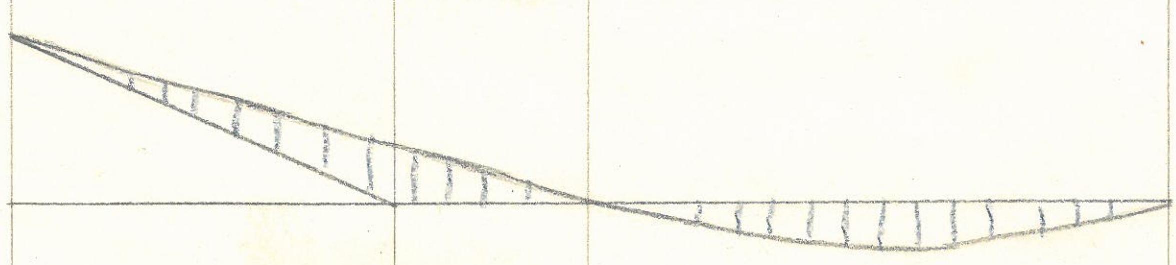
Combined influence line



Influence line for cantilever



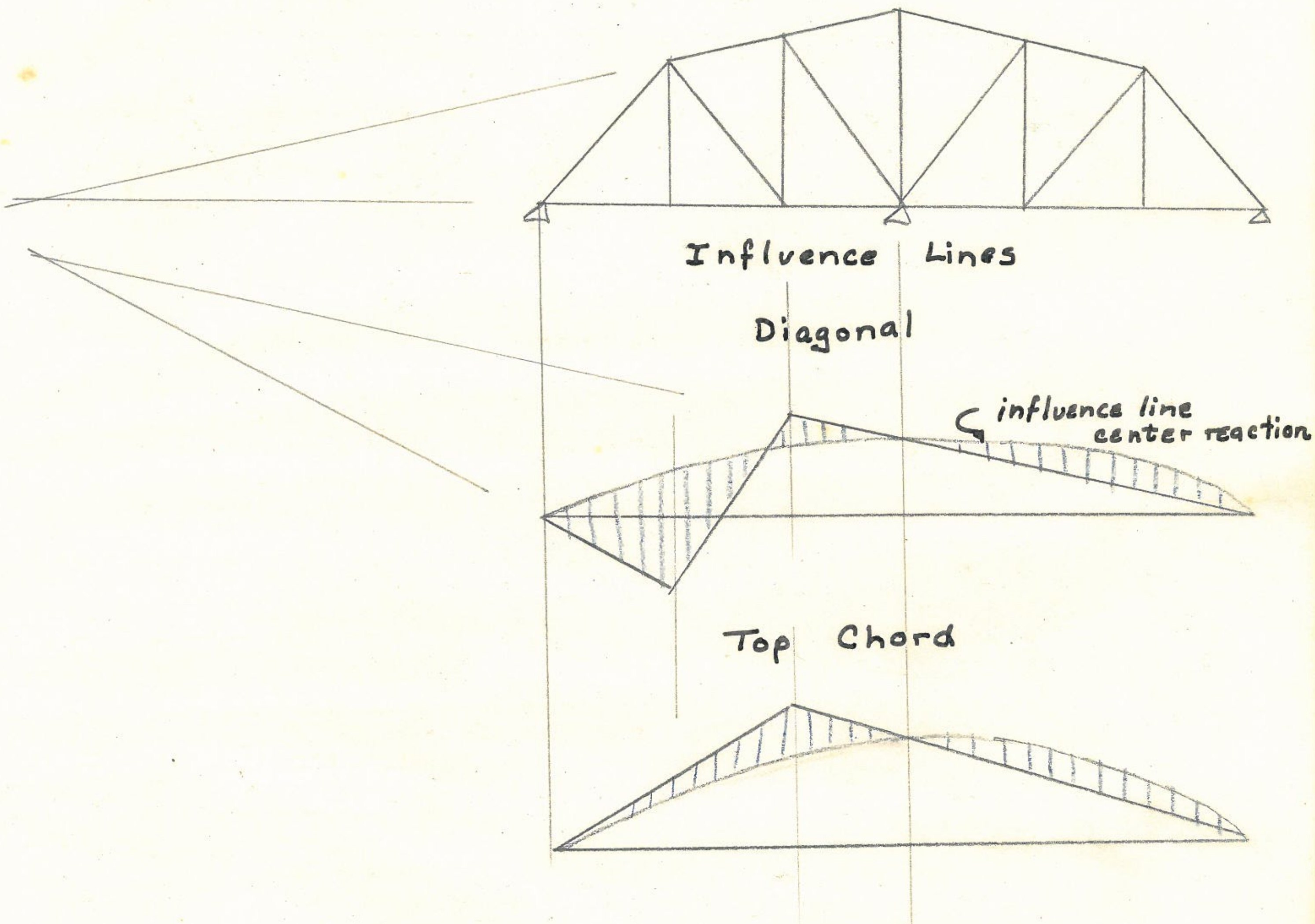
Influence line for end reaction



Combined influence line

L-32 (p. 7)
Prof. Cross
Feb. 24, 1953

Influence line for continuous curved chord truss.



The scale can be obtained by figuring the stress in the bar for a load in center, i.e., the ordinate at the center. There can be a variation in sketching of the influence line for the center reaction, but whatever the error is, it is probably unimportant.

In a truss, the web deformations are considerably larger than the web stresses in a beam. If the truss is deeper at the center, it is stiffer at the center and the influence line for the center reaction tends to flatten at the center and rise at the quarter points.

To design a truss, figure the dead load stress at time closure occurs. For the live load use influence lines and compile data to make up stress sheet. It must be emphasized that it is necessary to have truss to start out with. Texts give the truss and show how to analyze it, but fail to show how the truss is obtained.

It is important to remember that a truss has lots of bracing, so that the truss should be drawn three dimensionally.

The shape of the influence line can be obtained exactly. There are several ways of doing it. Put a unit load at the center and obtain the shape of the deflection by computing the deflection at each panel point by one of the following ways:

1. Virtual Work
   This is not a wise choice since there would be different "u" stresses for each panel. This would require much tabulation.

2. Compute each angle change.

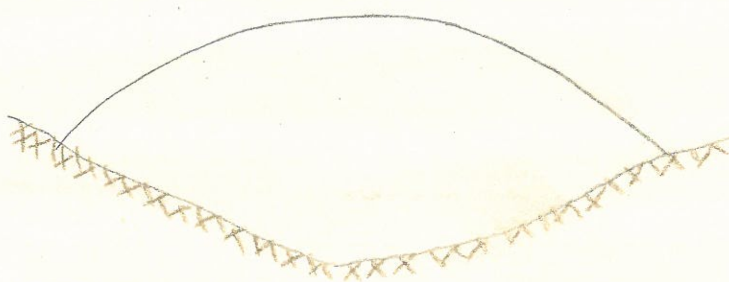
3. Displacement (Williot) Diagram
   Compute the change in length of each bar. Lay off graphically and measure deflection. As to whether Mohr's rotation diagram is important is very questionable.

There is some question about the change in length of bars. The interaction between members (participation stresses) do change considerably the stress in the members. The floor and the truss do not interact so easily.

Having obtained the deflection diagram for a load on the center, go back to the original influence diagram and correct. There will not be much change. So, draw the influence lines, design members, make corrections to the influence line, and then apply that to the completed structure.

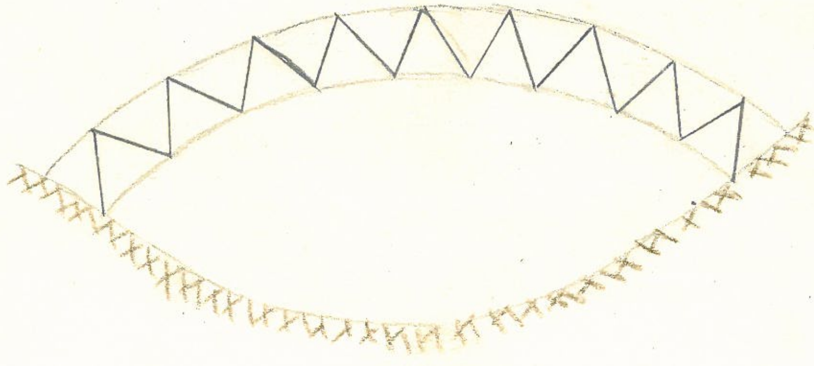
There is no point in extending continuous truss over three spans. With three, four, and five spans there is likelihood of trouble due to expansion.

If the truss is made deep enough there will be no need for chords, i.e., the chord stress is nominal. The earth acts as the lower chord.

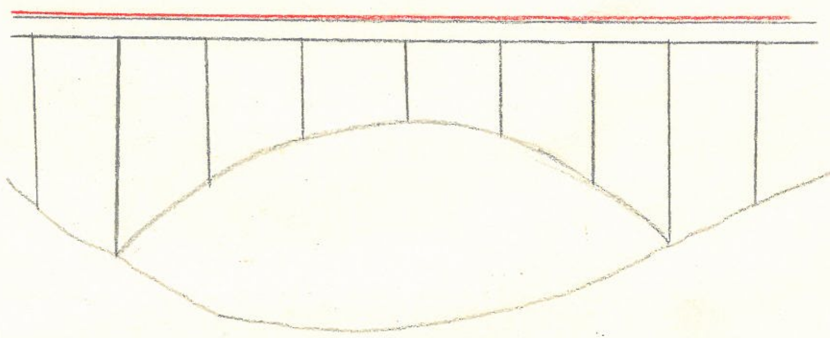


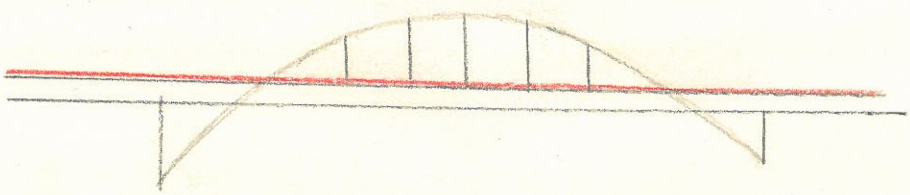
The economy, however, is not as great as it would seem. An anchorage abutment must be built. However, there is no web. The girder takes the shear -

Or a truss system.

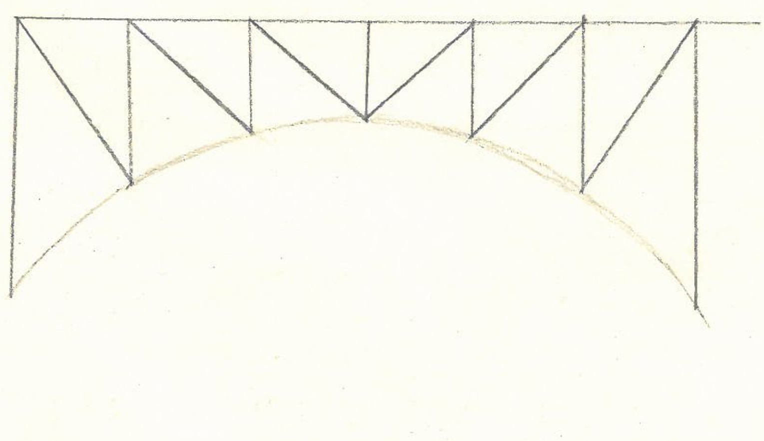


The deck may go anywhere, above or through. The location may depend on the location of the roadway -



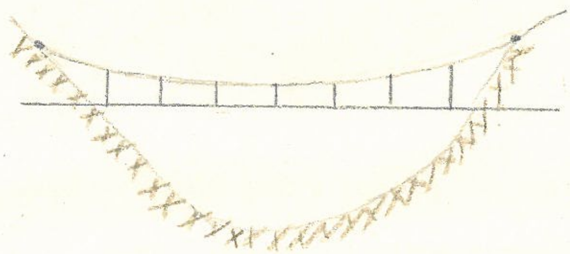


or on the bracing. The following is an example of a spandrell braced arch:



The diagonals take the shear. Note that the term bracing can mean many things. Here it means something to take the web stress. Without this bracing the arch will fall down.

The earth can be the compression chord in the case of the suspension bridge.

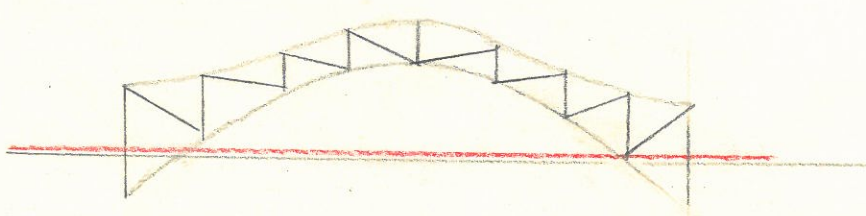


Must here consider the elements mentioned frequently this year:

1. Strength and Stability
2. Stiffness
3. Sa tisfactoriness

Since there are enormous changes in shape due to lack of stability, the above structure is not satisfactory.

Some forms do not split automatically into girders, trusses, or arches as is exemplified by the Hellgate and Cottage Farm Bridge. The roll of the top chord gives a nice effect.



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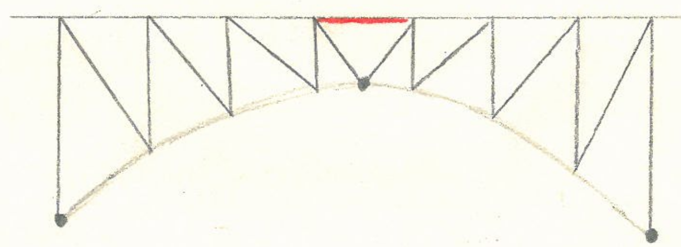
Whether an arch is to be built hingeless, two hinged or three hinged, it must, nevertheless, be erected three hinged.

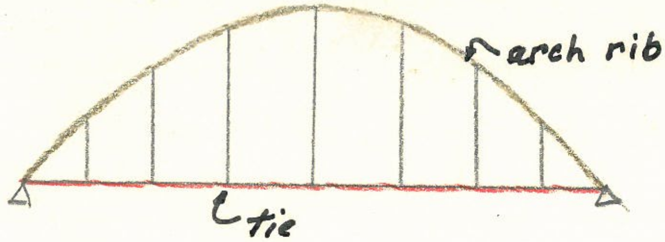


To make the arch pictured above two hinged, the bar marked in red must be put into place. (Of course, this is not true with concrete.) A stress is locked in in that member by jacking apart the upper chord. Up to where the stress is locked in, the arch is still three hinged.

To make the arch hingeless it becomes necessary to control the arch in three places. This must be done simultaneously. It is not easy to erect steel hingeless.

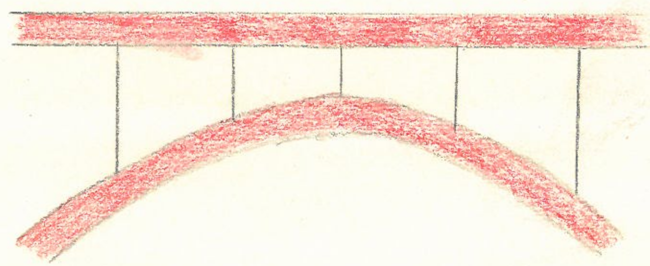
Is it worthwhile to make an arch two hinged if it is built three hinged? There are no parasitic stresses in a determinate structure, but there will be movement which means that some sort of expansion joint must be put in.

In arches there is always a problem of putting in abutments. However, the arch can be tied across the bottom.



If the rib, tie, and other components heat at the same time there will be no temperature stresses. Since something must take the shear the tie may be made a stiff girder or a truss, or the arch rib can be made stiff to share the shear. Bending resistance can be put in either or both the rib or the girder just as you choose. If both are made stiff, they will act together just as two parallel beams.

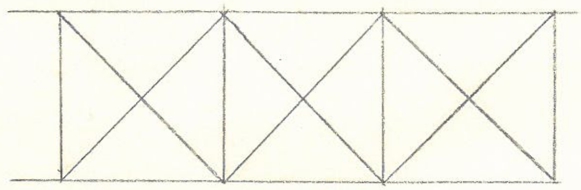
The following structure is not a tied arch but the rib and girder will share the moment.



Rarely, almost never, is a steel hingeless arch built. It is much too complicated to build. A notable exception is the Rainbow Arch Bridge in Niagra which was built hingeless but as a result had a very complicated erection problem. However, there was a special reason for building it hingeless. The tendency towards vertical buckling is less pronounced if hingeless than if two hinged.

It should be noted that concrete arches are hingeless because that is the easiest way to build them. In steel the easiest way to build an arch is three hinged. The exception in steel is the continuous truss which has been discussed earlier in which it is sometimes easier not to put in the hinge to make it a cantilever. There are many elements involved such as differential settlement.

Another problem is that of internal indeterminacy. The following diagram illustrates this:

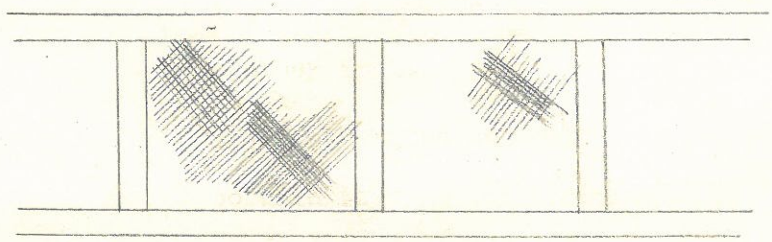


If the structure shown above were shallow, it would be called a lattice girder. If it were a truss, it would be called a double intersection truss. One diagonal is in tension, the other is in compression. Trusses are not usually built this way today. This truss has an advantage in that light sections (two angles back to back) can be used. If there were one diagonal it would be quite heavy. Which do you want--one heavy or two light ones? Today, everyone goes to the one heavier diagonal.

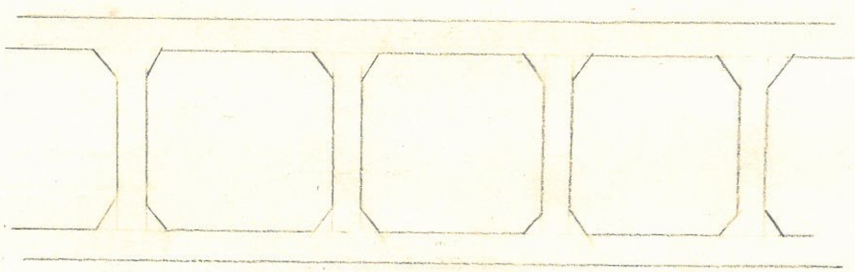
Books throughout the last century carry on this matter of indeterminacy. They are not sure how much goes to each diagonal. In practice, the railroads divide the stress equally between the two.

J. A. Van der Broek wrote a paper, "Theory of Limit Design," ASCE Transactions, Vol. 105. He seemed to be convinced that no one else thought about it. He said that one diagonal takes as much stress as it can and then yields and then the other takes it. This concept has been applied to the modern design of concrete columns. If the concrete yields the steel picks up the stress. If the steel yields, the concrete picks up the stress. Therefore, they act together.

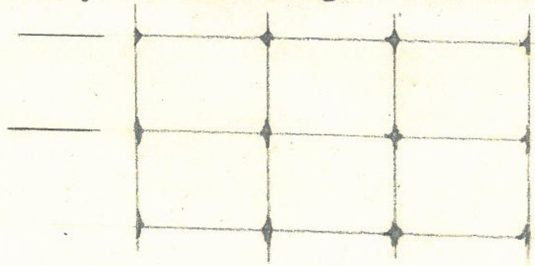
In the cases previously mentioned, the web took the shear. In the double intersection truss the diagonals take both tension and compression. However, there are other innovations such as the Wagner Truss. The Wagner Truss is a plate girder in which the web is very thin. The web buckles but since it is so thin it does not stay wrinkled. The web takes tension but no compression.

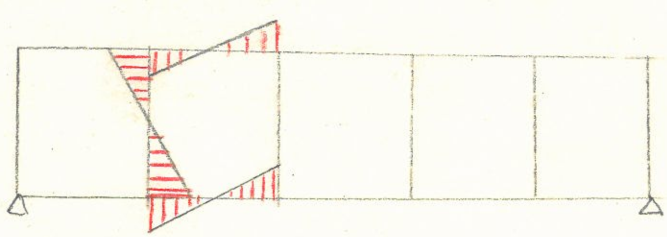


The Vierendeel Truss is an open webbed girder.



This is really the same thing as wind bracing.





The chords and verticals can take bending.

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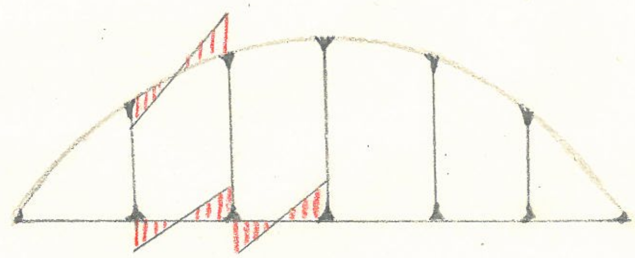
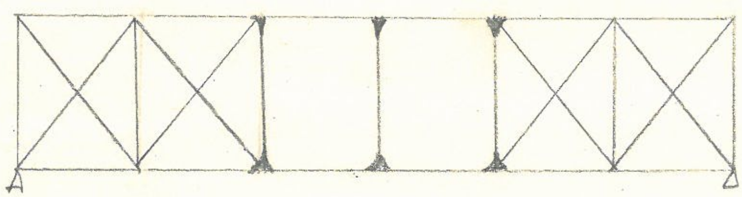
Prof. Cross
Feb. 24, 1953



With the curved chord Vierendeel Truss part of the shear is taken by the inclined chord.

Trusses can be built this way but there should be some reason for doing it. There are endless number of combinations. The following example shows the advantage of using this type in a case where conveyors are to pass through the truss.



Vierendeel advocates claim enormous economy. This is not so, but there may be advantages. Remember:

C onstruction

A ppearance

U se

S tructural (Stability
(Strength
(Satisfactoriness

E conomy

The advantage may, in certain cases, lie in "Use" since it permits the use of a big window, of conveyors, etc. The question of economy is economy for whom? It may be world economy, national economy, economy for the client, for the contractor, the sub-contractor, the man who erects the steel or the man who fabricates and sells the steel. If honest, the consulting engineer will be concerned with economy for the client.

SUSPENSION BRIDGE

American Society of Civil Engineers

Paper No.	Transactions	
1849	Vol. 98 (1933), p. 1880	"Suspension Bridges under the action of Lateral Forces" LEON S. MOISSEIFF, FREDERICK LIENHARD
2148	Vol. 107 (1942), p. 847	"The Suspension Bridge Tower Cantilever Problem" BLAIR BIRDSALL
2243	Vol. 110 (1945), p. 439	"Rigidity and Aerodynamic Stability of Suspension Bridges." D. B. STEINMAN

Proceedings

January, 1938, page 69. "Preliminary Design of Suspension Bridges"
SHORTRIDGE HARDESTY
HAROLD E. WESSMAN

February, 1941 "On the Method of Complementary Energy"
H.M. WESTERGAARD

Rensselaer Polytechnic Institute Engineering & Science Series No. 24
"Suspension Bridge Analysis by the Exact Method Simplified by Knowledge of its Relation to the Approximate Method."
ARVID H. BAKER Troy, New York, June, 1928

Journal of the Franklin Institute Vol. 235, No. 3, March, and No 4., April, 1943
"Theory of Suspension Bridges"
STEPHEN P. TIMOSHENKO

NAME

DATE

SUSPENSION BRIDGES (continued)

OREGON STATE HIGHWAY DEPARTMENT
Technical Bulletin No. 18 September, 1944
"Multiple-Span Suspension Bridges"
Development and Experimental Verification of Theory.
By
C. B. McCULLOUGH
GLENN S. PAXTON
RICHARD ROSECRANS

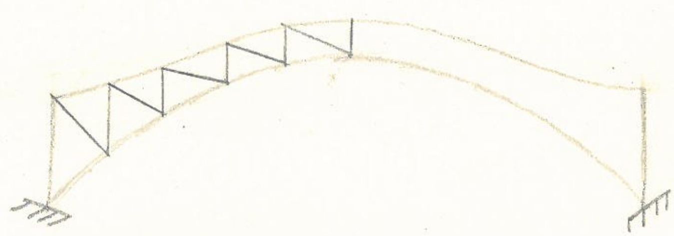
FORM A

YALE CO-OPERATIVE CORPORATION

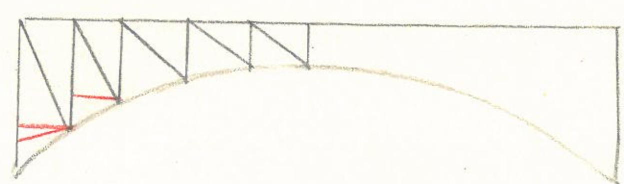
L-33 (p. 1)
Prof. Cross
Feb. 26, 1953

Erection stresses should not be omitted from any discussion of steel arches and trusses. In some cases erection stresses will be of considerable consequence. They will occur in all structures which are cantilevered and those erected by back tying such as the steel arch. There is not much literature on this matter and periodicals seem to be the main source of information.

The spandrel braced arch is an attractive structure that can be built either two hinged or three hinged.



However, if the top chord is straight the end verticals are long columns and require additional bracing.



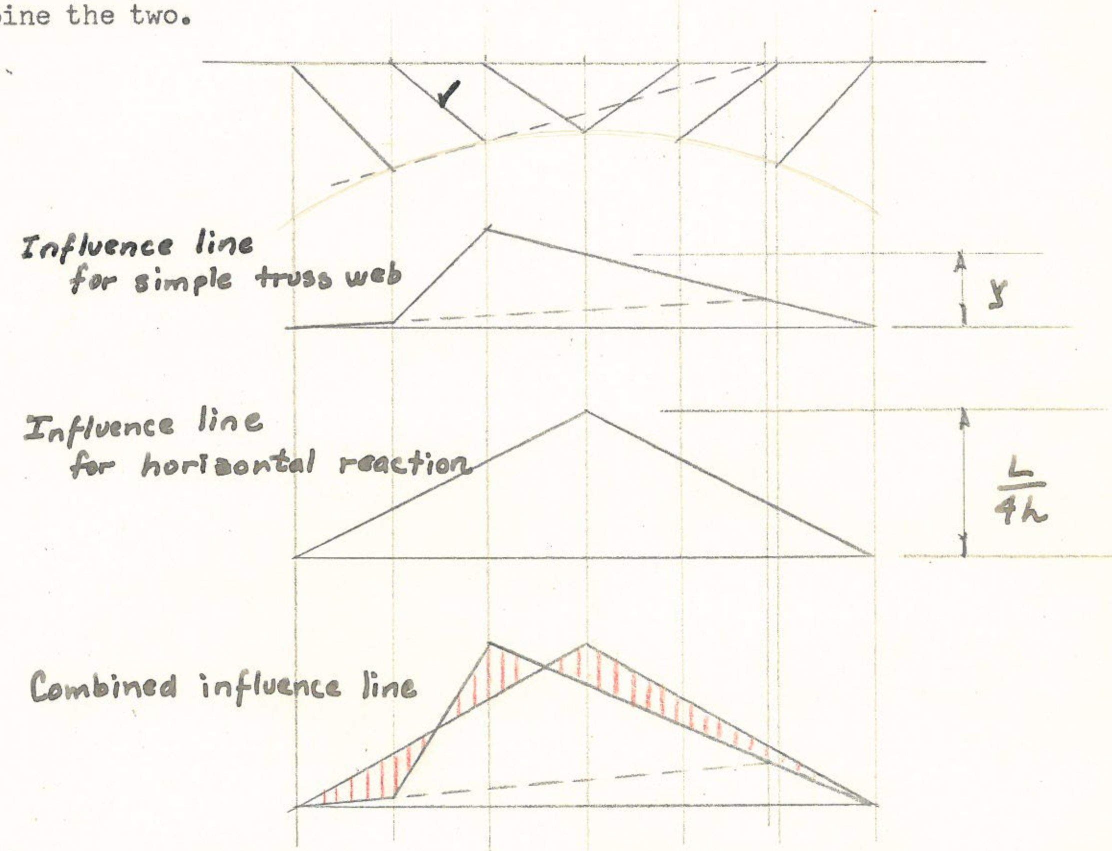
This additional bracing makes the structure very ugly.

The lower chord should be the string polygon for dead load. The arch is the lower chord and it will be quite a heavy member. The rest of the structure is incidental and it is probably heavier than it need be.

Is there an advantage to making the arch two hinged or three hinged?

Three Hinged Arch

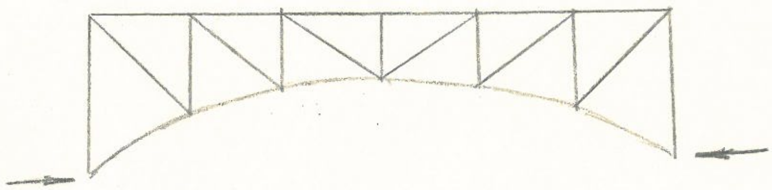
In a three hinged arch there are no temperature stresses. To draw the influence line for a structure think of the influence line for a simple truss, then the influence line for the horizontal reaction, and then combine the two.



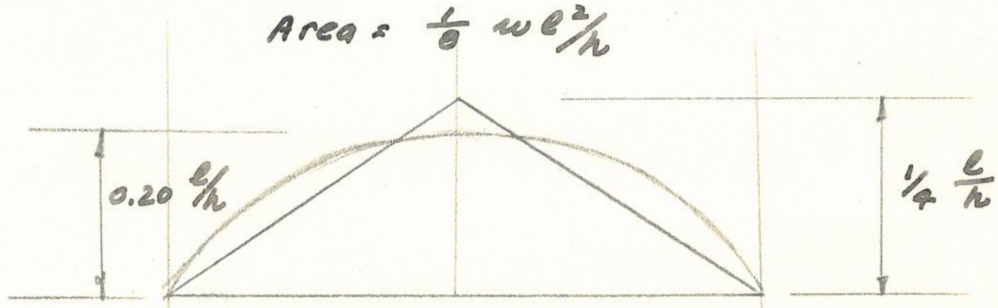
The scale can be obtained approximately since if the lower chord is the string polygon for the arch the area under the influence line will be zero.

Two Hinged Arch

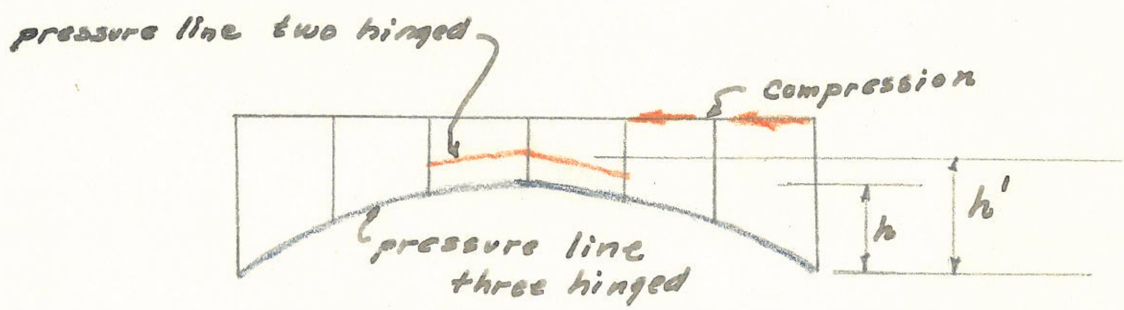
If the arch is two hinged the influence line for the horizontal thrust will have the same area as the triangle for the three hinged but will have a different shape. The shape is not known exactly but this is not important at this phase.



The arch is deep where the moment is small and shallow where the moment is large so that it will curve little at the ends and much at the middle.



If the arch is three hinged the pressure line goes through the hinge. If the arch is only two hinged the pressure line can go anywhere.



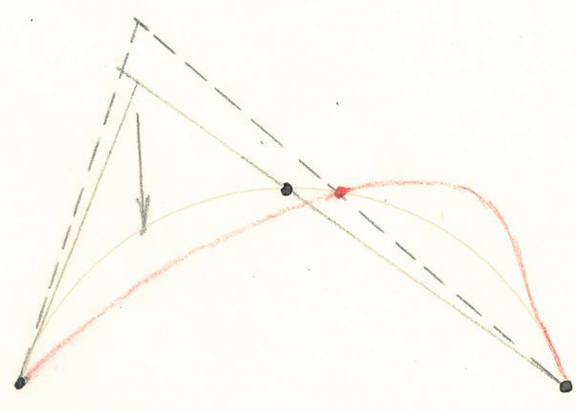
Since the position of the pressure line is not known how can the structure be designed? Design for h. Analyze by virtual work. Finding that h' is much higher. Repeat. It is not at all obvious that the results will converge rapidly. A good indeterminate structure in steel is a close approximation to a determinate structure. This is another time to criticize text books. They tell you how to analyze precisely but do not tell you how to get the structure.

The error in the influence line is great because the final influence line is obtained through the difference in two influence lines.

Temperature changes cause an added stress in indeterminate steel arches. For a complete analysis of a 350 ft. steel arch refer to pages 455 through 482 of "Movable and Long Span Steel Bridges" by Hoole and Kinne. For a satisfactory approximation see "Statically Indeterminate Structures" by Hardy Cross, page 143.

Additional stresses are another matter which, however, should not be stressed since there is some skepticism as to whether it means anything. Additional stresses merely means if a structure with a certain shape is figured and stresses are obtained, the stresses obtained are not really the right stresses because the structure has changed shape. If the structure is very flexible there will be a great change in dimensions.

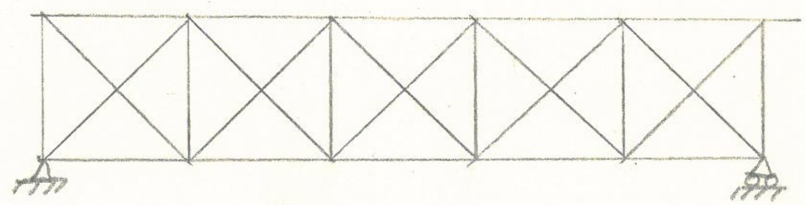
The following diagram illustrates what occurs if a very flexible three hinged arch is unsymmetrically loaded.



It is not very likely that anything of significance will happen since arches are not usually so flexible. However, it is quite proper to be suspicious if the arch with which you are concerned is flexible. Rowe in his thesis prepared at Yale analyzed three hinged arches for the effects of additional stresses.

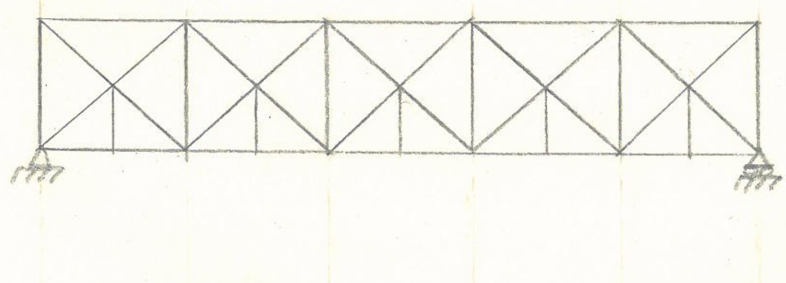
INTERNAL INDETERMINACY IN TRUSSES

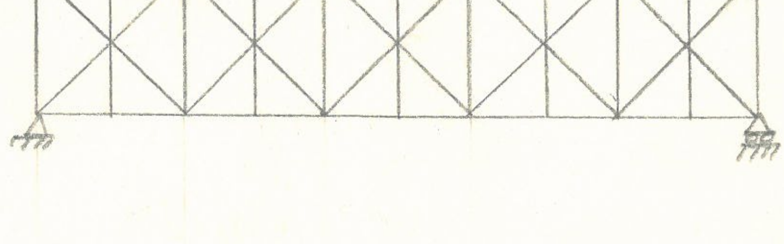
The Double Intersection Truss



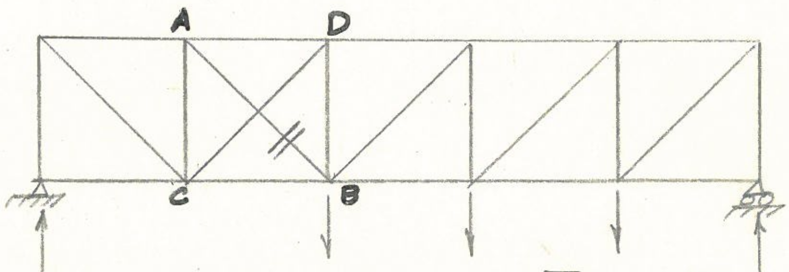
Students think that the more adept he gets at computing this "junke" the better a structural engineer he is. However, everyone goes through that phase. There is a vague analogy to shear in a web, not exactly like it, but it is there.

The Great Northern Railroad liked the following articulations:





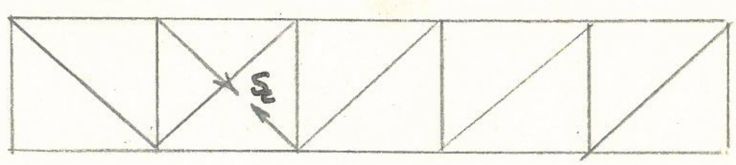
The stresses in the diagonals of the double intersection truss as drawn are pretty near equal. If the stresses in each diagonal were 15,000 psi then there would be nothing to be concerned about, but if one were taking 30,000 psi while the other took zero stress then redesigning would be necessary.



Guess at the stresses in the two bars $\overline{AB}$ and $\overline{CD}$. Then figure how much gap would be in bar $\overline{AB}$. Let S_0 be the stress guessed at. If S_0 is correct

$$\Sigma \frac{S_0 u l}{AE} = 0,$$

if not, correct with S_c.



The gap is equal to $\Sigma \frac{S_0 u l}{AE}$. To close (or open) the gap a movement $\Sigma \frac{S_c u l}{AE}$ is required so that

$$\Sigma \frac{S_0 u l}{AE} + \Sigma \frac{S_c u l}{AE} = 0$$

The S_c and the μ diagrams are alike so that

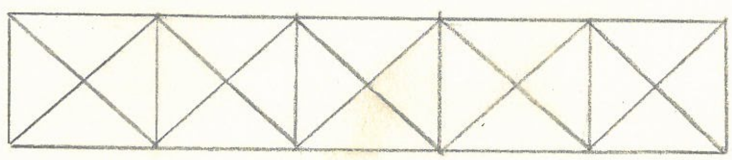
$$\Sigma \frac{S_0 \mu l}{AE} + S_c \frac{\Sigma \mu \mu l}{AE} = 0$$

or

$$S_c = - \frac{\Sigma \frac{S_0 \mu l}{AE}}{\Sigma \frac{\mu^2 l}{AE}}$$

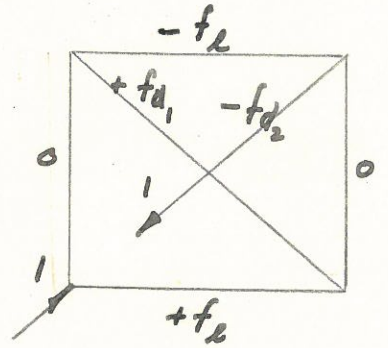
There is some question of signs. It is necessary to physically picture what actually happens.

According to Maxwell, a structure is determinate if $2j - 3 = b$ and indeterminate if $2j - 3 < b$, where j is the number of joints and b is the number of bars. The important thing if $2j - 3$ is greater than b is which bar or bars are indeterminate.



In the truss pictured, five bars are unnecessary. For analysis five equations have to be written. Even if they are written it is no small feat to solve them. "$2j - 3$" represents the number of equations. "b" represents the number of unknowns. If there are more equations than unknowns, there is something "screwy." If the structure is unstable the structure falls down. If there are more bars than equations the stresses cannot be found by statics alone and geometry must be used.

Assume the verticals have zero stress and use virtual work to analyze.

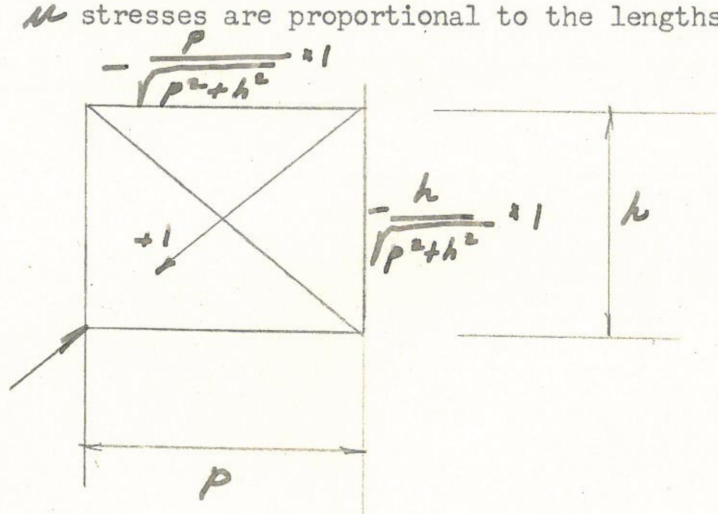


$\Sigma f u l = 0$

Formula for virtual work.

This would mean that the stresses are all right and the bar would close.

The u stresses are proportional to the lengths of the bars.

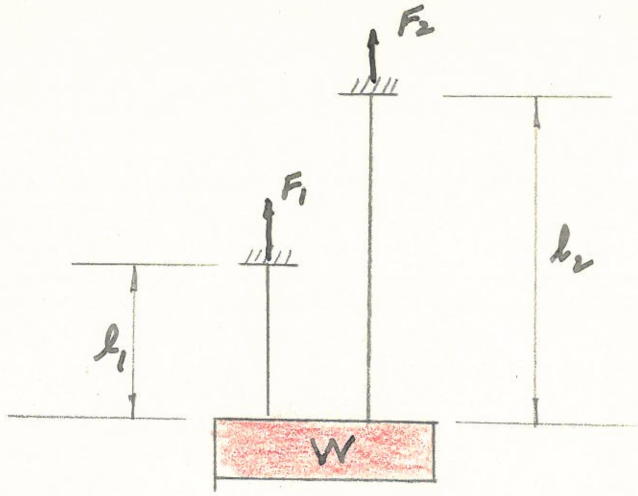


Therefore $\Sigma f u l = 0$

becomes $\Sigma f l l = 0$

The upper and lower chords and the diagonals would cancel out when combined with the u stresses in the fake truss. Any error in $\Sigma f u l = 0$ would be in the verticals. Therefore, if there were very small stresses in the verticals there would be very nearly equal stresses in the diagonals.

If the panels are not rectangular the stress would be taken in the short diagonals. The stresses are distributed proportionally to the stiffness. The short one will take more as the following analogy illustrates:



$$\Delta_{l_1} = \Delta_{l_2}$$

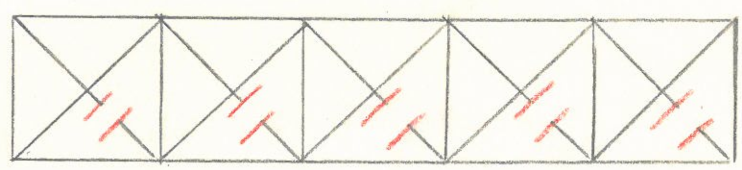
Assume A and E are equal

$$F_1 l_1 = F_2 l_2$$

$$\frac{F_1}{F_2} = \frac{l_2}{l_1}$$

or $F_1 > F_2$

If the truss is indeterminate many times proceed as follows:



Cut the bars as shown and figure the stress in the uncut bars, f_0 . Compute the change in gap in every cut bar. Place a unit load at every cut bar one at a time and figure closure. Then, the equations write themselves.

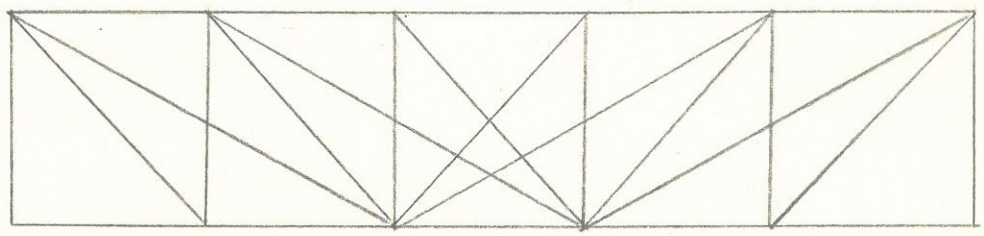
$$\Sigma f_0 u_1 l + X_1 \Sigma u_1 u_1 l + X_2 \Sigma u_2 u_1 l + X_3 \Sigma u_3 u_1 l + \text{etc.} = 0$$

$$\Sigma f_0 u_2 l + X_1 \Sigma u_1 u_2 l + X_2 \Sigma u_2 u_2 l + X_3 \Sigma u_3 u_2 l + \text{etc.} = 0$$

etc.

There will be as many equations as cut bars. Trouble arises when the coefficients of the simultaneous equations are to be worked out. Since they involve decimals, unless they are carried all the way out when it comes time to subtract the coefficients will all equal zero.

Indeterminacy may be even more complex than the examples used so far.



You are certain to have indeterminacy between the bracing and the truss. Although they are there for a different purpose, they will act together. This will be discussed in a later lecture.

The following questions illustrate the proper approach to the problem of indeterminacy.

1) How did the indeterminacy get there?

2) What is the danger of it?

3) What is the order or magnitude of the indeterminacy?

4) Is it of significant magnitude to matter?

5) What do you do with it if it is?

Secondary Stresses

The history of secondary stresses begins in Germany as far back as 1870. A German society put up a prize for anyone who could solve the problem of indeterminacy. Mandellar won the prize.

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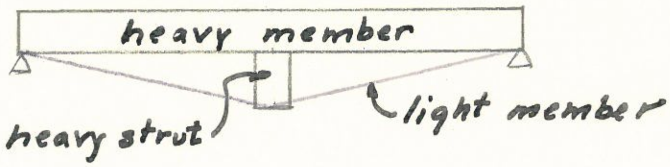
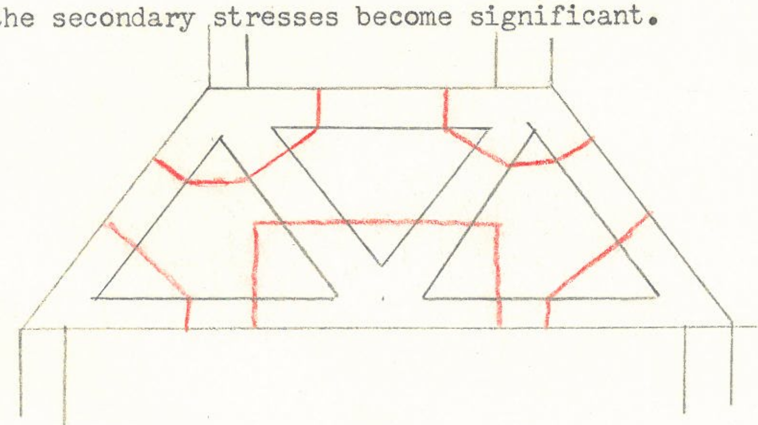
Prof. Cross
Feb. 26, 1953



There is considerable <u>strain</u> in the heavy upper chord because the middle support has settled. This takes the load off of the truss. Refer to page 189 of "Statically Indeterminate Structures" by Hardy Cross for a numerical example of the king post truss.

Secondary stresses do sometimes effect the primary. The matter is practically never of importance. However, if twenty or thirty stories are supported by a truss, the gussets of the truss are tremendous and the effect of the secondary stresses become significant.



In railroad trusses secondaries may get to be 30 percent but nevertheless no one worries about them. Architects usually "poo-poo" the idea of secondaries but still they keep them down even if they do not figure them.

1. When are secondaries so large as compared to primary stresses as to cause trouble?

Answer: A limit probably cannot be put on it but 1/2 may be all right.

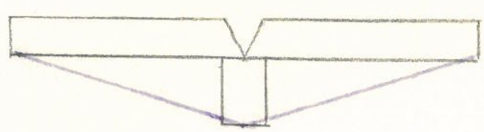
2. When do I look for these things? When am I getting into trouble?

Answer: Deep members, greater than 1/10 the length, should be looked into. Also, when the member must adapt itself.

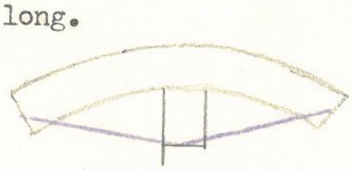
3. How can the secondary stresses be removed? What can be done about it?

Answer: (a) Make the member shallower. If the member is shallower there will be less strain. A common proportion may be 1/10 of the length.

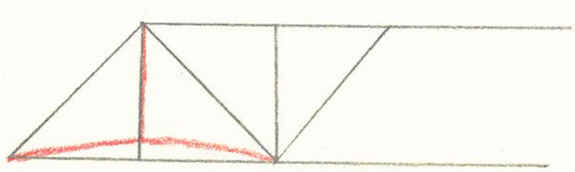
(b) If convenient, make a hinge.



(c) Lock in a stress or prestress. There are lots of variations to this. Lindenthal adopted this in the Sciotoville Bridge when he put initial bending in the members by jacking. In the king post truss make the strut too long.



On a truss, make the hangers too short.



(d) Put in bolts instead of rivets, apply dead load, and then remove bolts and rivet. The initial deflection, therefore, will not cause secondary stresses.

If it is desired, figures can be put on secondary stresses. Observe that when a truss is stressed by a load a strain is produced. It is primarily a matter of geometry. It is a question of overall proportions. The ratio of the depth to the length is a control. It controls the ratio of the secondary to the primary. The expansion of the arch is a matter of geometry, also. The ratio of the depth of crown to the rise of arch is a control. It is a matter of geometrical proportions.

