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Degrees of freedom of plane and space frames

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Synopsis

A unified general approach for the determination of the degrees of freedom or indeterminacy of plane and space frames is developed from basic concepts. The term ' frame ' is used in a general sense and includes structures as well as mechanisms.

The first step in any form of structural analysis is the determination of the degree of indeterminacy or freedom of the frame. Some of the papers and books[1–8] that deal with the subject give numerous formulae depending on the kind of members or type of joints and nature of structures, which are classified for this purpose as simple, compound, complex, composite and so on. These formulae are derived either from the number of 'releases'[2,3,5] necessary to obtain stable and determinate structures from the given frame or on the basis that all frames are developed from the basic pin-jointed triangular bar frames, each bar having only two joints[8].

The method outlined in this paper has the distinct advantage that it deals with the frame as it is and does not require any modification into an equivalent pin-jointed bar-frame by adding or subtracting imaginary ' effective members ', or by making a ' released ' primary structure. It is a general approach readily applicable to all types of frames (excluding critical forms, detailed information on which is given in reference 1). A number of examples have been worked out to illustrate the simplicity of the procedure.

Definitions

(1) A ' frame ' is an assembly of rigid members with joints (small elastic deformations are ignored); it may be either a structure or a mechanism. As is well known, a structure remains geometrically stiff or stable, whereas a mechanism is geometrically unstable, permitting relative motion. A just stable frame is known as a simple or determinate structure, and an overstable frame is indeterminate, while an unstable frame is called a mechanism. In ultimate load analysis a structure is said to have failed if it becomes a mechanism and it is necessary to know the degrees of freedom of the mechanism to determine the mode of collapse.

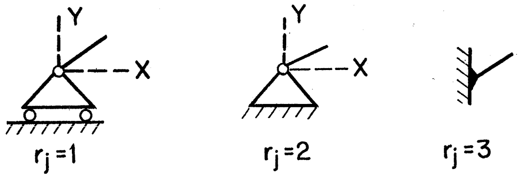
(2) ' Degrees of freedom ' denote the number of independent restraints necessary to determine the geometric stability of a member or of the frame as a whole, relative to some reference member or system of co-ordinates. Each co-ordinate may be looked upon as a restraint which removes one degree of freedom from the member or frame under consideration.

A member, freely movable in a plane, has 3 degrees of freedom or 3 possible movements, thereby requiring 3 co-ordinates or restraints in the plane to fix it in a stable manner with respect to the reference member or system of axes.

Likewise, a member movable in space has in general 6 degrees of freedom or 6 possible movements, 3 linear translations and 3 rotations.

(3) A ' joint ' provides the connexion between *any two* members; its essential property is to restrain or take away some of the degrees of freedom which one member would have relative to the other, if they were not connected. The number of restraints imposed by a joint between any two members is denoted by r_j.

Fig 1 shows the three types of joints for plane frames for which the r_j values range from 1 to 3 only as indicated.

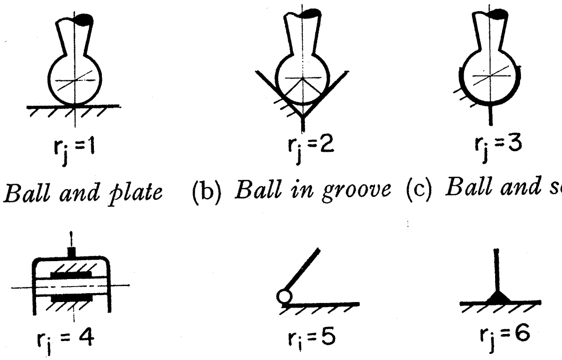


(a) *Roller* (b) *Pin* (c) *Rigid*

Fig 1—Typical joints in plane frames

The roller joint restrains against movement along the Y-axis; as such its $r_j = 1$. Likewise the pin-joint (Fig 1(b)) imposes two restraints against movements along both the axes, or $r_j = 2$. The encastre joint (Fig 1(c)) obviously has $r_j = 3$.

Fig 2 (a—f) shows typical joints for space frames for which the value of r_j ranges from 1 to 6 only.



