$r = 4$, and $j = 14$; for the structure in Fig. 2-47(b), $m = 59$, $r = 5$, and $j = 32$. Hence, both structures are statically determinate.

For space trusses, a total of $3j$ equations of equilibrium may be obtained by considering each of the joints as a free-body. Then, the degree of statical indeterminacy of a space truss is given by

$$d = m + r - 3j \tag{2-12}$$

while the criterion for statical determinacy is

$$d = m + r - 3j = 0 \tag{2-13}$$

Suspension bridges. The center span of a suspension bridge may be idealized as a cable that carries a uniform horizontal load from the roadway through a set of uniformly spaced vertical hangers. The roadway may be stiffened by a girder or a truss as shown by the structures in Fig. 2-48. For these cases, there are four known components of reaction in each system, if the stiffening girder or truss is subjected to vertical loads only. This is true because the directions of the cable reactions must be in line with the tangents of the cable at supports A and B, and the hangers are so flexible that they can transmit only vertical loads. Hence, each system is statically determinate because an additional condition of release is present in the system. In Fig. 2-48(a), the condition of release is provided by the center hinge C' in the girder, whereas in Fig. 2-48(b) no moment can be transmitted from truss $A'C'$ to truss $B'C'$. Thus, joint C' in Fig. 2-48(b) has the same effect as hinge C' in Fig. 2-48(a).

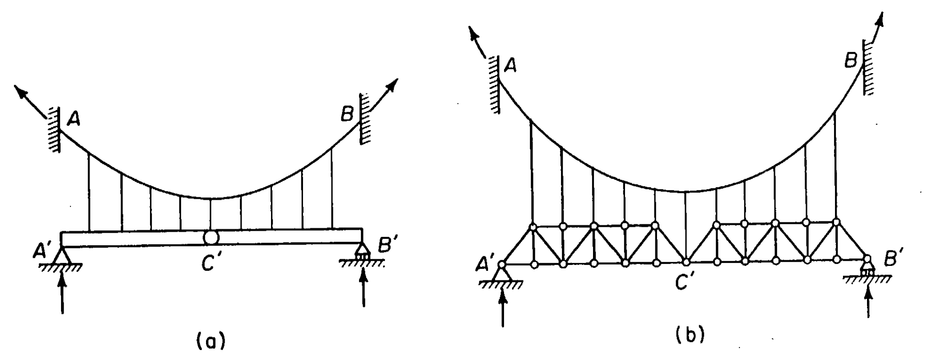


Figure 2-48

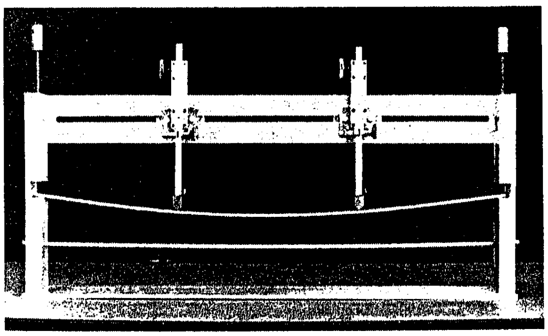
If no release is introduced in the stiffening girder or truss in a suspension bridge, then each system will be statically indeterminate to the first degree under the specified assumptions.

2-8 DEFLECTED SHAPES OF STRUCTURES

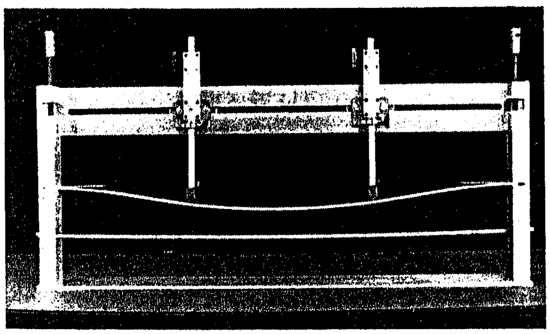
Structural members deform in accordance with the force-deformation properties of the material when they are subjected to internal forces and moments. Hence, the deflected shapes of structures are influenced by the constraints at supports and the applied loads. For the purpose of illustration, the deflected shape of a structure can be sketched in a scale that would exaggerate the significant features even though the deflections in the elastic range generally are too small to be visible. For example, before finding the moments at various sections of a beam, it is possible to detect the directions of these moments by observing the curvatures of the beam at these sections from its approximate deflected shape. On the other hand, if the moment diagram is found first, the deflected shape of the beam can be accurately sketched by noting the moment-curvature relationship.