When welding think judiciously about secondary stresses. Matters, unimportant in riveting, become very important in welding. The easy way out by bolting in riveted connections is not possible with welded connections.

<u>References</u> (Secondary Stresses)

"Statically Indeterminate Structures," Hardy Cross, p. 124.
"Continuous Frames of Reinforced Concrete," Cross & Morgan, p. 238.
"Modern Frame Structures," Part II, Turneaure.
"Secondary Stresses in Bridges," von Abo, Vol. 89, 1926, Transactions of the ASCE.

L-34 (p. 1)
Prof. Cross
April 7, 1953

Always lay out the basic picture-

DESIGN-BUILD-OPERATE

Design here is used in a different sense. It means to plan. No man ever does all those things on one job. Nevertheless, unless the engineer foresees the operation and all the implications of it, he cannot design. When laying out a bridge, where the roadway goes and how the connections will be made are essential elements. Figures must, therefore, be put down on how much it will cost so that the question of finances becomes quite pertinent. There are, of course, numberous other cases where planning engulfs the other phases.

The decision of whether to make a girder or truss depends on several preliminary sketches. We come on to the matter of building.

Build
- C onstruction
- A ppearance
- U se
- S tructural
  - Strength and Stability
  - Stiffness
  - Satisfactoriness
- E conomy

<u>Construction</u>

This concerns all the elements of organization, equipment, and erection.

<u>Appearance</u>

What do you want the structure to look like? What kind of bridge or building do you want?

Use

This concerns in the case of bridges elements like the volume and type of traffic. There are, of course, many elements of use in buildings. Strathcona Hall is a case where use was completely neglected, since there are improper lighting facilities and classroom facilities.

Structural Elements

Strength and Stability

Stiffness

Satisfactoriness

The relative importance of these elements depends much on the structure. A structure may be strong and stiff, but if it cracks or deteriorates it may become objectionable to the user. Most of the literature talks about strength--what is the moment in x-x, or what is the stress in bar A.

Economy

This is a very important element. Although much of the cost may be scraped off, care should be exercised so that none of the other elements of "building" are sacrificed. Strength is computed by:

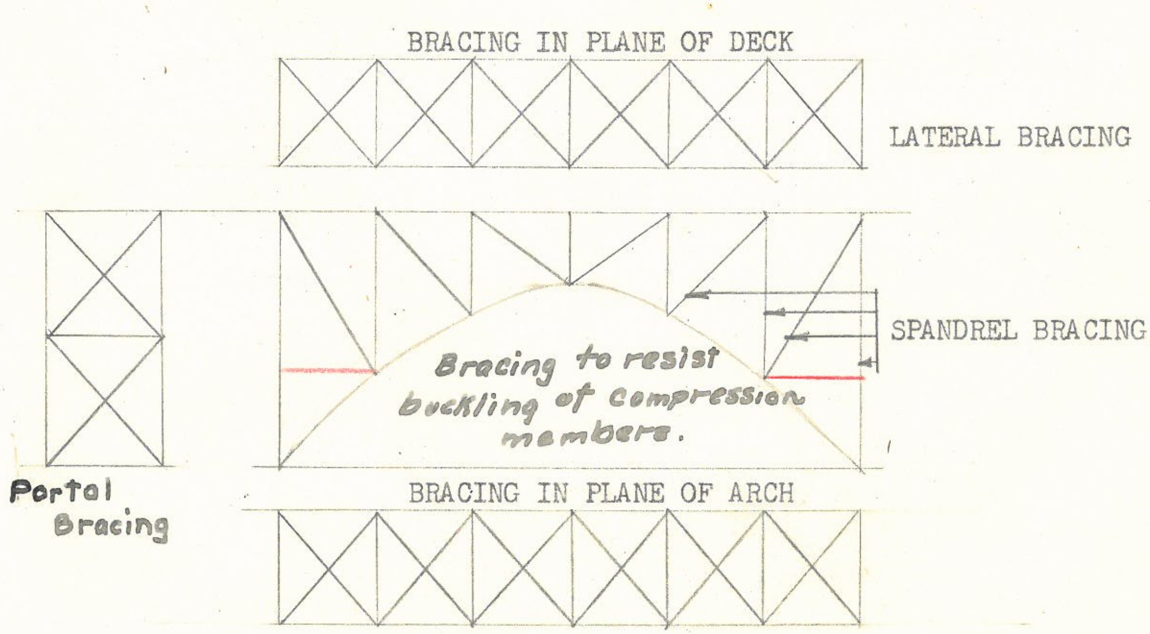
Statics--Proportionality--Geometry
of deformation
of the material

These must be related to the failure conditions. What constitutes failure; rupture, excessive moment, compression? Talbot tested some columns without cement, just gravel and spirals. Since the gravel could not move because of the spirals, it was forced to take load. Therefore, failure depends on how it can fail, where can it go.

If a structure is strong enough it is usually stable enough. The engineer acquires an uncanny faculty of judging conditions where a little change in physical value can completely change the picture. A one percent change in a property such as E can produce a 0.1 percent, a 1 percent, a 100 percent, or even 1000 percent variation in the result. Upon what variation it does cause depends whether or not the structure will be built.

In this lecture and in the next lecture the matter of stiffness will be discussed. There is not much on it in books. What there is is very fragmentary and not tied together. The term cannot properly be defined. It concerns motion. To control motion bracing is put in.

Bracing has several meanings. Lateral bracing is used to give strength against a secondary force, wind. Here we are talking about strength <u>and</u> stiffness, but more stiffness.



The term bracing, when talking about the spandrel braced arch, refers to subsidiary members. The arch takes the load. Why is the web system in a truss not called bracing, too?

Secondary members, called also bracing, are put in to resist movement such as buckling as shown in the illustration above. A structure is dangerous if it can move in any direction without setting up a resistance. Movement can occur in six ways: 1 & 2 - horizontally; 3 - vertically; 4, 5 & 6 - rotate in three directions. Intuitive synthesis, known as common sense, tells you or warns you whether movement can occur.

The codes say provision should be made against certain forces, 30 psf wind. Then "grandma" winks with one eye and says if this load is combined with gravity load a 30 percent increase in stress can be allowed. This is because of the improbability of both occurring at the same time. There is some doubt in most cases as to what or what not should be allowed. However, not only could evidence not be presented, but how to obtain such evidence is not known. In most cases it is not known whether the forces computed by the codes were chosen because they represent reality or because experience has shown that if the bracing is designed under these loads it will be satisfactory.

Experience has shown that not only does the bracing designed in such a way give satisfactory strength, but not much vibration or lateral movement results. The load, therefore, gives adequate stiffness and adequate strength.

It is not known how to find wind pressure against tall buildings. Many have tried. In th e early work the experimenter would put up a board and measure pressure on the board. The results were no good. Even in a steady wind the board will wave back and forth. Much of the effect was suction. People are inclined to forget the negative effect of wind on the leeward side.

Nosing and centrifugal forces on railway bridges are a source of lateral forces. Nosing is very serious and is large enough to throw off a person standing on the bridge. Centrifugal forces here are more complicated than mechanic's books make them seem. The point here is that there are several sources of lateral forces.

There is also a resonant effect, a vague thing we call impact. It is not a blow effect but a build-up due to vibrations. Wind also can cause a resonant effect. This has been quite serious as the Tacoma Narrows failure indicates. It acts like a wheel rolling across the bridge and falling off the side. The girder on the end of the wheel rolls off and goes down. When the floor is moving up, the wind makes it move up; when the floor is moving down, the wind makes it move down. The wind always aids the movement. It is really a common phenomena. Leaves flutter, but, even in what seems a calm, some much more than others.

Such a phenomena occurred at Tacoma Narrows. When the bridge failed the wind was not very high. The movement was about 13 or 14 feet and then the floor broke loose. It is interesting to note that much did not fail. The cables and towers were still there.

Transmission towers bring up the question of which members are main members and which members are bracing. One of the first things taught students is the importance of classification and definition. However, many things can be defined more than one way which is perfectly all right when used for different purposes. Main members do the main job while bracing does the incidental jobs. This becomes a puzzle. With bridges it is perfectly clear, but with a building it is not at all clear. Is the purpose of the roof truss of a building to hold up the snow, withstand the wind or what? The purpose of a transmission tower is to hold the transmission line off the ground but failure of transmission towers is due to wind effects. One of the most important factors is galloping of transmission lines. Transmission towers are a specialized field. For reference, see Ketchum's Handbook. In Den Hartag's book on vibrations, he discusses galloping of transmission wires during ice storms.

Stiffness is an elusive term. It can be discussed by considering three phases. Psychological, operational, and maintenance.

1. Psychological: At the opening of the Bronx-Whitestone Bridge, two girls leading the parade of cars over the bridge became frightened when the deck in front of them disappeared from view and they halted and ran off the bridge. Two hinged arches are said to have unpleasant vibrations. It is difficult to find out one way or the other. It depends mostly on whether or not we are used to the vibration. If you stand on falsework, the vibrations up and down will not bother you, but the horizontal movements will be very upsetting.

There seems to have been a mistake made by those investigating this effect. A person is not conscious of movement, or acceleration, but the irregularity of movement or acceleration. This of course is important to those interested in high speed elevators. It also is important to those interested in industrial buildings, because vibrations tire out operators without the operator being conscious of it.

2. Operation: This is entirely a metter of stiffness.

3. Maintenance: This is a matter of deterioration. If a chair wiggles and you continue to sit in it without stopping the wiggle it will break. The railroads had trouble with their train shed roofs. The cranes of mill buildings also cause difficulty.

There are several sources of evidence.

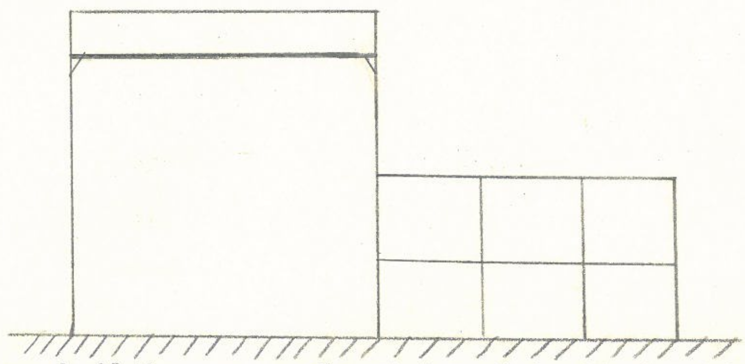
1. Analysis: Even though it may take a long time it is inexpensive.

2. Experiments: It is difficult to experiment on structures. A bridge cannot be built in a lab. Whatever type of experimenting is done it will cost a great deal.

3. Experience: This is the most valuable source in ways, but situations are all different because there is no two structures alike. Further, it is difficult to get information when failure occurs. During law suits everyone is silent.

4. Common Sense: This actually exists. Ketchum's Handbook though somewhat out-of-date is quite useful. Ketchum had experience on head frames for mine shafts. He recommends a minimum for tension members. He was thinking about resonance here.

We have not talked particularly about wind bracing in high buildings. There have been attempts to give mathematical eleastic theory in buildings high or otherwise. There has been also much research on this. The question is how much movement is bad.



The above building was analyzed for wind effects which took many months just because the designer was afraid not to. Small movements may be detrimental to intricate machinery. Strength and stiffness are not necessarily hand in hand as is exhibited in suspension bridges, where the deck deflects 8 or 9 feet.

However, you brace you get into indeterminacy, i.e., bracing is redundant to the main members.

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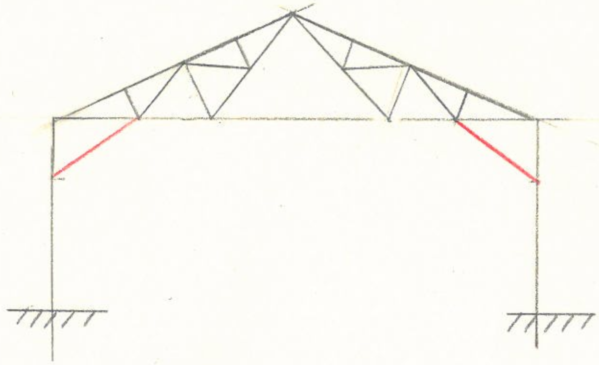
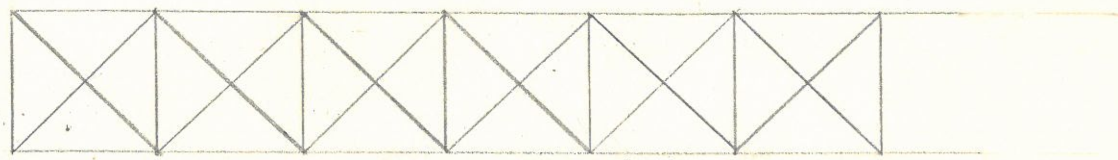
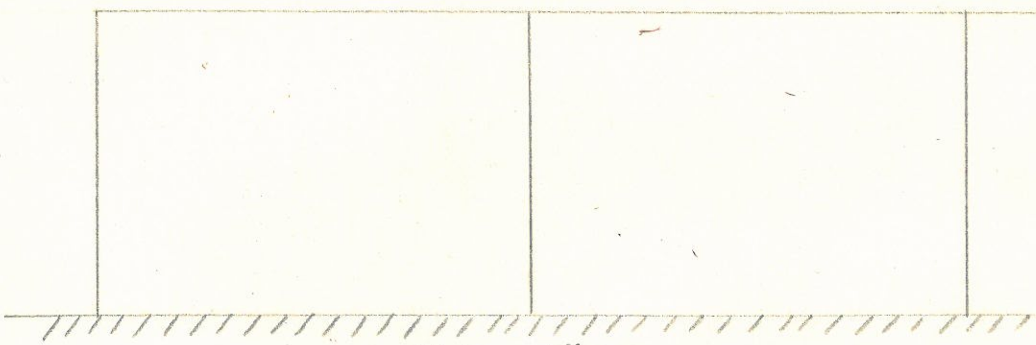
Prof. Cross
April 7, 1953

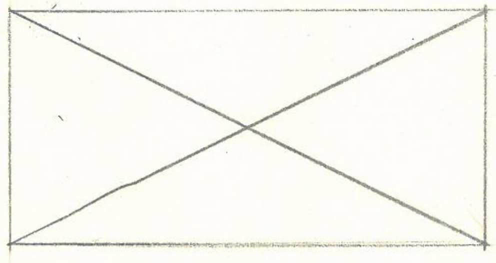




Lower Chord



Side View



End View

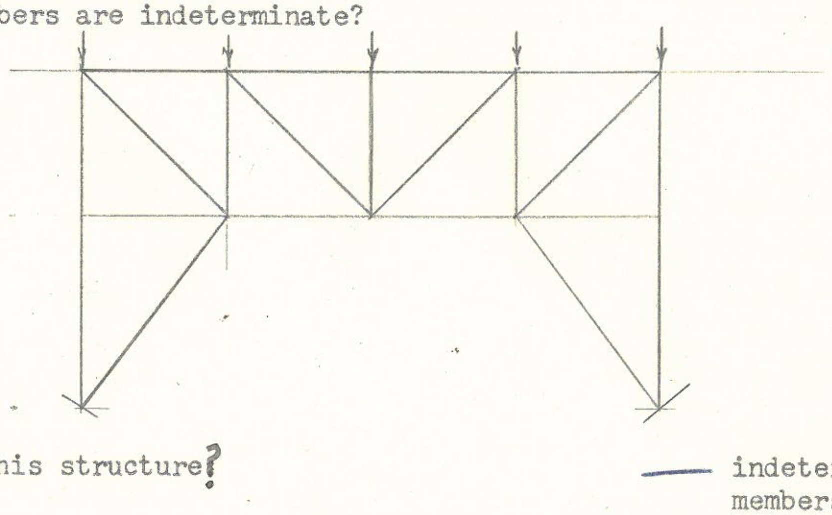
If there is no lateral bracing and but one knee brace the structure is determinate, but add another brace and the structure is indeterminate. The important thing here is not how to analyze but how did you get indeterminacy. If one brace buckles the other one will be in tension. Two questions must be answered:

1. What is the danger to the redundant member or element?

2. What effect does the redundancy have on the main structural members?

A little study will probably show neither one is very important. A few simple sketches show how the knee brace reduces the moment at the top of the column. However, some authorities use many pages to analyze this situation.

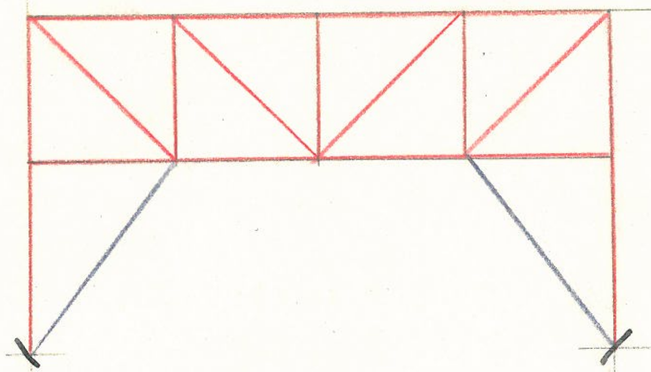
The way to handle indeterminacy is to design the main members to do their job and the secondary members to do theirs, and then to see if one endangers the other. However, as is illustrated below, which members are indeterminate?



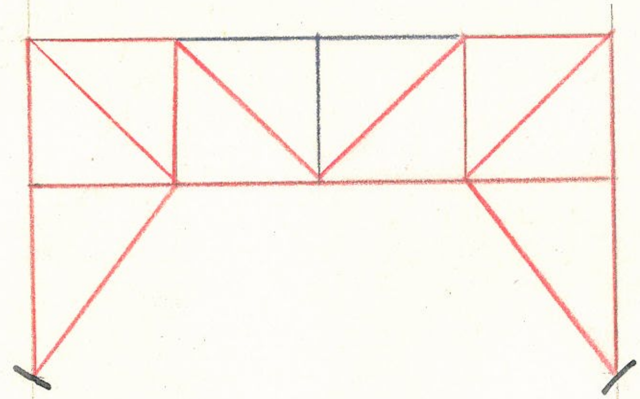
What is this structure?

—— indeterminate members.

see next page



A truss on columns with knee braces added?



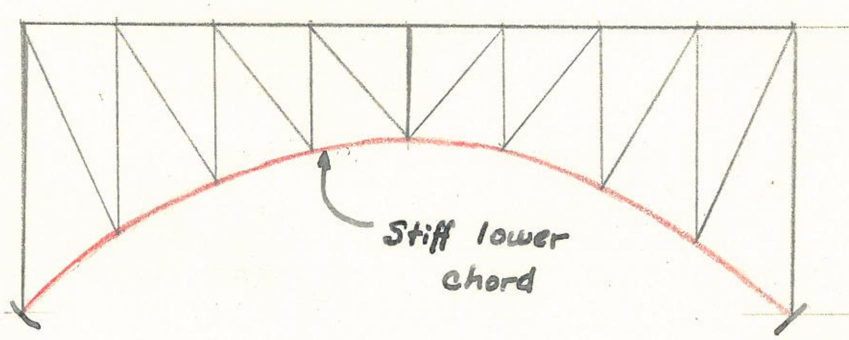
A three hinged spandrel braced arch?

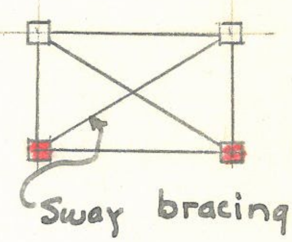
After the indeterminate members are selected then the question must be answered as to whether or not it is worthwhile to take advantage of the indeterminancy.

The trouble with text books is that they do not show how to determine the areas of the bars. In order to determine the areas it is necessary to assume what the members look like. The text also neglects to say much about failure after analysis is made.

It is best to brace along the stiff primary path since the codes allow the working stress to be raised by one-third.

Always try to get the lateral forces to the stiffest members. In the spandrel braced arch, the upper chord must be braced, but the lateral forces should be brought down to the stiff lower chord. This can be easily accomplished by means of sway bracing.





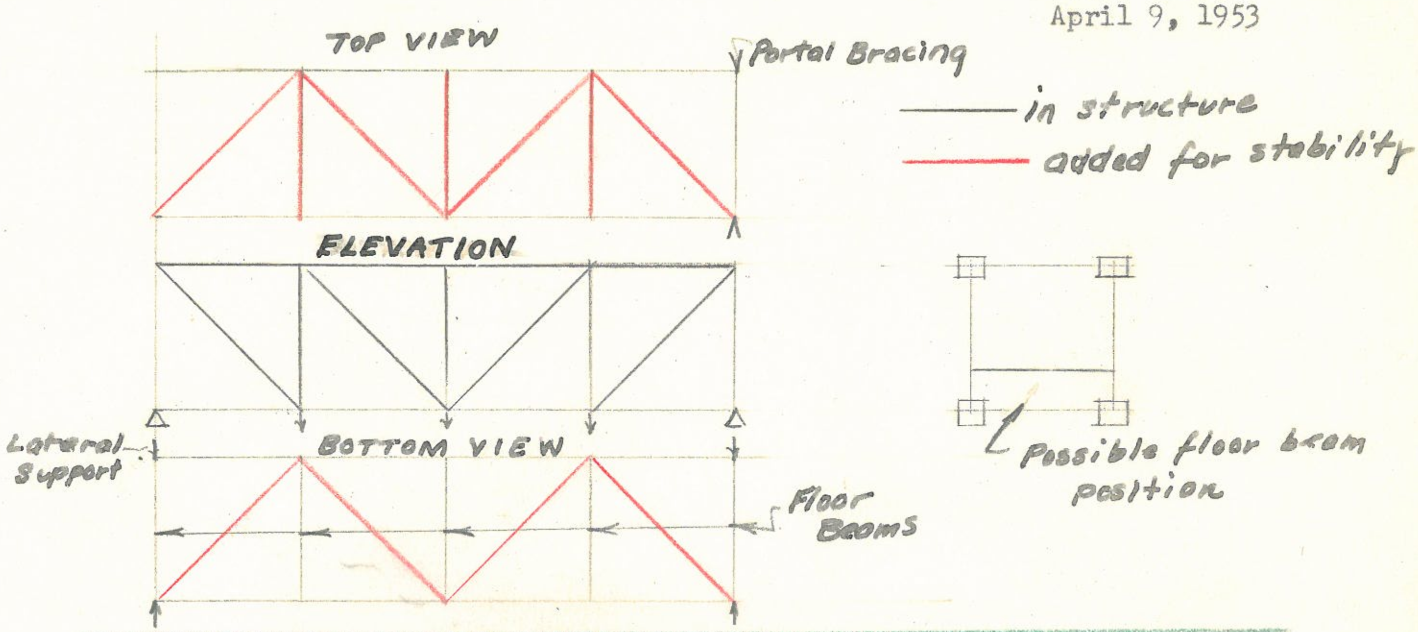
See Kuntz, "Steel Arch Bridges" for a discussion of lateral bracing in arch bridges.

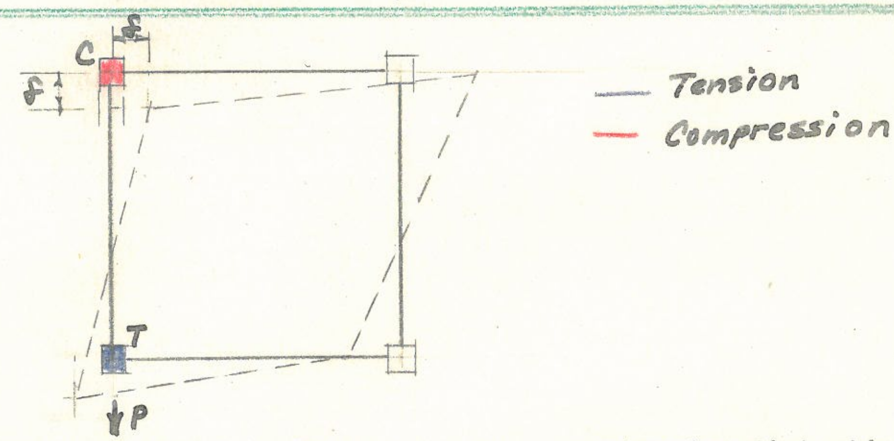
This lecture will continue the discussion of bracing. Do not underrate it. In steel structures and, perhaps, even concrete, very likely 1/4 to 1/2 of the steel is in the bracing and the details. As was indicated in the last lecture bracing is largely a matter of experience, although the codes do provide certain procedures to proportion bracing. These provisions are, however, based on experience, what has been proven by trial to be satisfactory from the standpoint of movement and vibration.

In order for a structure to be stable some bracing must be put in. However, after having put in bracing a fairly redundant system results, i.e., there are more members than statics call for. Just what size to use for bracing depends a good deal on experience. If you want to take steel out of a structure and not get caught doing it, take out the bracing. However, to remove bracing is just about the most dangerous thing to do. Outside of erosion and foundation failures, lack of bracing is probably the most likely reason for failure.

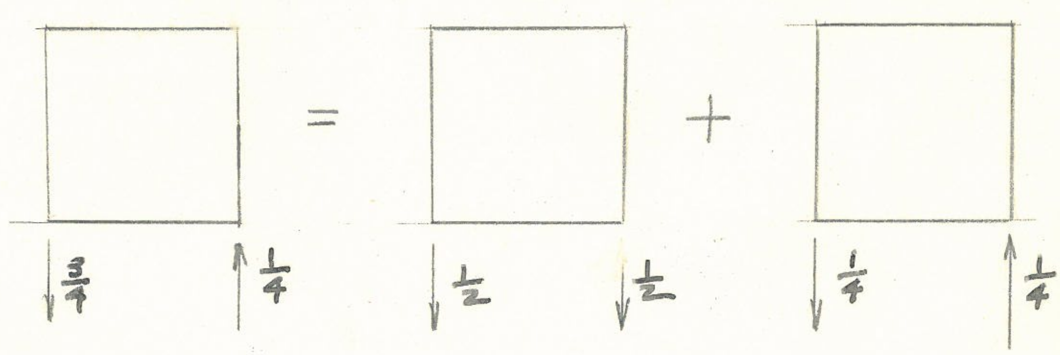
<u>Bracing Redundancy</u>

The structure must not be free to move in any direction without setting up an appreciable resistance.

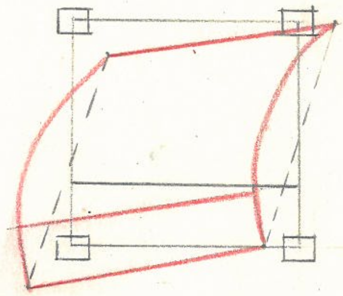




When one truss is loaded as illustrated above, a shearing distortion results. When the trusses are loaded unequally they should be figured as shown below.

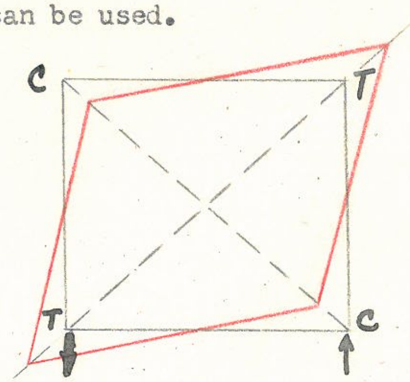


Since the floor is often made stiff and is rigidly connected to the verticals, a bending of the verticals will result.

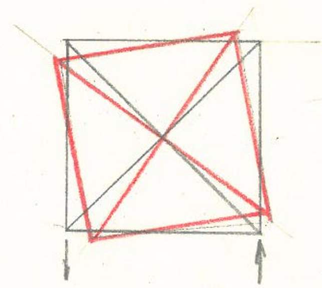


It is not likely that the effect of unequal loadings are very important. Remember it is only the little structures that are complicated. In most large structures many of these effects are insignificant.

There are, however, means of avoiding distortion if it should be considered serious enough to cause concern. On a deck truss transverse bracing can be used.

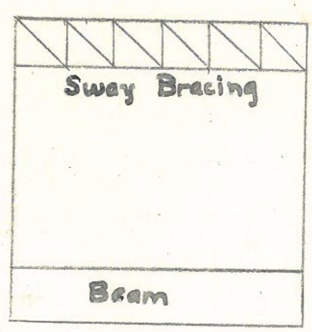


In this diagram there is no transverse bracing (or what there is is light enough to deform a great deal). There will be no torsional stress but there will be a torsional distortion.



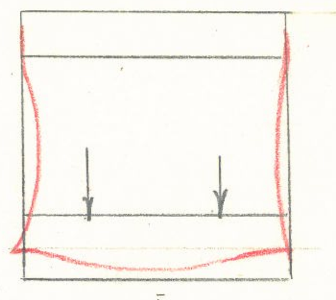
In this diagram there is substantial transverse bracing. There will be a torsional stress but no torsional distortion. Actually we are somewhere between this situation and the other type of movement. The transverse bracing stops somewhat the working of the joint.

In a through truss the cross bracing in the preceding illustrations cannot be used so sway bracing which is relatively flexible as compared to cross bracing is used.



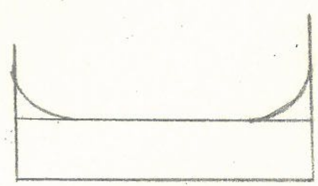
How much sway bracing should be used? The answer is not known. Some engineers will not put in any sway bracing. On the other hand, the tendency in highway bridges today is to put in very heavy sway bracing.

The wheel loads on a bridge will cause bending in the floor beams. This will, in turn, cause flexure in the verticals.



There is a temptation to analyze as a continuous beam.

There is a further temptation after using moment distribution on the continuous beam to try to take advantage of the continuity by making the floor beams wider. This tends toward an upside down arch. Lindenthal did this at Sciotoville.



What is the effect of the main member (the floor beam) on the bracing (the vertical)? Is it dangerous? It probably is not, but if it is how do you get rid of it? What does the bracing do to the main member? The answer is nothing because the floor beam is much heavier than the vertical.

In deck trusses, bracing is used as shown below.

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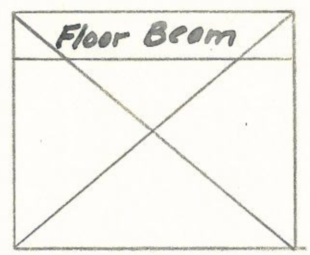
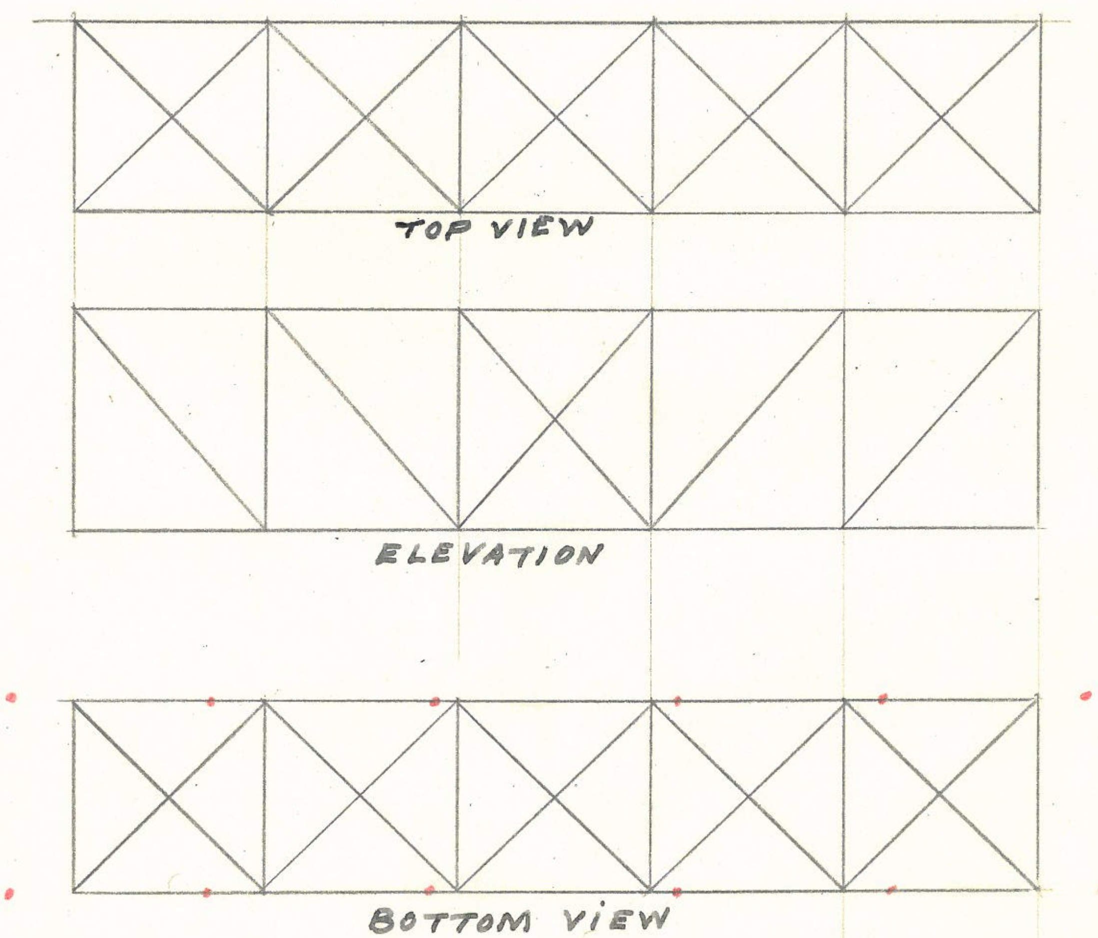
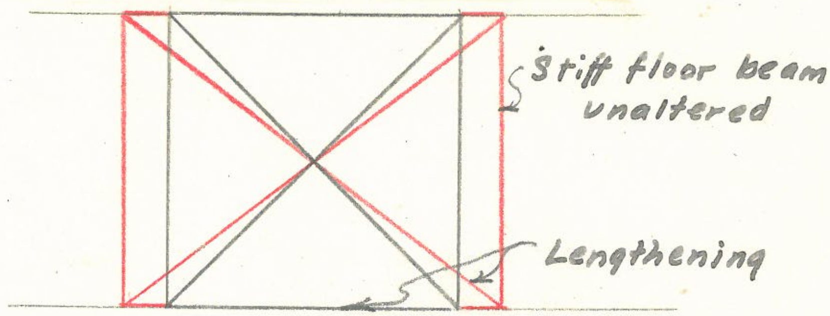
Prof. Cross
April 9, 1953



No lateral bracing is necessary in the plane of the lower chord, but in the plane of the top chord bracing is put in to prevent buckling of the compression member.



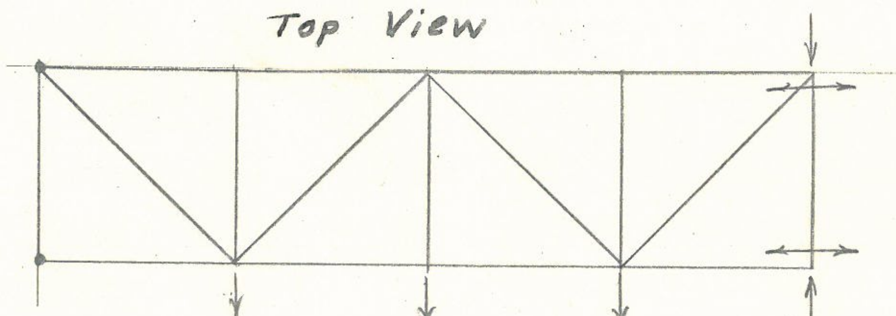
LOWER PLANE (one panel)



The illustration above shows that there is participation between the lateral system and the main truss. When there is tension in the chord, the main members will lengthen as will the diagonal lateral bracing and the two will share the tensile stress. Consequently, the stress in the lower chord will be reduced. In the plane of the upper chord, the chord members will be in compression and the diagonal lateral bracing will reduce the compression stress in the chord members.

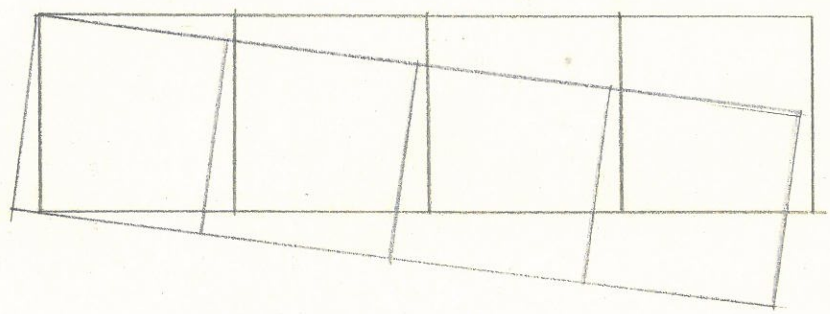
This participation can be avoided by detailing the laterals longer or shorter. This is really "prestressing." At any rate, the participation of the laterals with the chords is going to change the computed deflection of the chord. This may be a large factor.

Top View

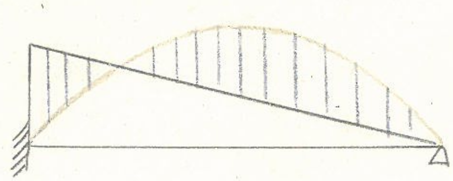


Assume that the truss above is fixed at one end and free to move longitudinally only at the other end. It would seem then that a beam fixed at one end and free at the other may be assumed.

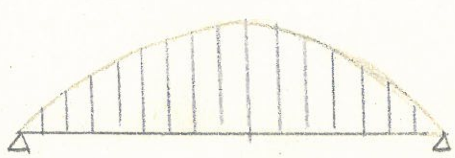
However, since the main members are large and the laterals are small, there is no continuity. The distortion is all shear. Continuity and shear do not go together, but continuity and change in shear do.



It is, therefore, correct to figure lateral bracing in a truss as though it were a truss itself simply supported at the ends.



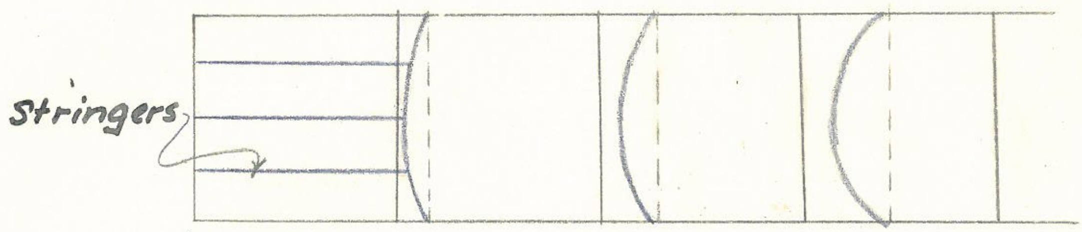
incorrect



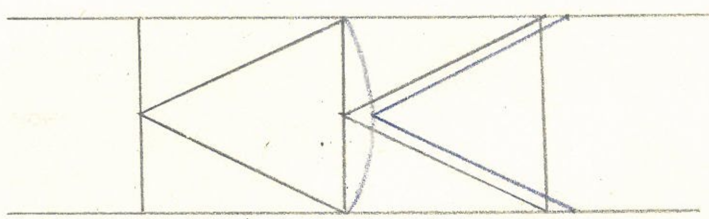
correct

This illustration is of a simple one horse truss. Even in it, it can be seen that there are many problems of interaction and participation.

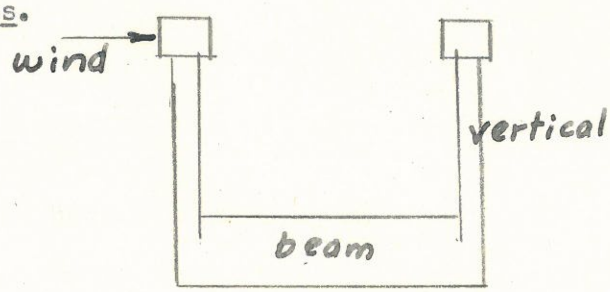
The chords elongate or shorten as the truss is loaded. However, the stringers do not. This causes some bending in the floor beams which, however, is not a very serious matter.



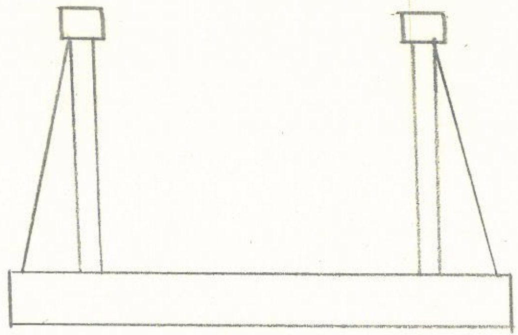
By using K-bracing the participation action between bracing and main members can be avoided. The floor will be distorted but the bracing will not be distorted.



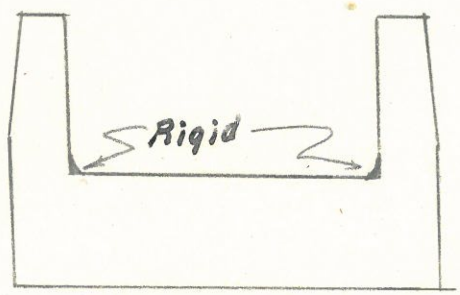
If we are inclined to put in sway bracing we are doing it to prevent distortion. There is a possibility of leaving out the upper lateral bracing and making the cross-section an inverted arch. This is called a pony truss.



It has the advantage of completely eliminating the clearance problem.

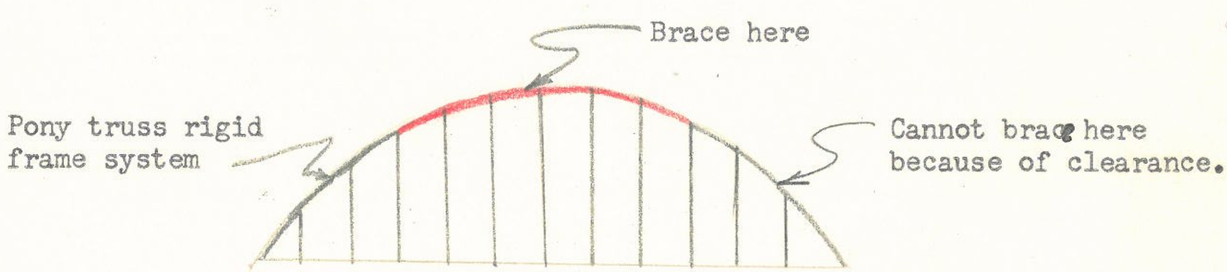


OLD TYPE PONY TRUSS



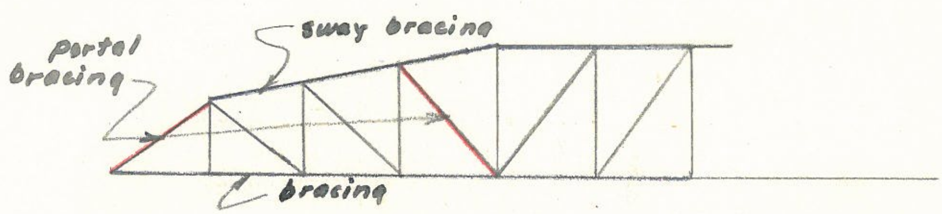
MODERN PONY TRUSS

The same situation occurs with the tied arch system where only part of the arch can be braced on top because of clearance.



The bracing and the layout of the bracing is the most important part of designing of a truss.

The redundancy introduced by the bracing cannot be avoided particularly if it is a continuous truss that is braced.



Even the bracing system itself is redundant. This may be done without realizing that it has been done and what the consequences are.

BUCKLING

Another aspect of bracing is buckling. Additional stresses caused by a change in shape of a member produces some more stress which causes more change in shape. This can pile up until a failure in buckling is obtained. Euler's formula expresses the general case:

$$P_{cr} = \frac{\pi^2 EI}{4 l^2}$$

For derivation, see Art. 87 "Elements of Strength of Materials," Timoshenko and MacCullough or any other similar text.

Although you can generalize and say you are investigating elastic instability actually the elastic phase is short since the buckling member quickly passes its yield. One way is to imagine every conceivable shape a buckling member could get.

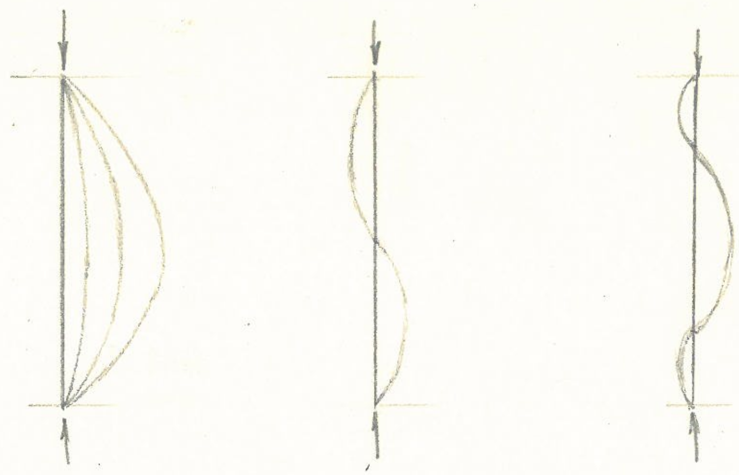


Figure how much the member bends and how much it bends because it bends.

$$f = \frac{P}{A}\left(1 + K + K^2 + K^3 + \text{etc.}\right)$$

or

$$f = \frac{P}{A}\,\frac{1}{1-K}$$

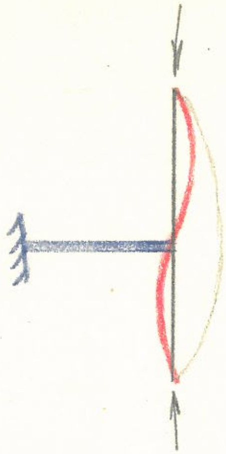
$$\text{if } K = 1 \quad f = \infty$$

How fast does this series converge?

Euler assumes that bending goes on and on until failure occurs.

The string is an example of positive stability. As the string deforms laterally, the tension forces tend to restore the string to its original shape. This is the reverse of buckling.

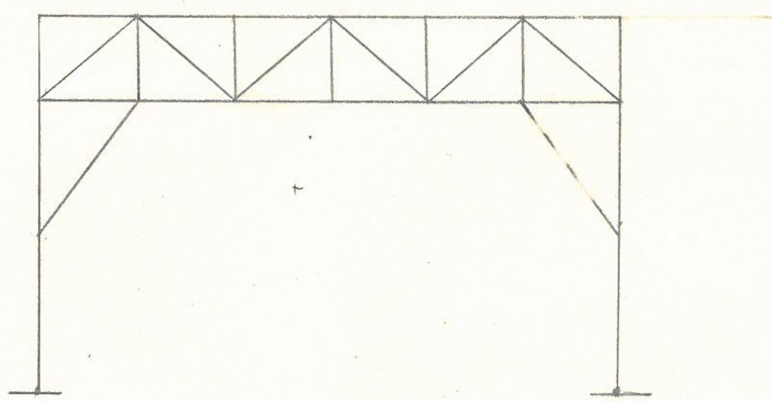
Braces are put in to prevent movement from even starting. A structure must not be able to move in any direction without causing appreciable movement. To prevent buckling tie back the compression member.



The bracing must be very stiff. It is difficult to arrive at the stress in the strut.

Bracing is a very elusive subject. What has been said so far about it is not too much and is probably not enough. The redundancy that is brought into the problem is very important.

Last time two cases were discussed.

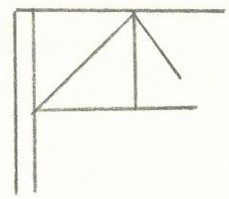


Without knee braces this structure is unstable. With both braces the structure is indeterminate. If the braces are tight it can be assumed that only one brace acts at a time, in tension.

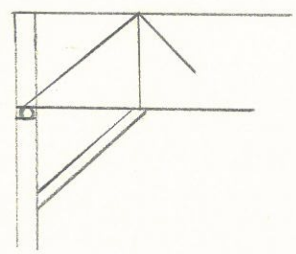
There is some buckling in the post. Some force goes from the braces to the truss. Will this do any harm to the truss? It is not likely. The braces will be all right if they are designed to do their job. The knee braces will not generally be endangered by the main action of the truss itself.

If the brace is omitted the column is made continuous for stability.

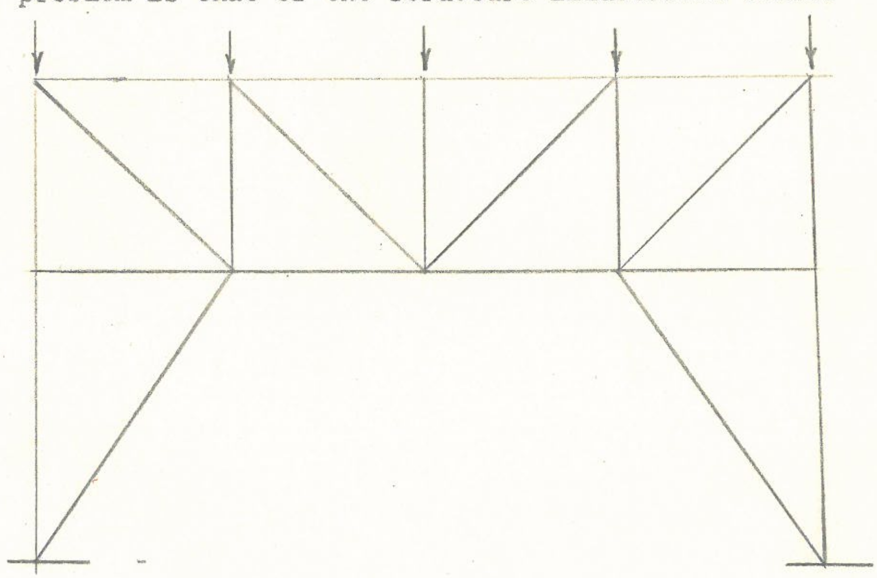
L-35 (p. 14)
Prof. Gross
April 9, 1953



If a brace is put in a hinge is used and there is less stress in the column.