(a) *Ball and plate* (b) *Ball in groove* (c) *Ball and socket*

(d) *Sliding pin* (e) *Pin* (f) *Rigid*

Fig 2—Typical joints in space frames

The joints shown are merely illustrative; many alternative designs of joints are clearly possible for the same value of r_j.

Development of the Method

In any frame having n members, one has to be chosen as the reference. Each of the remaining members will have three degrees of freedom in a plane frame and six‡ in a space frame, relative to the chosen reference, if not connected by joints. If f_m denotes the number of degrees of freedom of each free member, the total number of degrees of freedom of all the members (excepting the reference) is expressed as Σf_m (Σ denoting summation).

Every joint imposes r_j restraints between the members connected, r_j varying from 1 to 6 according to the nature of the joint used (Figs 1 and 2). The total number of restraints in the frame is equal to Σr_j. The resultant degree of freedom f of the frame as a whole may, therefore, be expressed as

$$f = \Sigma f_m - \Sigma r_j \quad . \quad . \quad . \quad . \quad (1)$$

‡ If a member in a space frame degenerates into a bar (rod), it has only five degrees of freedom, since rotation about its own axis has no significance.

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This simple relation decides the degree of indeterminacy or freedom of any frame (excluding critical forms). The three possible cases are:

(1) $f = 0$, the frame is statically determinate and stable;

(2) $f < 0$, the frame is indeterminate to the degree f;

(3) $f > 0$, the frame is a mechanism with f degrees of freedom.

The r_j values of the joints can be determined by mere inspection but the number of actual joints when several members meet at a point requires further elucidation.

It is clear that there must be a joint between *any two* members of a frame. When more than two members meet at a point, the following step-by-step procedure leads to a simple expression.

The four members OA, OB, OC and OD meet at the common point O, in Fig 3(a). Let OA and OB form the first joint (Fig 3(b)); OC is joined next as in Fig 3(c) and the number of joints becomes two. Finally OD requires a third joint (Fig 3(d)). The total number of joints to be considered in the calculation of Σr_j value at point O is, therefore, 3, i.e. (4—1). This conclusion can be generalized: if n members meet at a common point the number of joints to be counted at the point is (n—1) and not 1. This manner of counting is of primary importance in the method presented here.

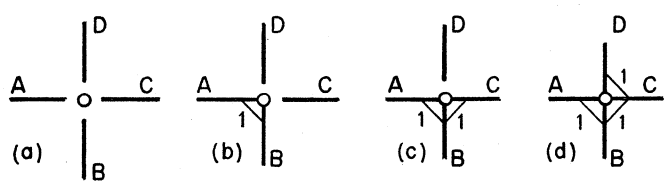

Fig 3—Counting joints at intersections

Example 1

The plane frame in Fig 4(a) can be analysed by two methods.

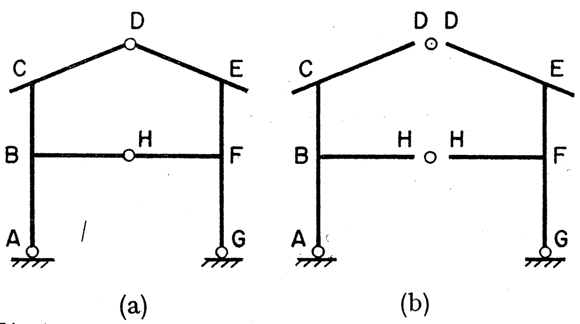

Fig 4

Method 1

The frame may be considered as made up of two members, as shown in Fig 4(b) (excluding the reference member, ground). Therefore $\Sigma f_m = 2 \times 3 = 6$ (at three degrees of freedom per member). Number of pin-joints = 4 (one each at A, D, G and H); $\Sigma r_j = 4 \times 2 = 8$ (at 2 restraints per pin-joint, Fig 1(b)). Hence, from equation 1, $f = 6 - 8 = -2$; this indicates that the frame is indeterminate to the second degree.