The exaggerated deflected shapes of beams can best be illustrated by laboratory models made of elastic material shown in the photographs from Figs. 2-49 through 2-56. In these models, the simply supported ends are allowed to rotate freely while the fixed ends are clamped. Concentrated loads are applied by lowering the pointers to produce the desired deflections.

In Fig. 2-49(a), a single-span beam with two concentrated loads is simply supported; by contrast, a single-span beam with the same loading in Fig. 2-49(b) is fixed at both ends which are characterized by the horizontal slopes at the supports. Note the single convex curvature of the simple beam in Fig. 2-49(a) and the change of curvature at the points of inflection of the fixed-end beam in Fig. 2-49(b). The deflected shapes of a single span beam with overhangs at both ends under different loading conditions are shown in Fig. 2-50(a), (b) and (c). The overhangs in Fig. 2-50(a) remain straight because of absence of bending moments; so are portions of the overhangs beyond the concentrated loads toward the free ends in Fig. 2-50(b) and (c). However, the deflected beam

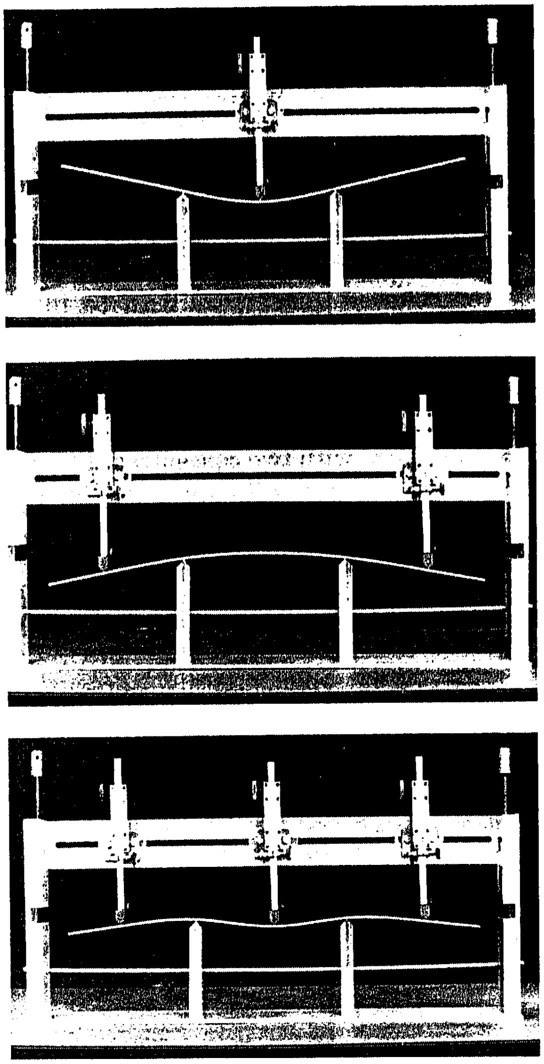


(a)



(b)

Figure 2-49



(a)

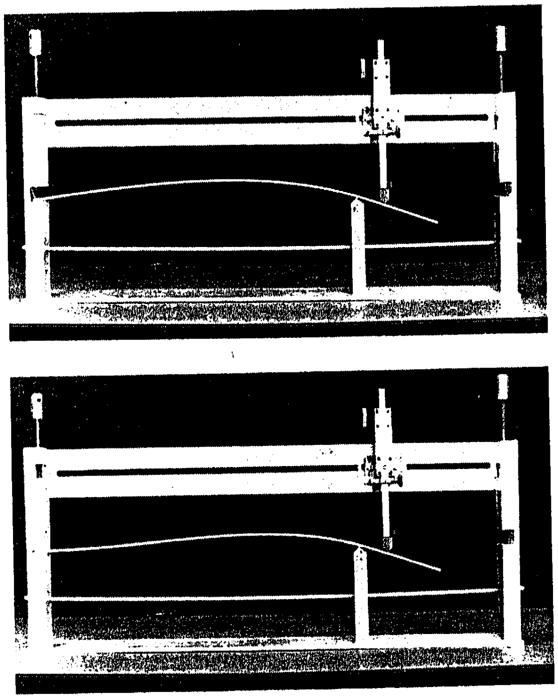
(b)