The other problem is that of the structure illustrated below.



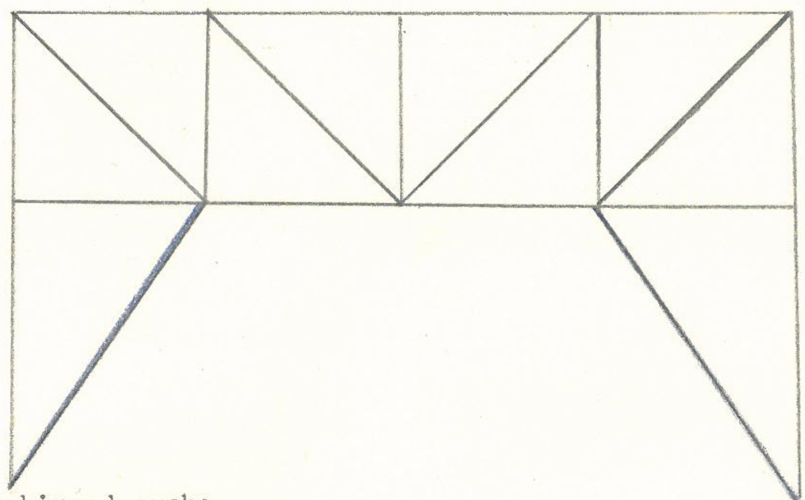
There is going to be an appreciable amount of effect of the "braces" on the truss. Two typical idiotic questions that go hand in hand with this structure are:

1. Design structure (dimension members).

2. Find stress in structure.

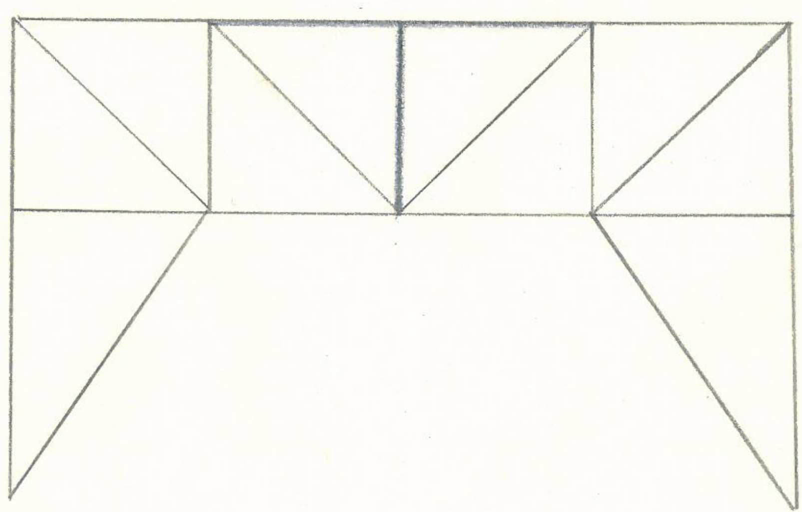
However, what kind of structure is it should be asked.

Truss on columns:

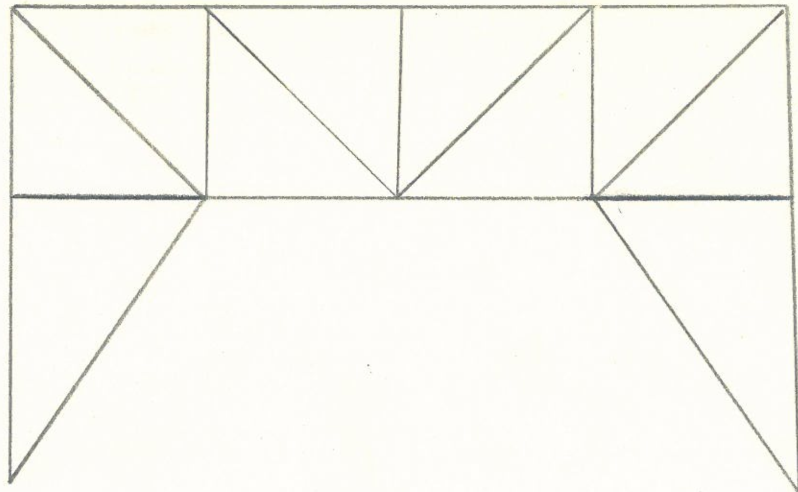


Redundant

Three-hinged arch:



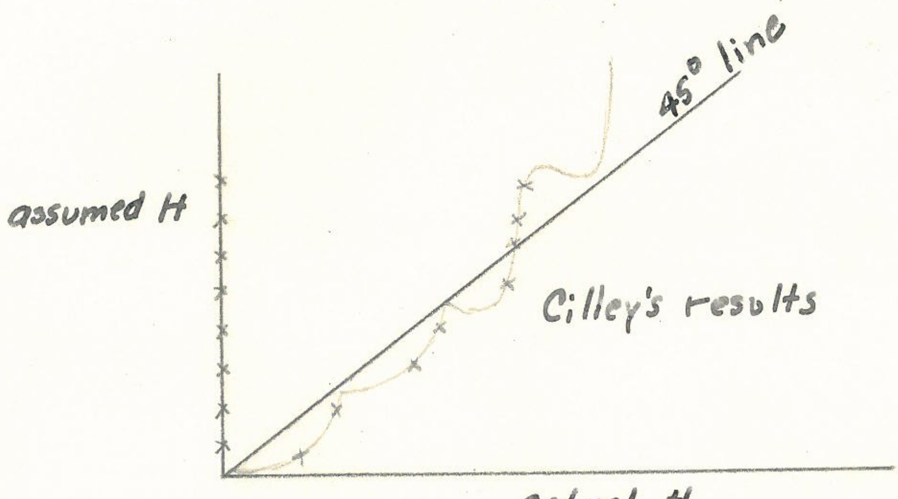
Truss:



To design assume a horizontal force. Compute the true horizontal force:

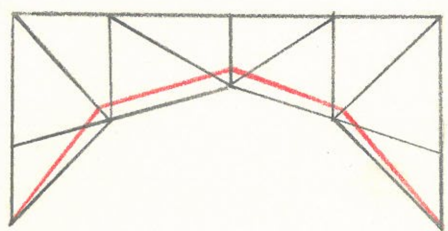
$$\text{True } H = \frac{\Delta_H \text{ due to loads}}{\Delta_H \text{ due to unit horizontal thrust}}$$

and if it equals the assumed H it makes a 45 degree line.



If you were to get the size of the member by assuming an action and then analyzing you would get either more or less than was bargained for. If the two were the same (see graph) the resulting structure would be statically determinate because at least one of the bars would have no stress in it. Which determinate structure obtained would depend upon where you started assuming H.

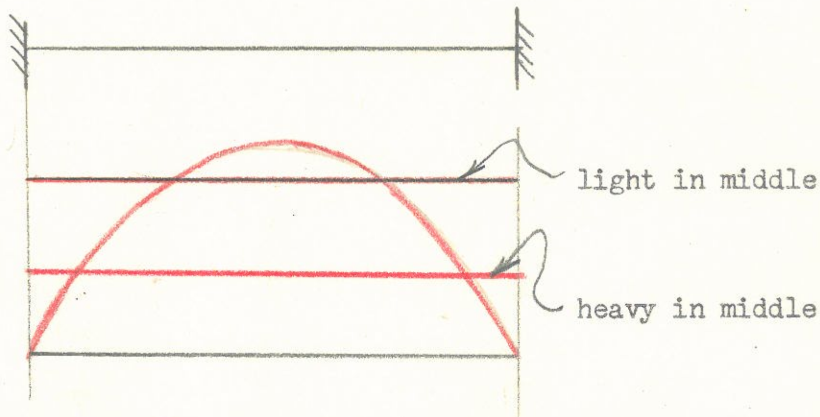
If the structure to be considered is as shown below where the line of pressure is very little above the hinge, it would be very close to a three hinged arch.



The general outline of the arch is ill-conceived. Cilley (see Vol. 33, Transactions of the ASCE, 1900) says that it is inefficient, that an indeterminate structure acts against itself. However, he lacked scale; the members were not proportioned at all to their stresses.

To some extent in steel you can in some cases predetermine the action of an indeterminate structure as illustrated in the sketch of the three hinged arch above where the pressure line hugs the frame.

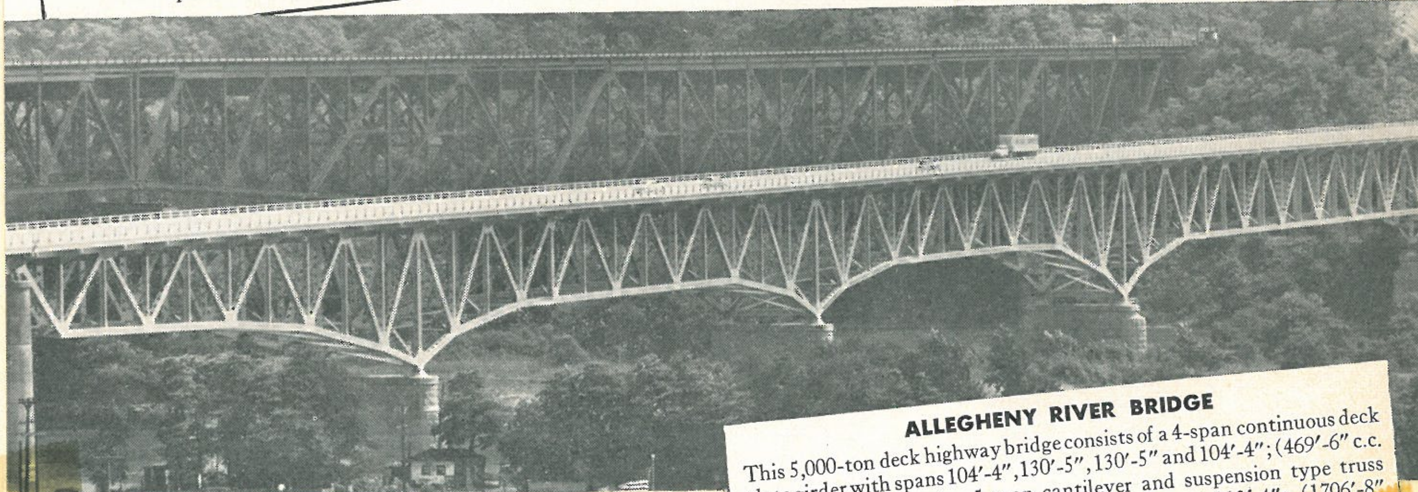
Another equally foolish question is to draw the moment diagram for a fixed beam.



The moment curve can be made anything. The moment can be thrown to the middle or the end. A skilled designer who has a scale on continuity can play with contintuity and use it. However, the fixed end beam above cannot be made all negative.

BEAVER RIVER BRIDGE

This 4,000-ton deck highway bridge consists of two 65′ beam spans; two 175′-6″ simple truss spans and one 3-span continuous truss, with spans measuring 321′-9″, 404′-6″ and 321′-9″ respectively.



ALLEGHENY RIVER BRIDGE

This 5,000-ton deck highway bridge consists of a 4-span continuous deck plate girder with spans 104′-4″, 130′-5″, 130′-5″ and 104′-4″; (469′-6″ c.c. of end bearings) and a 5-span cantilever and suspension type truss with spans 266′-8″, 346′-8″, 346′-8″, 533′-4″ and 213′-4″; (1706′-8″ c.c. of end bearings).

L-36 (p. 1)
Prof. Cross
April 14, 1953

The following two subjects will be discussed in this and the next lecture:

1. Buckling

2. Secondary stresses

This will tie in with the next lecture. There is a great deal of interaction, indeterminancy in any structure.

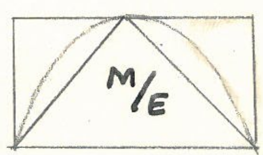
BUCKLING

Consider first a simple case, the straight column:

Assume a small initial deflection, Δ_1

The moment, $M = \Delta_1 P$

The additional deflection, $\Delta_2 = K \frac{M l^2}{EI}$



Shape	K
Rectangle	1/8
Triangle	1/12
Parabola	1/9.6
Sine curve	$1/\pi^2 = 1/9.87$
Average	1/10

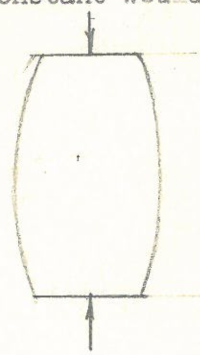
$$\Delta_2 = \frac{1}{10}\,\frac{P\Delta_1 \ell^2}{EAr^2} = \frac{1}{10}\,\frac{P}{A}\,\frac{\ell^2}{r^2}\,\frac{\Delta_1}{E}$$

$$\Delta_3 = \frac{1}{10}\,\frac{P}{A}\,\frac{\Delta_2}{E}\,\frac{\ell^2}{r^2} \qquad \text{Let } P/A = f_a$$

$$\Delta_{TOTAL} = \Delta_1\left(1 + \overline{\frac{1}{10} f_a \frac{1}{E}\frac{\ell^2}{r^2}} + \overline{\frac{1}{10} f_a \frac{1}{E}\frac{\ell^2}{r^2}}^{\,2} + \text{etc.}\right)$$

$$= \Delta_1 \frac{1}{1 - \frac{1}{10} f_a \left(\frac{\ell}{r}\right)^2 \cdot \frac{1}{E}}$$

The above is the usual approach except that π^2 is used instead of 1/10 in mechanics books. With a different shape such as a tapered member a different constant would be obtained.



If 1/10 $f_a\left(\frac{\ell}{r}\right)^2\frac{1}{E}$ equals 1 the column would buckle and would fail. Therefore, the critical stress, $f_x = \frac{10E}{\left(\frac{\ell}{r}\right)^2}$

The well-known Euler's formula is:

$$f_x = \frac{\pi^2 E}{(\ell/r)^2}$$

f_a = stress due to axial load

f_b = stress due to transverse load, i.e., the bending stress in column due to $P\Delta$

f_x = critical stress

f_t = total stress

$$f_t = f_a + f_b \frac{1}{1-\frac{f_a}{f_x}}$$

If f_a is small there is no intensification

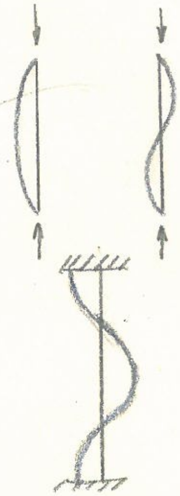
$$f_t = 0 + f_b \frac{1}{1-0} = f_b$$

If f_b is small there is nothing to intensify.

If $f = f_a + f_b$ is kept within reasonable limits f is mostly f_a and there is very little f_b to intensify. If a reasonable limit is put on the e/r ratio, it will be found that will not be over 6 or 10% additional stress.

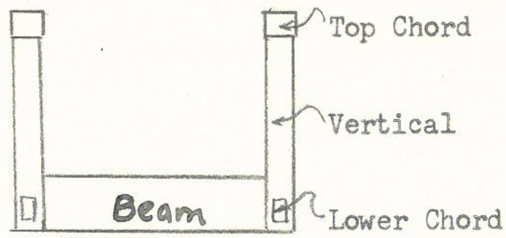
The Wagner Beam (see Lecture 32, page 15) is a plate girder with a very thin web which is permitted to buckle. However, when it buckles it does not set but returns to its original shape.

The column fails when the axial stress reaches the yield point. Books go overboard with Euler's Formula. We could investigate any case of buckling provided we could assume any shape the structure will take.

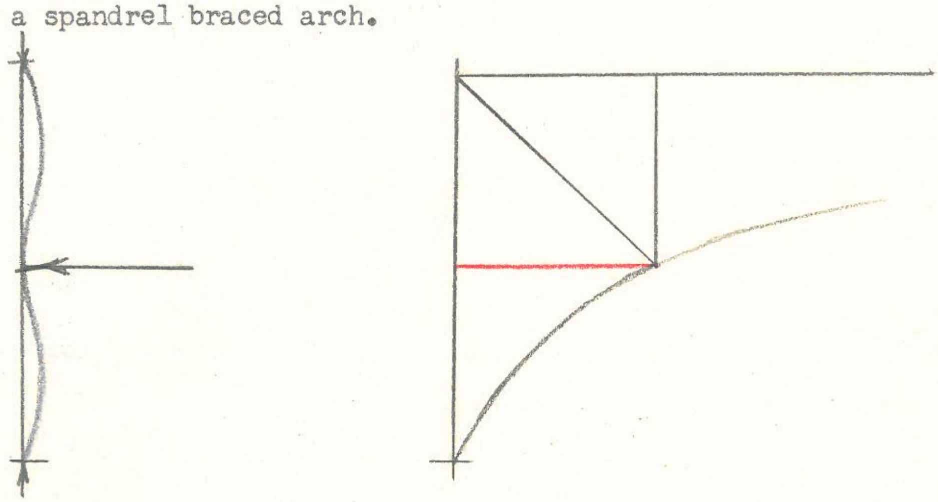


The column with two modes is stronger than the column with one mode. The length of the column buckling out is shorter. Therefore, the stress would be higher if bent in two modes. On a fixed end column the moment areas must balance.

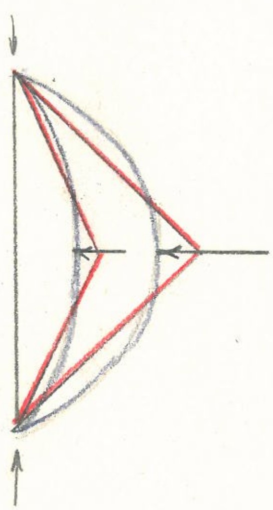
In a Pony Truss the top chord tends to buckle between panel points. How stiff must the verticals be to resist the buckling?



What size member should be used to prevent buckling of the vertical in a spandrel braced arch.



If the movement is small the answer can be seen quite easily.



—— String polygon

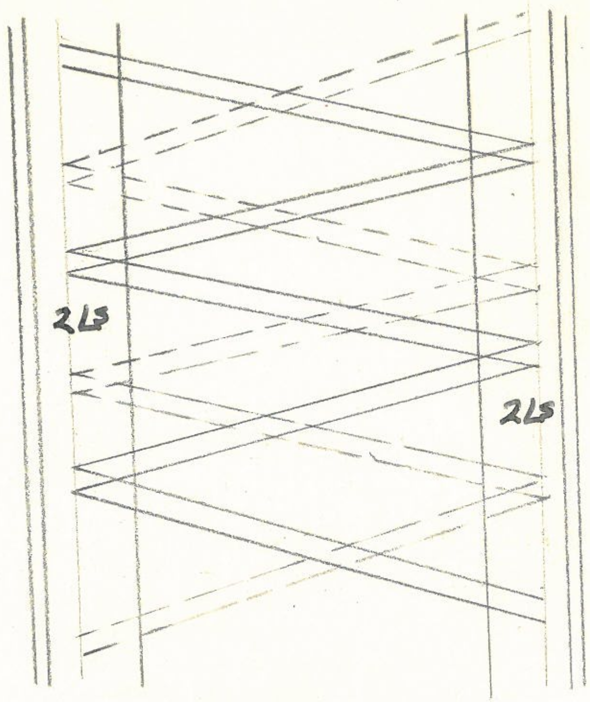
<u>Note</u>: the more the column moves the larger the force.

It is obvious from the above diagram that a very stiff member should be used for the brace because there must not be any movement at all. The diagram explains why the column could not be tied back with a string. A string would stretch which would permit more movement which would cause a greater stretch and so on.

The first thing wanted in such studies is the scale. This is what math forgets. Put a limit on $\underline{f}$. Keep f_x up to a reasonable figure and forget about intensification.

There has been much literature on column formulas. The initial interest was to get one formula for all circumstances. The importance of that is very doubtful. About the best way of settling this matter is to get someone to go solve it. Usually, a committee is appointed and they can go on and on discussing which formula to use without reaching a solution.

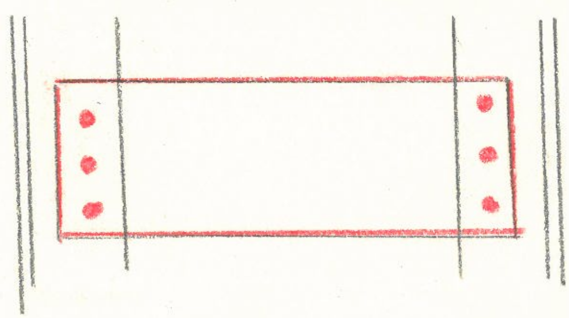
If the first curvature is not a sine curve it would not converge on 1/10. This, however, need not complicate things since it would converge on some other factor. Tapered columns such as in a crane boom are the same if it be thought out with just a variation in the constant. It is not necessary to go into a pretended exact analysis. When it comes to actual failure of columns, tests have to be depended upon. Columns fail at the yield point or at the details.



Lacing { single —
double - - -

If the lacing is too flimsy it may buckle. This was the primary reason for the failure of the Quebec Bridge. When the lacing buckled the column buckled.

Instead of lacing, D. B. Steinman advocates the use of batten plates. Whether they are effective or not is still questionable but they do have the advantage of being much easier to fabricate.



Structural failures in metals are most frequently failures in details because of the intensification of stress at rivet holes and gusset plates. Theory is useful only in a limited way. There have been tests but they are expensive and, because they were made on joints with two or three rivets while we are interested in thirty or forty rivets, they are mostly valueless.

<u>Static Tests</u>

"Tension Tests of Large Riveted Joints"
R. E. Davis, Glenn B. Woodruff, H. E. Davis
of the University of California
Transactions of the ASCE, Vol. 105, 1913.

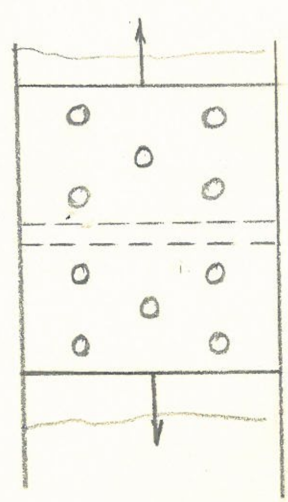
Testing was done in conjunction with other tests for the San Francisco Transbay Bridge Authority.

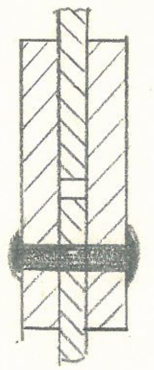
<u>Fatigue Tests</u>

"Fatigue Tests of Riveted Joints"
W. Wilson, University of Illinois
University of Illinois Bulletin, 302
(See also: Bulletin 327.)

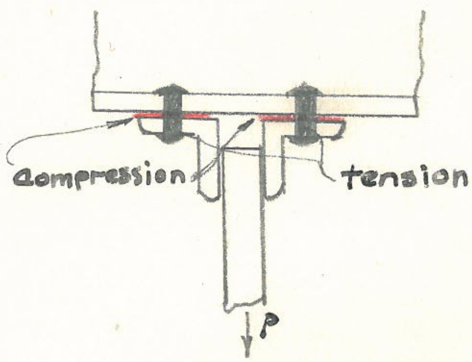
Wilson was compelled to devise a special heavy machine for conducting these fatigue tests.

Text books contain discussions of the theory of failure of riveted joints. They say that the rivet acts in bearing on the hole. This is true only when the joints are overloaded and slip occurs. Until this happens the friction between the two plates prevents movement.



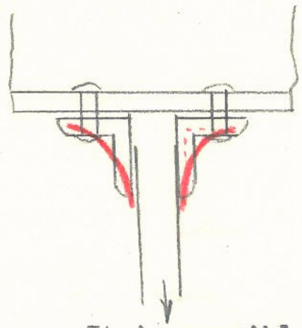


If the rivet is hot driven, it is already at the yield point. Take for an example a girder. There this matter is of additional importance. The rivets are already in tension and at the yield point. The flange of the girder and the leg of the angle are in compression. If stress is added as indicated on the end there is no increase in stress in the rivet until an added stress greater than yield is reached. The tensile stress must overcome the compression between the flange and angle and open up the joint.



Lately many have advocated bolting. Testings were made by Frank Baron at Northwestern University giving a comparison of action at joints between bolts, hot driven rivets, and cold driven rivets. This matter of rivets can be discussed for a long time. There is elaborate theory in texts on whether the first rivets take out some stress, the next some more, and so on. This is not an important matter and it may be disproved by further testing.

However, an important matter involving joints which receives considerable attention is the action of joints in effecting continuity between members. If continuity is desired much attention should be placed on detailing connections. In the connection diagramed earlier,

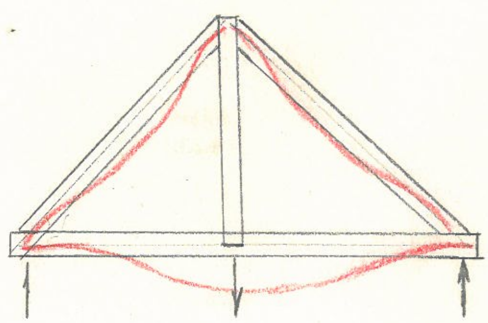


the angle itself does a good deal of bending. There is a possibility of fatigue cracking.

It is possible to go into all sorts of elaboration on detailing; to some extent in concrete, a great deal in steel, and even more in timber.

There is much theorizing about continuity. Some people think because two things are connected together there will be continuity. Polarized light may give some clue as to the action of joints.

SECONDARY STRESSES



The effect of the bending stress on the axial is very small. The fixed end moments necessary to cause that deflection can be figured by moment distribution. If the girder is stiff then the vertical stress is reduced but not very much.

The literature falls down on secondary stresses. More significant than the questions the texts suggest are:

1. Are stresses in the member important?

2. If of some importance is the member on the safe side?

3. If it is do you want to take advantage of it by making it smaller? If not, forget about it.

4. If the member is on the unsafe side, what can be done about it?

As can be seen from the above question, there is no interest in how big the stresses are.

There are three remedies for taking care of secondary stresses thatare too large.

1. Bolt up the end joints, allow them to slip under load and then rivet.

2. Set the lower chord high by detailing the hanger short. The load bends the chord down so that there are no secondaries. This is frequently done. The Sciotoville Bridge is an example of this.

3. Cut down the depths of the members on the ground. The deflection varies as the bending stress divided by the depth. The deflection times the depth is proportional to the fiber stress.

Two terms have been introduced: participation stresses, and secondary stresses.

<u>Participation Stresses</u>

The member participates in the action of another member. For example, where double diagonals work with the chords they produce some effect on the chords.

<u>Secondary Stresses</u>

These are additional stresses, unwanted but seldom important. An example is that of the truss which due to semi-rigid connections has bending in its members.

The secondary stress term can be extended and given a broader meaning. If a member has secondary stress and, because it is connected to another member, tends to make the other member move, if the other member is flexible it will move easily and not be stressed, and if the other member is stiff, it will not move and will be able to take the stress.

In the case of the two-legged bent if the columns are made stiff there will be action ofthe column on the girder and appreciable stress on the column. There is a somewhat important mutual action which can be called secondary stress. On a rigid frame, on the other hand, the action of the column is to take the moment and then it is called a primary stress. As was said when discussing the First Joint Committee's report on concrete, they did not talk about bending incolumns. The columns were designed to take only axial load. If the column is made to take the action of the girder it is a primary action. In the case of the moderately stiff column the stress that is caused by bending is a participation stress.

There is too much attention given in texts to precise analysis and not enough on interpretation. Uninterpreted moment distribution will yield nothing.

The main trouble concerning indeterminate structures when you look over literature about it is that the literature tries to talk about indeterminate structures as though they were determinate. The structure is given by the author. The questions asked are: What is the stress is bar A? What is the moment in section mm? -- instead of: How did the structure get indeterminate? What is the extra member? For example, the two legged bent with braces of the preceding lectures can be any one of four or five structures.

The structural engineer is continually confronted with the following questions:

1. What is the scale?

2. If it is serious or important, is it an advantage or disadvantage?

3. If it is an advantage, can anything be saved by it, can money be made because of it?

4. If it is a disadvantage, do you want to do something about it? If so, what?

It is true that exact analysis may have to be made if the various codes so specify. However, for many structures, particularly the large ones, there are no fixed codes, but just general recommendations.

There is a tendency to get things standardized (although this is probably not so very good) as a result of the influence of large commercial organizations such as the A.I.S.C., the Iron and Steel Institute, and others. The university is a principal subject for these industries' infiltration tactics.

Deformation Stresses

These are stresses produced in one member because another member to which it is connected deforms.

Secondary Stresses

These are stresses that occur in a bar because of rotation of joints in a truss when the truss deflects. It is an unwanted stress.

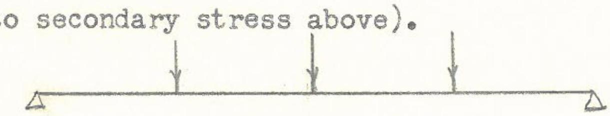
Participation Stresses

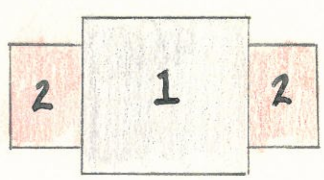
These are stresses that occur in a member because it is participating in the action of another member. An example is the diagonals of a truss which stretch and shorten because of the movement of the chord.

Primary Stresses

These are stresses for which the member is designed to take, i.e., the member is doing what you want it to do.

Consider two beams, a primary beam and a secondary beam (no relationship to secondary stress above).





If we were to analyze this beam arrangement the first thing to ask is how and why the secondary beam got there. It may be because some other member framing into the primary beam rests on the secondary beam.

Textbooks immediately ask that the maximum stress be determined. This can be done by finding the maximum moment, computing the moment of inertia of the entire section, and solving

$$f = \frac{Mc}{I}$$

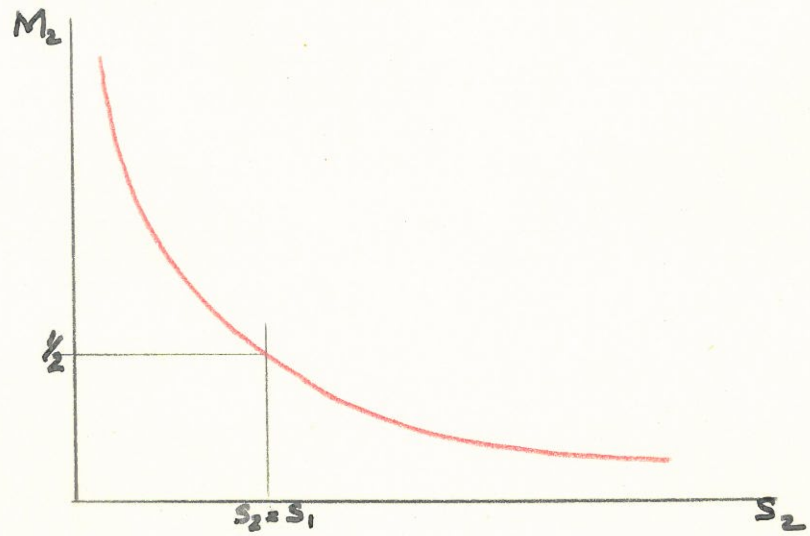
Analysis can be done by means of moment distribution. Find the moment in the whole member and distribute the moment according to the stiffness of the two beams. Stiffness is the ratio of force or moment and movement or angle change.

$$\frac{M_2}{S_2} = \frac{M}{S} = \frac{M}{S_1 + S_2}$$

$$M_2 \left(1 + \frac{S_1}{S_2}\right) = M$$

where M, the total moment as determined by statics, is constant

This expression is a hyperbola



Two cases could be considered:

(1) The area of the secondary beam is constant and the depth variable, or,

(2) The area is variable and the depth constant. We will consider the first one. If the area is constant, the depth varies and the secondary takes all of the moment.

$$f_1 = \frac{6M}{Ad}$$

Since the moment and the area are constant

$$f_1 \propto \frac{1}{d}$$

Remember that f_1 is the fiber stress in the secondary when the secondary takes all of the moment. This can be plotted as a hyperbola, too.

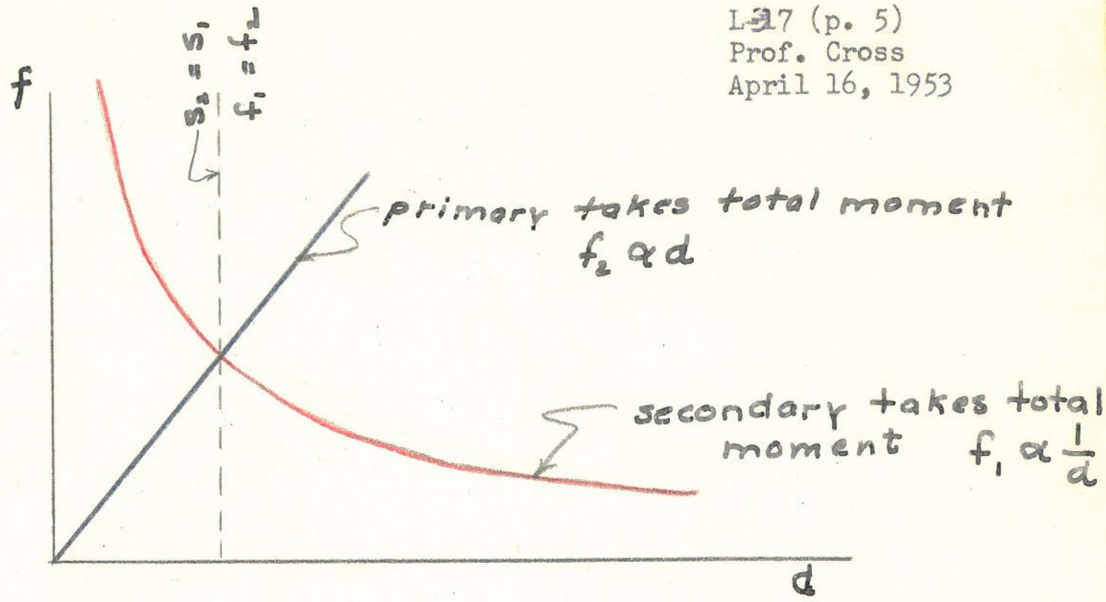
If the secondary takes no moment and the primary takes all the moment, the secondary will follow the beam,

$$\phi = \frac{\Sigma f}{Ed}$$

so that the fiber stress in the secondary, f_2 , is proportional to the depth.

$$f_2 \propto d$$

This can be plotted as a straight line.



The true stress in the secondary, $\underline{f}$, varies with the stress in the secondary if the primary took all the moment, f_2, and the stress in the secondary if the secondary took all the moment, f_1, and is expressed by the following equation:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

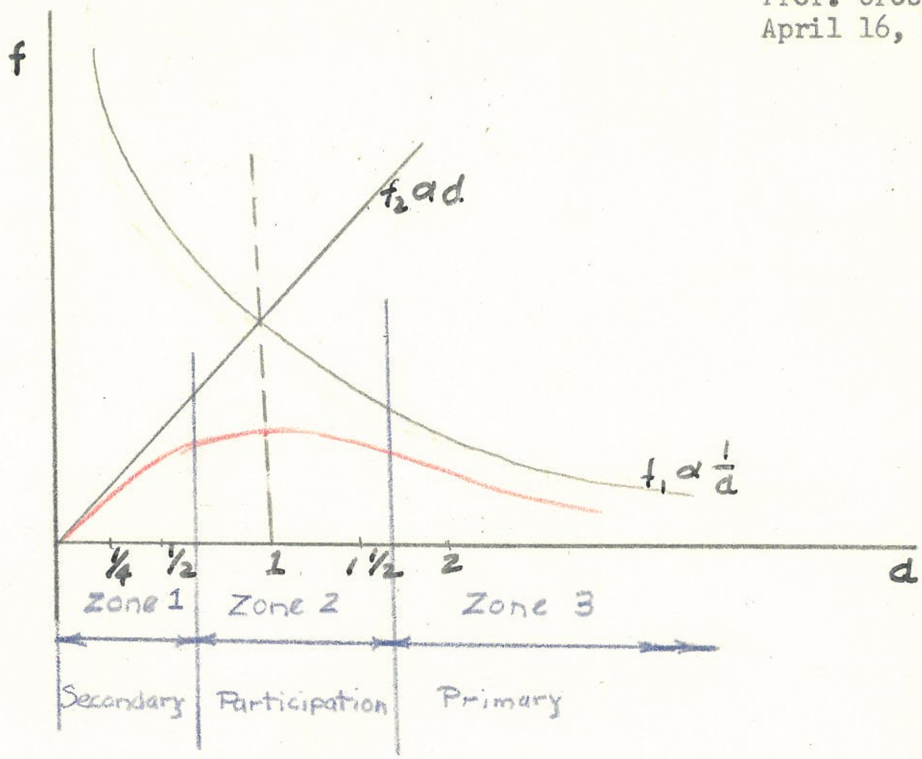
When the stiffnesses of the two beams are equal ($S_1 = S_2$) the fiber stress in the secondary, f_1 and f_2, are equal. Transform the equation.

$$f = \frac{1}{\frac{1}{f_1} + \frac{1}{f_2}}$$

A curve of some significance can now be plotted. Take the depth when the stiffnesses are equal as unity, and since $f_1 \propto \frac{1}{d}$, $f_2 \propto d$, and $f_1 = f_2$

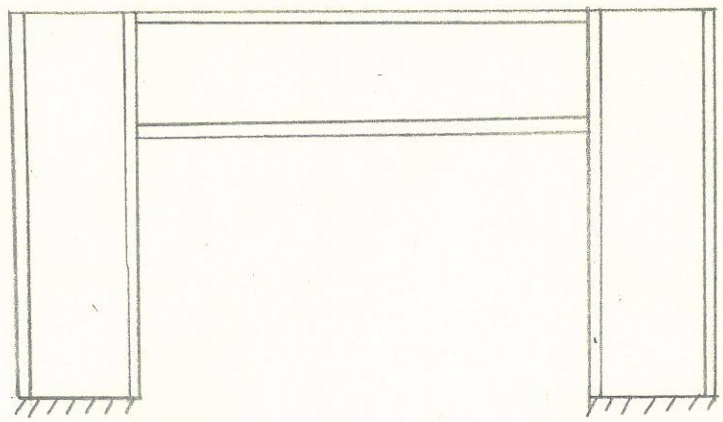
$$f = \frac{1}{\frac{1+1}{f_1}} = \frac{1}{\frac{1+1}{f_2}} = \frac{1}{2} f_1 \text{ or } \frac{1}{2} f_2$$

$d = 1$	$f_1 = f_2$		$f = \frac{1}{2} f_2$ or $\frac{1}{2} f_1$
$d = 2$	$f_2 = 4f_1$	$f = \frac{1}{1+\frac{1}{4}} f_1$	$f = \frac{4}{5} f_1$
$d = \frac{1}{2}$	$f_1 = 2f_2$	$f = \frac{1}{\frac{1}{2}+1} f_2$	$f = \frac{2}{3} f_2$
$d = 1\frac{1}{2}$	$f_2 = \frac{9}{4} f_1$	$f = \frac{1}{1+\frac{4}{9}} f_1$	$f = \frac{9}{13} f_1$
$d = \frac{1}{4}$	$f_1 = 16 f_2$	$f = \frac{1}{\frac{1}{16}+1} f_2$	$f = \frac{16}{17} f_2$



The true stress curve can be divided into three zones. In Zone 1 the secondary acts as a secondary and is stressed almost entirely because it deforms. Therefore, it can be said that here the stress in the secondary varies directly as the depth. If the stress in the secondary is too large cut down the depth and forget about it. In Zone 2 the stress is practically independent of the depth. Here the members are acting together to take the load. Since the depths would almost be the same, the possibility of making just one beam should be considered. If it should be decided that is not worthwhile to make the two beams act together, increasing the depth of one beam relative to the other will make the situation similar to Zone 1 or Zone 3. In Zone 3 the secondary acts like a primary.

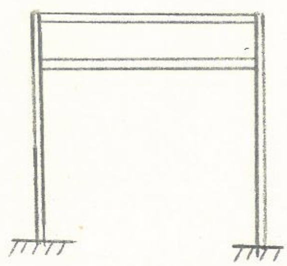
There is not much practical importance to this problem but it does illustrate an important principle. A more practical case is the two legged steel bent.

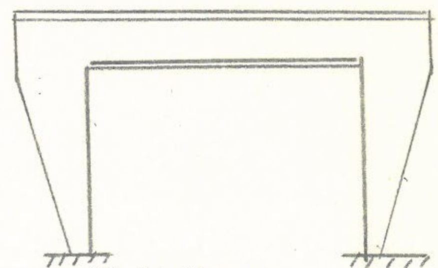


This bent is typical of those supporting elevated railways. The engineers who designed them designed them so that a girder was a girder, a column was a column. Unfortunately, though, the engineers went to college and found out that there were moments in the columns. If the girder is there to take the load to the column, and the column is a column, any bending in the column is secondary. However, the column gets to be such size that they develop appreciable bending resistance.

Assume fixed area and variable depth in the column. If the column is so wide the stiffness in the column is comparable to the stiffness in the girder, the bending stresses will be considerable. If the stresses in the column are big there is not much that can be done about it. If the column is made bigger and bigger it will be an indeterminate structure, a rigid frame.

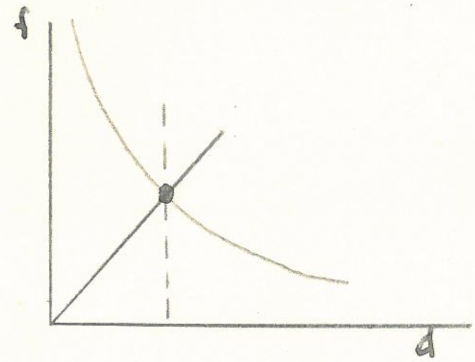
L-17 (p. 8)
Prof. Cross
April 16, 1953


Column and Girder

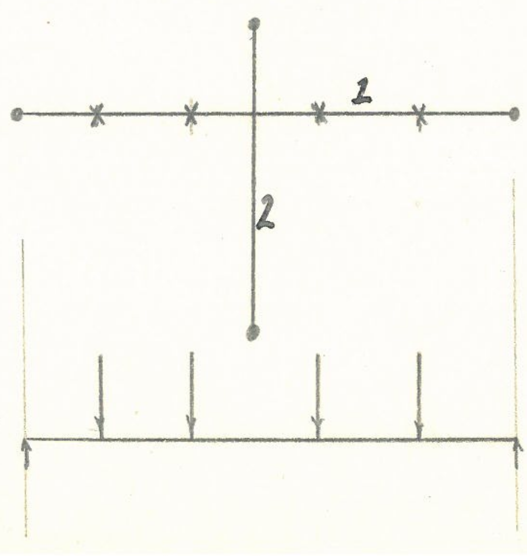

Rigid Frame

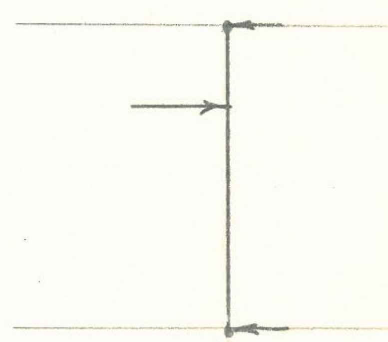
In between the column and girder, and the rigid frame is participation.

A scale on participation can be obtained. Assuming the area of the column to be fixed, at what depth of the column will the stiffness, M/ϕ, be the same in the column as it is in the girder? If the stiffnesses are the same the point indicated below would be the same.



Another problem is that of two intersecting beams shown below.





The primary beam (1) carries the load. The secondary beam (2) is a reaction for the primary beam making the primary beam a continuous beam. Is the secondary doing its job of carrying the reaction from the continuous beam? If the secondary is not satisfactory, i.e., too flexible it cannot carry the reaction and the continuous beam is not continuous.

The stiffness of the secondary relative to the primary member will determine whether the secondary member is doing its job of making the primary beam continuous (Zone 3), not doing its job and making the primary beam a simple beam (Zone 1), or someplace inbetween where it is merely helping out (Zone 2). It is not advisable to be in Zone 2, the zone of participation.

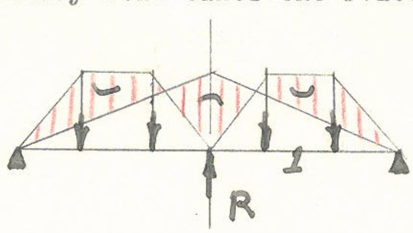
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

f - bending stress in secondary beam.

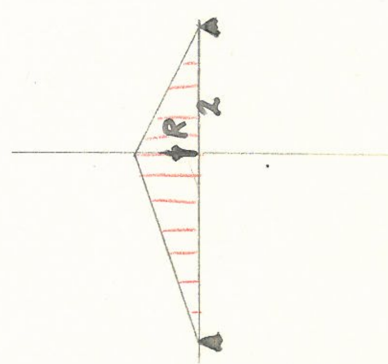
f_1 - bending stress if secondary took the reaction.

f_2 - bending stress if the secondary did not take anything.

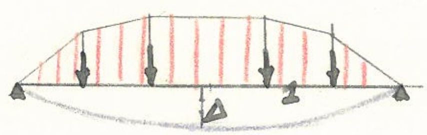
Secondary beam takes the reaction.



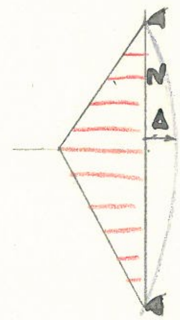
$$f = \frac{M}{Z}$$



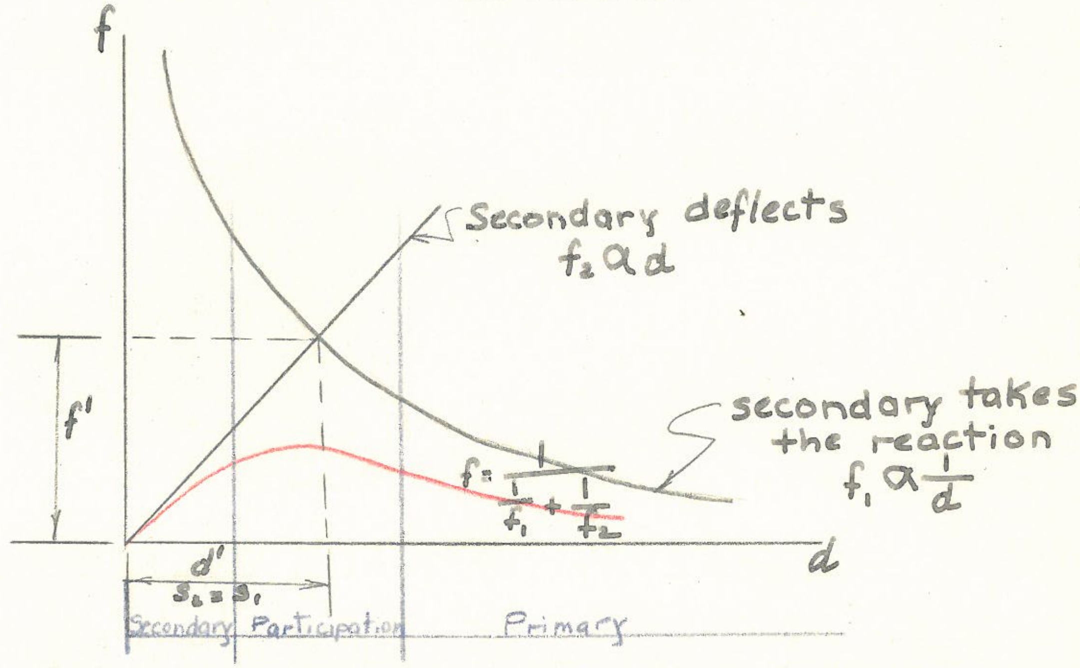
Secondary beam takes no reaction.



$$\Delta = \frac{\Sigma f}{E}\frac{l^2}{d}$$

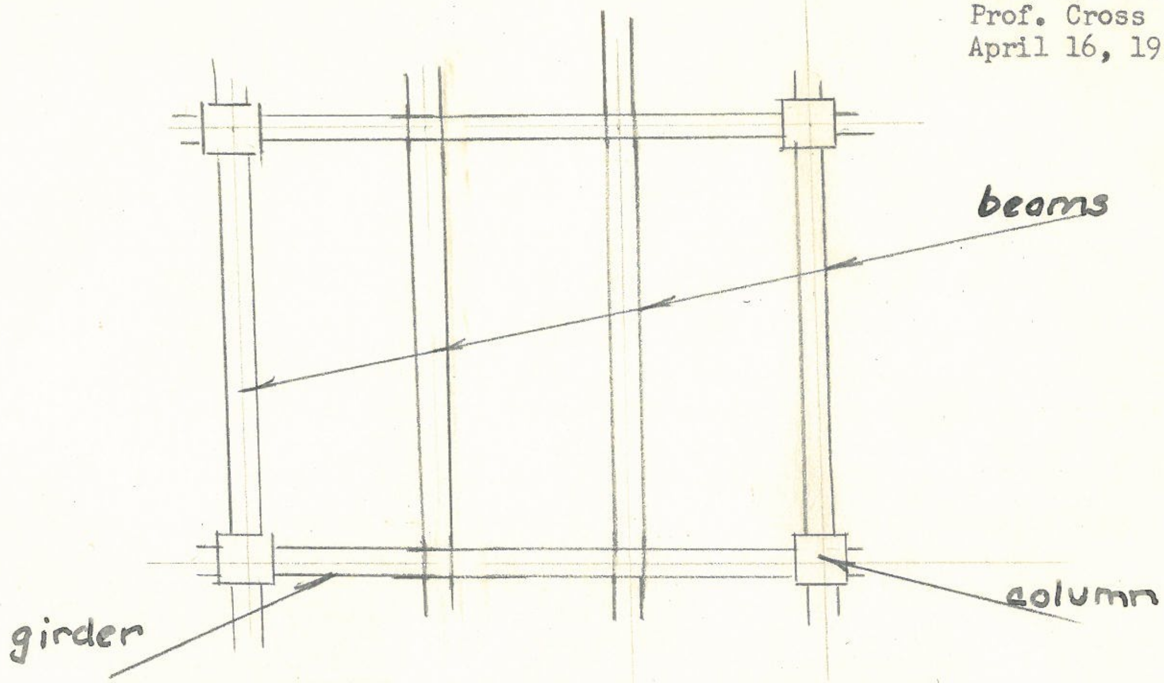


$$\text{Stiffness} \propto \frac{\text{Reaction}}{\text{Unit Deflection}}$$



If in the range of secondary action make the plank thinner to reduce the stress. If the secondary is doing the job of carrying the reaction from the primary, make it very stiff.