Method 2

The members AB, BC, CD, DE, EF, FG, BH and FH add up to eight numbers; $\Sigma f_m = 8 \times 3 = 24$. Number of pin-joints at A, D, G and H add up to four. Number of rigid joints:

at B and F, 3 members meet; hence the actual number of joints is (3—1) = 2, at each point B and F;

at C and E, 1 at each point;

total, 6 rigid joints.

Therefore, Σr_j for the whole frame $= 4 \times 2 + 6 \times 3$ 26; and $f = 24 - 26 = -2$, as before.

Either method can be used for the remaining examples.

Example 2

The plane truss (Fig 5(a)) is rigid independent of any foundation and any member, say AB, may be chosen as the reference member. Excluding AB, there are 6 members; $\Sigma f_m = 6 \times 3 = 18$. The number of joints, on the basis of (n — 1) joints for n members meeting at a point, is indicated on Fig 5(a); thus the total number of pin-joints is nine and $\Sigma r_j = 9 \times 2 = 18$. Equation 1 gives $f = 18 - 18 = 0$; the frame is therefore stable and determinate.

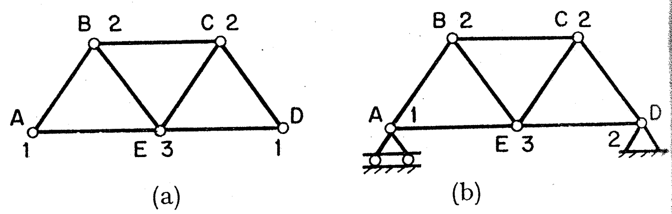

Fig 5

Fig 5(b) shows the truss of Fig 5(a) connected to the foundations. The total number of members, excluding the reference member, ground, is 7; $\Sigma f_m = 7 \times 3 = 21$. The number of pin-joints as indicated on the Figure amount to 10 and there is a roller joint also. Σr_j $10 \times 2 + 1 \times 1 = 21$. $f = 21 - 21 = 0$, as before.

Example 3

Fig 6 shows a rigid frame. The number of joints to be counted, on the basis of 1 for every 2 members meeting at a point, is indicated on the Figure. There are 16 members (excluding ground as the reference); Σf_m $16 \times 3 = 48$. Number of rigid joints = 20; and number of pin-joints = 2; $\Sigma r_j = 20 \times 3 + 2 \times 2 = 64$. Therefore, $f = 48 - 64 = -16$; the frame is indeterminate to 16 degrees.

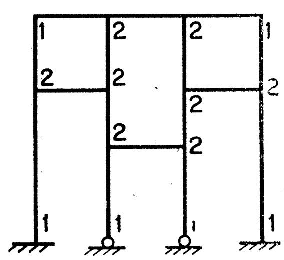

Fig 6

Example 4

Fig 7 shows a space bar-frame. Relative to the ground, there are 33 bars ($f_m = 5$ for each bar in space frames), 54 ball-and-socket joints and 3 ball-and-plate joints (at B, C and D); $\Sigma f_m = 33 \times 5 = 165$; $\Sigma r_j = 54 \times 3$ $+ 3 \times 1 = 165$; $f = 165 - 165 = 0$; the frame is stable and determinate.

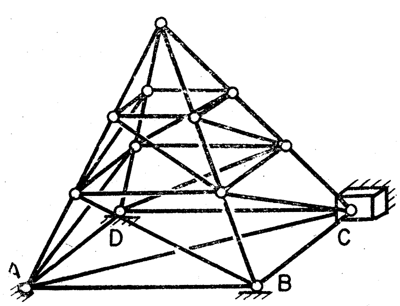

Fig 7

Fig 8 shows a rigid space frame; it also has been analysed[6].

Method 1

The frame has 56 members at 7 per portal, as indicated in Fig 8(b); or $\Sigma f_m = 56 \times 6 = 336$. The number of rigid joints are as follows:

3 at each of the 16 intersections marked B	=	48
1 at each of the 16 intersections marked A	=	16
1 at the foundation of each vertical member	=	16
Total	=	80

$\Sigma r_j = 80 \times 6 = 480$.