(c)

Figure 2-50

in Fig. 2-50(b) has a single concave curvature between supports while the deflected beam in Fig. 2-50(c) contains two points of inflection between supports. Both Figs. 2-51(a) and (b) show a single-span beam with a load on the right overhang, the difference being that the left support in Fig. 2-51(a) is simply supported while the left suppport in Fig. 2-51(b) is fixed. Note again the point of inflection and the horizontal slope at the left support of the deflected beam in Fig. 2-51(b).

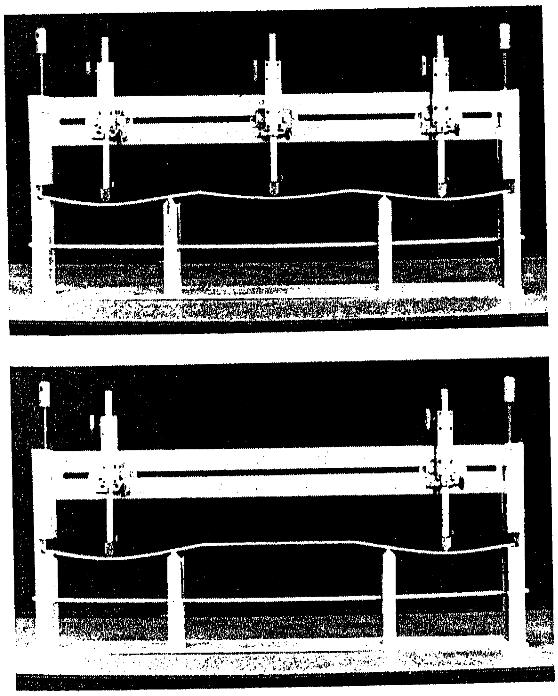
A three-span beam with two internal hinges in the center span is shown in Figs. 2-52(a) and (b). Note the kinks in the deflected shapes of the beam where the internal hinges are located. In Fig. 2-52(a), these kinks are accentuated by the load in the center span, whereas in Fig. 2-52(b), the portion of the



(a)

(b)

Figure 2-51

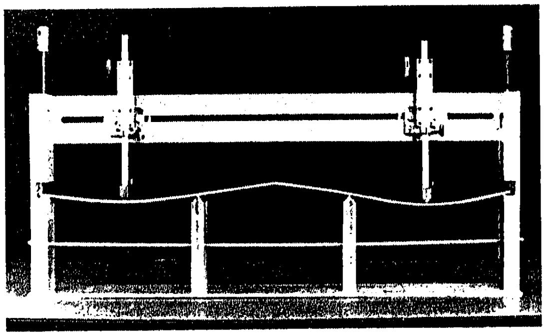


(a)

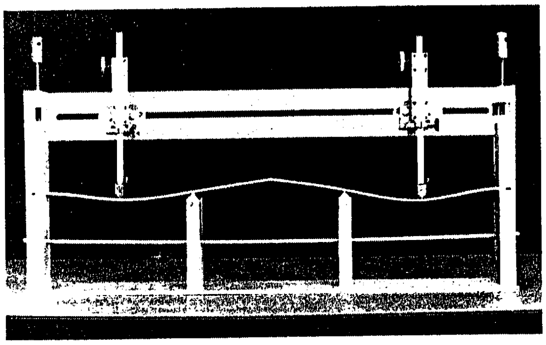
(b)

Figure 2-52

beam between the internal hinges is horizontal since it is not affected by the loads on the side spans. The three-span beams in Figs. 2-53(a) and (b) are, respectively, simply supported and fixed at both exterior supports. However, each beam has an internal hinge at the center span so that a kink occurs in the deflected shape of the beam at that point.