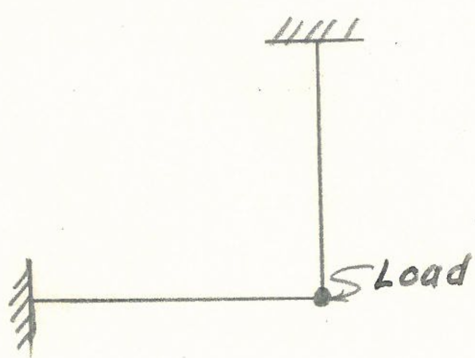
A similar situation occurs in the slab, beam, girder system in concrete.



The question is whether or not the interior beams which are somewhat larger than the girders can be called continuous if its supports, the girders, yield. Do we have here a ribbed flat slab or a slab-beam-girder affair. The girder must be a lot stiffer than the beam for it to be continuous. Looking at the layout we can put some judgement as to where we are.

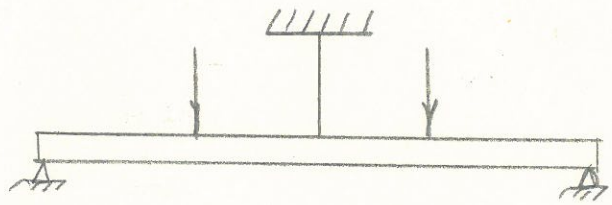
The following problem can be found discussed in the transactions of the ASCE.

Plan View



If the two members are of comparable stiffness the distribution will be equal. If one is flexible it will give no support to the other and the other will act as a cantilever.

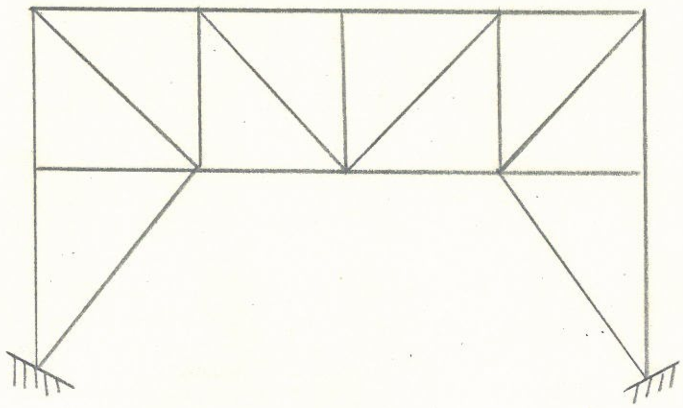
Another common problem is shown below:



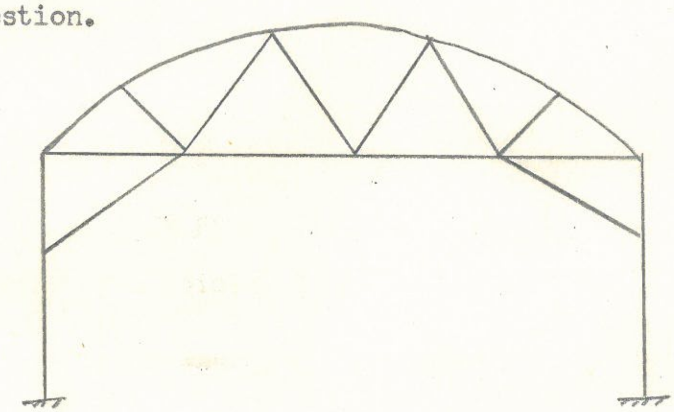
If the vertical member is very stiff then the horizontal member will be continuous. If the vertical member is a thin wire it will deflect with the girder, giving no support at all. We are not interested now in the algebra of the thing. Just plot a scattering of points between both extremes. Somewhere, perhaps, there is a "golden mean" between stress if it did the whole job and stress if it just took the strain.

The reason for all this is to put reality into the three classes of structural action. These can be extended to include varying of the area while keeping the depth constant, varying the depth and keeping the area constant as was done here, and varying both the depth and the area.

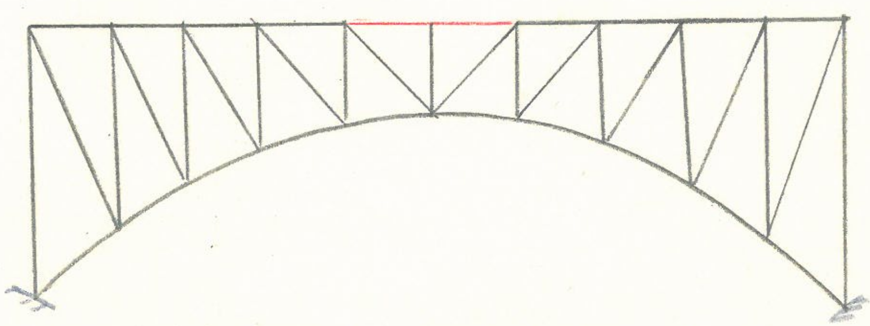
These concepts may be applied to the structures discussed in the lectures concerning bracing.



If only one brace is put in the structure it is still determinate, but if two braces are put in, as shown, the structure is indeterminate. There will be considerable compression and interaction in the members in question.



Ordinarily, there is no reason to be concerned about the knee braces. If the knee braces are slender enough they will buckle when the truss deflects. This is not now a wind stress problem.



next page

(pg 13)

Originally this was a three hinged arch but the member is put in to prevent movement of the deck. This member will affect all the other members of the structure. This is usually not important for dead load but is important for unsymmetrical live load.

This discussion is leading to the subject of proportioning, designing, and dimensioning of stiffening trusses for suspension bridges.

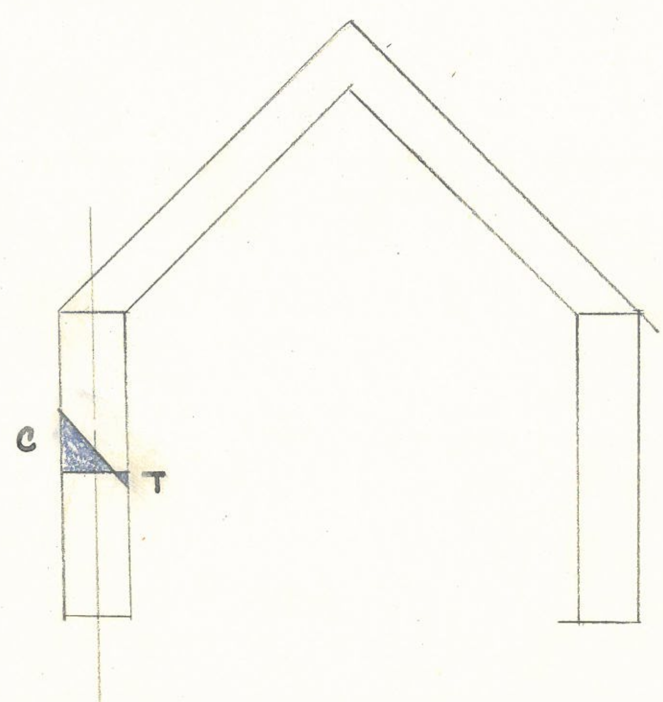
L-38 (p. 1)
Prof. Brennan
April 23, 1953

PRESTRESSED CONCRETE

This subject has been very much in prominence lately. In fact, there seems to be more literature than structures. It got its impetus because of the shortage of money and of material particularly in other countries.

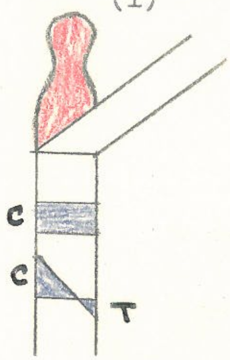
What is prestressing? It is a means of locking in a stress that will offset a detrimental stress. If by some means (usually patented) you lock in a tension then when compression comes into play the tension will be counterbalanced. Usually prestressing is done in reverse. It is compression which is locked in because of a detrimental tensile stress.

Actually, there is nothing new to prestressed concrete. Prestressed concrete was patented 60 years ago. Many masonry (usually stone) structures would not have been constructed except for prestressing. Consider the buttress.



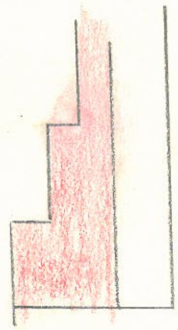
There were three methods of remedying the situation above:

(1)



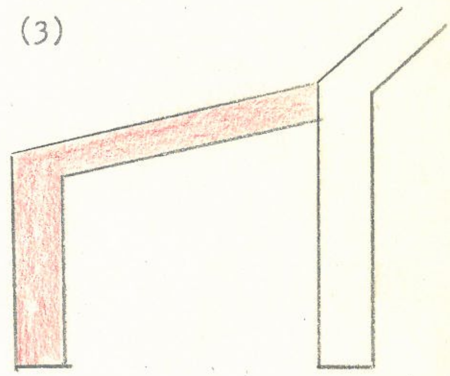
Increasing the height of the wall by adding stone forming fancy figures

(2)



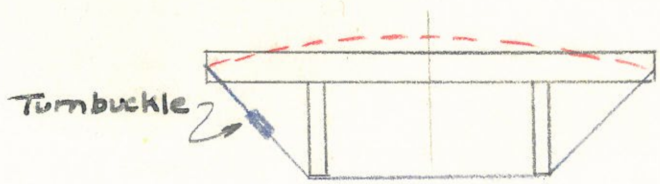
Stepping the wall.

(3)



Using a flying buttress.

Prestressing was used in early steel and timber structures, too. Examples are the king and the queen post truss.



The turnbuckle was used to prestress. With the dead load on, the deformed structure would return to its original position. This is as old as the use of wrought iron.

Even in modern steel structures this idea is utilized. Lindenthal at Sciotoville expected large secondary stress so he bent the members opposite to what he expected. He put in enough bending to offset the bending caused by dead load.

However, the obvious thing to prestress are materials that cannot resist both kinds of stress. This includes all standard masonry: brick, stone, concrete, and tile. It should be remembered that if it is applied to concrete there is no relationship between this prestressed concrete and reinforced concrete. Application of prestressing to concrete had a push because of the shortage of steel and the high cost of material. Elsewhere, labor is cheap, material expensive, so that they want to use as little material as possible. If a long span is what is desired reinforced concrete will not work but prestressing will work. This prestressing is capable of reducing stresses and in so doing reduces the volume.

In reinforced concrete as soon as the steel gets into tension there will be some cracking. Cracks cannot be avoided. If the strength of concrete is increased say from 3000 to 5000 psi and the same steel stress is used, the strength of the member is still limited by the bond, the tying together of the steel and concrete, or by the stress in the steel. If the strength in steel is increased, the strength of the member will still be limited by bond since following cracking the bond breaks down and the member fails in diagonal tension. So, in reinforced concrete, even if you want to use high stress in steel and concrete, it cannot be used to advantage since you will always be limited by imbedment.

With prestressing on the other hand, high strength concrete and high strength steel is used. This means that less cross section area is required so that there will be less dead load, more live load can be permitted, and longer spans are possible.

Arguments concerning prestressed concrete have been between the reinforced concrete concerns and the prestressed concrete interests so far, but soon steel interests will act up more. There already is such a thing as prestressed steel. Prestressed concrete concerns have carried on a vigorous propaganda campaign, but they have not justified any saving of money. An advantage to prestressing is the potential use of precasting. This offers the advantages of assembly line production and elimination of formwork and falsework. The pieces can be ordered by catalogue, shipped by truck and erected very quickly by crane.

Interest in prestressed concrete is from four standpoints:

1. To eliminate cracking.

   Light posts are prestressed to eliminate unsightly shrinkage cracks and to prevent deterioration.
   Note: it is not used for strength.

2. For handling purposes.

   Concrete piles are prestressed so that they can be picked up longitudinally and transported. Often, before driving, the prestressing is removed.
   Note: it is not used for strength as regards the purpose for which the pile was intended.

3. To make structure impervious to fluid.

   Concrete tanks are not very good because there is some leaking. So, wire is spun around in tension putting the concrete in compression, preventing cracks from opening up and in so doing making the structure impervious.
   Again prestressing is not used for strength purposes.

4. Prestressing of concrete flexural members.

   In such a member, three things are wanted:

   a) not to have at any time tension in the concrete (or at least not more than very, very little).

b)Not to have the concrete overstressed in compression.

c)Not to have the steel overstressed in tension.

Consider a girder.There are four steps:

1. Proportioning.

   How deep? What shape? Where will the cables be placed? What force-should be put in the cables? What are the end conditions?

2. Analysis.

   This step is relatively simple. What are the moments, shears, and stresses?

3. Interpretation.

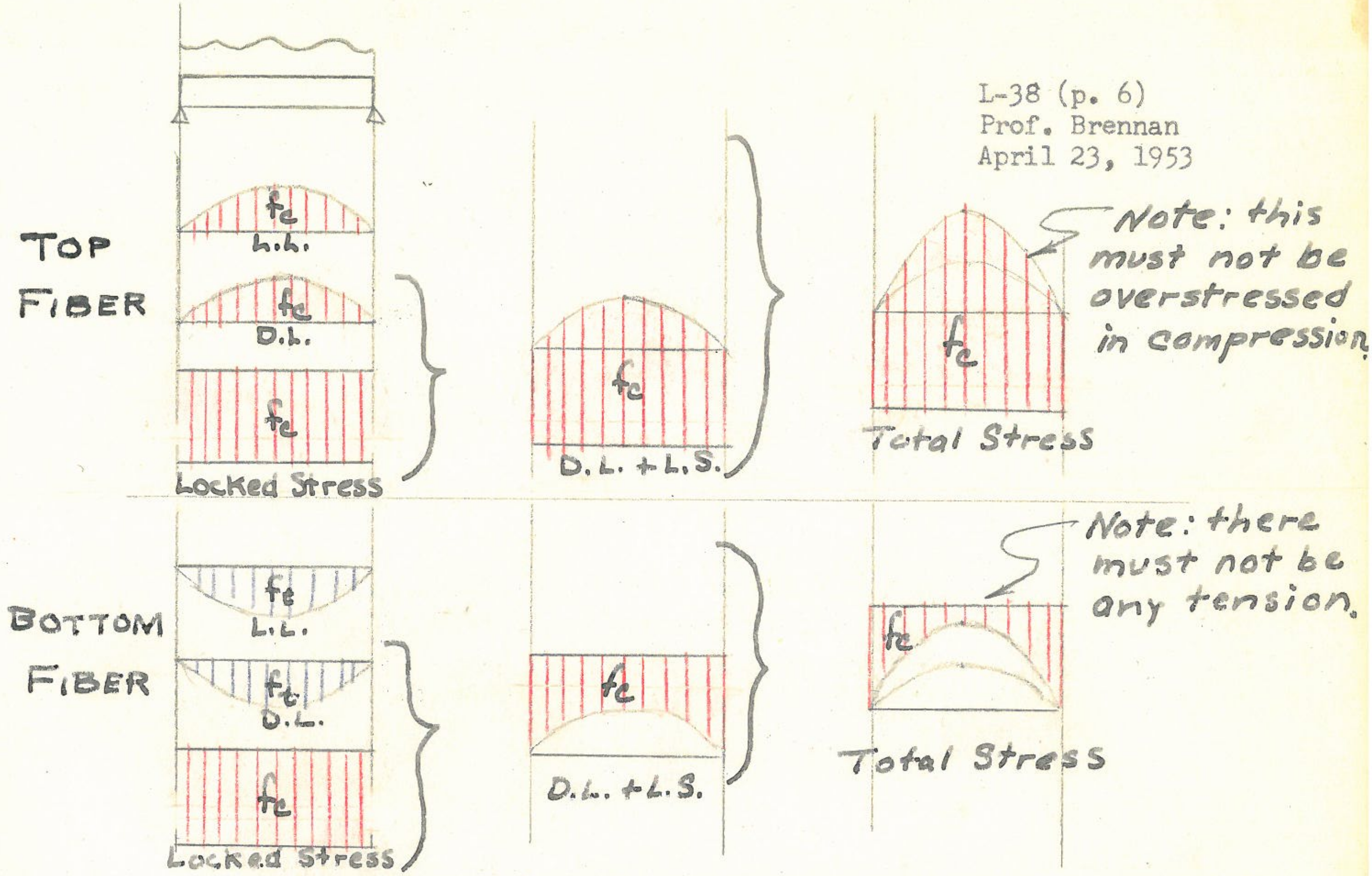
   This is very difficult. The stress conditions must be interpreted but there is very little knowledge concerning this.

4. Construction.

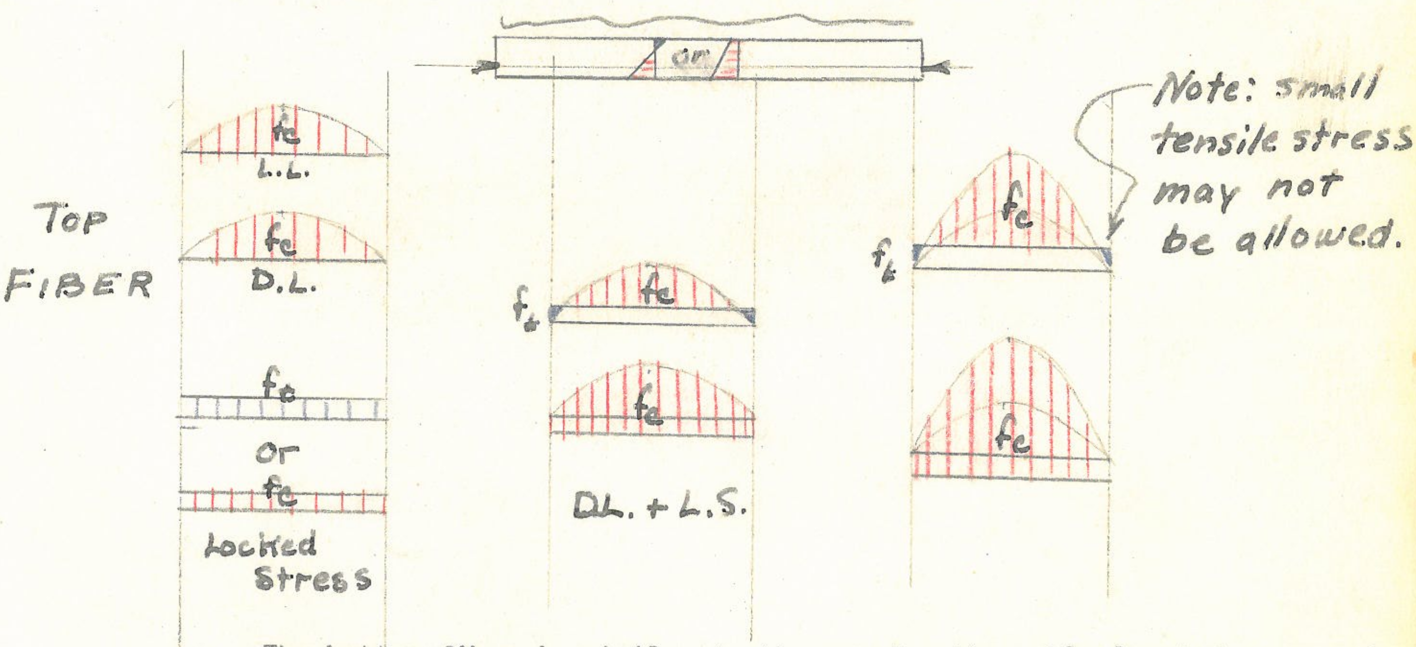
   Construction techniques are very difficult.

Consider the analysis of a simple prestressed girder.

Prestressing can be accomplished by means of jacks on both ends, or better still cables put inside so that it will be self contained.



The location of the cable can be moved to make it more efficient.

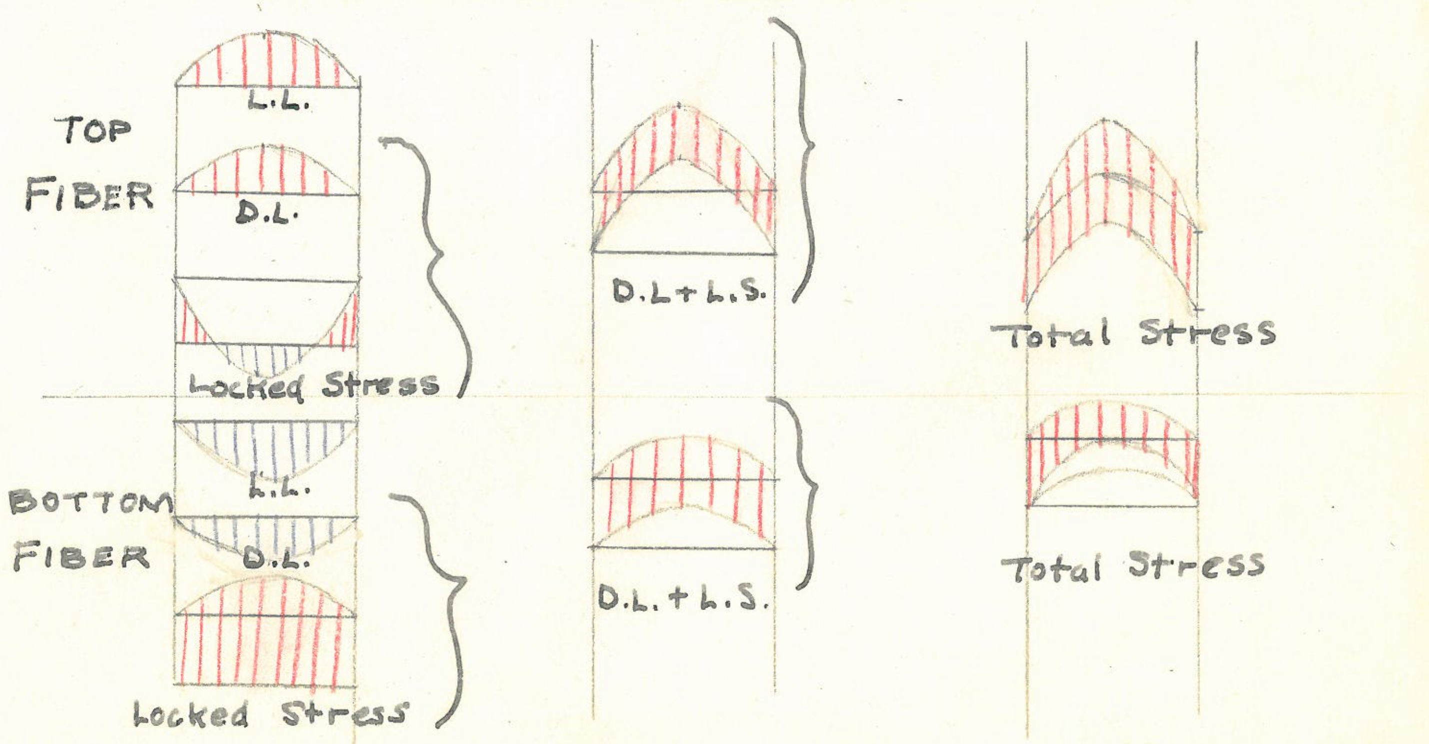


The bottom fiber is similar to the one for the cable located in the middle except the compressive stress will be larger.

Going further a curved cable can be put in. If the cable takes the shape of a quadratic parabola, it will carry a uniform horizontal force. Since this is true the moment will vary as the distance from the center of the beam.

Consider the top fiber. The axial load due to the cable is a rectangle. The eccentricity of the cable gives a parabolic distribution.

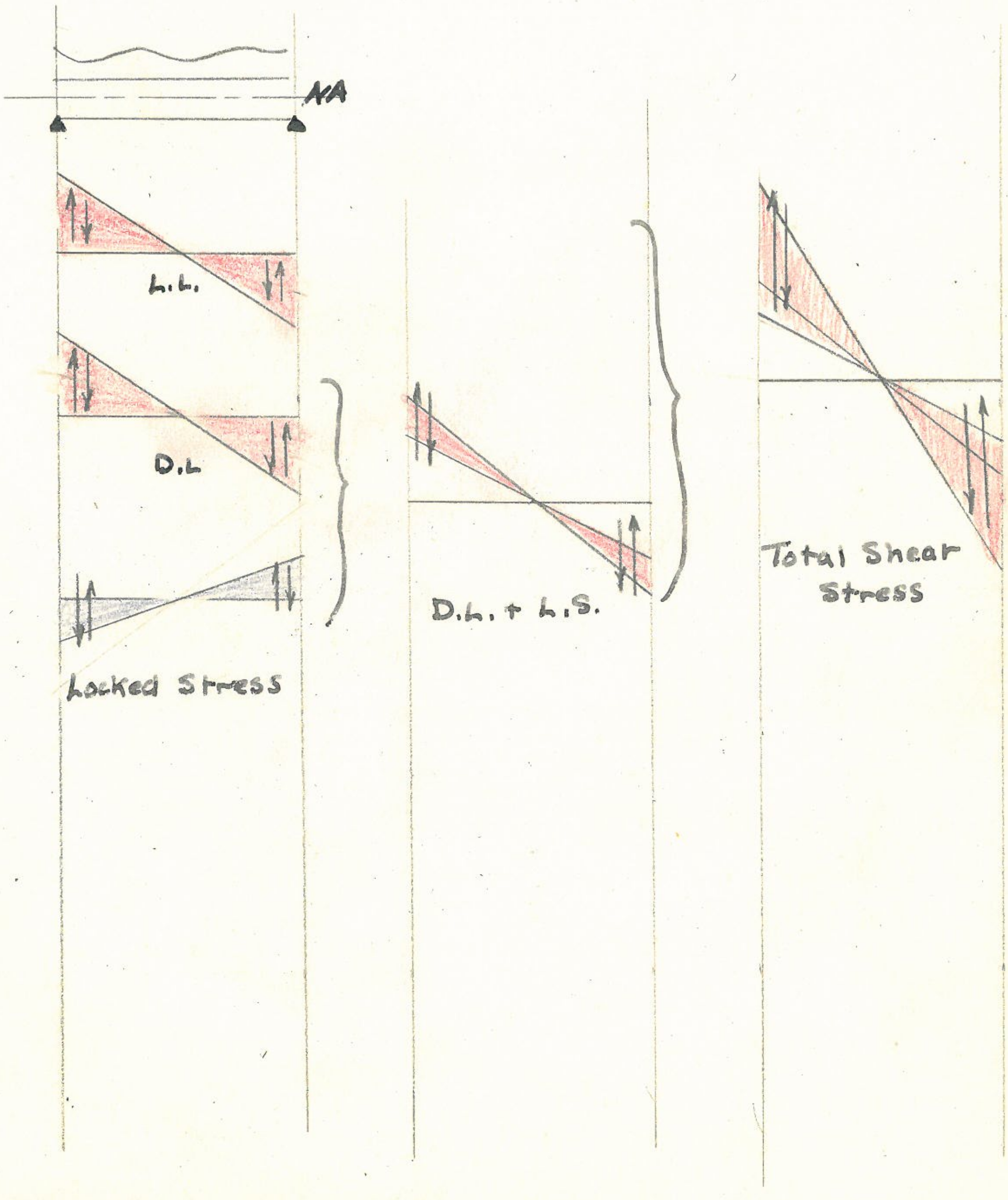
It can be seen now that the curved cable eliminates the tension at the ends that the eccentric cable alone discussed before causes.



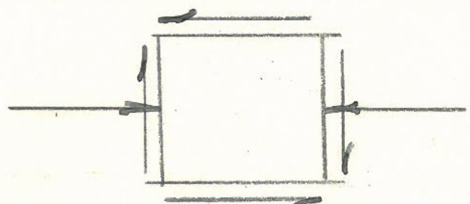
The question of shear must be considered in this problem too. The shearing stress distribution along the bridge varies as:

$$\gamma = \frac{VQ}{Ib}$$

$\frac{Q}{Ib}$ is constant along the neutral axis. Consider here the same beam with the cable through the center.



At the center there is no shear and the only thing to consider is the diagrams for fiber stress. However, at the ends of the beam a different situation occurs which requires some interpretation. Consider an element near the end of the beam.

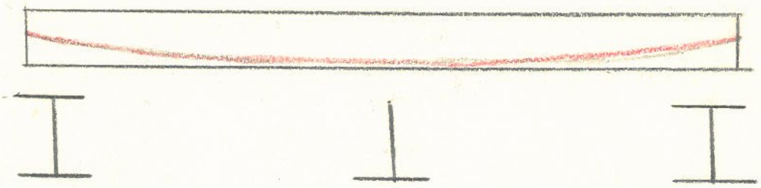


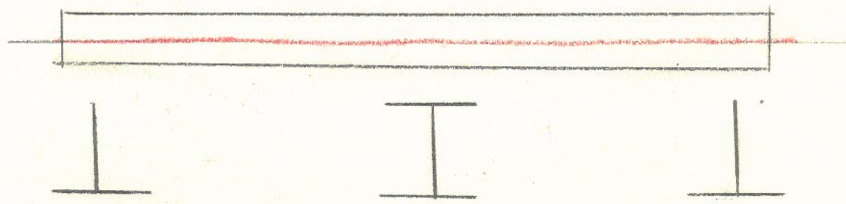
The maximum principal stress must be obtained by means of Mohr's Diagram or the Diadic Circle.

The analysis is simple from now on. Once the live load, dead load, and the location of and force exerted by the cables is known the stress can be determined. Because the beam is being compressed from both ends there is a possibility that buckling will occur and it may be necessary to support laterally. The stress in the steel can be obtained from the horizontal force and the area of the steel, and it should then be checked against the allowable.

The locked stress can be varied by moving the cable or changing the neutral axis of the beam. Instead of curving the cable, the beam can be made curved.

Changing the section throughout may give a better stress distribution.



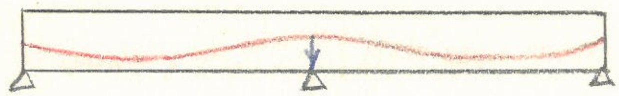


This principle can be extended to continuous structures. The problem is in the location and shape of the cable so that it follows the curve of maximum moment. There has been much talk on the shape and location of the cable. Books tend towards involving very high mathematics. However, it can be done by drawing moment diagrams and pressure lines.

There are two problems in the analysis of continuous prestressed concrete beams.

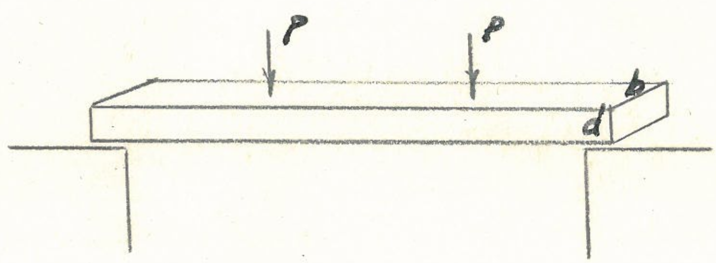
1. Where is the maxima where there is a moving load? With a prestressed beam there will be a high ratio of live load to dead load, because the prestressing cuts down on the dead load. There will be some difficulty with live loads.

2. When the cable is put in on a curve there will be difficulty with the reaction. The cable will tend to push down on the center support.



The analysis is relatively simple. It is the interpretation that is difficult.

Interpretation



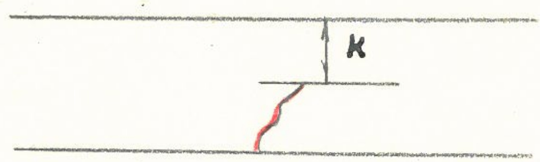
Suppose the beam above is put in a testing machine and loads are applied. Suppose when the first crack opens up is called failure and the shear and stress at that point is figured. Then a formula for failure is selected.

$$P = V$$

$$v = \frac{V}{bjd}$$

Many readings are taken and v is determined to be 60 psi. Using another strength concrete, v is determined to be 90 psi. This formula can be used and compared against laboratory results so that using the formula is perfectly safe even if it is entirely wrong.

Now suppose some one cam along and said that after the beam cracked the effective area would be less,



so that the formula should be

$$v = \frac{V}{bkd}$$

Now there is nothing to go on. Thirty years of experimenting is thrown out. This new theory is worthless unless it is tested in the laboratory.

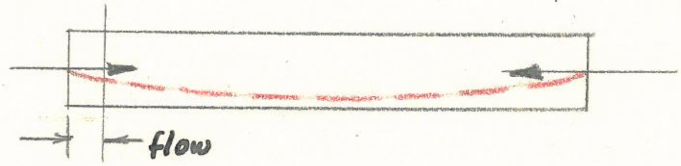
The same thinking is extended to prestressed concrete. Very few structures have been built of prestressed concrete. It is quite expensive to build strctures and test them to failure. In Europe they built structures to failure but this was not done very extensively. As a consequence there is not much data on what allowable stresses to use on prestressed concrete. Stresses developed for reinforced concrete cannot be used for prestressed concrete. Nevertheless prestressed structures are used even if nothing is known about it. There have been many tests run to determine the allowable f_c or f_t but it will be many years until good allowable stresses will be developed. Remember, work on tanks cannot be used for this.

Also, there is the problem of impact. Steel stretches and then returns to its original shape. Brittle materials with high stresses locked in may shatter under impact.

The fatigue problem may be important. No testing has been done on it. P.C.A. recommends the use of prestressed concrete but its specifications are still very conservative.

The most important thing which nothing is known about is <u>shrinkage</u> and <u>time yield</u>. Some venture to say how much it is but they do not agree at all.

Why are we worried about time yield? Suppose we put in a cable a certain stress, say 60,000 psi.



If the concrete flows the steel loses stress. Suppose the flow is 0.001 inches per ~~year~~ inches per year

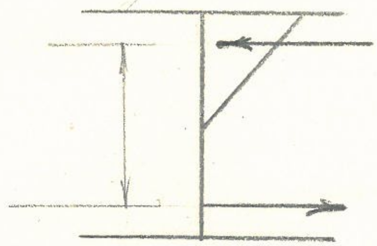
$$S = Ee = 30{,}000{,}000 \times 0.001 = 30{,}000 \text{ psi}$$

Therefore, the loss in the steel is 30,000 psi and only 30,000 psi is left. Suppose the stress in the steel originally is 150,000 and the time flow is the same (which it is not), then the stress that is left is 120,000 psi. Time yield and shrinkage cannot be separated. Time yield is not linear both as to load and time.

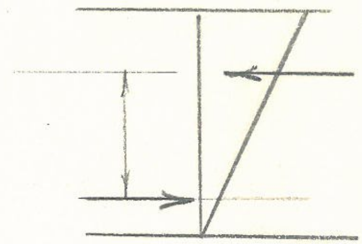
It has been advocated that a percentage be taken of the working stress to allow for time yield. Specifications by an engineer must be made for the structure. It must be said how much stress in steel and in concrete is allowable and the percentage reduction.

To get high strength, a very small wire must be used since the process of cold working to increase the working stress can be done only in small diameter wires. Galvanizing reduces strength of steel.

Concrete Beams



Not prestressed



Prestressed

If the same working stress is used in both beams more steel is required in the prestressed concrete beam than in the other because the lever arm of the prestressed beam is smaller.

Construction

Construction of prestressed concrete involves very fussy work. The forms are quite fancy particularly with beams of variable cross-section. High strength concrete requires a low water to cement ratio so that the mix is very stiff. This means that vibrating is necessary and vibrating causes segregation.

There are two kinds of prestressing.

1. Pretensioning

   The wires are stressed before the concrete is poured.

2. Post-tensioning

   The wires are stressed after the concrete is poured.

If it is post-tensioning that is done the problem is how to put the hole in the members. If a rubber tube is used it may deform. The end blocks must be fancy so that there is no leaking out of the stress. There are many patents on this.

The object is to have the same stress on each bar. Some bars are curved others will be straight. There are quite a few bars and only a few at a time can be stressed. Further they must be stressed symmetrically so that a moment will not be induced into the member. In order to have the same stress in each wire the value of the modulus of elasticity must be determined. A "sonar" method can be used to determine an average value for this modulus.

It is difficult to evaluate prestressed concrete on the basis of economy. Prestressing is competing with steel. Many comparisons have been made but they do not mean much unless the whole structure is compared, not just the girder, and if the best steel designer does the steel, and the best prestressed concrete man does the concrete, and the two are of equal ability.

Prestressing may be competing even more favorably due to precasting of entire members and also segmental members which can be used to advantage sometimes.

Suspension Bridges

Not much is known about the suspension bridge. The suspension bridge is like an arch, particularly a tied arch or bowstring truss. In an arch the earth furnishes the tension chord. It cost a good deal of money to put in unyielding abutments or a tie. The arch could be one hinged, two hinged, or three hinged. Something, however, must take shear. That is why we have spandrel braced arches, lattice arches, or stiff ribbed arches. Under unsymmetrical load an arch will deflect away from the pressure line and will be subject to buckling.

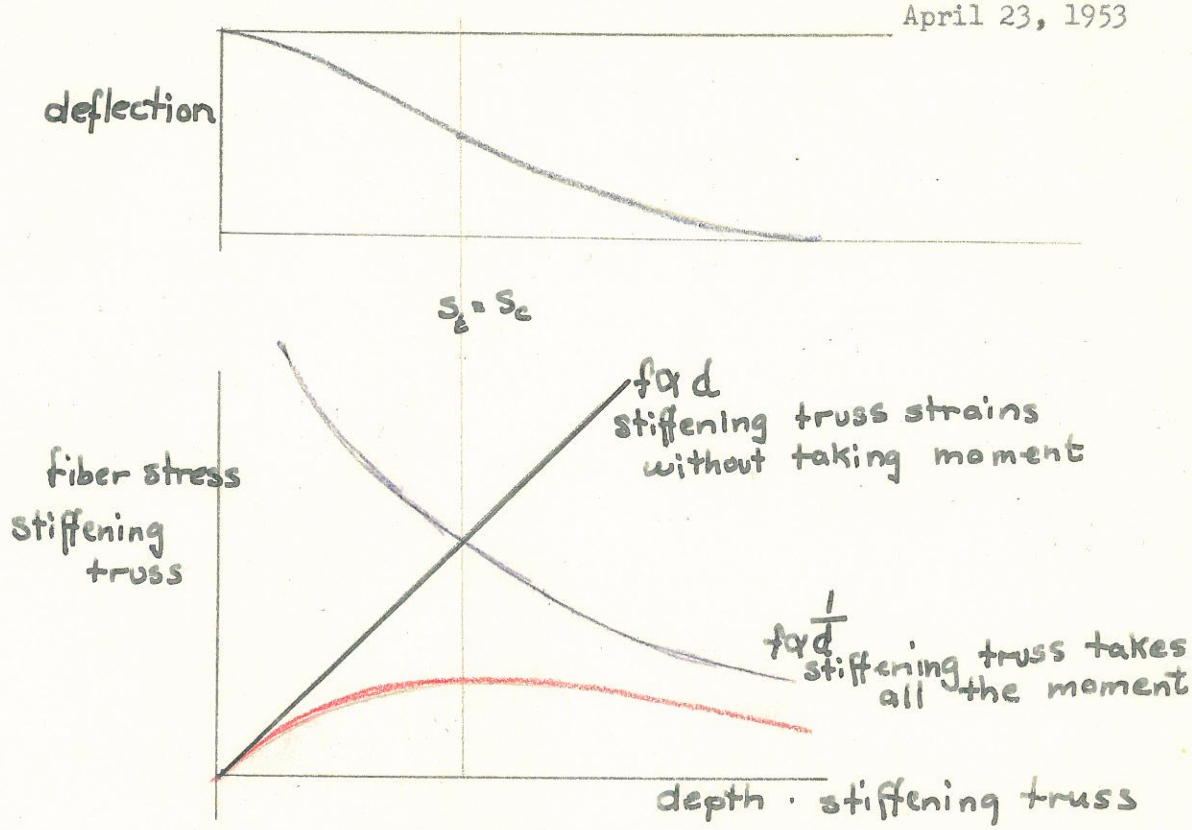
The suspension bridge is just the opposite of the arch since the curved chord is the tension member and the earth the compression chord. In some instances the floor or bottom chord is made the compression chord, but this cannot be done on long spans.

Large suspension bridges have spans almost a mile long plus approaches. From this the complexity of traffic and wind problems can be seen.

A suspension bridge consists of two approach spans, a channel span, anchorages for the cable, towers, the cable itself, and the floor. The towers and anchorage cost a great deal.

The cables of a suspension bridge automatically balance, i.e., they take the shape of the pressure line. If the cable is fairly tight because of a large dead load, the live load will not cause much deflection. However, this is not the usual case.

The original reason for the stiffening truss was to cut down the deflections the cable would have if it tried to adjust to the live load. Milan of Dresden suggested that it was not necessary to design the stiffening truss to take all the live load. Originally the stiffening truss was designed to take all of the live load like an arch upside down. The concept, then, of design was to design as though designing for moments in a two hinged arch. The arch rib took the thrust; the cable took tension. The truss, however, interacts with the cable. This is a situation similar to the elementary picture of the interaction of two beams bending together. The cable is not a beam but it does resist distortion because of the dead load. Milan apparently had no intention of making use of this. However, Moissieff took advantage of this effect and cut down the amount of steel in the stiffening truss of the Manhatten Bridge.



The deflection diagram can be called Moissieff's method. He did not actually know the diagram but he did figure stresses and work out deflections for various depths which constituted enormous mathematical work. Later, Hardesty and Wessman suggested by starting with stress due to strain variation working out the interaction curve.

If you want to save material you are going to lose stiffness. To make it stiff make it deep. On a long span it is very expensive to put in a deep truss and you cannot save material and get stiffness unless you make a <u>deep</u> truss. About twenty years ago they began to shift from the truss. The Port Authority investigated to see what would happen if there was no stiffening truss. The George Washington Bridge does not have a stiffening truss and yet there does not seem to be great flexibility. Shortly after that the trend was for relatively shallow plate girder stiffening trusses such as were found in the Tacoma Narrows before it fell down, and the Bronx-Whitestone as it was originally built. The Bronx-Whitestone had large deflections due to live load so it was stiffened by putting in trusses. The Golden Gate Bridge has a deep truss. The old Tacoma Narrows Bridge which had a plate girder lacked stiffness and had large deflections under live load. The Deer Island Bridge in Maine and the crossing at the 1000 Islands have a small span of about 800 ft.; had stiffening girders which are relatively shallower than a stiffening truss.

The serious trouble with these bridges came because of air effects, not just wind itself. The trouble did not occur necessarily a t high winds. The Tacoma Narrows failure occurred in a 35mph wind which is not much of a gale.

Since the suspension bridge is not new neither is the history of failure in suspension bridges. One of the earliest recorded failures is that of a foot bridge in Scotland. It was a 260 ft. span with a 4 ft. walkway suspended from inclined chains. Built in 1817, it was completely destroyed in a gale about 6 months after it was completed. Another Scottish Bridge built in 1820 had a 12 ft. roadway with two 3 ft. walks. Its span was 449 ft. and the suspended system was supported by 12 chains of eyebars, each 2 inches in diameter. It also failed in a violent wind after about 6 months' use. Details of these failures are not available.

Much better information is at hand concerning the difficulties experienced by the Brighton Chain Pier in Sussex County on the English Channel. This structure was built in 1823 and consisted of four spans of 255 feet each, supported by four chains of two inch round eyebars on each side of a 13 foot ± roadway. Ten years after construction it was partially destroyed by storm. The incident was repeated about three years afterwards only this time it was witnessed by Lt. Col. Wm. Reid of the British Army whose report and sketches were published. This failure was similar to the failure at Tacoma.

Sometimes dynamic failures of suspension bridges have resulted from causes other than the action of the wind. An English bridge built in 1829 sustained the weight of both foot passengers and carriages for 2 years without difficulty until a party of troops marched across the bridge, the rhythmic impact of their marching feet causing the structure to collapse.

A somewhat different experience occurred in the case of a 432 ft. structure at the South Esk River in Scotland which was built in 1829. Shortly after it was built about 700 people assembled upon it to witness a boat race. As the boats passed beneath the bridge, the people rushed simultaneously from one side of the roadway to the other. The resulting impact caused the chains on one side to break, and the loss of life was considerable. The bridge was repaired only to be destroyed 8 years later by wind. Rebuilt once more but more strongly it lasted 90 years until replaced by a more modern structure.

A somewhat larger structure, 580 ft. main span, was built in Wales in 1826. The bridge was damaged by gales during construction but only slightly. In 1836 it was again damaged and observations by a witness indicated double amplitudes of 16 ft. occurred. Again, three years later, it was damaged but more severely so that it was reconstructed and strengthened so that it remained in service for another 100 years.

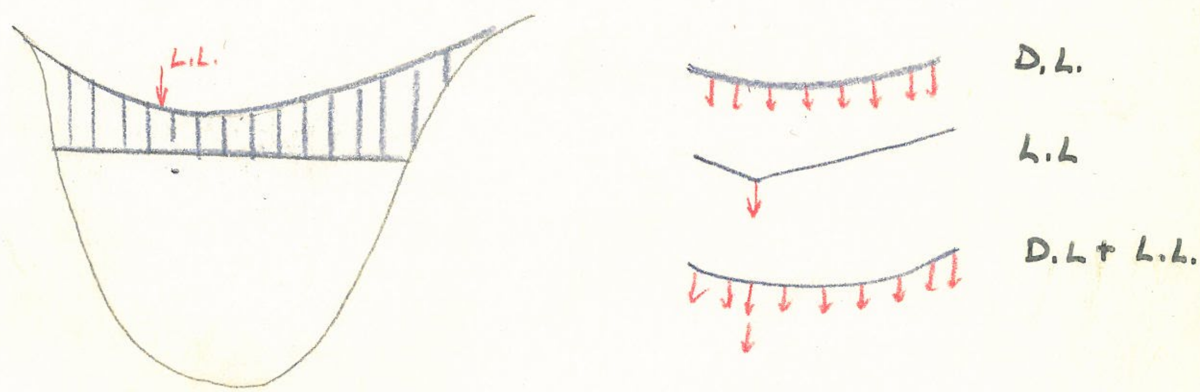
In Germany failure occurred to a 245 foot span built in 1830; in France, a 640 foot span built in 1840, and in the United States to 1,010 foot span over the Ohio River at Wheeling built in 1848.

A record breaking span built in 1850 of 1,043 feet over the Niagara River between New York and Canada was destroyed when guy cables which were put in to stabilize the structure were removed temporarily. It was feared that when an ice jam broke up stream it would foul on the guys and damage them. Then a heavy wind occurred so that the whole suspended system was lost. The cables remained in place for 34 years, but were never used again.

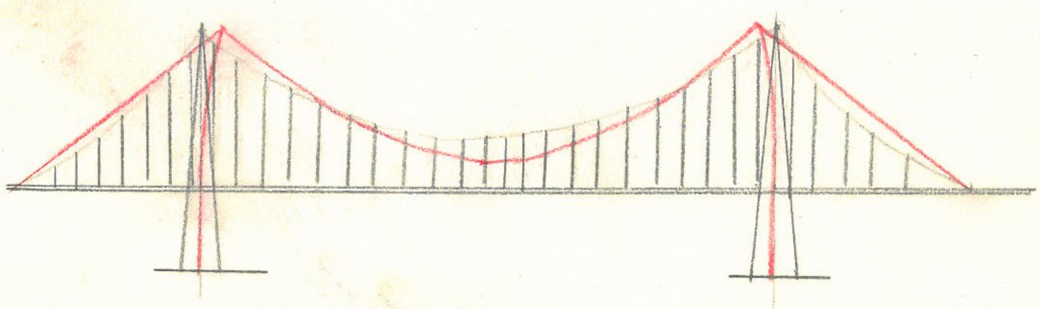
Of course the climax occurred in 1940 when the 2800 ft. Tacoma Narrows Bridge was destroyed by a 38 mile wind. Its instability was noticed even during construction so that several remedial measures were undertaken during construction and afterwards right up to the time of failure. (Ref.: The Mathematical Theory of Suspension Bridges," Chapter 1, McCullough, Rosecrans and Vincent.)

Returning to the stiffening truss there is another element, that of torsional resistance. If the stiffening truss has both upper and lower lateral stiffening systems and sway bracing there would be a torsional resistance preventing distortion of the section because one cable is loaded more than the other. There is also a disadvantage to having more than two cables. Unless there is more than one bridge each acting independently there will be some sort of unbalance.

What produces deflection of the main span?