Therefore, equation 1 gives $f = 336 - 480 = -144$; the frame is redundant to 144 degrees.

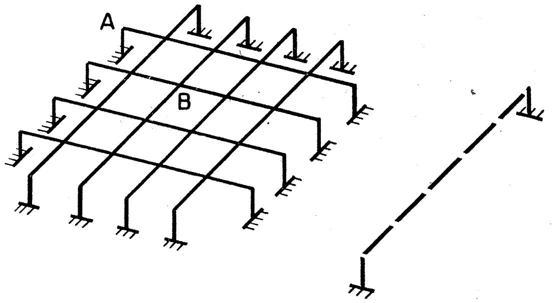

Fig 8

Method 2

The frame may be considered to have only 8 rigid portals, relative to the ground, interconnected by 1 rigid joint at each intersection B, together with 16 rigid joints $32 \times 6 = 192$; and $f = 48 - 192 = -144$, or indeterminate to 144 degrees, as before.

Example 6

Fig 9 shows a plane mechanism with three members, three pin-joints and one slider-joint relative to the ground. $\Sigma f_m = 3 \times 3 = 9$; $\Sigma r_j = 3 \times 2 + 1 \times 2 = 8$; and $f = 9 - 8 = 1$; or the frame is a mechanism with one degree of freedom.

Fig 9

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Book Reviews

Lessons from Failures of Concrete Structures, by Jacob Feld (Detroit, Michigan: American Concrete Institute Monograph No. 1, 1964) $9\frac{1}{4} \times 6$ in, 179 + xi pp., $6.

The author is a New York consulting engineer who has specialized since 1926 in the investigation of structural failures and he introduces this American Concrete Institute Monograph by defining failure as ' nonconformity with design expectations '. This unusual definition allows endless scope for reporting but the author has taken a few examples in each of four main categories to draw the main lessons to be learnt:—

(1) design deficiencies leading to failure;
(2) construction problems with a special section on formwork;
(3) problems of durability and compatibility of materials;
(4) foundation problems and special structures.

The reports are very short general descriptions, a great many being taken from *Engineering News Record*, an American weekly publication.

Examples of design deficiencies range from calculation errors, neglect of secondary stresses, thermal and shrinkage effects, detailing errors, to errors in assumed loading.

The section on construction problems deals with examples of failure due to:—

(1) inadequate supervision;
(2) poor mixing and placing;
(3) cold and hot weather;
(4) erection problems as with lift slab and precast units;
(5) formwork deficiencies.

That on durability and compatibility deals with examples of corrosive agents, abrasion and chemically reactive aggregates.

Finally there are a few pages on foundation problems, such as soil and ground water movements, expanding soils, frost action, pile foundation, retaining walls and abutments, tanks, bins and dam foundations.

The object of the book is to warn the uninitiated, the careless and the optimistic designers and builders, and although the reports are short, the photographs and sketches speak forcibly enough.

A final section on professional and legal responsibility has a most interesting section on the trends in America today and a plea for the introduction of a European-type private organization to control designs and construction methods.

No engineer can loose anything but sleep by reading this book. G.H.M.

Collected Papers, Vol. 1, 1963, Vol. 2, 1964 (Department of Civil Engineering, Faculty of Engineering, University of Tokyo) $10\frac{1}{4} \times 7$ in.

Some of the papers included in these are original contributions, others are papers submitted to various societies and some are reprints of publications. The first volume contains fifteen contributions including papers on the calculation of the effective rigidity of one-side stiffeners in welded plate girders, strength and stress distribution of haunchless reinforced concrete rigid frame corners, the lateral motion of suspension bridges, lateral stability of a suspension bridge subjected to foundation motion, as well as papers on the use of fly ash in concrete dams, on submerged breakwaters, coastal protection works, wave action, etc.

Volume 2 contains sixteen papers and includes research on three-dimensional consolidation of clay, compaction of sandy ground by vibration, vibroflotation, etc., earth pressure measurements in subway construction, lateral rigidity of arch ribs, programming for digital computation of suspension bridges under vertical, horizontal and torsional loadings, a fundamental study on allowable tensile stress in reinforcement