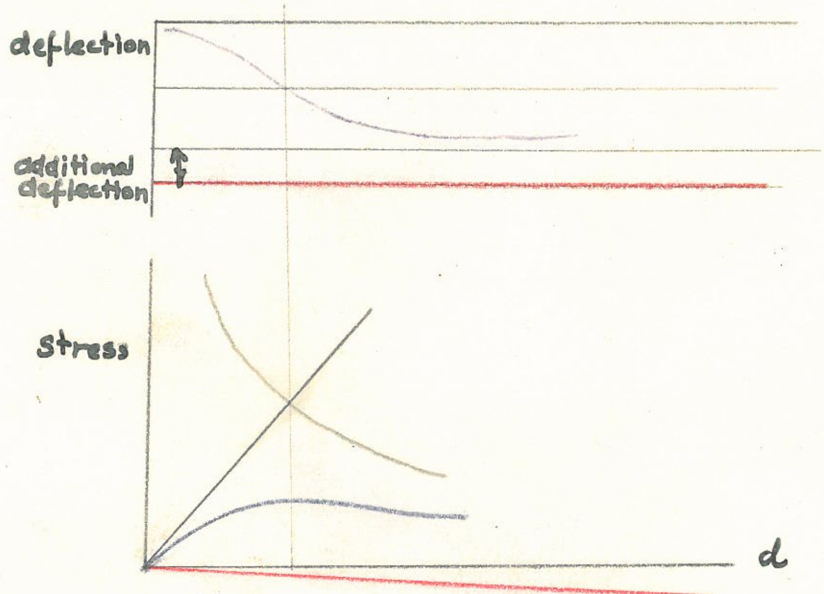


In a suspension bridge the heavier the dead load the better. The heavier the dead load the more the sag, the more the strength needed in the cable, and the more force the towers must take.



The sag ratios must be proportional to the span length for the horizontal thrust in the tower tops to balance. The cables would deflect under live load. As a result of the increased thrust in the tower tops, the towers deflect causing the cable to sag more. A temperature change in the cables will cause a deflection and will make the towers deflect more, adding to the deflection.

The following picture shows the effect of parasitic stress in the girder, i.e., when the temperature changes or a support settles. Something other than a load deflects it.



There is, therefore, a limit to the depth of the stiffening truss due to this additional stress. The deeper the truss the bigger the stress.

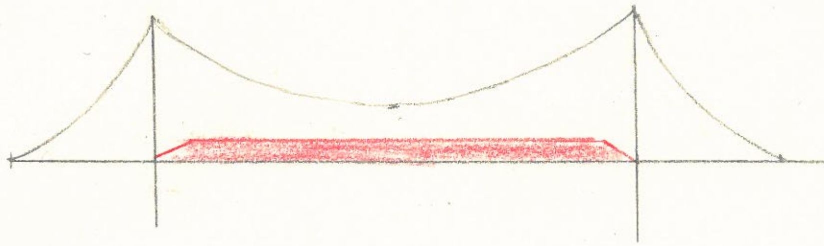
It seems perfectly obvious that the main purpose of the truss is for stiffness not strength. Nevertheless, if the stress is too high due to deflections the truss would crack and not be good for anything. There may be another function for the stiffening truss and that is to distribute the load over two or more members. If a hanger goes the load would go to the adjacent hangers which would cause them to go and so on. It acts like a seam.

The Tacoma Narrows Bridge never really fell down, only the floor did. The cables were not very seriously damaged and the towers and anchorages were undamaged.

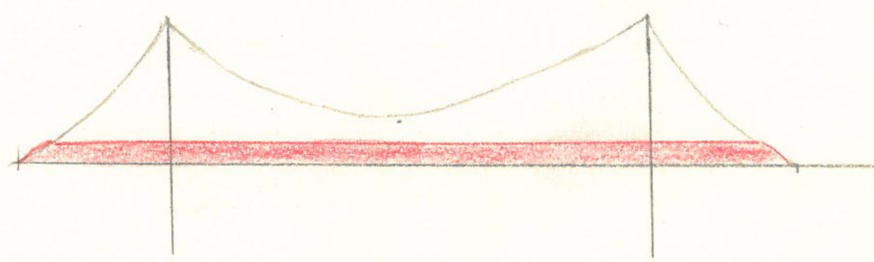
There are many problems in suspension bridges. One of these is that of anchorage. The anchorage must supply the vertical and horizontal reaction to the cable. Anchorage is conveniently accomplished if in solid rock. Another problem is that of the towers. The towers are enormous things and carry a large wind force at the top. Bending of the tower is usually not important. There is a question of whether to pin-connect or to fix the bottom. Generally they are fixed because the towers are very flexible since they are 4, 5, or 6 hundred feet high and about 25 feet at the base. Still another problem is that of the cable itself and the method of constructing it. Invariably they are made of cold drawn steel wire of small diameter. The individual wires are strung over one at a time and then bound together. In some cases wire rope or even eye bars are used. There are also details about the floors and hangers. How are the hangers attached to the curved cable so that they do not slip. How are the hangers attached to the floor beams? Then, there are the details of the stiffening truss.

A stiffening truss can be 1, 2, or 3 hinged, hingeless or continuous. The Brooklyn Bridge is three hinged, but most of the bridges are two hinged. These trusses are free at the end and just swing. Steinman wrote papers advocating continuity of trusses.

Two hinged



Continuous



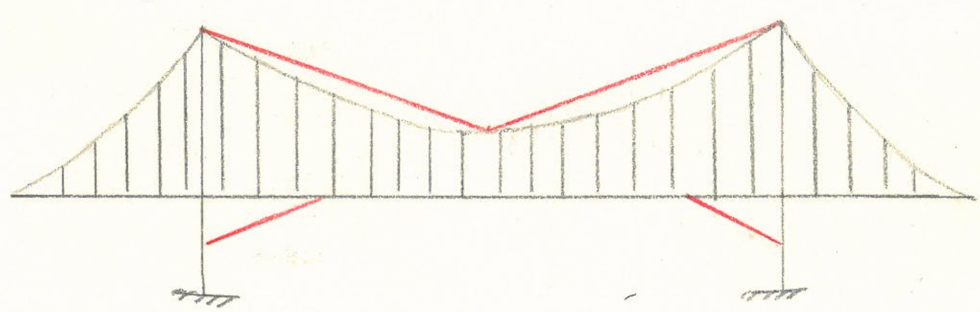
Of course there are numerous problems of erection.

As was indicated earlier in this lecture, there were a good many wind failures of suspension bridges. Many of the bridges gave trouble but all of them did not give the same trouble. The failure at Tacoma Narrows was a spectacular failure, but other difficulties occurred elsewhere that were not so spectacular such as at Deer Island, Maine and at the 1000 Islands. Movements at Tacoma that were recorded were as great as 15 ft. up and 15 ft. down. Immediately following the failure a committee was appointed to investigate the failure. They have given out with endless reports on the matter. Further, many field observations followed on other bridges that no one had been suspicious of but which they became suspicious of after these measurements were taken.

There were some disturbances noted with the Golden Gate. It seemed to be all right but since it was more or less on the border line as regards wind some alterations were made. The Golden Gate had an upper lateral system, and transverse bracing but no lower lateral system, so the lower lateral system was put in.

The Tacoma Narrows failure was a matter of autoresonance. When the bridge is going up a horizontal wind pushes it up, when going down the wind pushes it down. This resonance can go on and on. Damping, however, reduces this motion. There is not much damping in the bridge itself but in the air the bridge moves through.

There were efforts made in many suspension bridges to put in some mechanical device to damp the oscillations. In the Tacoma Narrows Bridge they tried dash-pot damping but it was not very successful. There are other devices.

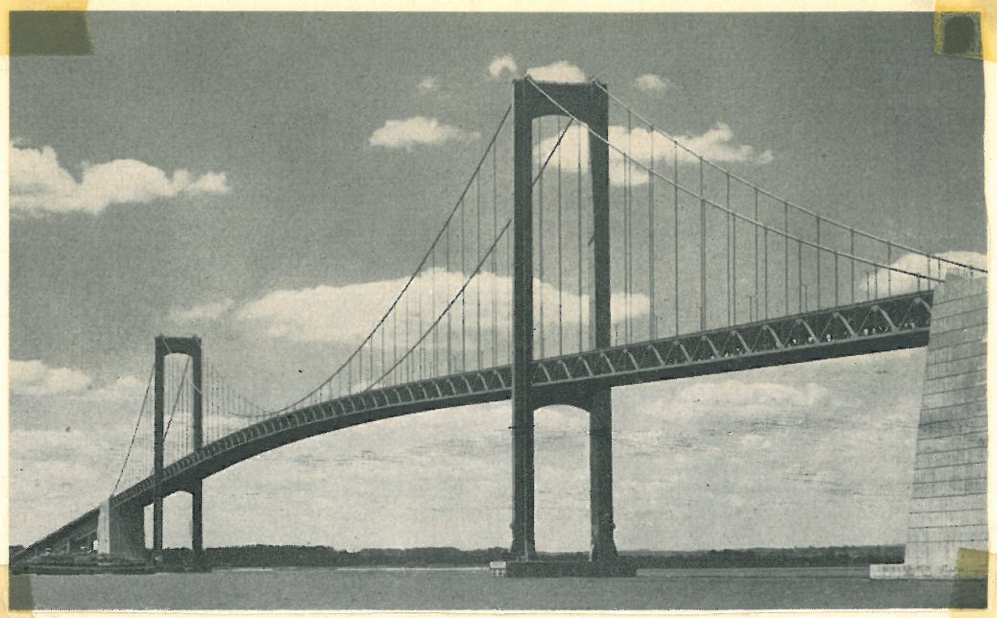


With unsymmetrical loads the cable would deflect and the cable stays tore the wires. The ties to the truss were not very effective either.

There are an unlimited number of modes of vibrations taking place: one half of the middle span going up while the other half goes down with some appreciable movement in the end spans; the center span cable going up while both side spans go down; and so on. At the same time there is a torsion or rolling motion.

Wind tunnel tests were made at the University of Washington in connection with the rebuilding of the Tacoma Narrows bridge. These tests have been reported for the benefit of the Bridge Committee. Just how to interpret wind tunnel tests is open to a great deal of discussion.

Suspension bridges are not used for railroads. Lindenthal has proposed a suspension bridge for the railroad over the Hudson. This would be a great boon to Jersey railroads. The Manhatten Bridge has els on it which may be the reason for its deterioration.



DELAWARE MEMORIAL BRIDGE

Located three miles south of Wilmington, this bridge spans the Delaware River. Owned by the State of Delaware, the structure was designed by Howard, Needles, Tammen, and Bergendoff, New York and Kansas City. The consulting engineer was O.H. Ammann; the consulting architect, A. Gordon Lorimer; and the fabricator, the American Bridge Company. Approximately 4,000,000 motor vehicles will use the bridge annually.

Total length of bridge--10,765 feet

Length of Main Span--2,150 feet

Length of each suspended side span--750 feet

Height of towers above piers--417 feet

Clearance of main span above water--175 feet

Size of wire rope suspenders--2 inches diameter

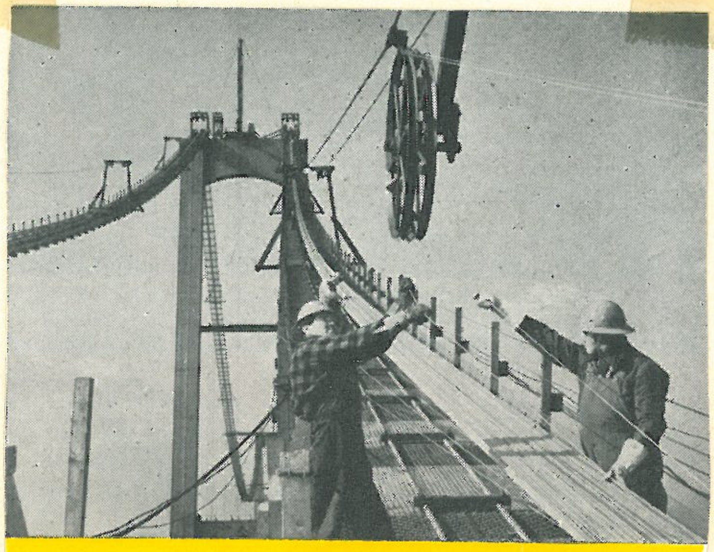
Size of main wire cables--19-3/4 inches diameter

Total length of suspenders--56,000 feet

Estimated weight of superstructure--73,141,500 pounds

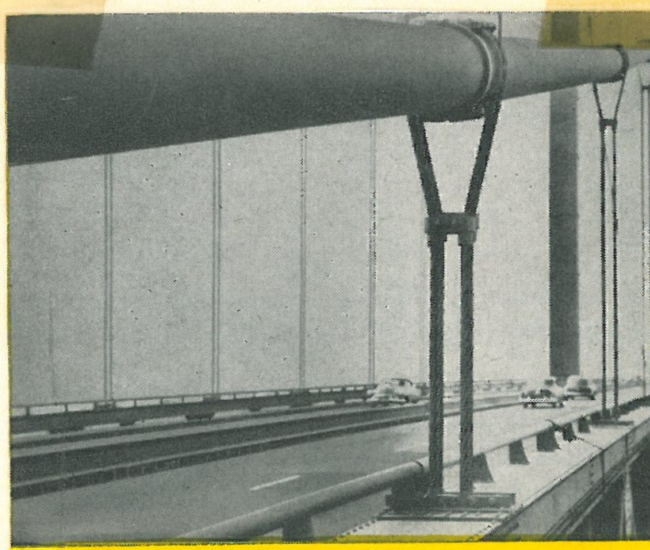
Total estimated cost of bridge--$43,900,000

BEAUTY IN STEEL. The Delaware Memorial Bridge represents the combined efforts of many different divisions of the United States Steel Company. The steel for spans and towers was made by the United States Steel Company, Cables and Wire Rope by American Steel & Wire Division, cement by Universal Atlas Cement Division and the bridge itself was erected by American Bridge Division.



SUSPENDING THE FIRST CABLES. Before any spinning of the main suspension cables could begin, four 2-inch American Tiger Brand Wire Ropes were strung from anchorage to towers and from tower to tower. On these were placed Cyclone Chain Link catwalks with wire rope railings to provide safe footing for the workmen.

Next a wire rope tramway system was installed to carry the hauling cable for the spinning wheels. The photo shows workmen picking the individual wires from the spinning wheel to form the main suspension cable. The 436 wires in each strand of the main cable are, in reality, two continuous wires which are connected to the anchorage at both ends by means of eyebars. The end of the wire from each successive reel was spliced to the end of the wire from the next reel and the ends of the last pair of wires of the strand were spliced to the ends of the first pair. It took 19 of these strands to form each of the main cables.



56,000 FEET OF TIGER BRAND WIRE ROPE SUSPENDERS. These two-inch wire rope suspenders were made from the rope which was originally used to support the catwalks. This unique method of construction saved many thousands of dollars on the job. It was originated by the American Bridge Division of United States Steel Company and is now used in building most suspension bridges of this type.

In this lecture certain phases of suspension bridges will be discussed; some of which have been discussed in the last lecture. Last time the anchorage, bridge cables, towers, and stiffening truss were mentioned. All of them are important, and, in large bridges at least, all expensive. Later, the small suspension bridge will be discussed.

The history of the suspension bridge is a long one. In the last lecture there is a brief history of failures of suspension bridges. For an excellent list of references refer to:

> "A History of Suspension Bridges"
> in Bibliographical Form, by
> A. A. Jakkula, Bulletin of the Agricultural and
> Mechanical College of Texas, 1941.

An excellent graphic study of details of suspension bridges is the following source:

> "Construction of Parallel Wire Cables for
> Suspension Bridges"
> John A. Roeblin's Sons Company.

This booklet contains valuable details driven home by photographs of the suspension bridge. It also contains a brief history of the bridges the Roebling's built. These people have had many years of cable experience. They have been associated with suspension bridges from the building of the aqueduct across the Allegheny River at Pittsburgh in 1844 until now, the latest mentioned in this booklet being the Bear Mountain Bridge over the Hudson River, 1924.

Bridge	Span, ft.	Date	
Pittsburgh Aqueduct	162	1844	
Lackawaxen Aqueduct	120	1848	
Delaware Aqueduct	134	1848	
High Falls Aqueduct	145	1848	
Rondout Aqueduct	170	1850	
Neversink Aqueduct	170	1850	
Niagara Falls Bridge	821	1854	(Combination rail-road & highway)
Allegheny Bridge	344	1856	
Cincinnati Bridge	1057	1867	(Longest suspension bridge in existence in 1867)
Brooklyn Bridge	1596	1883	(First use of steel for bridge wire. Wire protected by galvanizing. Refer to Mt.Hope and Ambassador Bridges)
Williamsburg Bridge	1600	1903	
Manhattan Bridge	1470	1909	
Parkersburg Bridge	775	1916	
Rondout Creek Bridge	705	1922	
Bear Mountain Bridge	1632	1924	

As can be seen from the above the career of the Roebling's has been an outstanding one.

The Bear Mountain Bridge is of the parallel wire cable suspension type with straight unloaded back stays. Suspended from the cables is a reinforced concrete highway 38 feet wide with a five foot sidewalk on each side. The bridge was designed to provide a floor system capable of carrying 15 ton and 20 ton trucks, and live load of 70 psf which is equivalent to a total of 216 10 ton trucks, or four lines of 54 trucks each. The towers are 350 feet high and rest upon concrete piers, carried on solid rock foundations. They have a width of about 90 feet at the base and 61 feet at the saddle.

The suspended structure is carried by two cables having a deflection of 200 feet under live load. There are a total of 7252 galvanized wires in each main cable which have a minimum ultimate strength of 215,000 psi. The cables are supported on cast steel saddles at the tower tops and are rigidly fixed to the tower saddles by means of steel casting clamps. Each of the cables is anchored into solid rock at both ends of the bridge. The anchorages are made up of 74 I-bars for each cable, the wires being carried around cast steel shoes.

The floor system and stiffening trusses are suspended from the main cable by means of wire rope suspenders. These ropes are 2-1/4 inch diameter and have an ultimate strength of approximately 420,000 pounds. A steel casting cable clamp is attached to the main cables at each panel point and is provided with a properly designed groove over which is passed the bight of each suspender, having both parts connected, by means of open sockets, to transverse beams at the bottom of the stiffening truss. These sockets are pin-connected to an equalizing bar which balances the stress in both parts of the suspender rope.

The stiffening truss, which are calculated for the required live load of 3160 lbs. per ft. of span, are of silicon-steel with a specified ultimate strength of 80 to 90 Kips. per sq.in. These trusses, spaced 55 ft. apart, have a depth of 30 feet, and are placed below the roadway.

The first charter for a bridge at this site was obtained in 1868. This brings out that a bridge is designed, redesigned, and redesigned over and over again until it is actually built. Steinman has been working many years on a bridge over the Narrows in New York. Eventually, perhaps, there will be a railway suspension bridge built over the Hudson River.

Returning to the Brooklyn Bridge, note that there was the first time galvanizing was used on wire cables. The failures at Mt. Hope Bay, Rhode Island and of the Ambassador Bridge between Detroit, and Windsor, Canada was attributed to a hot galvanized wire. When the deck was being put on at Mt. Hope it was noticed that some of the wires were broken at the anchorage. Wires were also broken elsewhere. Majeski, who represented the bankers, faced with a difficult decision decided that the cables had to come down. The cables were cut and dropped into the river. There were three or four investigations made but no answer given. So, it can be seen that millions of dollars were lost even though there was figuring and more figuring, testing and more testing, and all sorts of theories. You know the laboratory properties of the material, which is something that you wished on the material to conform with the test that you are able to make. In between the properties of the material and the computed stresses is a gap. This gap is failure and polarized light has not closed the gap.

In this book by Roebling there are a number of illuminating photographs of the Bear Mountain Bridge in construction.

Photographs include: the pier foundations in place and the tower being erected; excavation of the anchorages; erection of the foot bridge for spinning the cables; details of spinning of the cables and anchoring them; squeezing of the cables; details of the saddle; grounting of the eyebar chains; the anchorage houses; erection of the superstructure; concreting of the roadway; details of the cable and suspenders; wrapping of the cables; and the finished bridge.

This bridge is unusual in that the anchorage is in solid rock and the anchor spans are very short. The saddles do not move. On the Brooklyn Bridge a roller net was put in under the saddles but they soon froze and probably never moved. There is considerable stress in the towers. The towers are very flexible. The bending in the towers do not interfere with the main action of the bridge.

The Brooklyn Bridge is Prof. Cross' pick as the most beautiful bridge in the world. A faraway view like that from the Manhattan Bridge will show its magnificent grace. There is much discussion on the esthetics of suspension bridges and particularly tower design. Originally it was intended to veneer the steel towers of the George Washington Bridge with masonry. However, it was left off on the suggestion of the Port Authority to see what it would look like and, since it looked fine as it was and saved a great deal of money, it was never veneered.

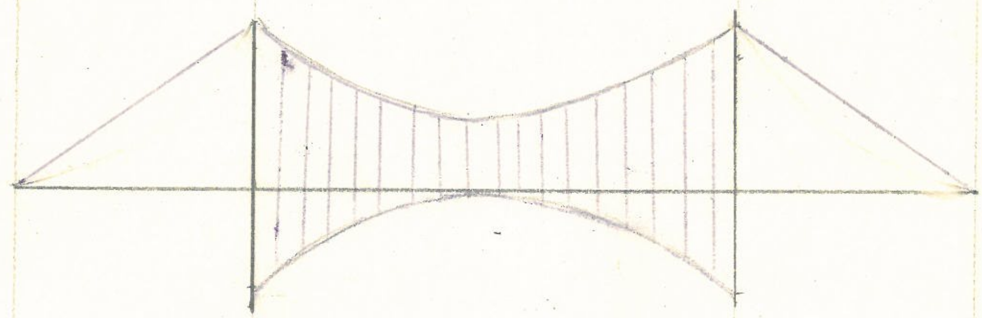
There was a little trouble with the anchorage of the Bronx-Whitestone Bridge. Also the Lion's Gate Bridge at Vancouver, Canada, a bridge of considerable span, which is supposed to be quite flexible, may be on the border line. Another bridge of interest, but somewhat smaller, 1000 feet, is the Toledo Bridge where one of three theories of stiffening trusses were introduced.

Changing the scale, small suspension bridges will be considered. There was at one time a rash of little suspension bridges built. An example of one is the bridge over the Royal Gorge in Colorado. There is another one across the Red River which separates Texas from Arkansas. The population in that area was very small. The Red River is a dirty stream with a 30 or 40 foot range from high to low water. This made it difficult to get across. The cars had to drive down to the water's edge to go across by ferry. When the water rose and then receded the road became muddy. The local bankers decided to build a suspension bridge. The cables were "telegraph wires" bundled together and not compacted. There was no stiffening truss.

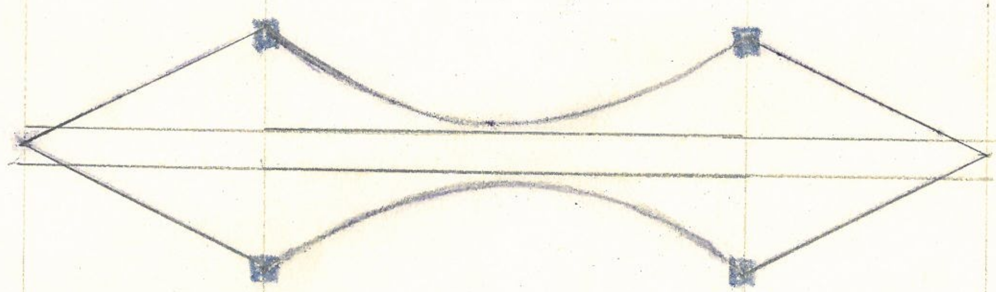
There were many failures in these bridges. In one, a pier washed out. In another it was said that a herd of sheep went on the bridge and the bridge collapsed. In still another, because the wires were carried back into the concrete and anchored, the wires corroded and there was an anchorage failure. These bridges cost about $150,000 apiece.

Small suspension bridges serve another purpose, to carry gas or oil pipelines over a river. Some have been built across the Rio Grande and then there may be some across the Mississippi.

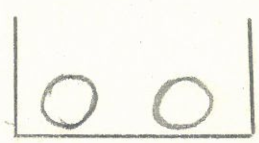
Elevation



Plan



Section



The above bridge is stiffened vertically by putting a flat cable in on the bottom. It is stifened laterally by putting the cables on the side as is shown in the plan view and by keeping them tight. There does not seem to be any failures of these bridges reported. If they do fail there is not much to lose.

However, it would seem that in main lines going to a large city such as Chicago it would be very important to keep the line in continual service. It is for that reason that lines are put in trenches passing under the river.

Last time the matter of stiffness was discussed. What is the actual amount of stiffness provided in actual structures? How much would the deflection be under live load with and without the stiffening truss? Hardesty and others tried to classify the bridges, those that had given trouble, had not given trouble, and that failed, based on the stiffness provided. The stiffening truss on the Manhattan Bridge cut down the computed deflections by 50 or 60 percent while only 2, 3, 4, or 5 percent on the Tacoma Narrows.

The interaction diagram has much to do with this. It is sort of a correlation of an enormous mass of theoretical analytical literature. It can be traced back in the Transactions of the ASCE to the 1880's.

There are three ways of attacking this problem:

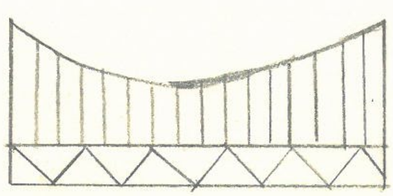
1. Arch or Elastic Theory

2. Deformation Theory or Melan-Moissieff Method

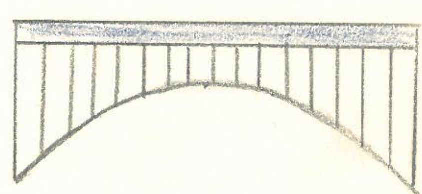
3. Hardesty-Wessman Approach.

1. <u>Arch Theory</u>

The stiffening truss takes the moments in the same way as the stiffening truss (girder) of an arch.



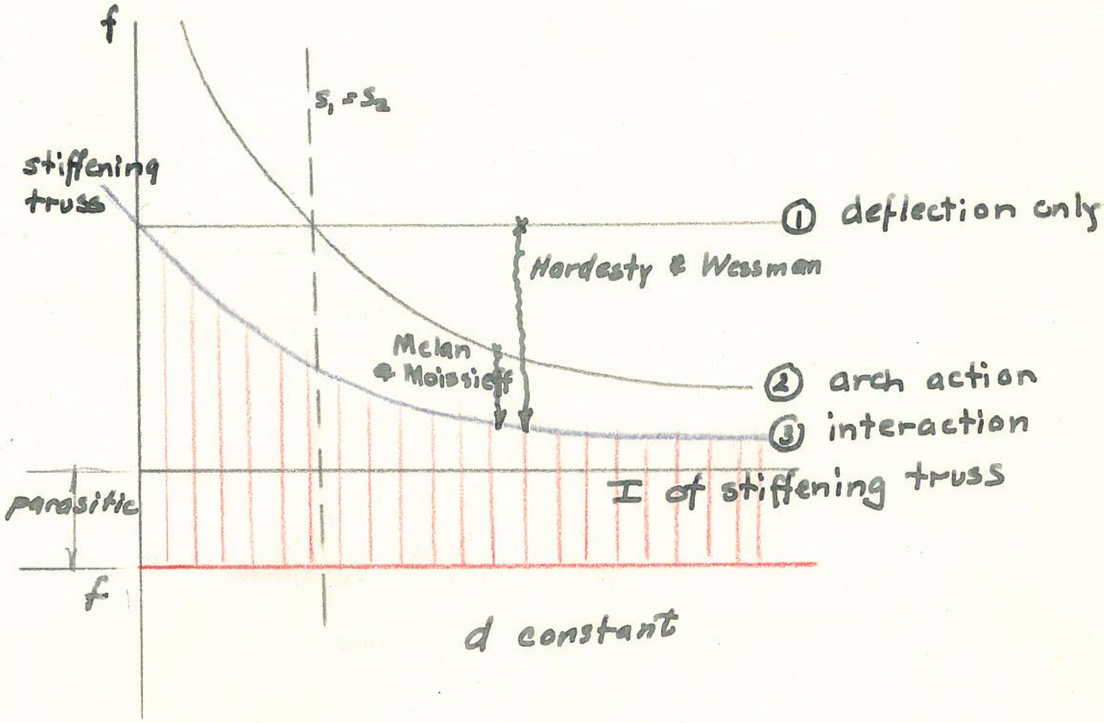
Suspension Bridge



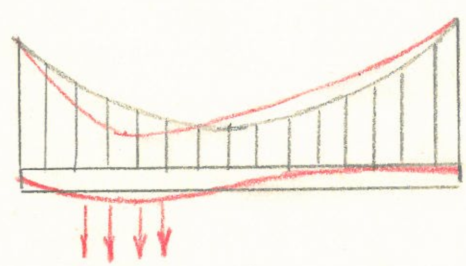
Arch

2. Deformation Theory

This theory is attributed to Moissieff. It had its beginning when applied to the Manhattan Bridge.



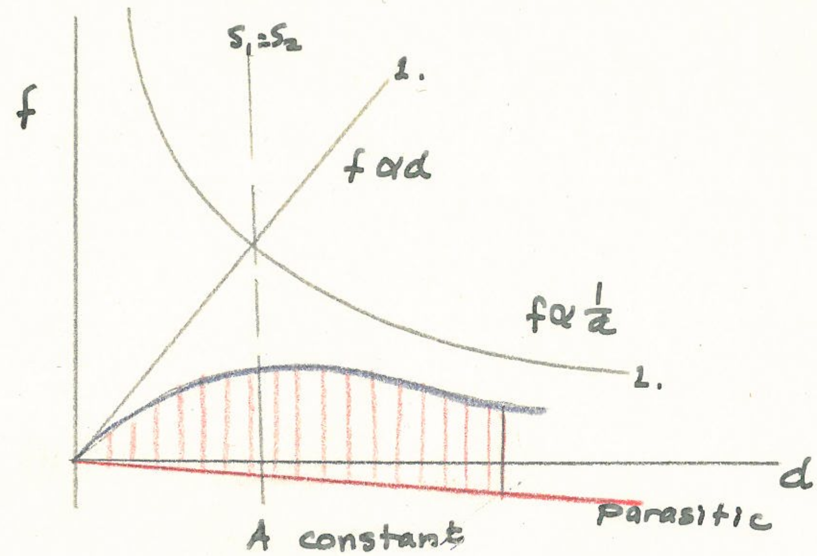
The stiffness of the stiffening truss is defined as the amount of deflection if it were acting as the stiffening girder of an arch compared to the amount of deflection if it just follows the deflection of the cable. If the stiffening truss does follow the deflection of the cable under live load as shown below, the truss will deflect as the cable deflects.



For a constant depth of the stiffening truss, the stress due to a given deflection is constant for any value of I. The stress due to the deflection of the stiffening truss can be figured and is represented by the horizontal line 1 . If the stiffening truss took all the moment then the stress for a constant depth can be computed from $f = \frac{Mc}{I}$. Therefore, the stress varies inversely as I and is represented by the hyperbola 2 . If there is interaction, it will be represented by the hyperbola 3 . The area, A, may also be made the variable.

By the Moissieff approach starting with the curve 2 , picking out an I by trial and error, determining how much is carried by the cable, it is possible to work down to the interaction curve 3 . See the diagram. Moissieff used elaborate mathematics. On the other hand, Hardesty and Wessman began with curve 1 figuring the deflections in the cable with the truss following along and then, by successive approximations working down to the interaction curve 3 . This approach was easier. Using the sag ratio and the live load to dead load ratio as variables, Hardesty worked out coefficients for the deflections of a cable. This can be found in the Transactions of the ASCE, Volume 104, 1939. This method was used first in the design of the Toledo, Ohio bridge. Although it is probably quicker than the Moissieff approach it gives the same answer.

A similar curve can be drawn but with the area constant and depth variable.



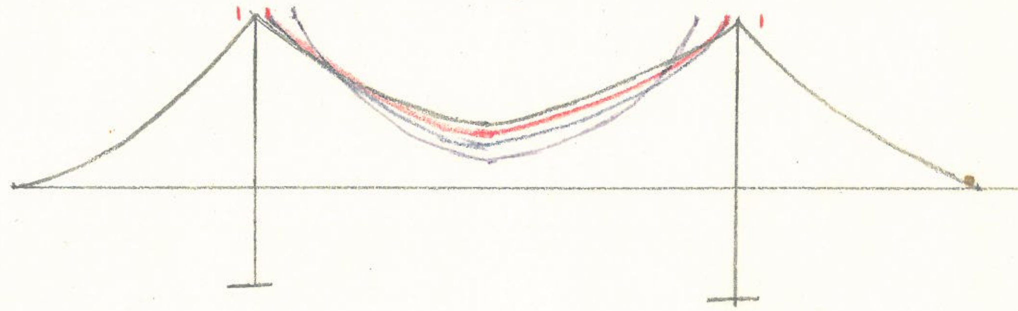
1. If the cable takes all the moment, the deeper the truss the more stress for a given deflection.

2. If the truss takes all the moment, the deeper the truss the less the stress.

Add to these a parasitic stress. The stress is proportional to the depth. In the first interaction diagram, where f vs I is plotted, since the depth is held constant, this is represented by a horizontal line. In this later interaction diagram, this proportionality is shown. The total stress in both cases is indicated by shading. It becomes apparent, therefore, that if there is a parasitic stress further stiffening may cause an overstress.

The term "parasitic" is misused here somewhat. The cable on a suspension bridge can be let down three ways:

1. Temperature change
2. Cable elongation
3. Straightening of side spans



1. Temperature Change

This is truly a parasitic stress. If everything expanded, earth and all, the towers and anchorages would move out, but the anchorages more than the towers. This is because the towers move because of expansion of the main span; the anchorages because of expansion of the main span and the side spans. However, the anchorages do not move so that the towers are pushed back to a position closer to each other than they were before. Therefore, the cable sags.

2. Cable Elongation

This has the same effect as temperature changes. It is analogous to rib-shortening in an arch. If a live load is on the main span it stresses and, therefore, the cable elongates. In arches it is customary to add about 15 percent to temperature effects. It is not very much of a factor in arches. It probably is not very important in suspension bridges.

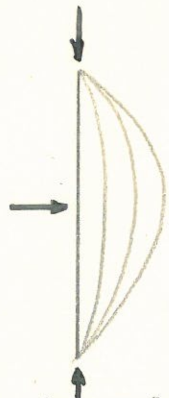
3. Straightening Out of Cables on Side Spans

With a load on the main span, the cables on the side spans will tend to straighten out permitting the tower tops to move closer together. This will cause the main span cable to deflect. Terming this a parasitic effectis a gross misuse of the term. This effect may be remedied by using straight backstays with simply supported end spans or by using stiff side span stiffening trusses.

The scale on the parasitic stresses shown in the diagrams is probably very much exaggerated. The line should not be really straight either because the deflections of the cable are not truly proportional to the live load. This is really a refinement and for our purposes it is a straight line.

It was the failure of the Tacoma Narrows Bridge ("Galloping Gertie") more than anything else that prompted the studies of aerodynamic stability. Such studies were made by Bleich of an analytical and mathematical nature, and F. B. Farquharson of the University of Washington who made wind tunnel test. Add to this the independent studies of D. B. Steinman. Still, we do not know much about it.

The symposium of the Rainbow Arch Bridge over Niagara Gorge which was printed in the 1945 Transactions of the ASCE covers certain important aspects of big bridge building. Phases it covered were planning of the bridge, failure of the old bridge, choice of bridge, erection procedure, deflections, and buckling. Although the preliminary design called for a two hinged arch, the arch was finally built hingeless even though erection was harder because they feared buckling. This problem would not occur in a suspension bridge. While the arch deflects away from the pressure line, the suspension bridge does not. Consider the following example:



Away from p-line

Toward p-line

Tension members tend toward stability; compression members tend away from stability.

Another thing of interest in the paper was that of erection procedures. The erection men are more theoretical than would ordinarily be thought.

Of interest and worth looking at is a report by Paine, Ammond, and Andrews on stiffening of the Golden Gate Bridge. The men concerned were of the highest quality. There ~~was~~ were millions concerned. The men refer to analysis, to wind tunnel tests, and to field data and then say that none of these give an answer. They were still guessing and gambling. The interest is in the point of view of the report.

Up to now torsion has been mentioned only casually. Time cannot be spent to go more thoroughly into it but some people make a mess of it. Take a box braced in four planes, i.e., there are two trusses and two lateral systems. If it is so braced that it will remain a rectangle for a torsional condition the web alone is stressed. It makes no difference what the depth is. This is analogous to twisting of a rectangular shaft where there will be no stresses at the corner, just on the faces.

An engineer has all sorts of things to do. He builds dams, retaining walls, bridges, buildings, towers, and many more things. The imagination can run wild over the things a man can build.

The engineer is interested in loads. Railroad and trolley bridges are both light and heavy. There are all sorts of loadings of highway bridges, too. In the building field, too, there are all sorts of loadings due to the types of manufacturing, to people, to light machinery, to heavy cranes, or to heavy or light storage. Also the probability of certain combinations of loadings must be considered. For example, what are the probabilities of all three stories of a building being loaded to capacity?

Then there are other loading such as earth pressure, water behind and under dams, and breaking waves against breakwaters. When considering the earth resistance to the structure, the structure should not be thought of separate from the earth or the earth separate from the structure.

The question of material is a large and important one. At first there was a great deal of timber available but it began to dwindle. It has had a revival with composite timber beams. Reinforced brick was advocated once and although it died out it may be revived. Then there is concrete which may be mass, reinforced, or prestressed.

Other materials include wrought iron, cast iron, steel, and alloy steels. These materials lead to details of fabrication: welding, bolting, riveting, and others. There are also details in concrete (anchorage of prestressing material) in timber (tension splicing).

Not only are there many variations of material but also of combinations of the materials.

These preceding problems are followed by that of manufacture, and availability, and then by fabrication. There is an advantage to doing things in the shop. It is more expensive to fabricate in the field. This brings in the problem of transportation of materials, particularly fabricated material which hinges on clearances, type of transportation and relative cost of transportation. It is the relative cost matter that influences the use of concrete rather than steel in distant places like the southwest.

There is also the matter of availability of workmen. Usually lower efficiency goes hand in hand with lower labor costs. All these matters must be thought over very carefully when translating that which is done in America to that which may be done in some other part of the world.

Then we come to what is known as "properties of materials." "Properties" are something that was invented in the mind to fit what is obtained from laboratory procedures. Take brittleness. Everyone who can read or write knows what brittleness is. Try then to define brittleness so that it can be measured and found in a laboratory.

Considering analysis, if the properties of the material can be defined, then the structure can be analyzed. Further, something can be set up to do the same thing such as a mechanical brain like photoelastic analysis. Still, there is a gap between the properties of the material and analysis. This thing called failure is very elusive. What is it? Is it when something falls down, or when something cannot be used well, or what?

An equally elusive term is "stiffness." Almost nothing is known about it. Definitions involve the psychological aspect, the operational aspect, and the maintenance aspect. The maintenance aspect involves, along with other things, another elusive term, fatigue, the deterioration of a material due to use. An example is the liberty ships during World War II which split in two when ductile plates broke without deformation.

Plasticity further complicates analysis. Whether to use it depends on how seriously you take the properties of the material.

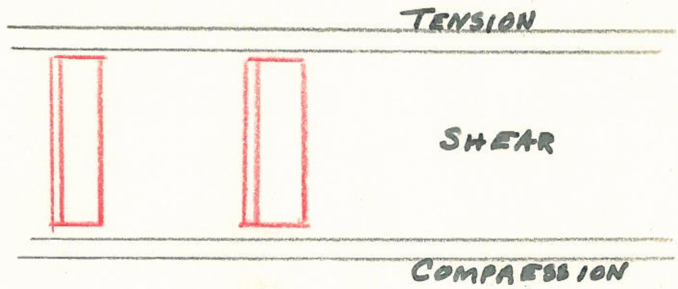
All the materials can be combined in an unlimited number of ways. Arch dams (have obvious advantages in certain sites but are more difficult to build), slab dams, retaining wall dams, mass dams all can be made. There is nothing standardized, at least other than temporarily.

The question of deterioration is of the greatest important. Huge sections of steel have been known to have been eaten away by rust.

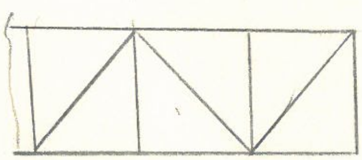
Arches - Trusses - Girders

An arch is nothing but a truss; a truss nothing but a girder, and vice-versa. Separation is a matter of teaching.

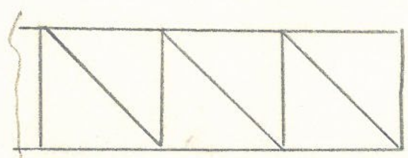
Take the girder:



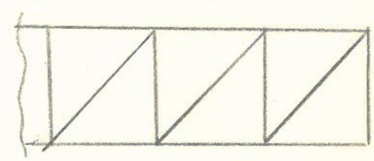
The flanges take tension and compression, and the web, shear. If the web is deep and thin, stiffeners are put in to prevent buckling. In the Wagner Truss used by the airplane industry the web is made deliberately thin and permitted to buckle. If the web is taken out altogether then it is an open webbed girder or a Vierendeel Truss. The lateral bracing in buildings is an open web system. If the web is taken out and verticals and diagonals put in then the Pratt Truss, Howe Truss, Warren Truss, or double intersection truss can be made.



Warren

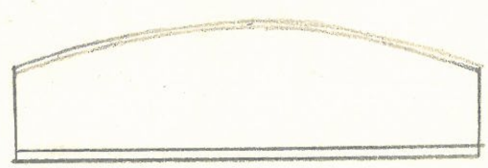


Howe



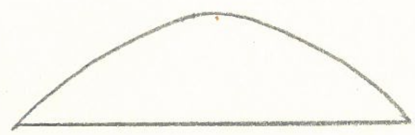
Pratt

The girder can also be arched, i.e., the shape can be changed.

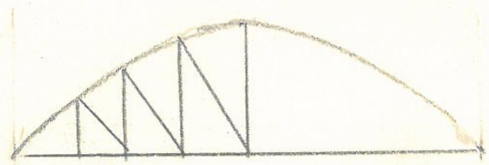


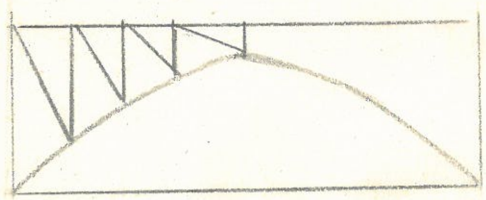
Then, the web can be taken out and be replaced by any combination mentioned above. With trusses an unbalance of load causes a reversal of stress. This was often avoided by putting in flexible diagonals which could not take compression but could take tension. These are known as counters.

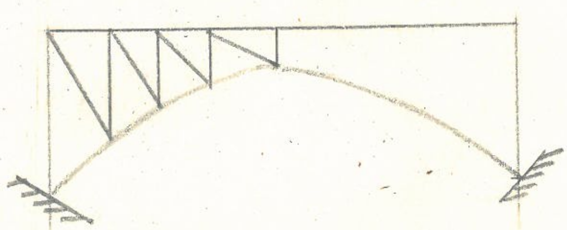
If the structure is all loaded it does not need a web.



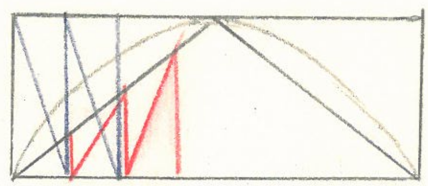
This is a tied arch. However, as soon as an unsymmetrical load is put on the structure a web is needed but not as much as before.





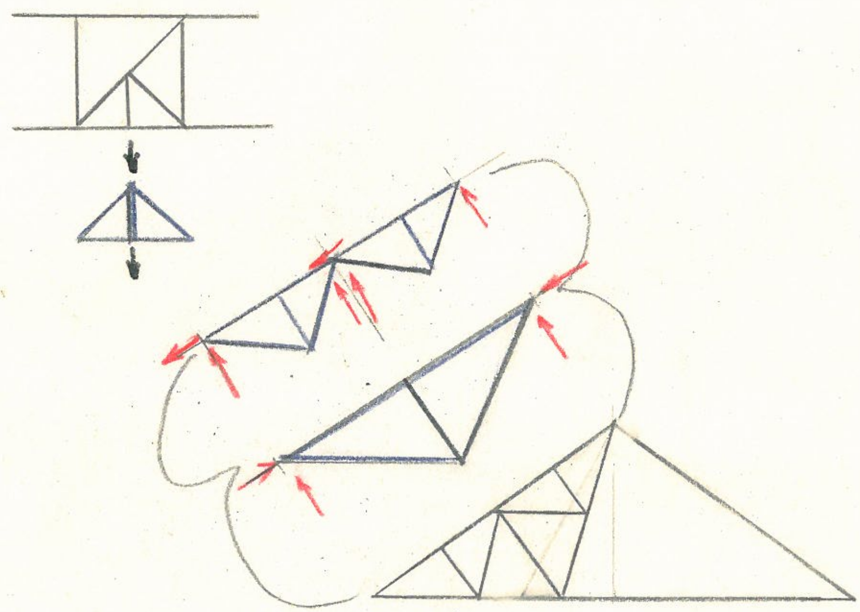


Turning the arch upside down and making the earth the compression chord results in a suspension bridge.



Resistance to shear must be provided for. With the parabolic curve no web reinforcement is required.

With any structure it can be broken down to its fundamental parts so as to see just what the thing is doing.



There are of course endless variations. A fink truss is an outstanding example of a truss in a truss in a truss. This gives some visualization of the functions of the members.

Too often students are required to find the stresses on a plane. Actually everything is in space. There are six possible movements. Even the slab which is in one plane nevertheless is not in the plane of the loads. The approach to domes is very simple. Refer to the Handbook of Kidder. He uses the Rankine approach. There is no simple solution that exists for shells and the whole general problem of structures in three dimensions.

The problem of correlating all that has been done is extensive. It is always further complicated by construction problems and new materials. A wise man said the more changes there are the more it stays the same.

Probably even more time could have been spent on codes. The rules in codes were drawn up by committees which were composed of representatives of everyone who is interested in the game: the theorists, the laboratory men, the fabricators, the manufacturers, and the contractors. It was made up of an honest crowd trying to do an honest job. The revision of just one joint committee for reinforced concrete takes five or ten years and involves a large amount of travel and time. Some of the committee give their own time and money. Then when they do some compromising at 2:00 A.M. the college professor criticizes that the compromise is not perfect. Nevertheless, "grandma" is a must. It is true she changes her views all the time. The standards of today may not be the standard of tomorrow.

PIPELINE BRIDGE STABILIZED WITH DIAGONAL ROPE STAYS

by

D. B. Steinman

Continued vertical oscillations at low wind velocities threatened the life of a suspended pipeline bridge crossing the Coosa River near Clayton, Alabama which has a 700 foot main span with straight backstays. It was stiffened transversely by wind cables located in the horizontal plane of the floor beams but, when built, there was no vertical stiffening provided, apart from the minor stiffness of the 30 inch steel pipeline itself.

Various poorly advised, ineffective attempts were made to stop serious and persistant vertical oscillations of about 3 feet. The contractor added another dummy pipe which, although it tended to increase the rigidity by its deadweight, served no purpose because it also increased the lift. Then the contractor attached makeshift sea anchors each time wind and oscillations seemed serious. They had to be disconnected after each emergency in order to permit free navigation. Then the contractor connected a spring damper at midspan. This damper is effective theoretically only when the frequency of the spring damper is resonant with that of the impressed oscillating force. This was not the case here since the impressed frequency varies with the wind velocity and the spring damper frequency remains constant.

D. B. Steinman was then called in to stabilize the bridge. The recommended and adopted installation for stiffening and stabilizing the bridge consists of a system of light, 1/2 inch wire-rope stays. This is equivalent to forming a vertical truss with the two main cables of the bridge forming the top chord and the bottom 30 inch pipe constituting the bottom chord. A single Warren system was used, with each stay having an initial tension of about 3000 pounds. This initial tension enabled each stay to act equivalent to a compression member since it was sufficient to prevent reversal of stress. The installation cost only $7,368.00 but reduced the amplitude of the oscillation to less than 1/2 inch.

Abstrated from <u>Civil Engineering</u>, March, 1952.

The Puzzled Centipede

A Centipede was happy quite,
Unit a frog in fun
Said, "Pray, which leg comes after which?"
This raised her mind to such a pitch
She lay distracted in the ditch
Considering how to run.

Author Unknown

If you look over the catalogues of various schools you will see that the centipede has become a millipede--from 100 to 1000 legs.

A coordinator is a man who has a desk between two other men who are looking for a phone. These men are called expeditors.

Beyond a certain level a person is no longer a civil engineer, a mechanical engineer, or a ceramic engineer. He is just an engineer and not even that. An engineer is the one who plans, builds, and operates. A few do all three, but if he does only one he must know the others. All engineers get into these phases.

An engineer tries to meet elementary human needs: dryness and warmth from clothing and shelter; eat and drink from agriculture, husbandry, and water supply; waste disposal; and transportation for getting food and water. If you obtain food and water and build yourself a home you are all set, but in order to live comfortably heat and power are needed. With power and more power the load is taken off of the arm. Clothing, cooking utensils and many other things are needed for which ceramic and chemical engineering and other fields are created--or, as DuPont's slogan says, "Better living through better things in engineering."

As has been said, in order for an engineer to do one thing well he must know all the others. He has to begin early to wake up and ask questions:

Do you want something?

What do you want?

Can you get it?

Have you got it?

What are you going to do about it?

There are examples where someone works up some scheme that nobody wants.

It is not necessary to take courses about climate, disease, or people, but these things should be thought about. Someone should write a book on the question of locating industry. For example, consider where to locate a steel plant. What will the taxes be? Is sufficient labor and power available? Where is the source of the raw material? What is the cost of and the type of transportation? Where will the finished product be distributed?

Of local concern is the question of what took the textile industry out of New England. Was it availability of raw material, taxation, labor costs? It is something to think about.

When we speak of "design" here it is used in a different sense. It is the kind of design used in the planning course--how many buildings and where should they be, where should the railroad come in,where should the power source be situated, etc. The planning course is an outstanding part of Yale education.

"Dimensioning" is also used somewhat differently. Look at something and say this should be a 30 inch girder. Then can it be made smaller or is it big enough? If you cannot and it is then it is all right. Remember that engineering is a gamble. Insurance companies insure suspension bridges and flood area properties but do they know what they are doing? The dimensioning can be followed through by making a tentative decision on the size of reinforcements to use.

Think about construction procedures. If in business, many people will be anxious to give you information but they are all trying to sell you, except the large concerns. The large concerns know that if they sell you a bill of goods everyone will be through with them. Bethlehem stole many a job from under U.S.Steel's nose by giving detailed engineering information to its customers who had problems.

Construction is made up of many elements:

Material

What can be fabricated?

Equipment

It is difficult to keep abreast of the latest equipment.

Labor Supply

- Custom, Law, Uses (in the particular area, or in general).
- Speed of Construction--Procurement.

Perhaps a course in Cost and Values should be included in an engineering education. This involves many things such as initial cost, depreciation due to use and to going out of fashion, and the like. This is not a course in economics.

Then there is the element of coordination. We should take an interest in these things. Much can be learned by talking to men in other seemingly remote fields. The railroad probably has more coordination than anywhere else. By referring to the Proceedings of the AREA it can be seen how much area they cover in just one part of the industry.

Just before the war a ship carrying many nervous Europeans, was coming to our shores. On it was a magician who entertained these people by making all sorts of things disappear, and a parrot who called him "fake" time after time. However, the ship was torpedoed and the magician and the parrot ended up on a raft together. After sitting there awhile the parrot broke the silence and said, "O.K. wise guy what did you do with the ship."

This can be applied to the colleges. What did you do with the education? At a big university they have 100 or more graduate courses. The courses were called the science of this, the science of that. What did they do with engineering?

An engineer has to be proficient in three languages.

1. Graphic Language

These do not look like real pictures but like primitive Indian pictures. An engineering picture of a reinforced concrete bridge does not look like a reinforced concrete bridge.

2. Symbolic Language

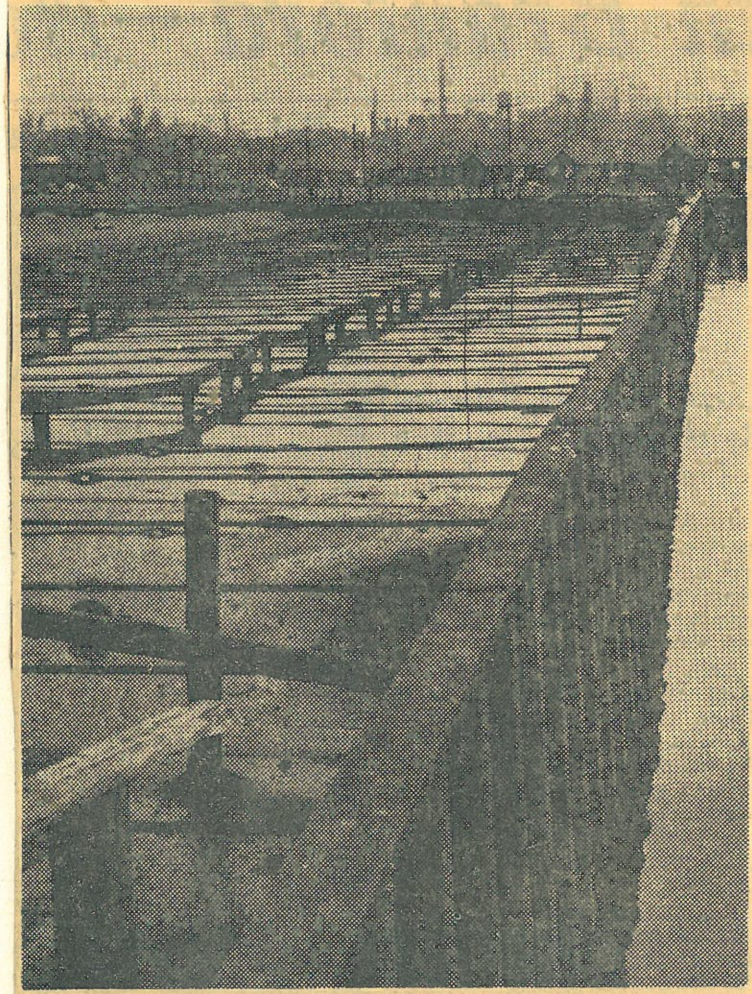
Algebra and differential equations. An engineer makes this language up as he goes along. It looks like hieroglyphics.

3. Ordinary Verbal Language

An engineer can be forgiven if he is not proficient in this language if he knows the others. However, he is damned by the "liberal arts" people. Yet, he can write better than the scientist or any army colonel. What the engineer should be damned for is his inability to read. Give him ten books, he will check everything on the first ten pages of the first book. He cannot seem to look at it with enough perspective--with enough scale.

SCALE

Bulkhead Must Be Rebuilt For $65,000



This sagging bulkhead in New Haven Harbor must be reconstructed at a cost of $65,000 to hold back sand fill. The sheathing called for in the State Highway Department's plans was too thin to support the weight and crumpled. Thicker sheathing must be driven to correct the situation off the Sperry & Barnes Company plant. This picture shows a considerable length of the bulkhead, with units of veterans' housing in Waterside Park in the background. The engineering error was disclosed Wednesday in the State Senate. The engineer responsible for the mistake in calculations has been demoted.

APRIL, 1953

ROUTE 1 GIVES HIGHWAY DEPT. NEW PROBLEM

Lack of Firm Footing to Delay Bridge Just North of City Point

The State Highway Department has a second problem on its hands in the preparation of filled in sections along the west side of New Haven harbor for relocation of U. S. Route 1, it was learned yesterday.

Work on the construction of a bridge which will carry the new highway over an underpass to permit traffic to reach the shore line will be delayed for several months beyond the original schedule because of difficulty in securing firm footing for the structure.

'Mud Waves' Created

To assure a satisfactory base extra weight, in the form of sand, was piled at the point of the structure. The overload caused shifting of the soft sub-soil and numerous "mud waves" were created immediately off shore.

The site of the latest problem, which engineers do not consider serious except for the loss of time in construction, is just north of City Point. It is designated on the plans as Station 72.

The C. W. Blakeslee & Sons Company, which has the contract for the bridge, has suspended operations while special tests are conducted. Gauges which show the supporting strength of the subsoil are read daily. State officials said these daily readings show steady improvement.

16-Foot Clearance

The bridge will carry the expressway over an underpass with a

Film Star Rewards

Since it was en route to New Delhi, to the northeast, indications were that it was forced off course by a near hurricane which swept the Calcutta city area last night in the onset of the summer monsoon.

The plane's British operators and the U. S. Consulate in Singapore reported the three Americans had booked passage for New Delhi in the plane. It stopped en route at Bangkok, Rangoon and Calcutta.

NAME

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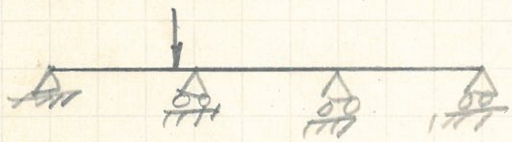
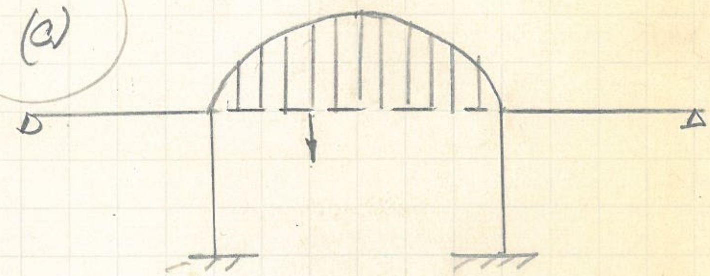
1. Draw the deflected structure (or component deflections) and accompanying moment curve (or pressure line) with SCALE. Supply your own dimensions. Assume no longitudinal deformation of members.

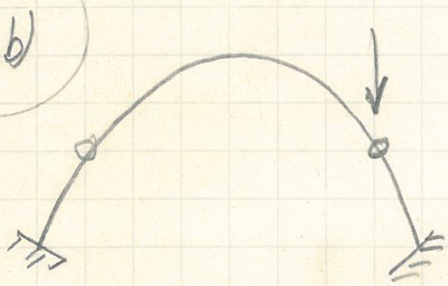
(a)



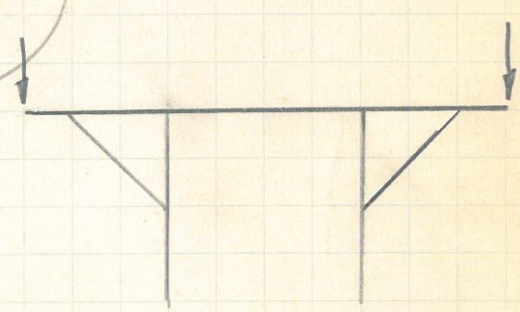
(c)



(b)

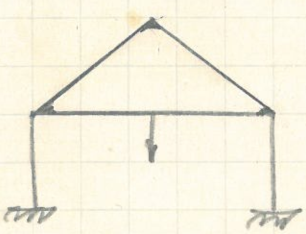


(d)

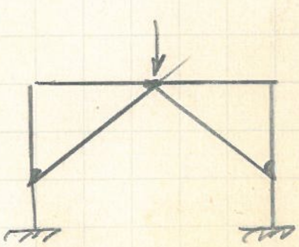


(e)

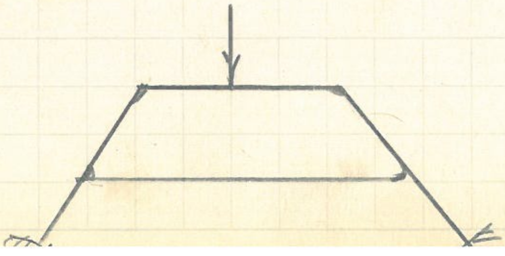
(f)



(g)



(h)



Prob 1. NAME R. A. Heller

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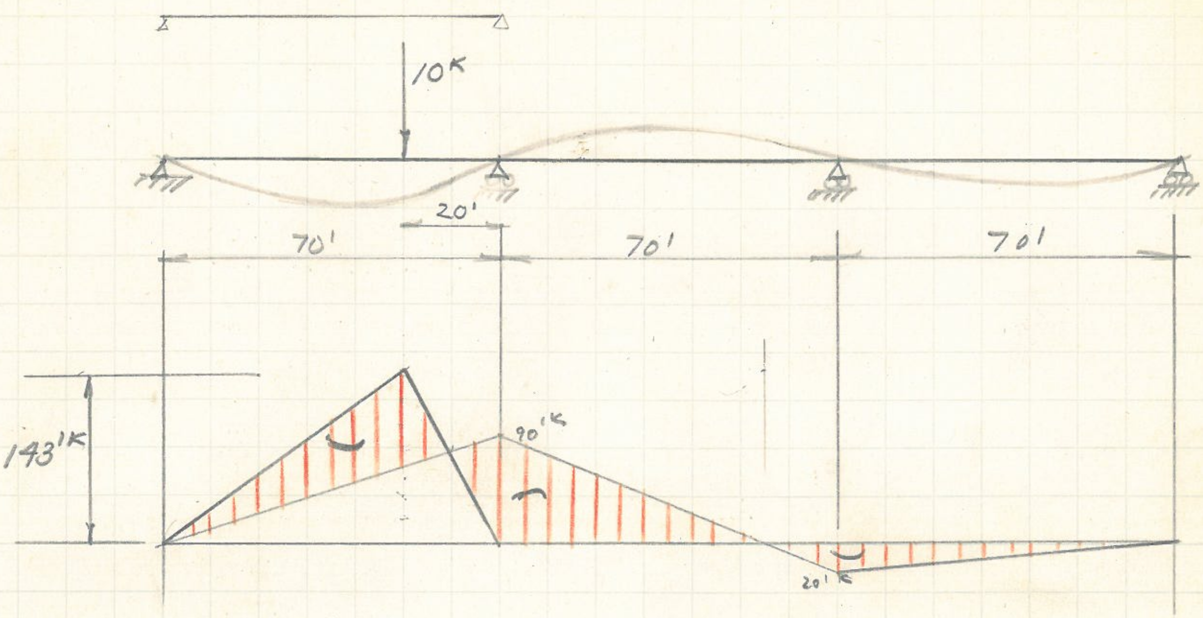
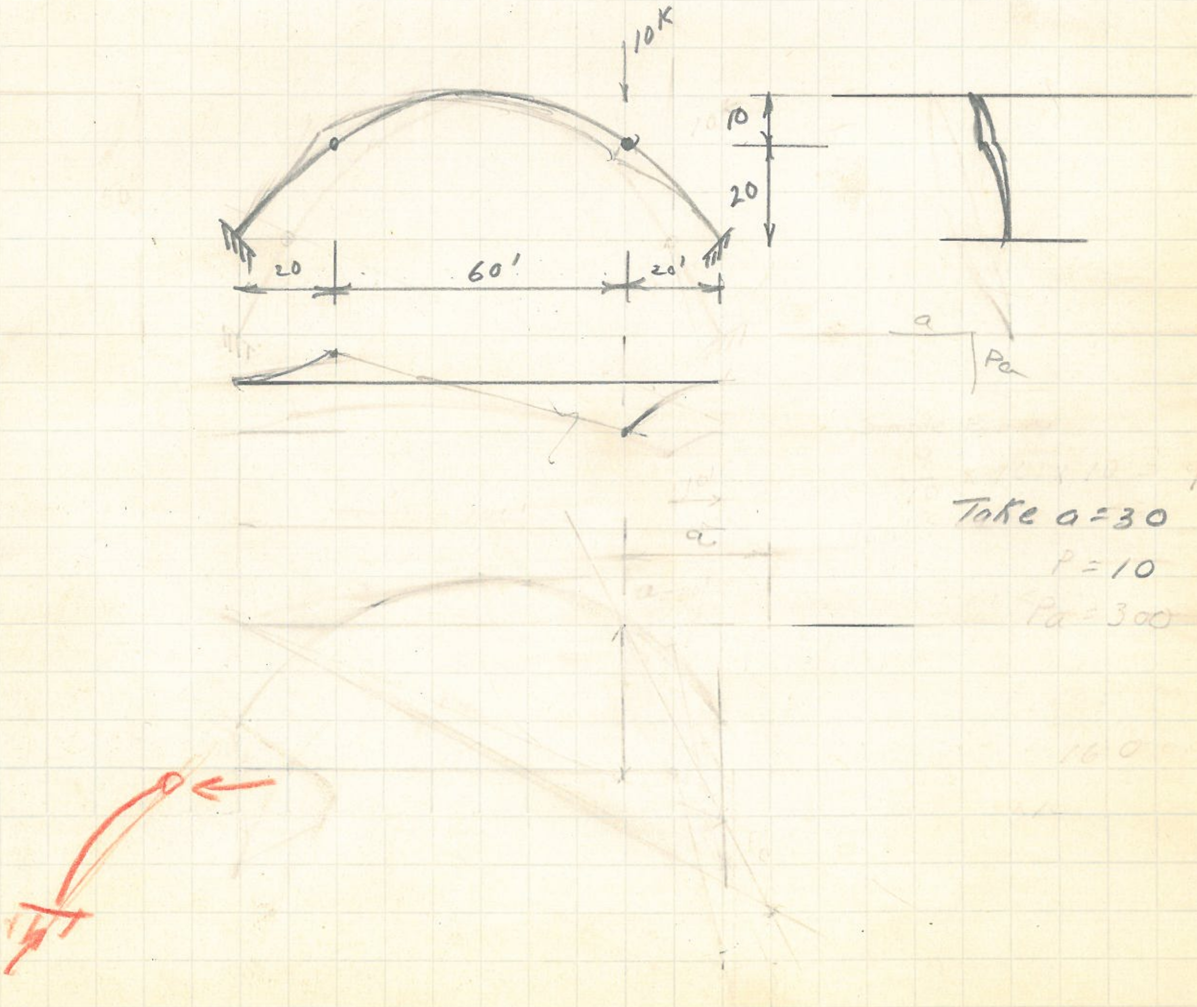
(a)

$\frac{2}{7} \times 50 \times 10 = 143^{'K}$



(b.)



Take a = 30

P = 10

Pa = 300

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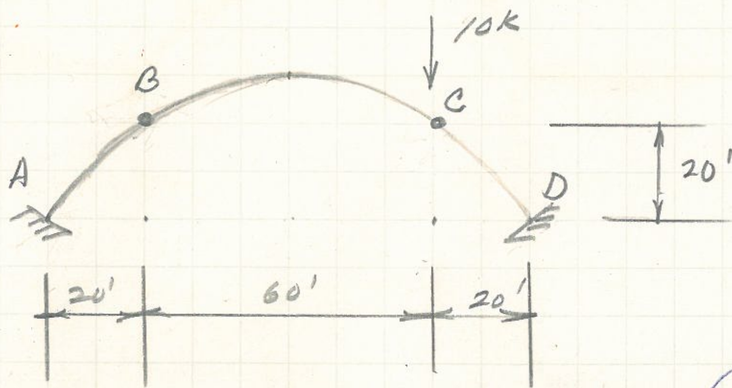
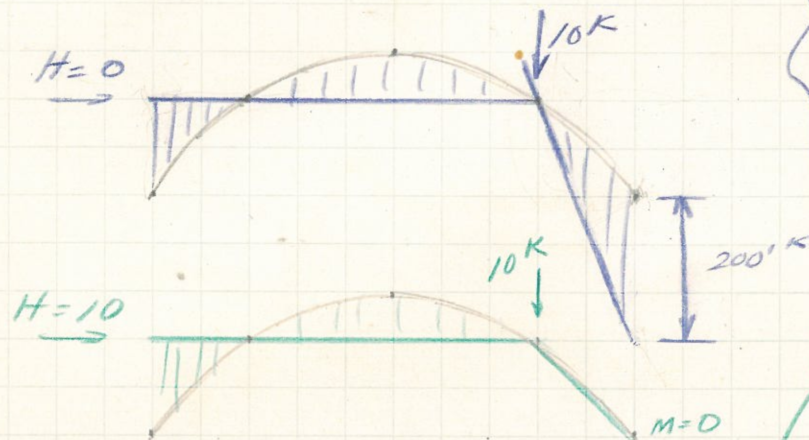
(b).

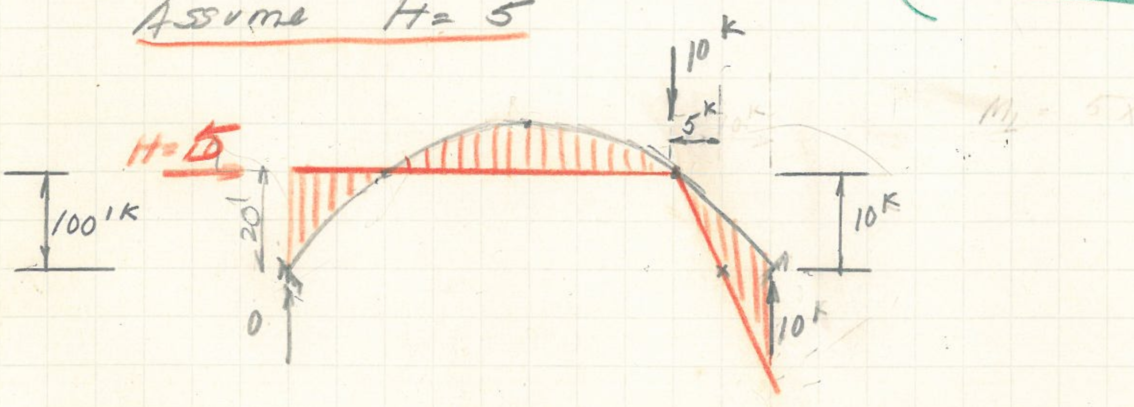




ABC very flexible
CD rigid
End moment =
10×20 = 200'k

ABC very rigid
so that the moment
@ D = 0
End Moment
= 0 = H·20 − 10·20
∴ H = 10

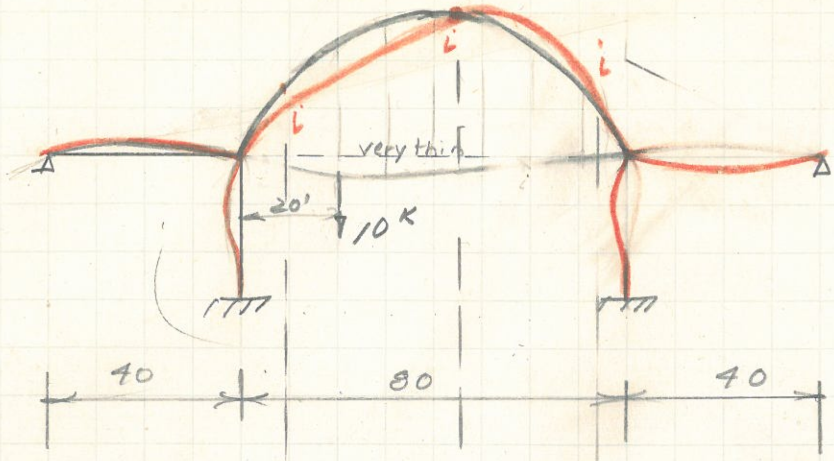
Assume H = 5



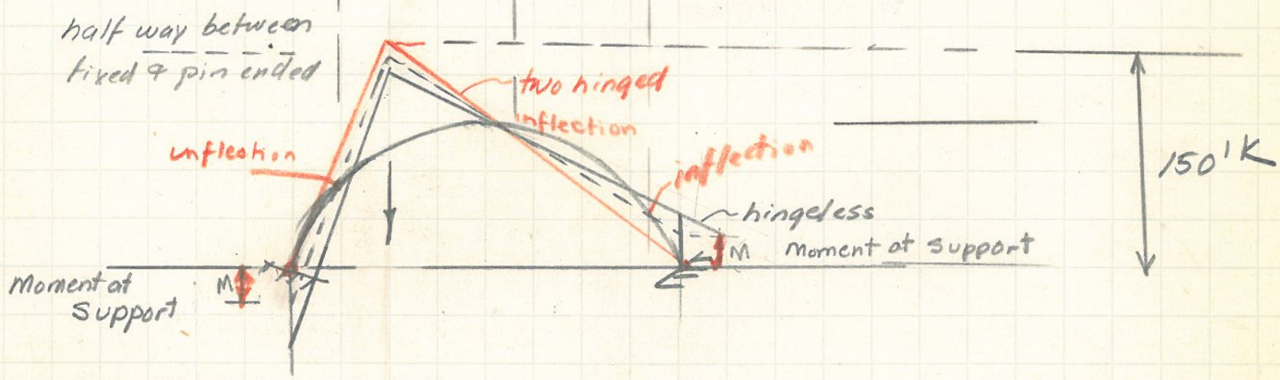
This makes the moments on both ends about equal — although not far from wrong since it could be designed that way nevertheless if of equal stiffness throughout the right hand moment is probably larger than the left hand as the deflected structures bears out

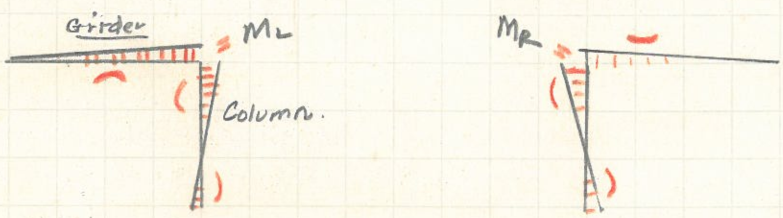
Prob 1

(c)



$\frac{3}{4} \times 10 \times 20$





Left side M_L > Right side M_R

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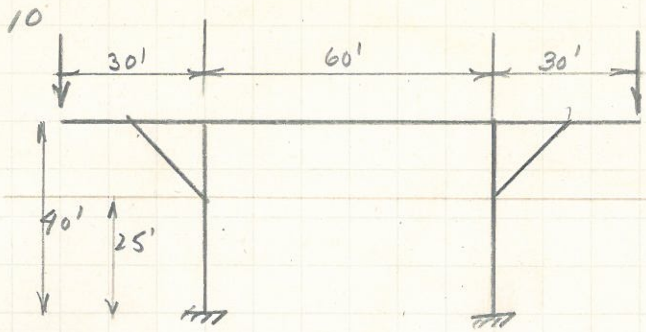
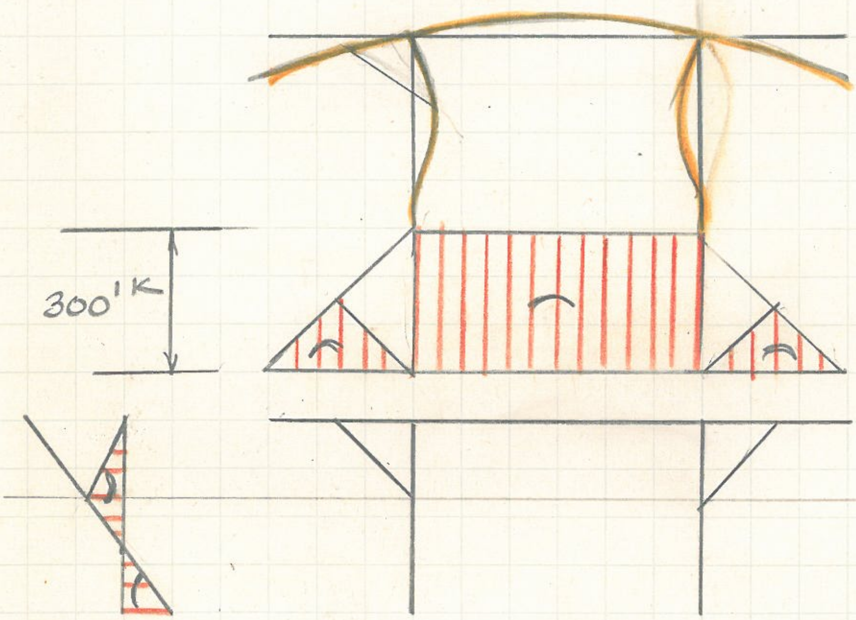
(d)



$30 \times 10 = 300$



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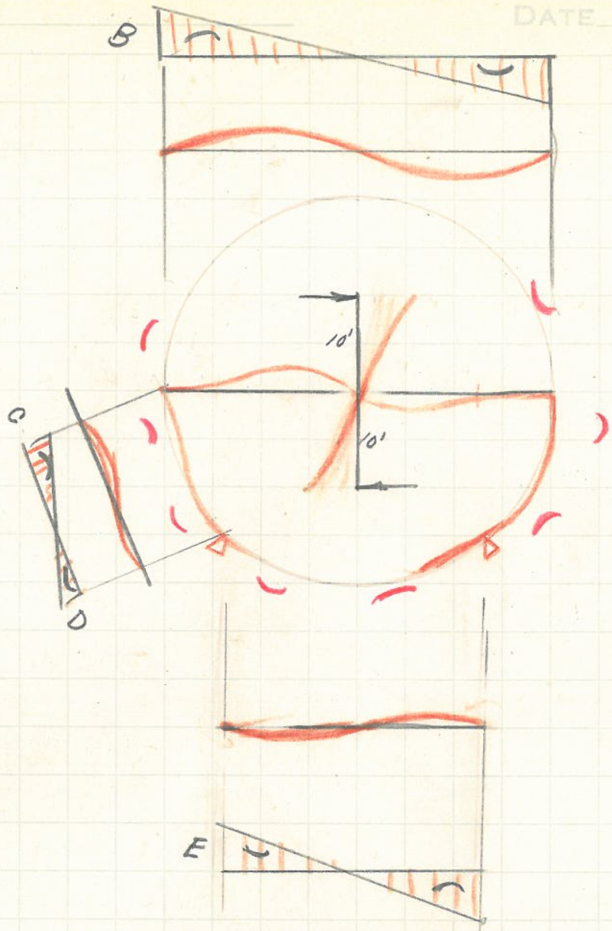
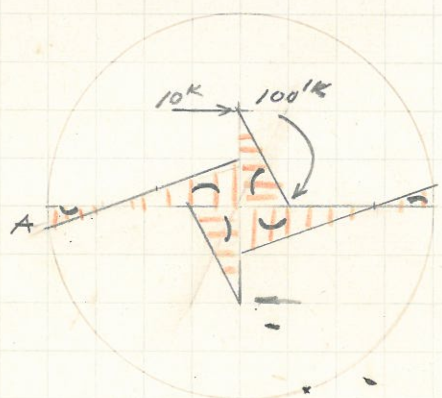
(e)



Moment A = Moment B + Moment C

Moment D = Moment E



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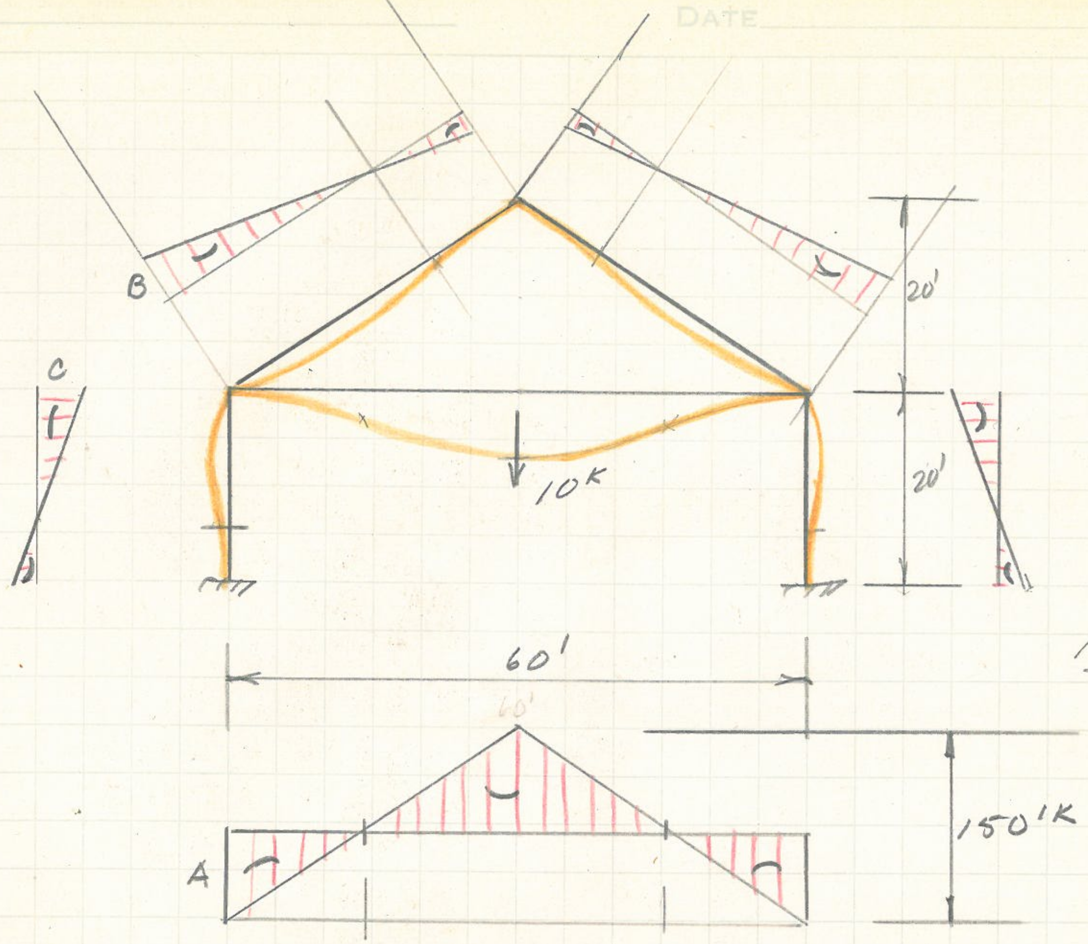
(f)



$\frac{10 \times 60}{4} = 150$

Moment A = Moment B + Moment C

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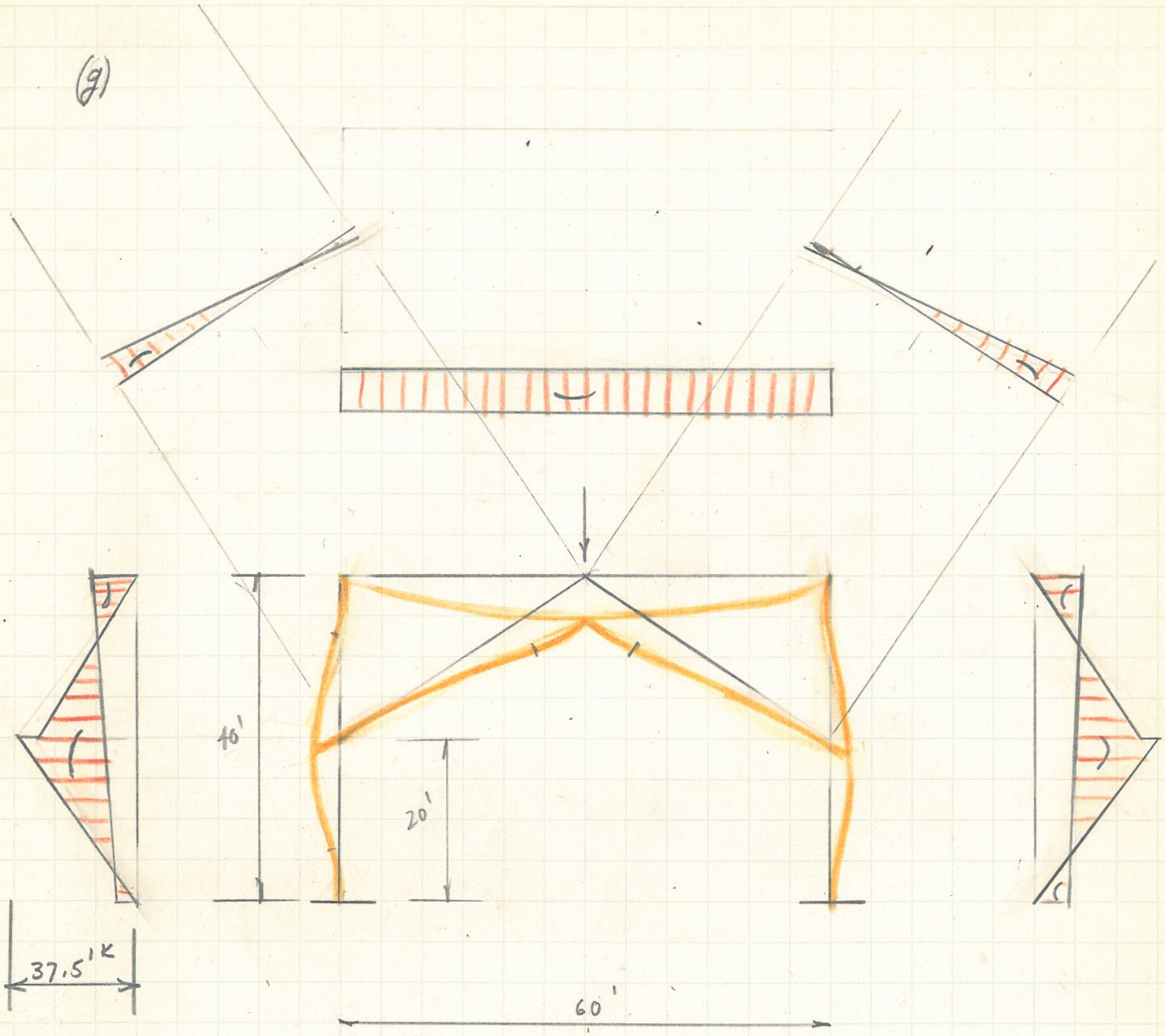
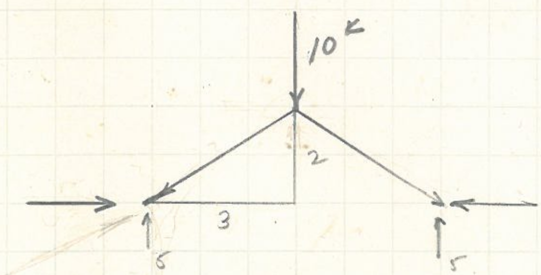
(g)





$\frac{3}{2} \times 5 = H = 7.5^K$

$M = \frac{7.5 \times 20}{4} = 37.5$

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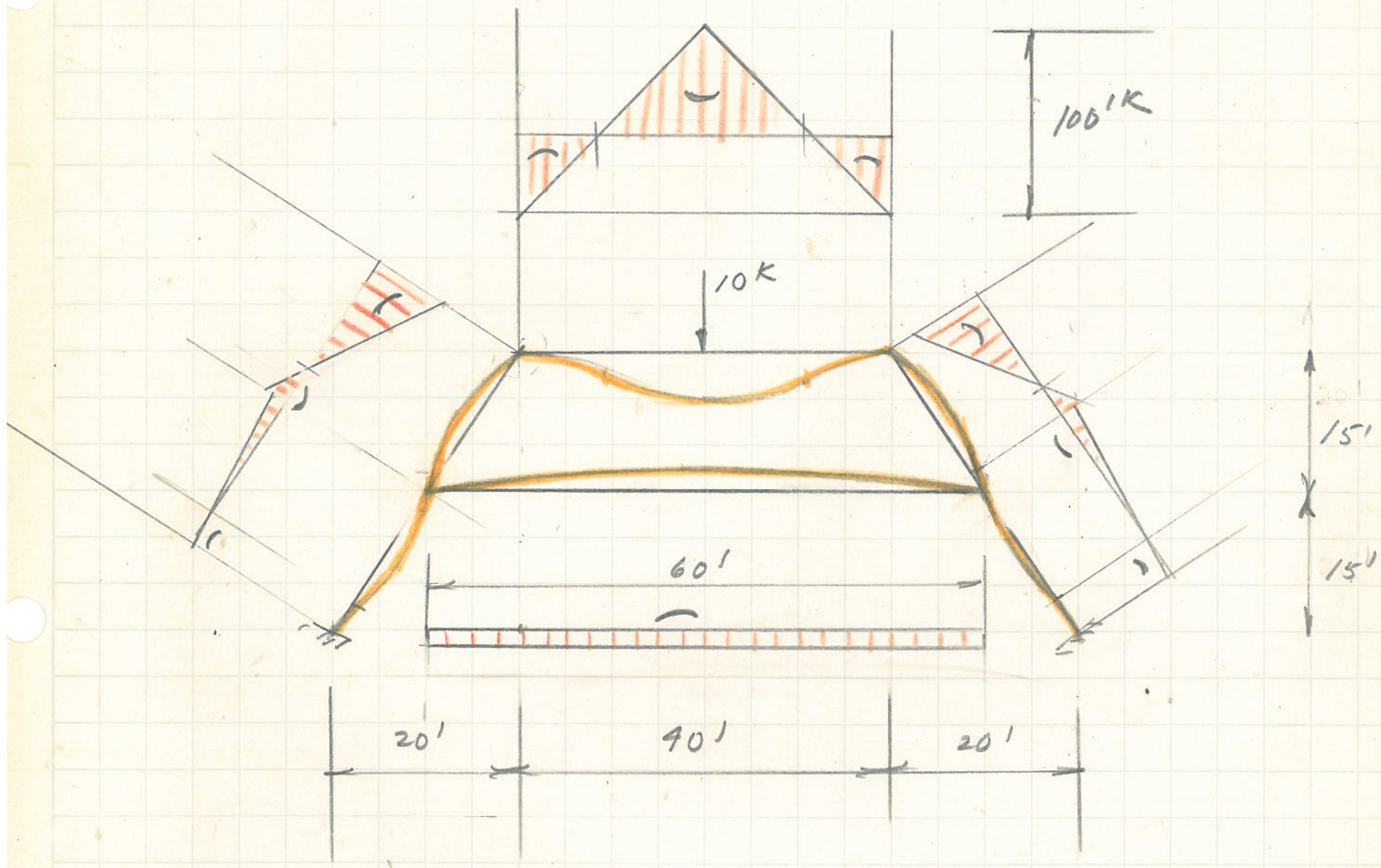
h.



$$\frac{10 \times 40}{4} = 100'^{K}$$

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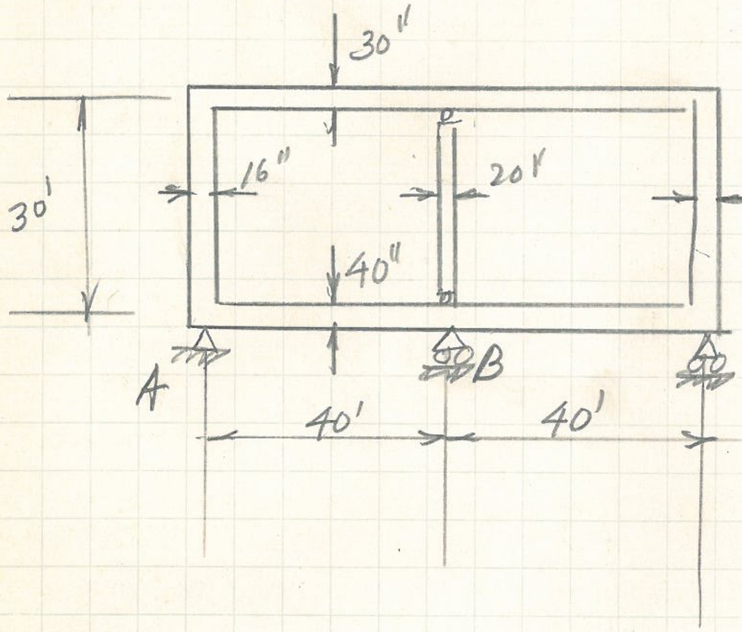
2. (a) Determine the maximum bending stresses in the vertical members due to a 2" settlement of support B.
Assume no "Kickback" in horizontal members.

(b) Calculate maximum bending stress in each chord.



Prob 2.

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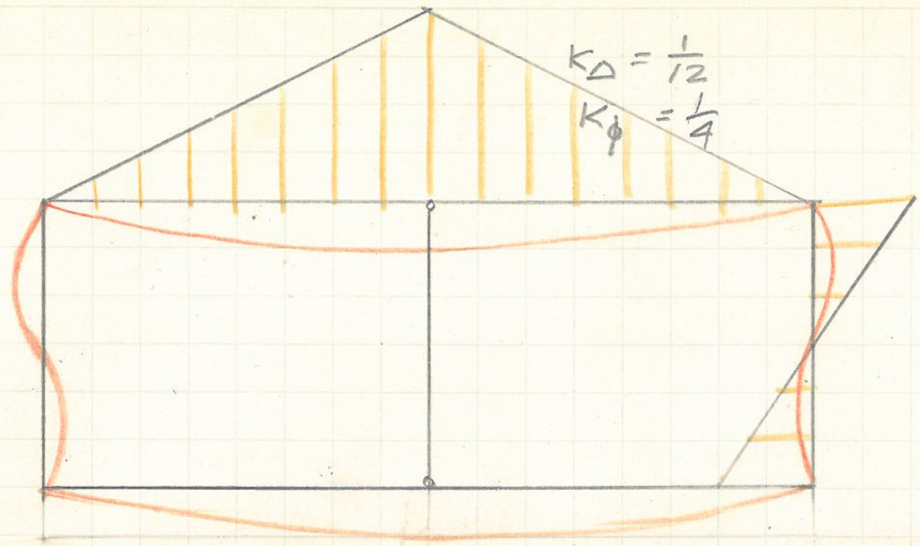

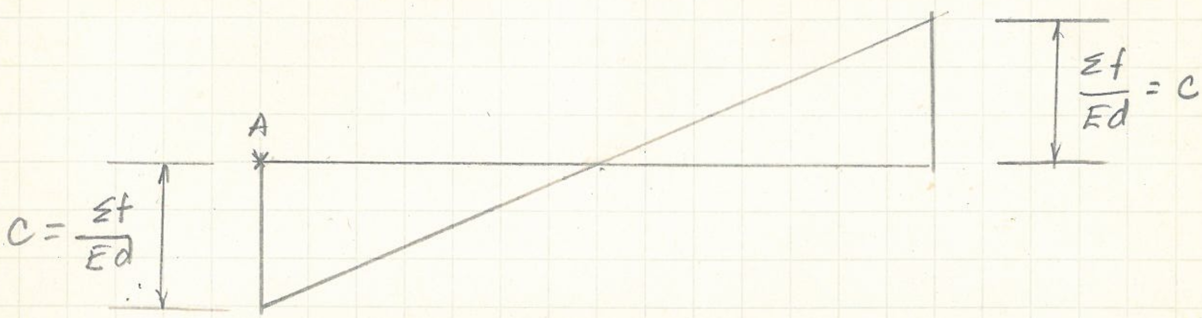

ΣM about A

$$\Sigma M = \frac{1}{2} \times C \cdot \frac{l}{2} \; \frac{5}{6} l - \frac{1}{2} \cdot C \cdot \frac{l}{2} \cdot \frac{1}{6} l$$

$$= \frac{2}{3} \cdot \frac{C l^2}{4} = \frac{1}{6} C l^2$$

$$R = \frac{1}{6} \frac{\Sigma f}{Ed} \frac{l^2}{l} = \frac{1}{6} \frac{\Sigma f}{Ed} l$$

$$\therefore \; K_\phi = \frac{1}{6}$$

Prob 2.

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(a) $\Delta = \frac{1}{12}\frac{\Sigma f_h l_{80}^2}{Ed}$ $\qquad \phi_{hor} = \frac{1}{4}\frac{\Sigma f_h l_{80}}{Ed}$

$\phi_{hor} = \phi_{vert}$ $\qquad \phi_{vert} = \frac{1}{6}\frac{\Sigma f\, l_{30'}}{Ed}$

$$\Sigma f_{80}\begin{cases}30''\\40''\end{cases} = \frac{12\,\Delta\,Ed}{l_{80}^2} = \frac{4\,Ed\,\phi}{l_{80}}$$

$$\phi_{\substack{hor\\ \text{or vert.}}} = \frac{3\Delta}{l} = \frac{3,2}{30\times12} = \frac{1}{160}$$

$$\phi_{vert} = \frac{1}{6}\frac{\Sigma f\, l_{30'}}{Ed}$$

$$\Sigma f = \frac{\phi\, Ed\, 6}{l_{30'}}$$

$$\Sigma f = \frac{1}{160}\cdot\frac{30{,}000{,}000\times10\times6}{30\times12} = 31{,}200 \text{ psi}$$

$f_{10''} = 15600$ psi

$$\Sigma f = \frac{1}{160}\cdot\frac{30\,000\,000\times16\times6}{30\times12} = 50\,000$$

$f_{16''} = 25\,000$ psi

(b) $\Sigma f = \frac{\Delta\, 12\, Ed}{l^2}$

$$f_{30''} = \frac{2\times12\times30{,}000{,}000\times30}{2\times80\times12\times80\times12} = 11{,}700 \text{ psi}$$

$$f_{40''} = \frac{2\times12\times30\,000{,}000\times40}{2\times80\times12\times80\times12} = 15{,}600 \text{ psi}$$

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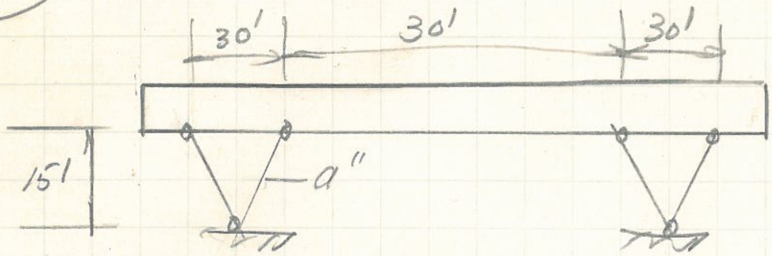
(3.) Draw influence lines (with scale) for vertical loads.

(a)



1) Horizontal drift

2) Force in bar "a"

(b)

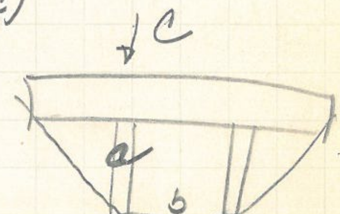
(1) Vertical Reaction at A

(2) Horizontal Thrust

(3) Moment at A

(4) Moment at Crown

(c)



(1.) Force in "a"

(2) " " "b"

(3) " " "c"

(4) Moment " c

Prob 3.

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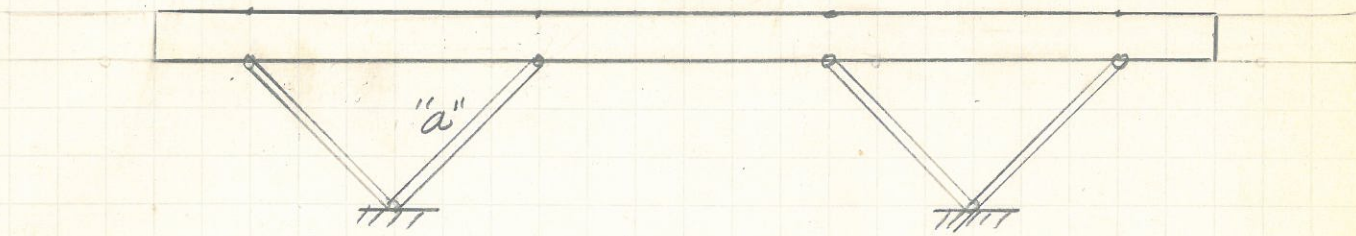
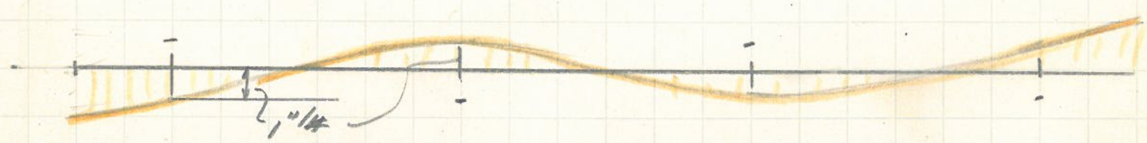
(a)



1) Horizontal Drift

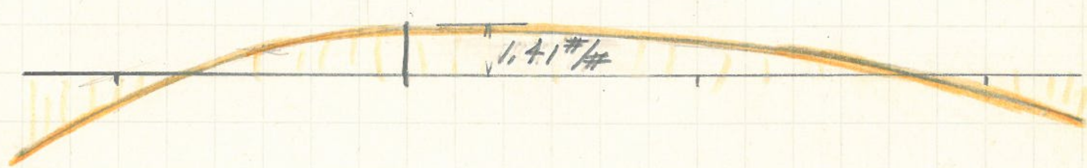
Put it back over supports - represents vertical displacement



Bars are at 45° so that a horizontal displacent will cause a vertical displacement that is equal to it.

(2) Force in bar "a"

Elongate bar "a" .. Check points of loading to see whether they go up or down.



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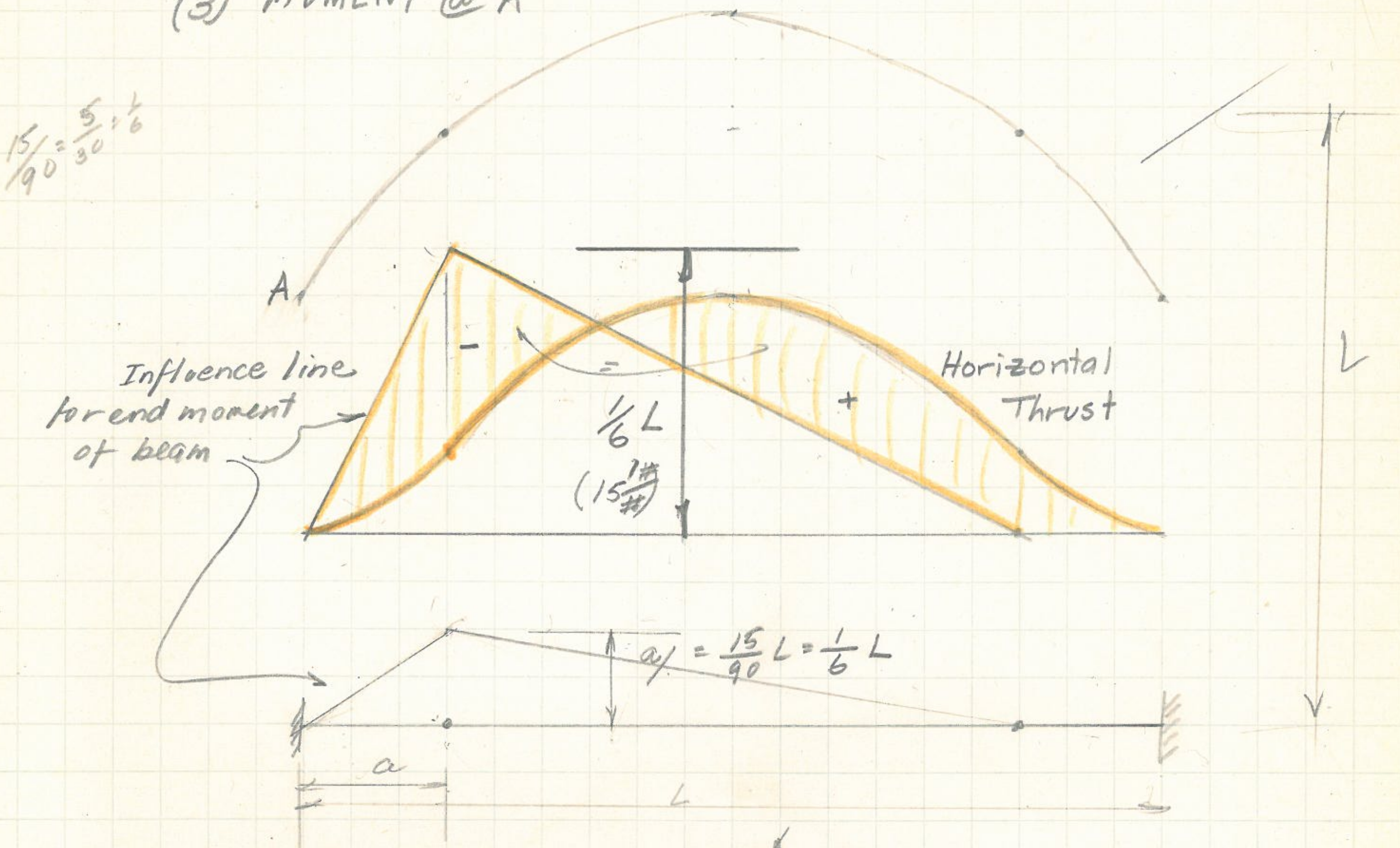
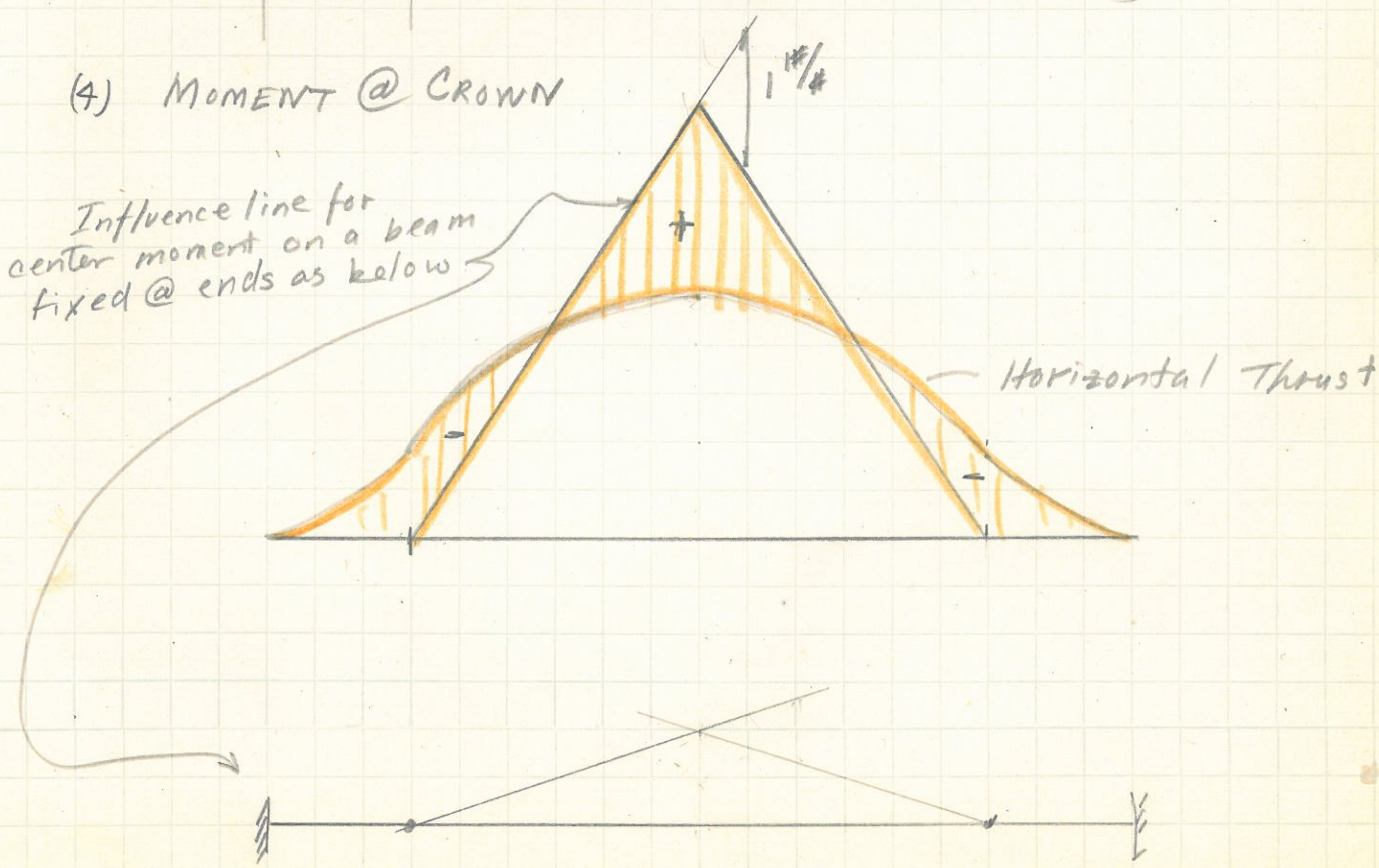
(6.) cont.

(3) MOMENT @ A



(4) MOMENT @ CROWN



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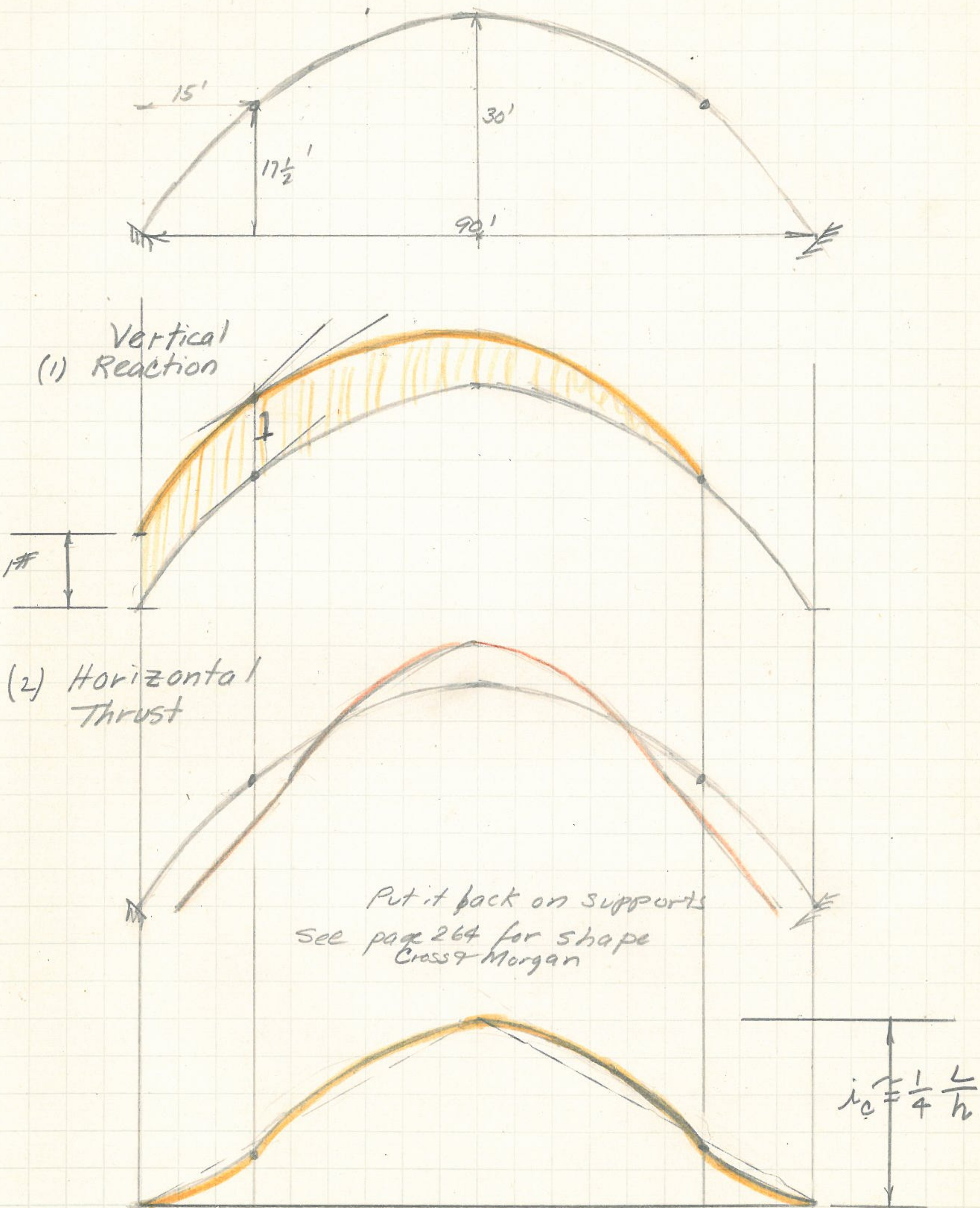
(b.)



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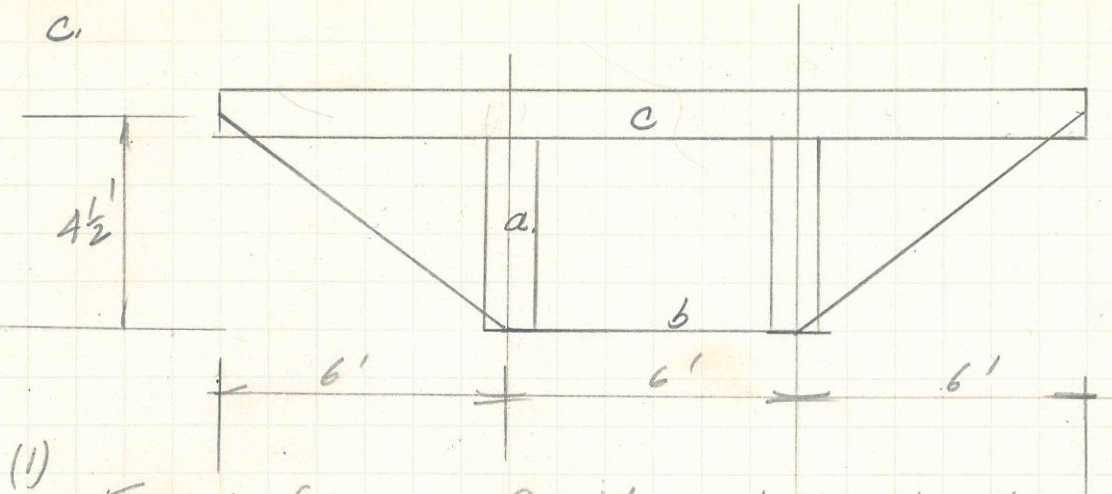
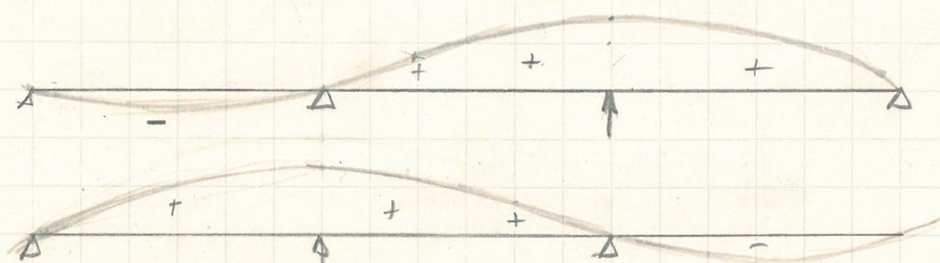
c.



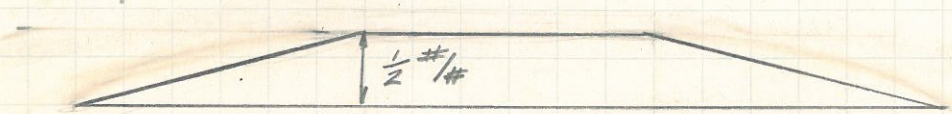
(1)

Force in (a) Consider two simple supports one @ a time

Force in one post = force in other



Total = straight line distribution



For scale see "Modern Framed Structures" Part II pg 359 Johnson, Bryan, & Turneaure. If at is loaded over the support with a load of $1^{\#}$ the stress in each of the verticals will be $\frac{1}{2}$

(2) Force in "b"

Same shape as part (1), Scale: horizontal component. — in this case $\frac{1}{2}$.

$$\frac{\frac{1}{2}}{4\frac{1}{2}} = \frac{(b)}{6}$$

$$(b) = \frac{6 \cdot \frac{1}{2}}{\frac{9}{2}} = \frac{3}{9} \cdot 2 = \frac{2}{3}\ \#/\#$$

see ~~over~~ next page

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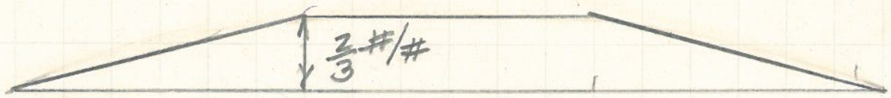
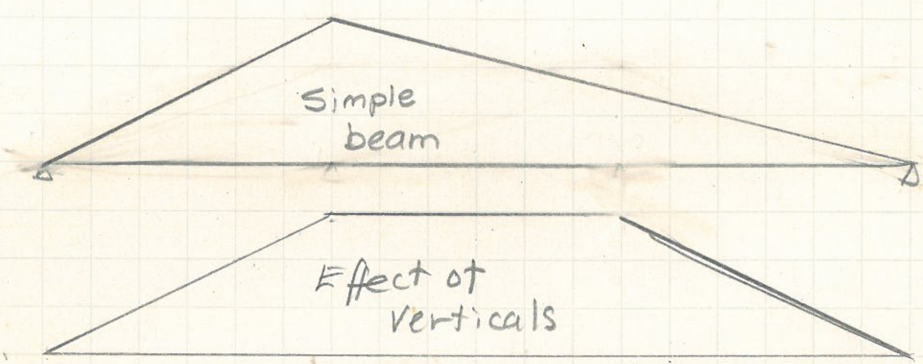
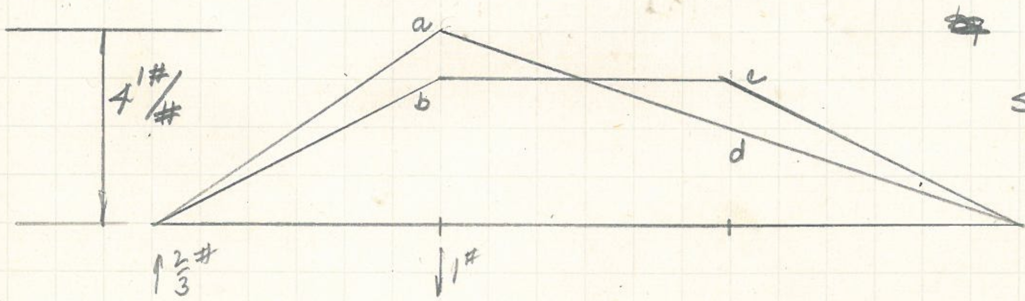


(3) Force in "c"
same shape and same scale as part "b"

(4) Moment in "c"
Add two curves -



Total



ab = cd
see prob 15

NAME

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4. Compare maximum downward deflections of the 2 designs.

(a) 3 simple span plate girders
$L = 80'$ $d = 8'$

(b) 3 continuous plate girder spans.
$L = 3 @ 80'$ $d = 6'$

Both designed for D.L 1000#/ft uniform
L.L. 2000#/ft "

Allowable Stress 18000#/in²

PROB 4.

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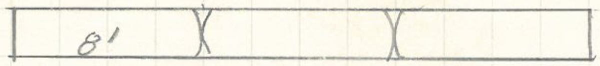
Simple Span



$K_\Delta = \frac{1}{9.6}$

$$\Delta = K_\Delta \frac{\Sigma f l^2}{E d} = \frac{1}{9.6} \times \frac{36\cancel{000} \times 8\cancel{0} \times \cancel{8}\cancel{0} \times 12 \times 1\cancel{2}}{30\,0\cancel{0}\cancel{0}\,\cancel{000} \times \cancel{8} \times 1/2}$$

Simple

$\underline{\Delta_{TOTAL}} = \underline{1.2}''$

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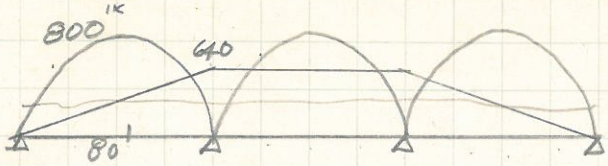
Continuous

D.L.



$F.E.M = \frac{wl}{12} = 533^{D.L.}$

$1066^{L.L.}$

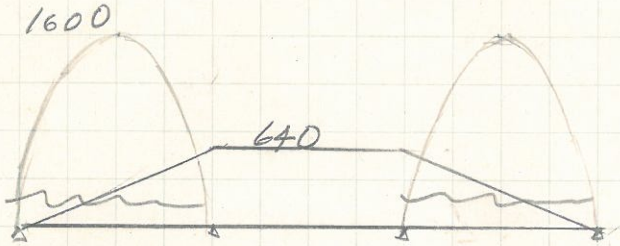
D.L. - Mom. Dist. Coeff. = 1.2.

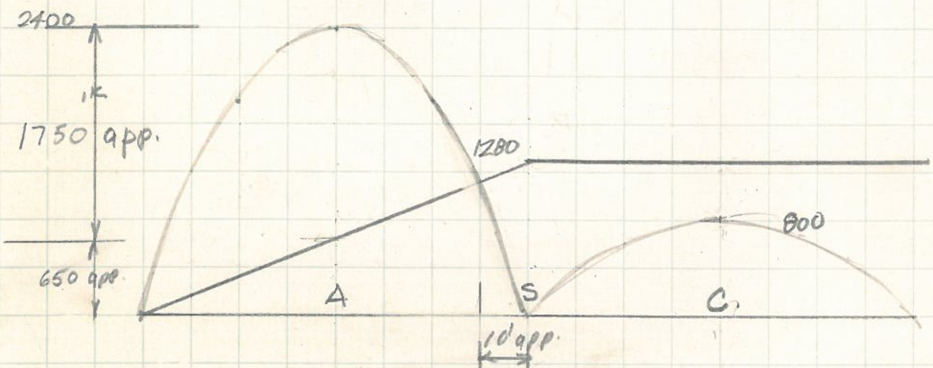
L.L - Mom. Dist Coeff. = 0.6

$\frac{wl}{8} = 800^{D.L.}$

$1600^{L.L.}$

L.L.





Assume max moment = 1750^1K @ A

Max fiber stress = 18,000 psi ; $\Sigma f_A = 36,000$

∴ fiber stress over the support

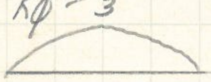
$= \frac{1280}{1750} \times 18,000 = 13200$; $\Sigma f_s = 26400$

Σ fiber stress at $C = \Sigma f_s - \frac{800}{1750} \times 18,000 \times 2 = \Sigma f_s - \underline{16500}$

Slope @ support S

Separate the 2 parts

$K\phi = \frac{1}{3}$



$\boxed{K\phi = \frac{1}{2}}$

$\phi = K\phi \frac{\Sigma f}{Ed} l$

$\phi = \frac{1}{2} \cdot 26400 \frac{l}{Ed}$

$= \frac{1}{3} \times 16500 \frac{l}{Ed}$

$\phi_s = (13200 - 5500) \frac{l}{Ed} = 7700 \frac{l}{Ed}$

Prob 4

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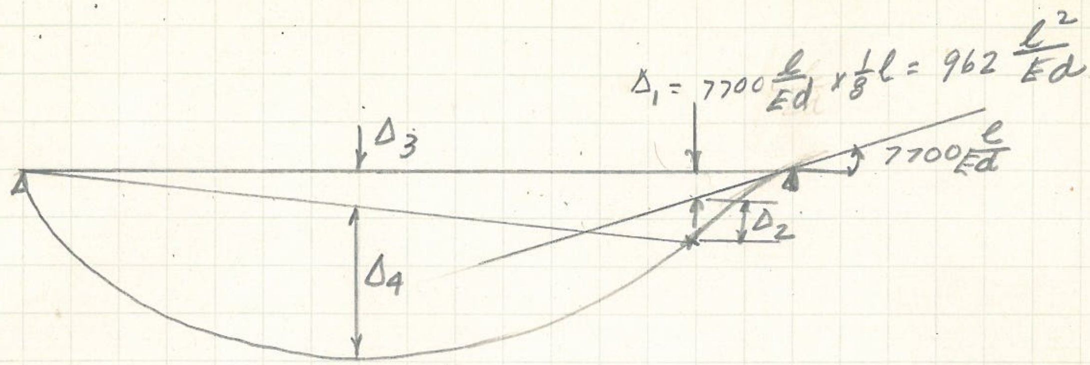

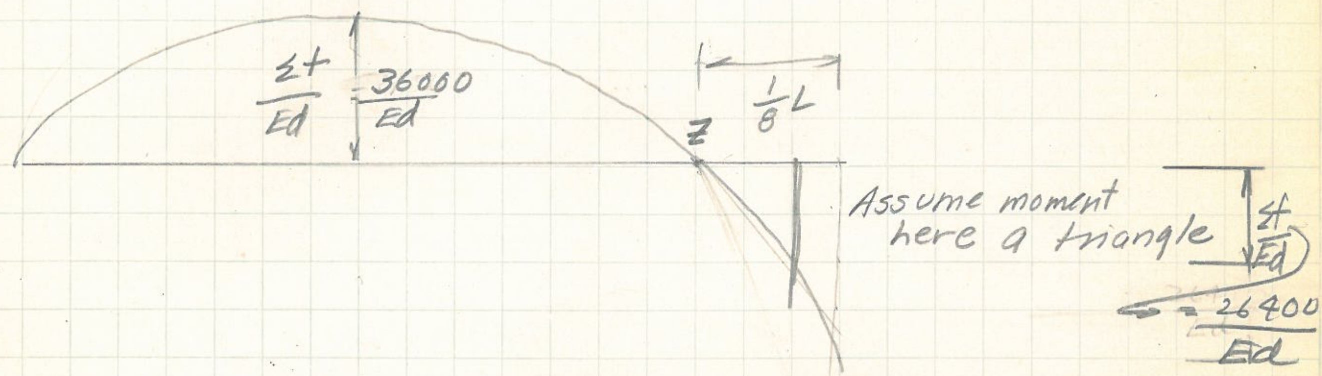

$$\Sigma M_Z = \Delta_2$$

$$\Delta_2 = \frac{26400}{Ed} \cdot \frac{1}{8} l \cdot \frac{1}{2} \times \frac{2}{3} \frac{1}{8} l$$

$$\Delta_2 = 138 \frac{l^2}{Ed}$$

$$\Delta_1 + \Delta_2 = 1100 \frac{l^2}{Ed}$$

$$\Delta_3 = 550 \frac{l^2}{Ed} \qquad \Delta_4 = \frac{1}{9.6} \times \frac{36000}{Ed} \left(\frac{7}{8}\right)^2 l^2$$

$$= 2870 \frac{l^2}{Ed}$$

$$\Delta_{TOTAL} = \Delta_3 + \Delta_4 = 3420 \frac{l^2}{Ed}$$

$$\frac{l^2}{Ed} = \frac{80 \times 80 \times 12 \times 12}{30{,}000{,}000 \times 6 \times 12} = \frac{64}{150{,}000} = .000426 \frac{in^3}{\#}$$

Continuous

$\Delta_{TOTAL} = 1.5''$

Simple

$\Delta = 1.2''$

NAME

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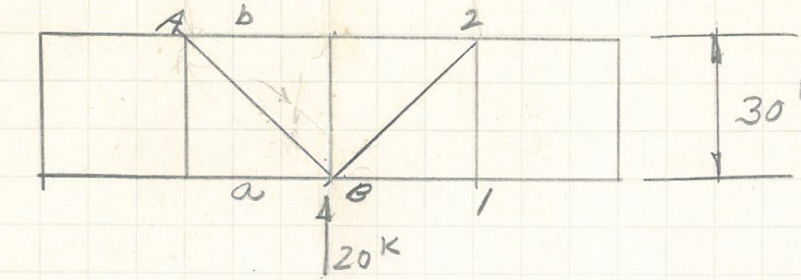
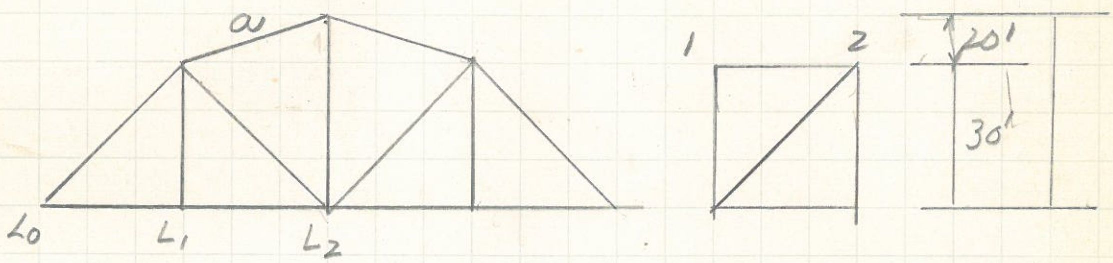
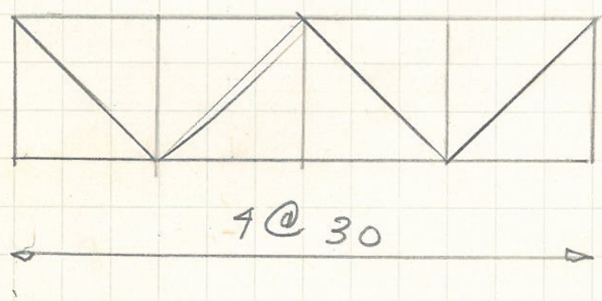
5. Find stress in bar $\underline{a}$ due to horizontal load shown.







No Portal
No Transverse
Bracing

PROB 5

NAME R. A. Heller

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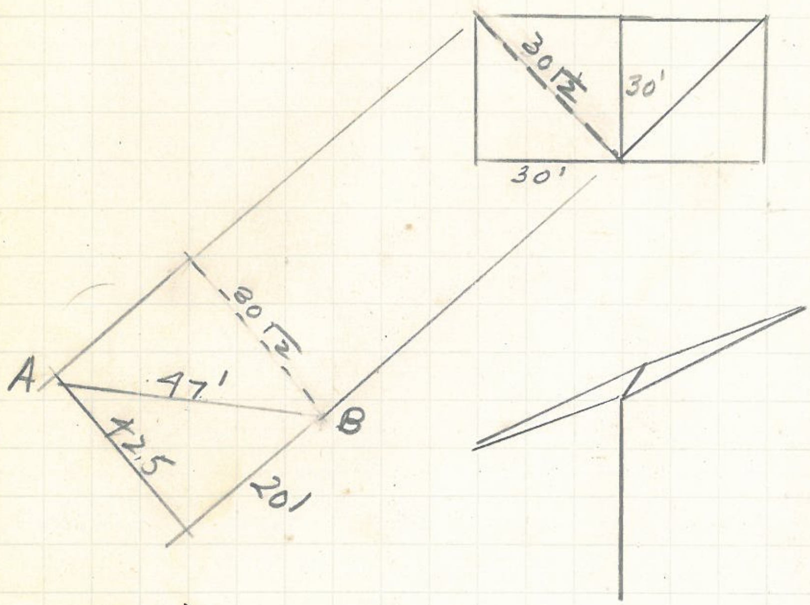
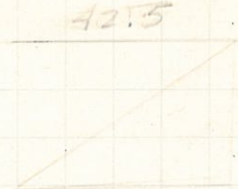
$30 \times 1.414 = 42.5$





$42.5^2 = 1800$

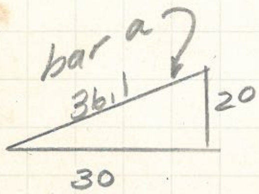
$20^2 = \underline{400}$

2200

$\sqrt{2200} = 47$

Length of member AB = 47'

Length of member a = 36.1'



Stress in AB = $\frac{47}{30} \cdot \frac{20^k}{2}$ = not needed

$\frac{400}{900}$
1300

Stress in bar a = $\frac{36.1}{3\cancel{0}} \cdot \frac{2\cancel{0}}{2} = 12^k$

Stress is proportional to length

36.1

It was assumed here that the stress is brought over to lateral frame 1-2 and that bar "a" resists in tension the upward vertical component of bar AB which is $\frac{20}{30} \times \frac{20}{2} = 6\frac{2}{3}^k$

NAME

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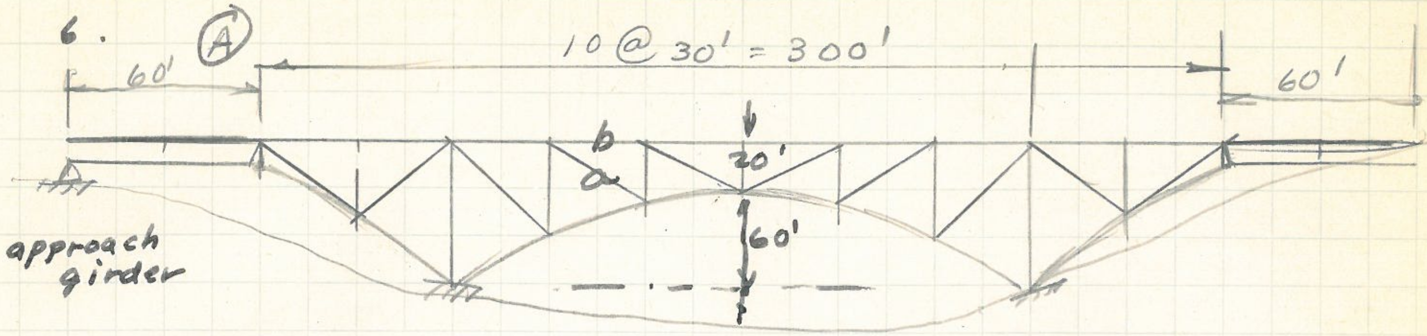
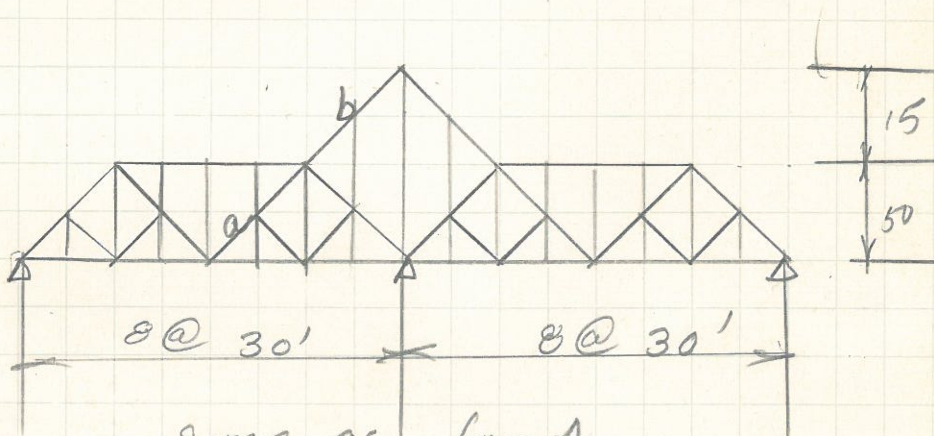
6. A



Panel points of lower chord on a parabola.

Draw approximate influence lines for stresses in bars "a" and "b" and from them compute approximate live load stresses in these bars for a load of 2^k/ft per truss.

B.



Same as for A.

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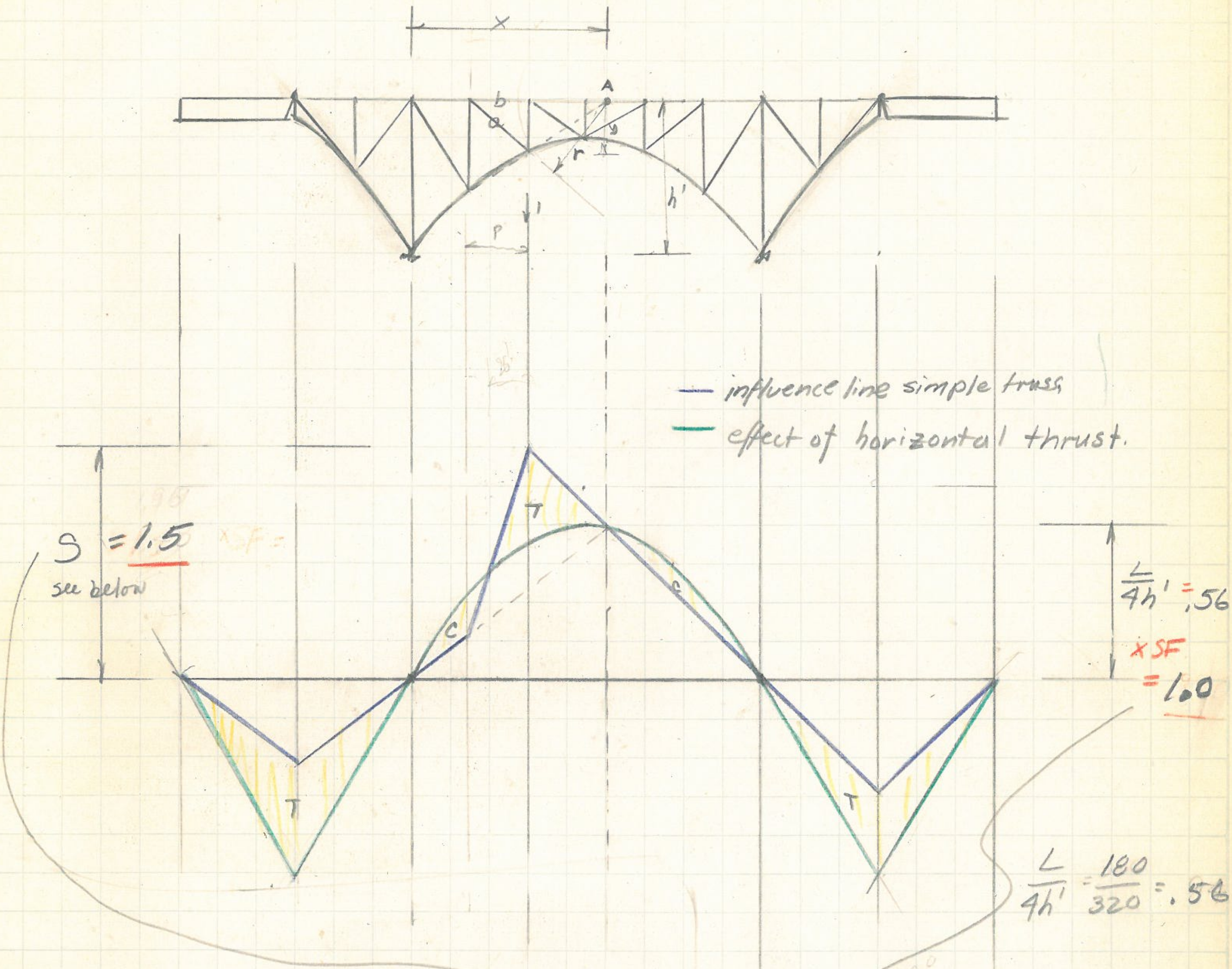
A (a)



Simple truss

$R = \frac{2}{3}$ $r \cong 45'$

$Sr = Rx$

$S = \frac{\frac{2}{3}\,100}{45} = 1.48$ say 1.5

Scale factor $= \frac{h'}{r}$

$= \frac{80}{45} = 1.78$

$.56 \times 1.78 = 1.00$

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A. (a) cont.

Live Load Stress in bar "a"

Approximation of areas of curves

Tension –

$2 \times 3 \times \frac{1}{4} \times \frac{60'}{2} = 45$

$2\frac{1}{2} \times \frac{1}{4} \times \frac{20'}{2} = 6\frac{1}{4}$

$2\frac{1}{2} \times \frac{1}{4} \times \frac{40'}{2} = 12\frac{1}{2}$

$2 \times 2 \times \frac{1}{4} \times \frac{60}{2} = 30$

94

$\times 2$

Max L.L $\longrightarrow$ 188^{K} T

Compression –

$1 \times \frac{1}{4} \times \frac{30}{2} = 7\frac{1}{4}$

$1 \times \frac{1}{4} \, \frac{5}{2} = \frac{5}{8}$

$\frac{3}{4} \times \frac{1}{4} \times \frac{40}{2} = 3\frac{3}{4}$

$2 \times \frac{1}{4} \times \frac{10}{2} = 2\frac{1}{2}$

14

$\times 2$

$\longrightarrow$ 28^{K} C

Stress when loaded thruout = 160^{K} T

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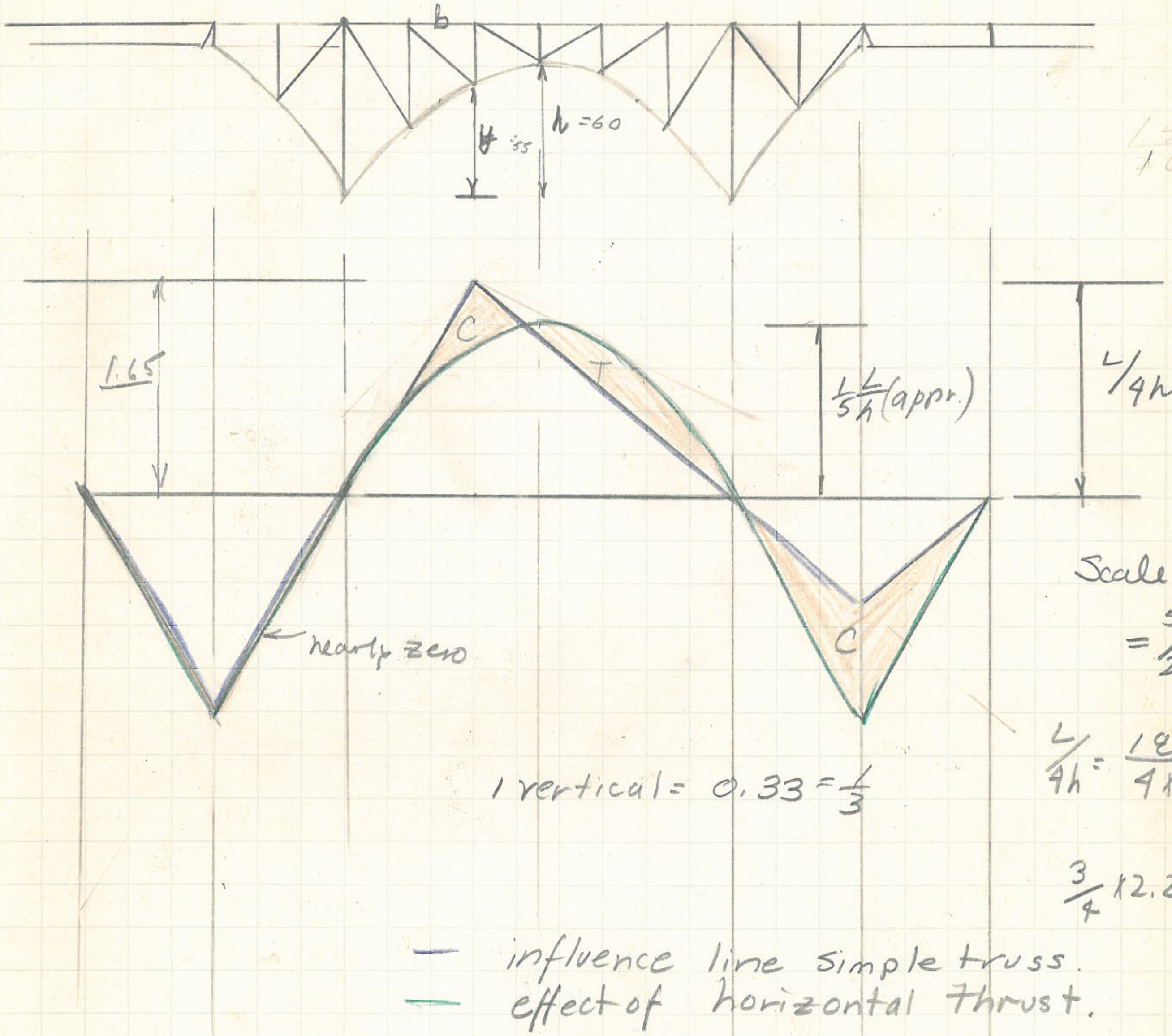
A (b)



150
150
165

Scale $= y/r$

$= \frac{55}{25} = 2.2$

$\frac{L}{4h} = \frac{180}{4 \times 60} = \frac{3}{4}$

$\frac{3}{4} \times 2.2 = 1.65$

1 vertical $= 0.33 = \frac{1}{3}$

— influence line simple truss.

— effect of horizontal thrust.

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A 6 cont.

Live Load Stress in bar "b"

Comp.

$\frac{3}{2}\ \frac{35}{2} \times \frac{1}{3} = 8\frac{3}{4}$

$\frac{3}{2} \times \frac{25}{2} \times \frac{1}{3} = 6\frac{1}{4}$

$2 \times \frac{5}{2} \times \frac{60}{2} \times \frac{1}{3} = 50$

65

2

130^k C

Tens.

$30 \times 1 \times \frac{1}{3} = 10$

$\cancel{1}\cancel{4} \times 20 \times 1 \times \frac{1}{3} = 20$

$1 \times 15 \times \frac{1}{3} = 5$

35

2

70^k T

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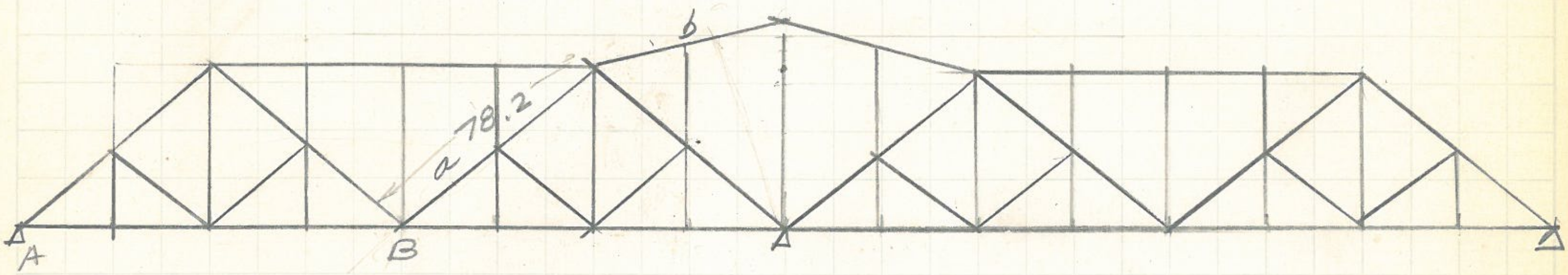
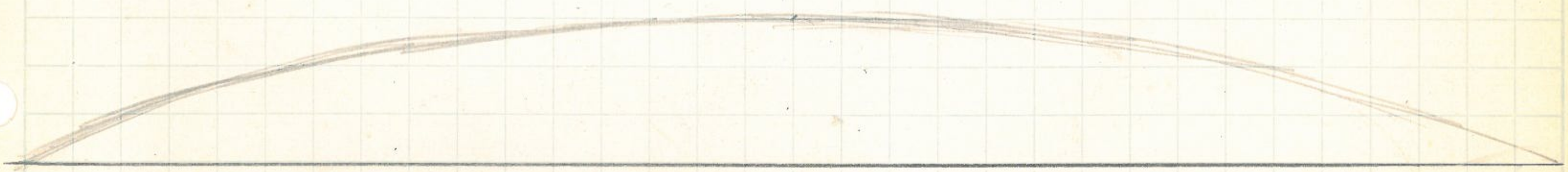
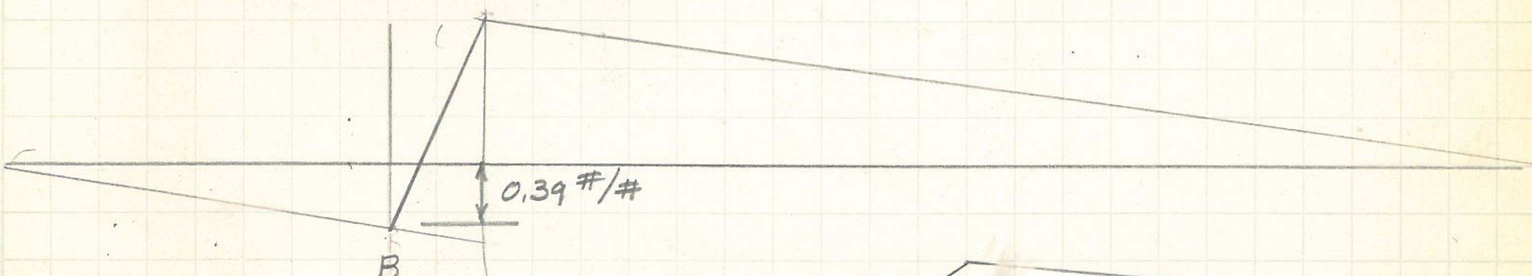
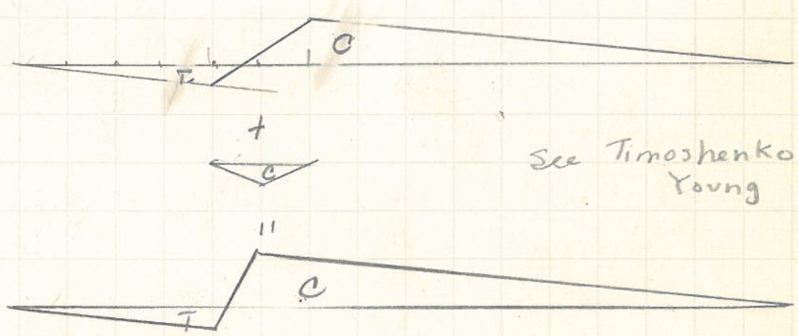
B (a)









See Timoshenko & Young

Put a one lb. load on at B. The shear in panel "a" is

$\frac{3}{4} - 1 = \frac{1}{4}$ T

The stress in bar "a" is $\frac{78.2}{50} \cdot \frac{1}{4} = .39$ #/#

over

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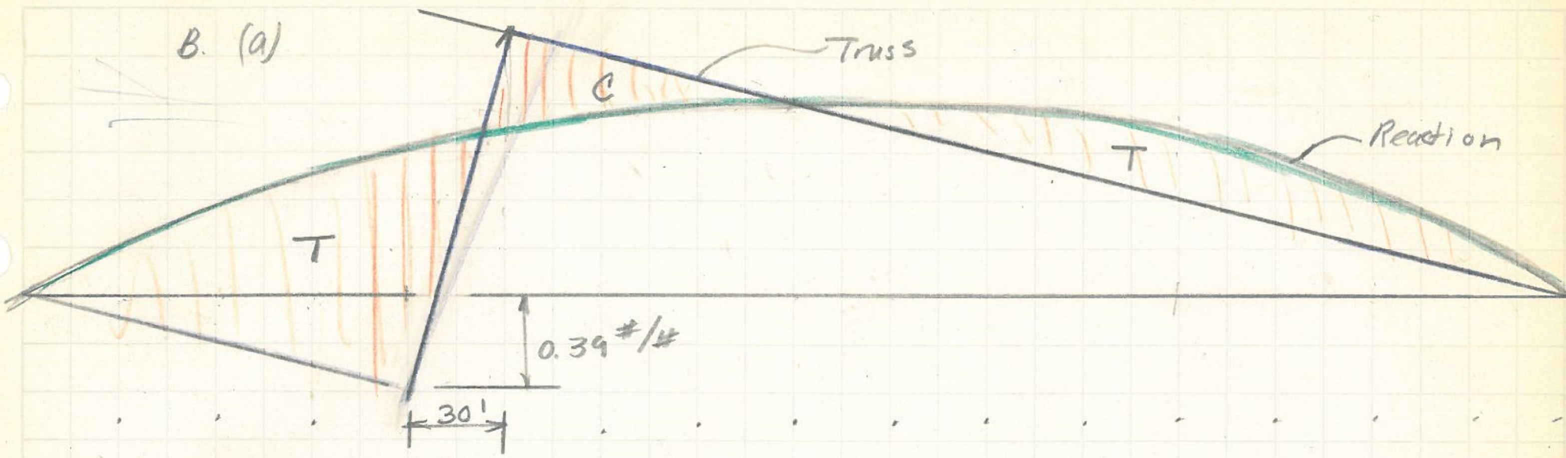
B. (a)



Tension

$\frac{7}{2} \times .2 \times \frac{130}{2} = 45.5$

$2 \times .2 \times \frac{130}{2} = 26$

$\frac{3}{2} \times .2 \times \frac{240}{2} = 36$

107.5

2

215^{K} T

Compression

$2 \times .2 \times \frac{100}{2} = 20$ C

× 2

40^{K}C

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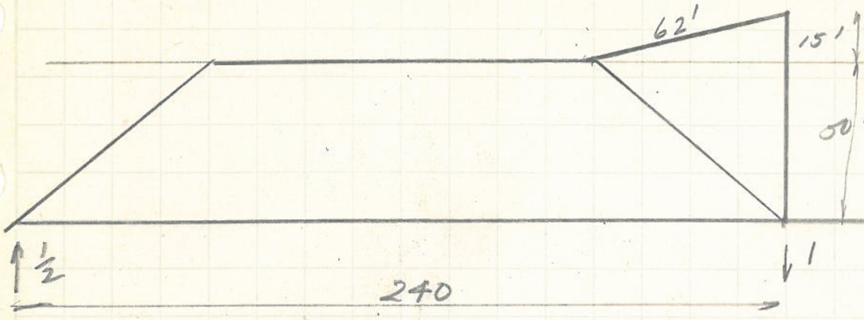
B b. Influence line for "b"

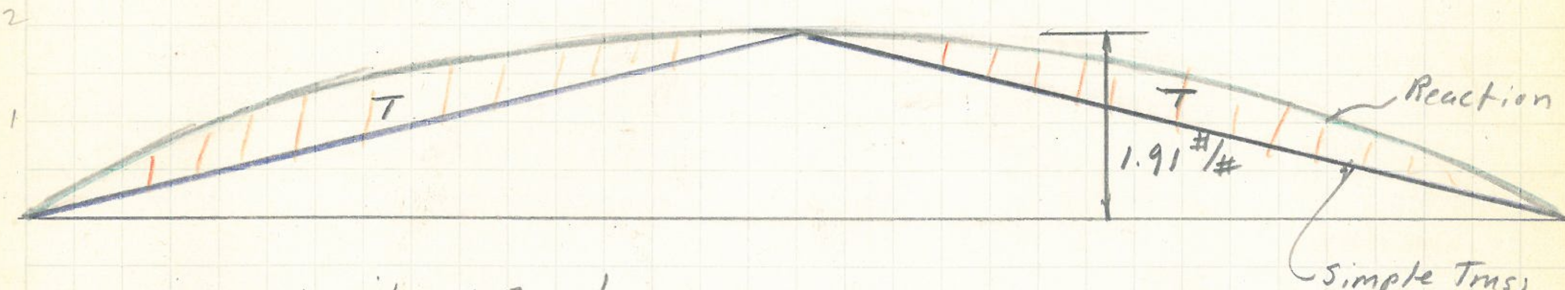


$\Sigma M = 0$

$\frac{1}{2} \cdot \frac{240}{65} = H = 1.85$

Stress in "b" $= \frac{62}{60} \times 1.85$

$= 1.91$

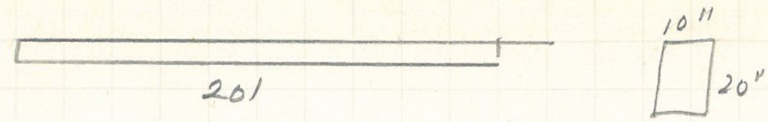


1 unit $= 0.5 = \frac{1}{2}$

$2 \times \frac{2}{3} \times 240 \times \frac{1}{2} = 160$

$\frac{\quad 2}{320}$ K T

7)



steel

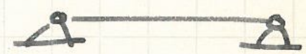
Temperature change top fiber +100°F
" " bottom " + 20°F

Compute maximum fiberstress at top and maximum deflection relative to ends if:

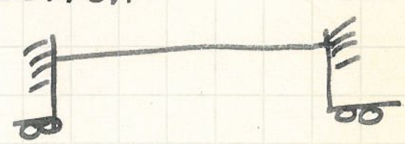
a) ~~Both~~ ends are free in position & direction

(b) Both ends are fixed in position & direction

c) Both ends are fixed in position
free in direction



d) Both ends are free in position
fixed in direction



Prob 7

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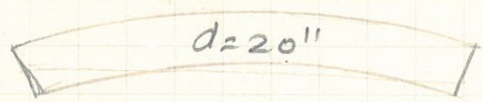
7(a)



See Roark
Formula's for Stress & Strain
pg 322

(1) No stresses $\Delta T = 80^{\circ}$

$$\text{Radius} = \frac{d}{\Delta T \alpha}$$

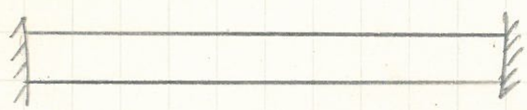
$$R = \frac{20}{80 \times .0000065} = 38,5000 \text{ in.}$$

$$M \simeq \frac{c^2}{8R} = \frac{(240)^2}{8 \times 38,500} = 0.187'' \text{ deflection}$$

say

0.2''

7(b)



Ends fixed - bar will not deflect - will be held straight by end couples $EI \Delta T \alpha / d$ and the max. stress will be

$\frac{1}{2} \Delta T \alpha E$ where ΔT is the difference in temperature

but there is also a 20° rise in both flanges so

(1) 20° change both flanges

$$S = \Delta T \alpha E = 20 \times .0000065 \times 30,000,000$$

$$= 3900 \text{ psi C.}$$

(2) 80° change top flange

$$S = \tfrac{1}{2} \Delta T \alpha E = \tfrac{1}{2} \cdot 80 \times .0000065 \times 30,000,000$$

$$= 7800 \text{ psi C.}$$

Total Compression = 11700 psi top flange

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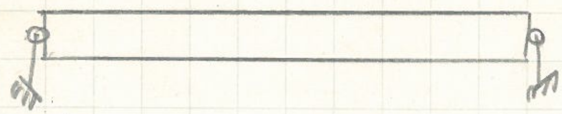
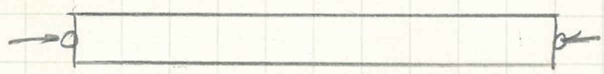
7 (c) Ends cannot move out — but can ~~not~~ rotate



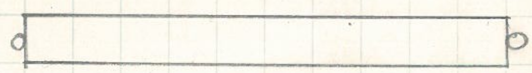
Take the first 20° change



$S = \Delta T \alpha E = 3900$ psi

$P = 3900 \times 10 \times 20 = 780{,}000$ #

Take the next 80° change on top



There will be no stress. The beam rotates

$\Delta = 0.19''$ as in 7(a)

<u>but</u>!

The bending of beam increases the stress, ie. if you had a bent beam and put a load on it the stress, $S = \frac{P}{A} + \frac{Mc}{I}$

$$S = 3900 \pm \frac{780{,}000 \times 0.19 \times 10}{\frac{1}{12}\, 10 \times 20 \times 20 \times 20} = 226$$

$S = 4125$ psi bottom fiber — compression

<u>$S = 3675$ psi</u> <u>top fiber</u> — compression

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7c (cont)

The total deflection can now be found

$$\Delta = \frac{1}{10}\frac{\Sigma f \ell^2}{Ed} = \frac{1}{10}\frac{.450 \times \cancel{120}^{240} \times \cancel{120}^{240}}{30{,}000\,000 \times 20}$$

$= .00432$ or negligible so

$\Delta_{TOTAL} = 0.195''$

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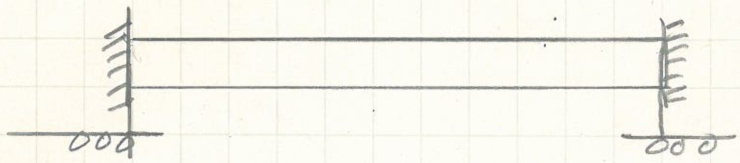
7(d) Beam can move but can not bend



The only stress will be that due to differential temperature change, $\Delta T = 80°$
See 7(b)

$S = 7800$ psi C in top flange
There will be no deflection

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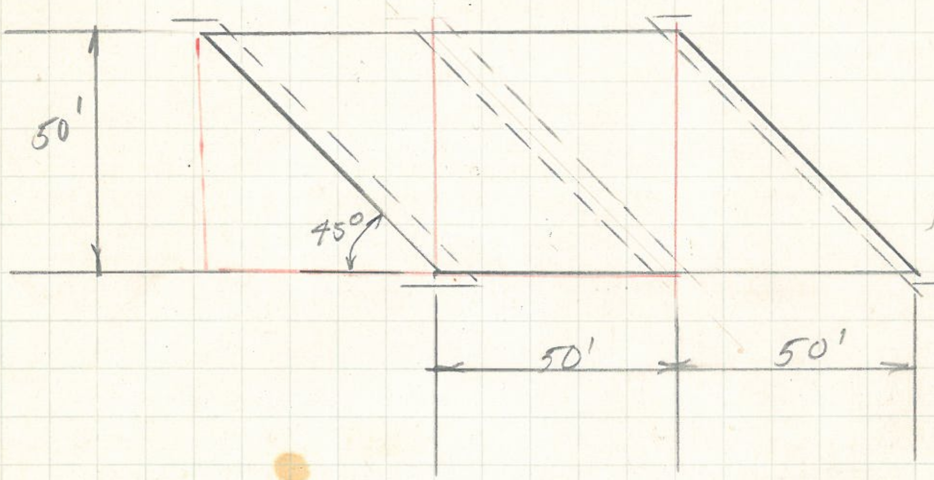
8. Design a skew slab highway bridge in reinforced concrete. Make any assumptions you think applicable. Provide for any loading you think feasible

Done in collaboration with Sam Marcus



AASHO

H-20 Loading

Uniform Load 640 lb per lin ft of lane

Concentrated Load { 18,000 for Moment ; 26,000 " Shear }

Assume 4 lanes

Moment

C.L. $\frac{Pl}{4} = \frac{18,000 \times 50}{4} = 225,000$ 1#/lane

U.L. $\frac{wl^2}{8} = \frac{640 \times 50 \times 50}{8} = 200,000$ 1#/lane

425,000 1#/lane

I

$\frac{50}{50+125} = \frac{50}{175} = \frac{1}{3.5}$

I 121,000 1#/lane

Total 546,000 1#/lane

M = 45,500 1#/ft width

assuming 12' lanes

PROB 8.

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D.L.

$$\frac{wl^2}{8} \qquad w = 1\tfrac{1}{2} \times 1 \times 1 \times 150$$

$$= 225^{\#}/\text{lin ft}$$

$$\frac{225 \times 50 \times 50}{8} = 70300'^{\#}/\text{ft width}$$

Total Moment (D.L. + L.L.) = $115{,}800\ '^{\#}/\text{ft width}$

$= 1{,}390{,}000\ ''^{\#}$

AASHO 1949 $f_s = 18{,}000\,psi$

$f_c' = 3000 \qquad n = 10$

$f_c = 1350$

From Dunham pg. 534

$$K = 248 = \frac{M}{bd^2}$$

$$d = \sqrt{\frac{M}{248b}} = \sqrt{\frac{1{,}390{,}000}{248 \times 12}} = 21.6''$$

Assume $D = 27''$

" " $d = 24''$

$$A_s = \frac{M}{f_s j d} = \frac{1{,}390{,}000}{18{,}000 \times \frac{7}{8} \times 24} = 3.74$$

$$A_{s\,@45^\circ} = 1.414 \times 3.74 = 5.3^{\square''}$$

Use #11 @ $3\frac{1}{2}$

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Bond

$$u = \frac{V}{\Sigma o j d} \qquad j = \frac{7}{8}$$

$$\text{D.L. Shear} = \frac{300 \times 50}{2} = 7500$$

$$\text{L.L. Unif. Shear} = \frac{640 \times 50}{2 \times 12} = 1330$$

$$\text{L.L. Conc. Shear} \quad \frac{26000}{12} = 2160$$

$$10990^{\#}$$

$$u = \frac{11000}{15.2 \times \frac{7}{8} \times 24} = 34 \text{ psi OK.}$$

Shear

$$v = \frac{V}{bjd} = \frac{11000}{12 \times \frac{7}{8} \times 24} = 44 \text{ psi O.K.}$$

Distribution Reinforcement

A.A.S.H.O. 3.3.2.(C)

$$\% \cong \frac{100}{\sqrt{50}} = .14 \text{ of main reinforcement}$$

$$.14 \times 5.36 = 0.75^{\square''}$$

Use #8 @ 12"

Prob 8.

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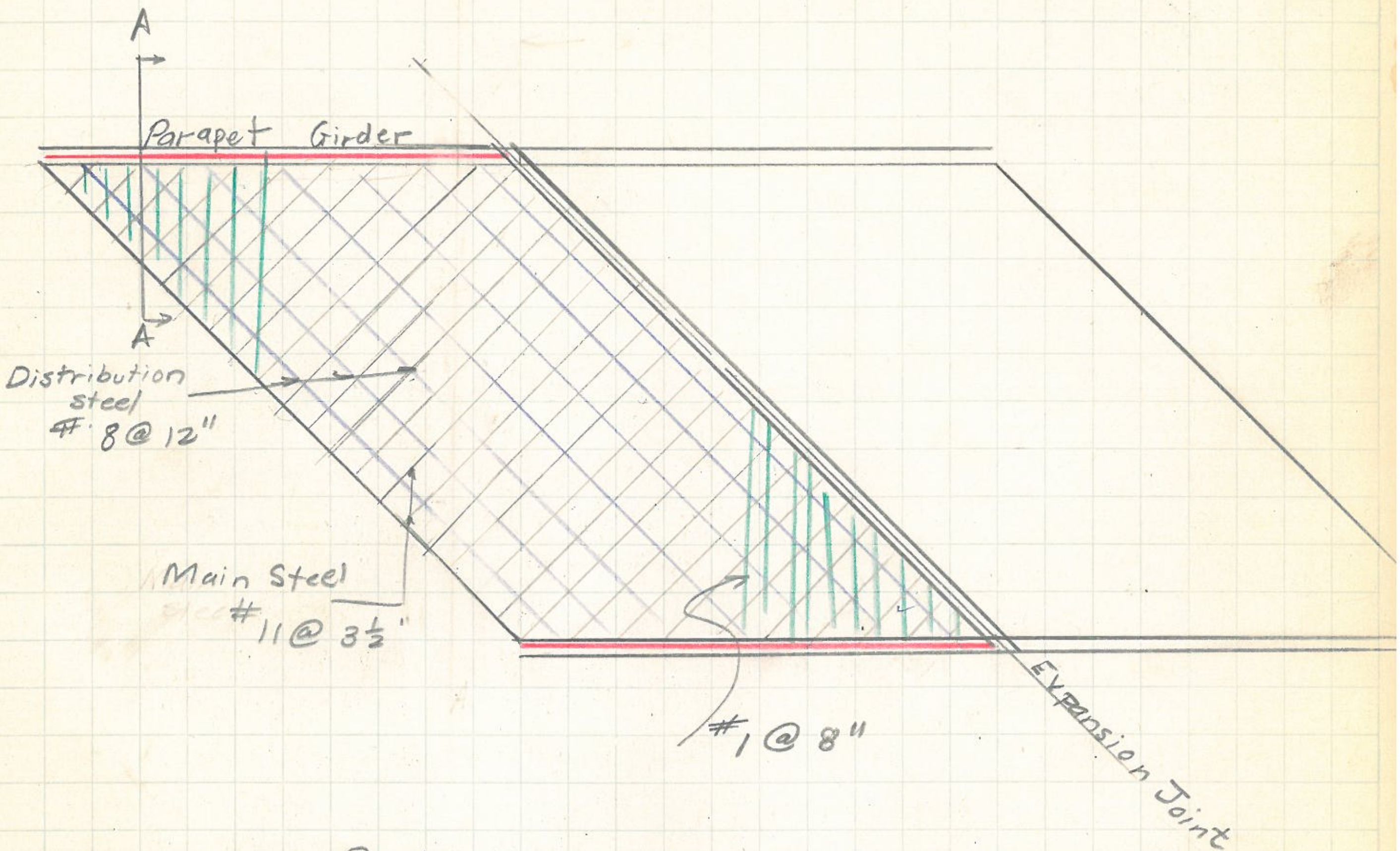
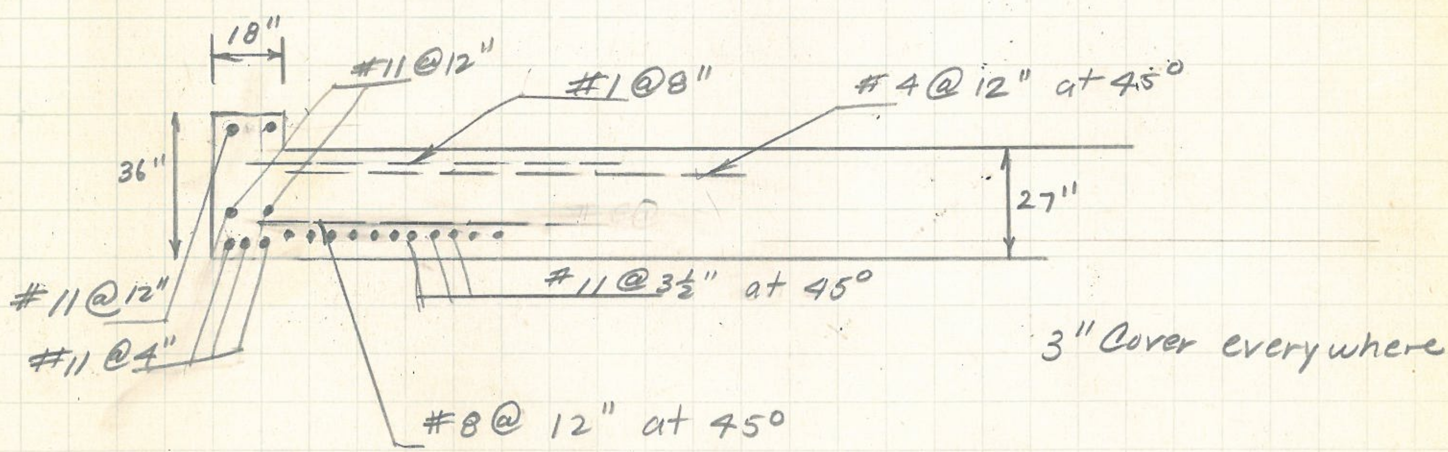


Part
Section A-A



Prob 9 NAME R. A. Heller

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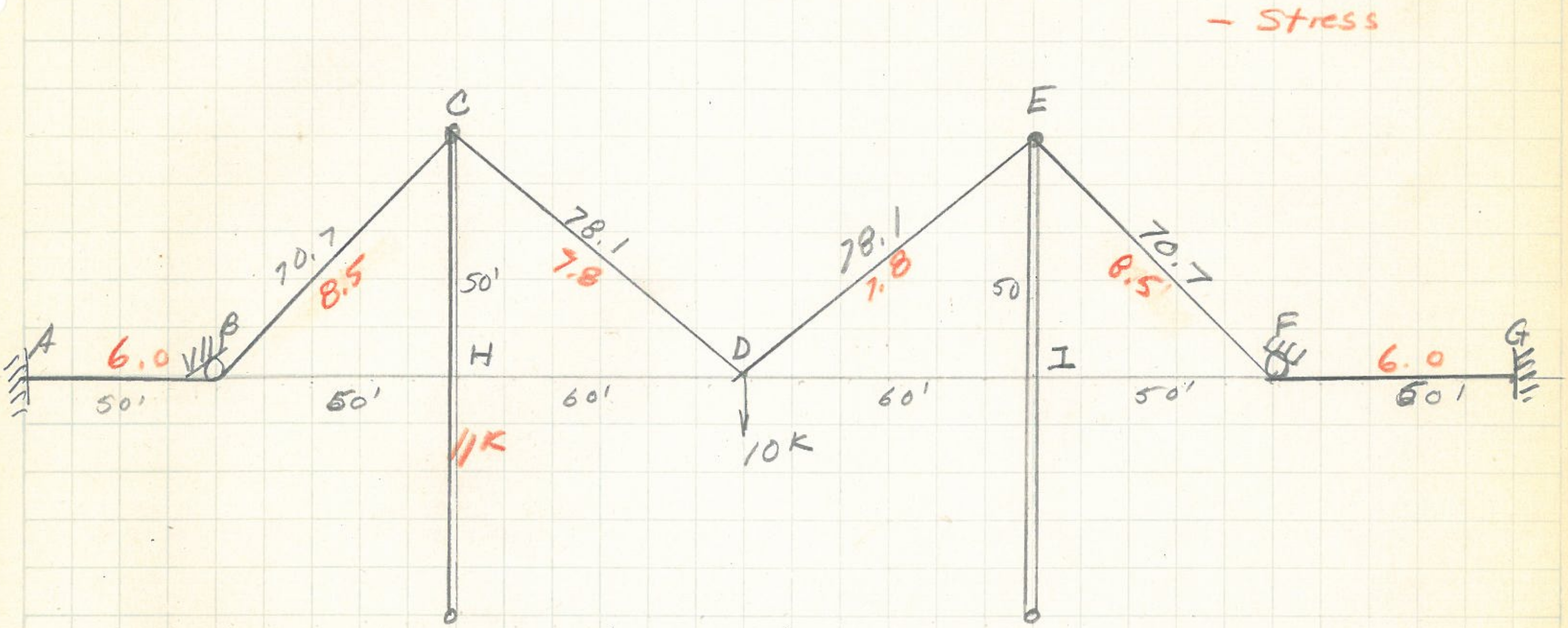
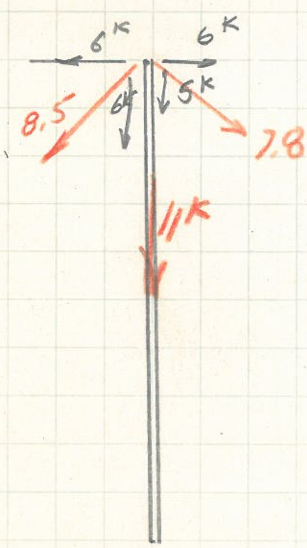
9\. (a) Find stress in towers

(b) Find maximum stress in cable.





(a) Stress in towers = 11 kips

(b) Max. stress in Cable = 8.5 kips

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10 Explain in detail how to compute the rise of the crown of 6A due to a temperature rise of 50°F.

Remove the redundant by putting one of the supports on rollers. Then, permit all the members to expand due to the 50°F temperature rise. The elongation of each member due to this temperature rise is $\delta_t = \alpha \Delta T l$. The structure will deflect and the crown will rise and the support move out. Compute the movement of the crown and the movable support by means of a Williot Diagram.

Now, put a unit horizontal force on the displaced support. This will cause the crown to rise. The deformation of the members can be compute by $\frac{Pl}{AE}$. The deflection at the crown can be figured by another Williot Diagram.

The total rise in ~~arch~~ the crown will be the sum of the rise computed by the first Williot Diagram and the rise computed by the second diagram multiplied by a factor. The factor is the movement of the support under temperature rise divided by the movement of the support due to the unit horizontal thrust.

That is.

$$\left(\Delta_{TOTAL}\right)_{CROWN} = \left(\Delta_{TEMP}\right)_{CROWN} + \left(\Delta_{UNIT\ HOR.\ FORCE}\right)_{Crown} \frac{\left(\Delta_{TEMP}\right)_{SUPPORT}}{\left(\Delta_{UNIT\ HOR\ FORCE}\right)_{SUPPORT}}$$

NAME

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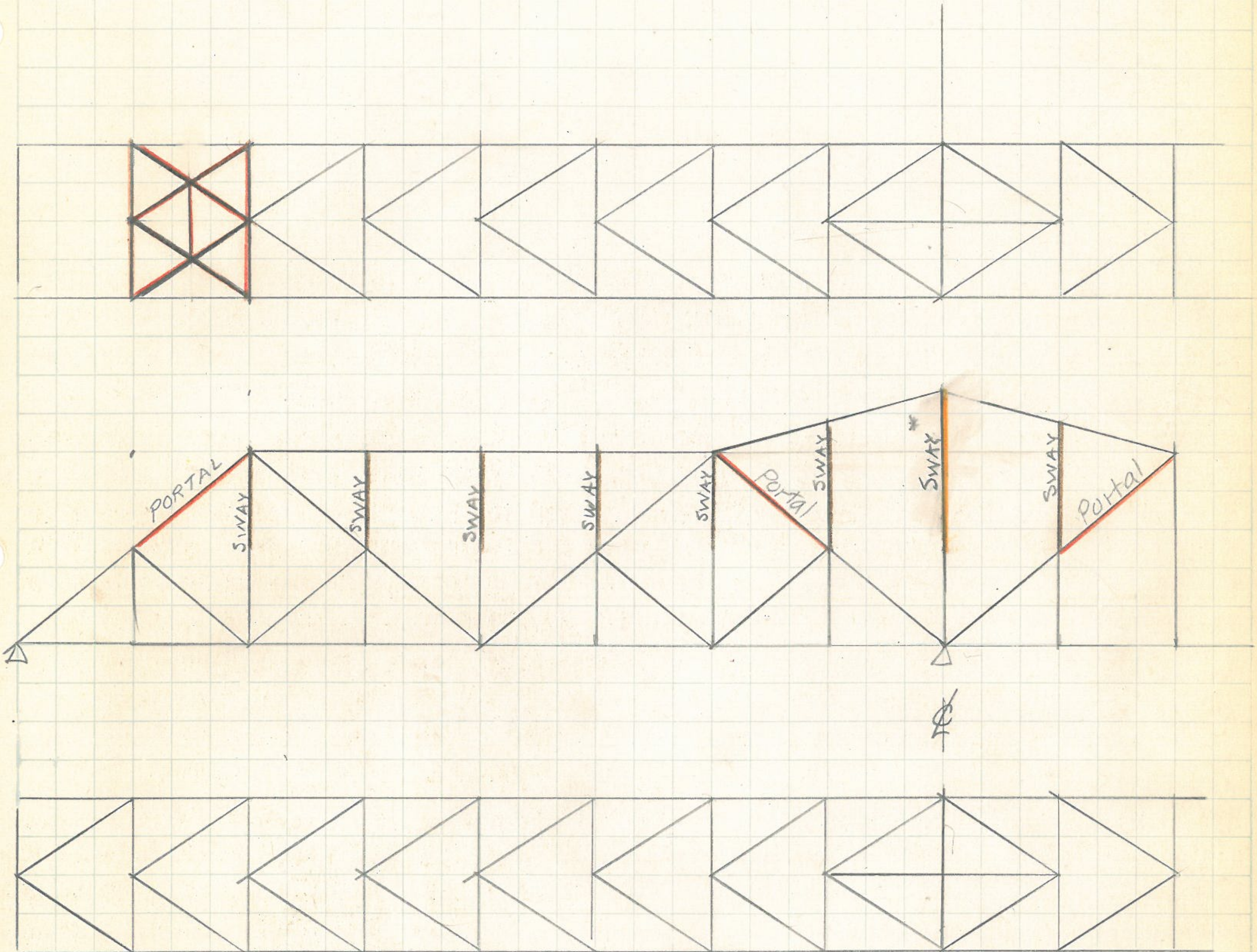
11.

Layout all bracing for 6 B of a 2 lane highway bridge.



See next page

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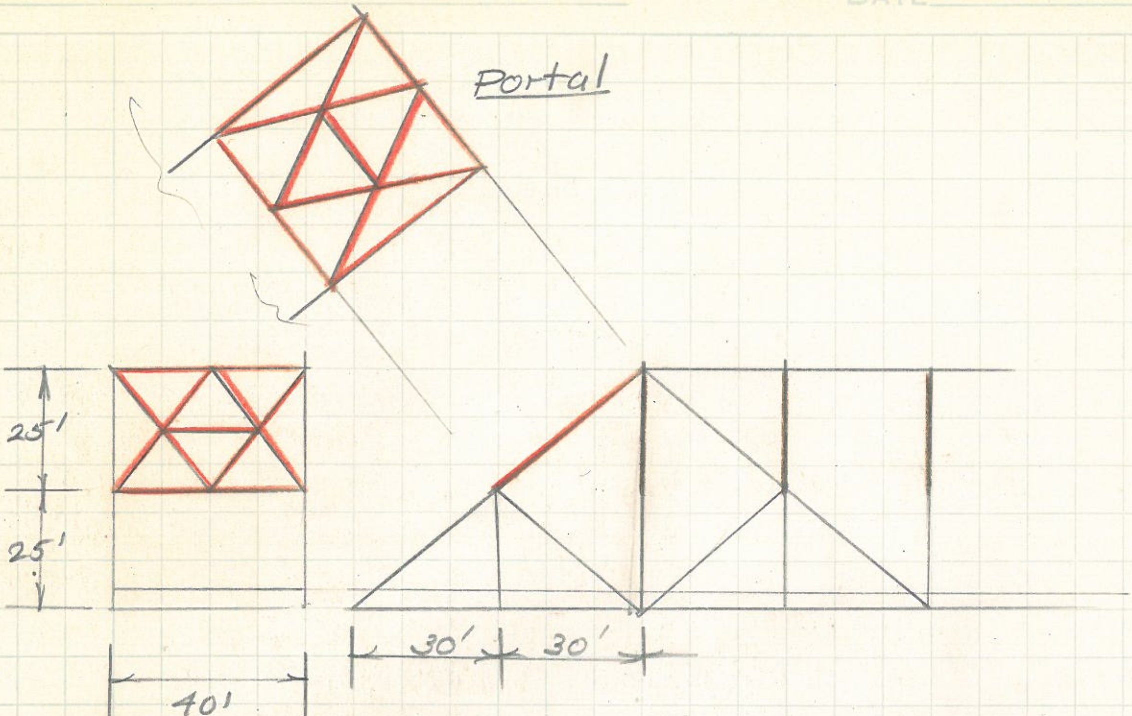
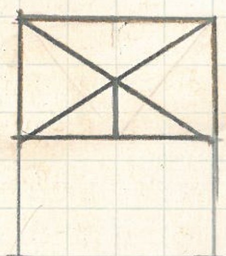
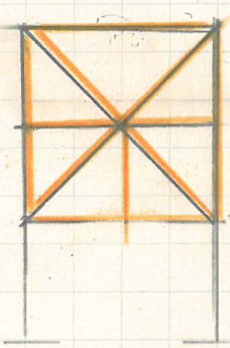


Portal (not true length)



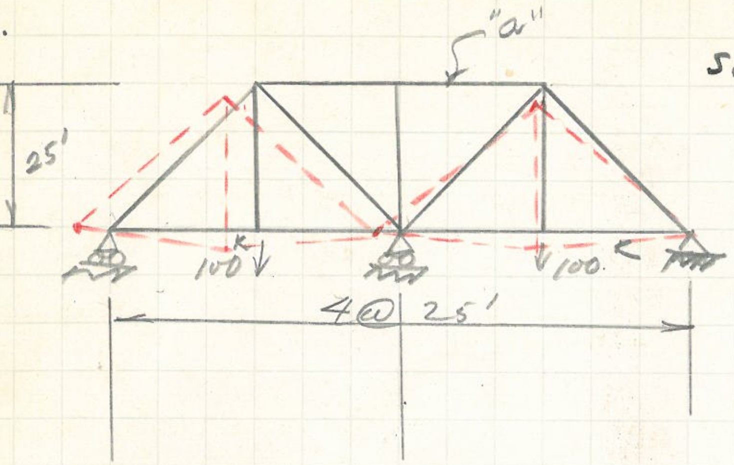
Intermediate Sway Frame



*

Sway Frame at Center of Bridge

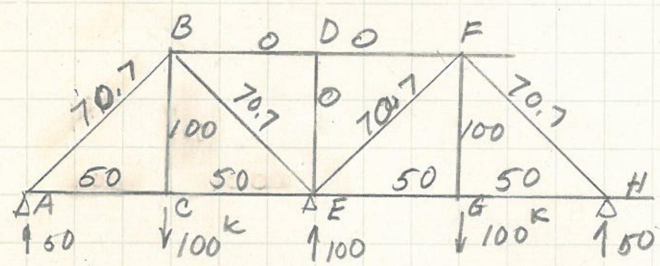
12.



Supports on same elevation

Is it possible that unit stress in bar "a" is zero for loads shown? If so find areas of other bars for "f" not over $10^{k}/in^{2}$?

It is possible that the unit stress in bar "a" is zero for the loads shown if the top chord at the center (bar "a") is left out, the loads applied, and then the bar is replaced. That is, the structure will deflect as shown and bar "a" will have to be detailed long. Then, since the stress in bar "a" will be zero the area of the other bars can be found.



MEMBER		AREA
AC, CE, EG, GH	$\frac{50}{10}$	5 □"
AB, BE, EF, FH	$\frac{70.7}{10}$	7.1 □"
BC, FG	$\frac{100}{10}$	10 □"
DE	nominal	

From the areas the deflections can be determined so that the length of bar "a" can be found.

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13. State briefly what you consider the most pronounced advantages of indeterminate structures if there are any. What, if any are the disadvantages?

There are both advantages and disadvantages to indeterminacy. It is necessary to mention case, ie. define the structure we are taking about.

For one thing, there is a determinate structure that is compreable to an indeterminate structure as regards economy of material. For example, in a two or three span truss, the contilever compares very closely to the continuous. If there is a saving of ten percent in steel, the overall saving would only be 1 or 2 percent. Therefore, there is no advantage as regards material. It is the construction element that dictates what kind of structure to make.

The pronounced advantage to the continuous structure is that if a structure is built continuous it would be inefficient to make it determinate. The notable example is that of a two span continuous truss such as that at Sciotoville which was <u>built</u> continuous. The cutting of one of the top chord members would be an unnecessary step. In concrete, of course, to make a continuous span discontinuous would require a great deal of detailing. Putting in hinges in concrete is a messy job.

It would be, on the other hand, of some disadvantage to making steel arches indeterminate. An arch is built three hinged. However, any advantage obtained by leaving it three hinged is probably counterbalanced by the resulting movement of the deck at the crown, for which an expansion joint would necessarily have to be put in the deck.

In the case of the continuous viaduct it is a disadvantage to making it that way becouse the detailing of the end connections is a herculean task. It is much easier to make it determinate. Determinate structures have the additional advantage of not being affected by parasitic stresses.

It is difficult, therefore, to make a <u>general</u> statement as to the predominate advantage or disadvantage of indeterminacy. From the foregoing, it can be seen that the <u>predominate advantage</u> of indeterminacy is <u>some</u> structures is that it is built that way and it might just as well be left that way. The <u>predominate disadvantage</u> in <u>some</u> structures, on the other hand, of indeterminacy is that it is built determinate and must be made indeterminate.

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14. Name 2 books on structural theory that you have looked over.

(a) one that seems promising for further study. Why?

(b) one that does not seem promising for further study. Why?

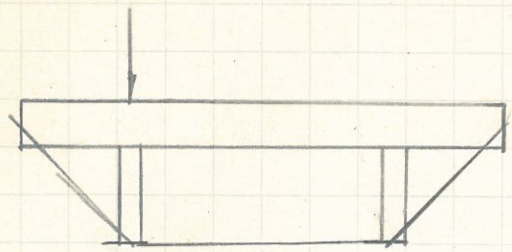
a) "Statically Indeterminate Structures"
by
Cross & Morgan

This book is more promising for further study than any other book that I have looked over. The contents cover a very wide range of topics. Although many topics are covered few, if any, are completely discussed thus leaving many paths which may be followed. It is in this, in that it is broad and suggestive, which makes it worth further study. There are also many references that help to direct the reader to more extensive literature.

b) "Theory of Simple Structures
by
Shedd and Vawter

Although this book may be useful as a reference, and can serve as a text for undergraduates, it does not seem promising to me for further study. It commits the sin so many books of this kind do in that it shows how to find the stress in bar "a", the moment at "B", and so on, but it does not consider <u>real</u> structures. It makes structural theory seem standard and, worse yet, entirely settled, which means there will be very little to nourish any ideas or questions I might have.

15.



Choose dimensions

Draw Moment curves for the Girder.

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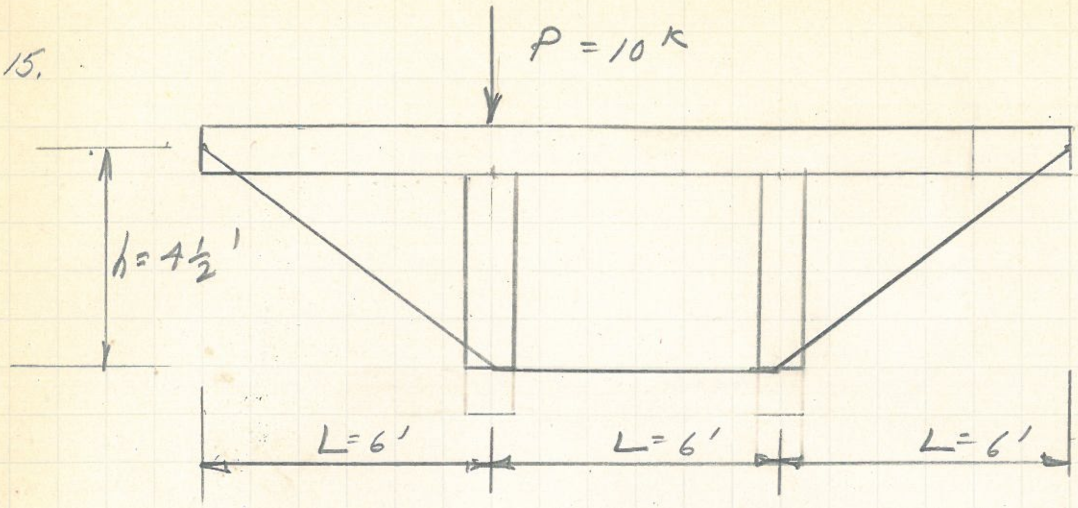
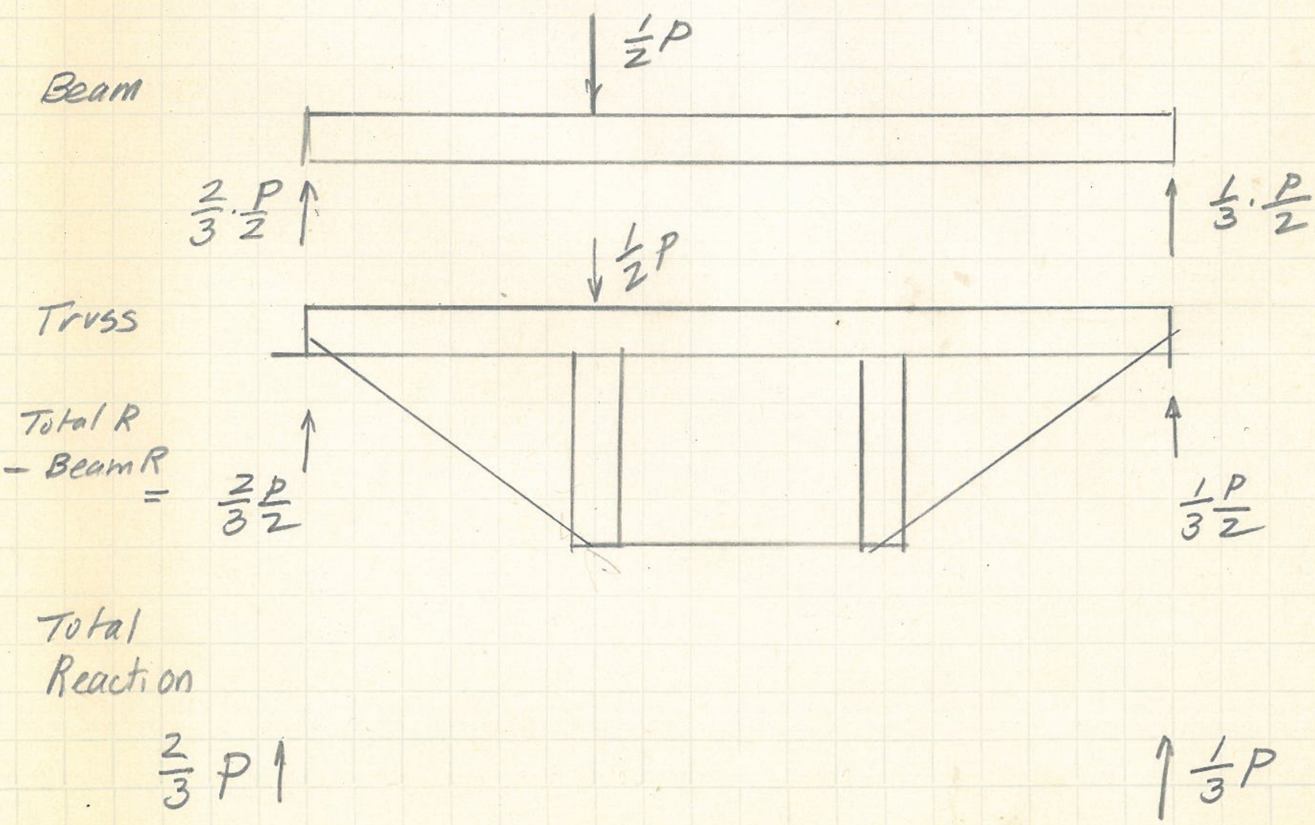
15.



Refer to "Modern Framed Structures" Johnson, Bryan, & Turneaure Part II pg 359 For a structure, proportioned as above, with a load as shown, the stress in each vertical $= P/2$ – the truss carries one half of the load, the beam the other half. The point of inflection will be at the center.



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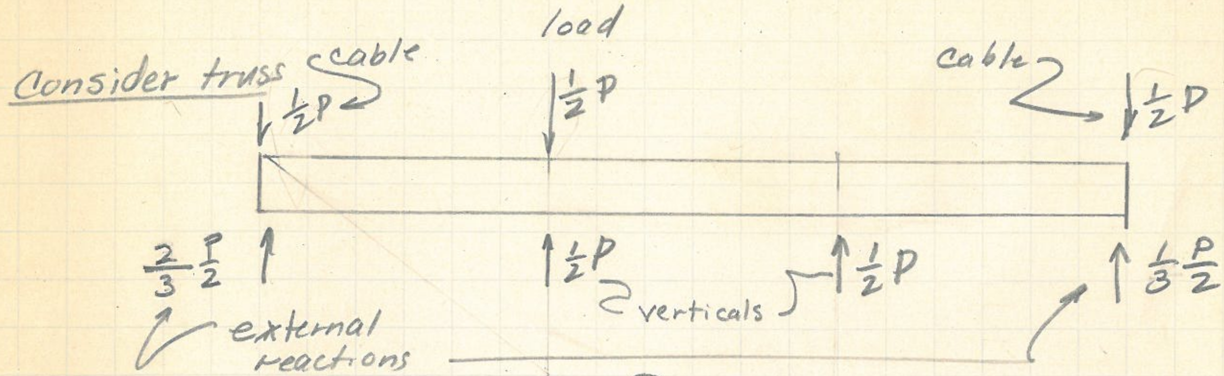
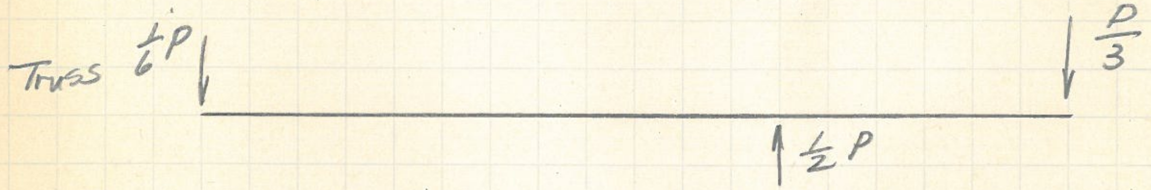
continued.

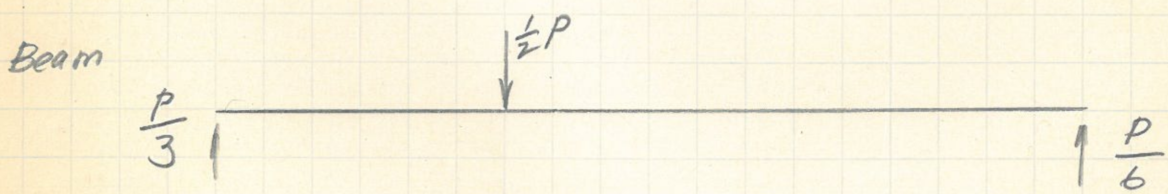
Consider truss

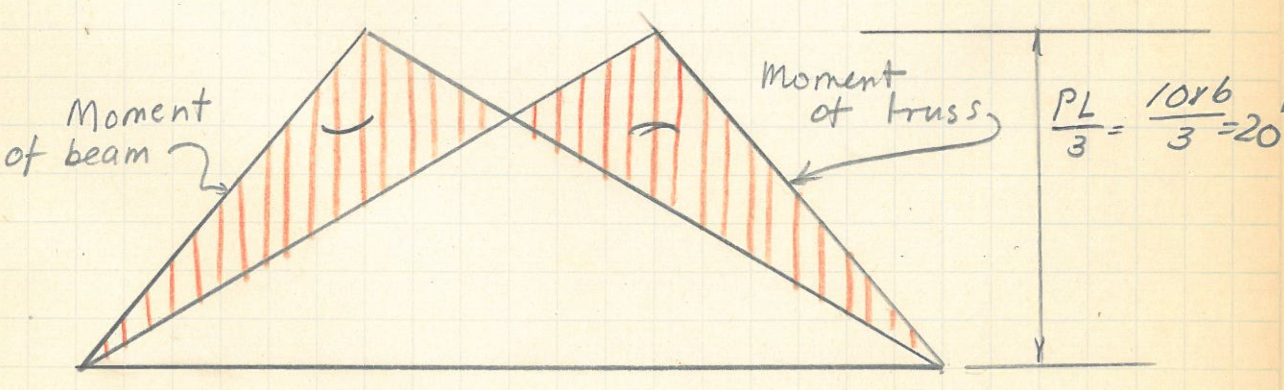


$\Sigma F_y = 0$
$\Sigma M = 0$ } statics balance

$\frac{1}{2}\cdot 2P + 3\frac{P}{\frac{6}{2}} - \frac{1}{2}P\cdot 3 = 0$







Note: Total length = 3L