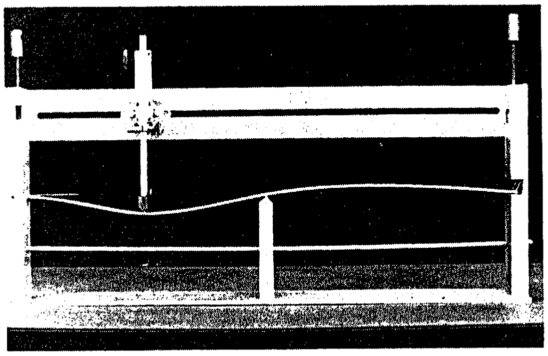
(a)



(b)

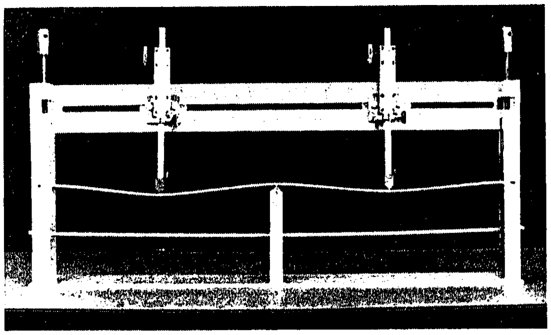
Figure 2-53

Figure 2-54(a) shows a two-span continuous beam with a fixed exterior left support and a simply supported exterior right support. When a load is applied to the left span only, the deflected shape of the right span is concave upward, and that of the left span is affected by the fixed condition of the exterior left support. Fig. 2-54(b) shows a two-span continuous beam which has



(a)

Figure 2-54

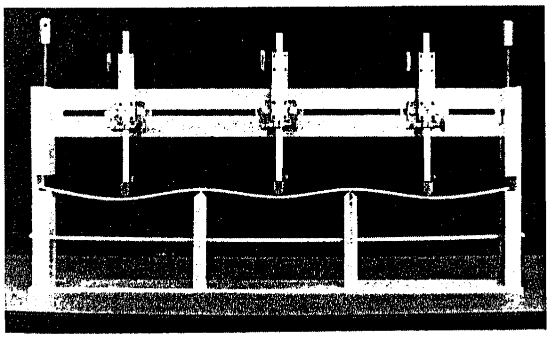


(b)

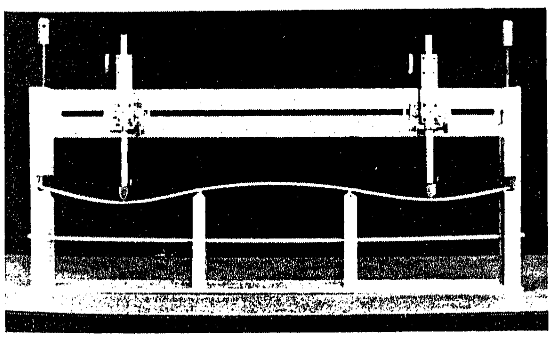
Figure 2-54 (cont.)

fixed supports at both exterior ends. Because of beam geometry and loading conditions, the deflected shape shows symmetry with respect to the center support.

Figures 2-55(a) and (b) show a three-span continuous beam with simple exterior supports under different loading conditions, while Fig. 2-55(c) shows a three-span continuous beam with fixed exterior supports under a load at the center span. In each case, the deflected shape reflects the symmetry of beam geometry and loading conditions. Note also the deflections of the unloaded spans and the effects of the fixed supports on the slopes of the deflected shapes. In Fig. 2-56, the four-span continuous beam with simple exterior support is loaded at the right span only. It can be seen how the deflected shape of the unloaded spans is affected by the load on the right span.

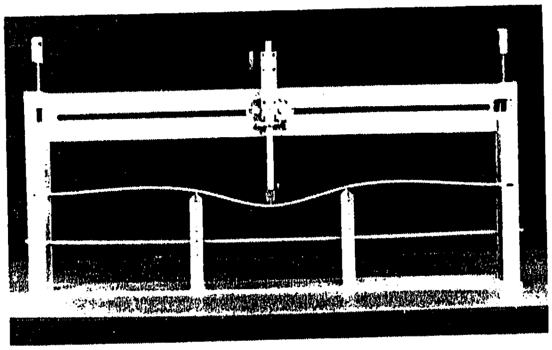


(a)



(b)

Figure 2-55



(c)

Figure 2-55 (*cont.*)

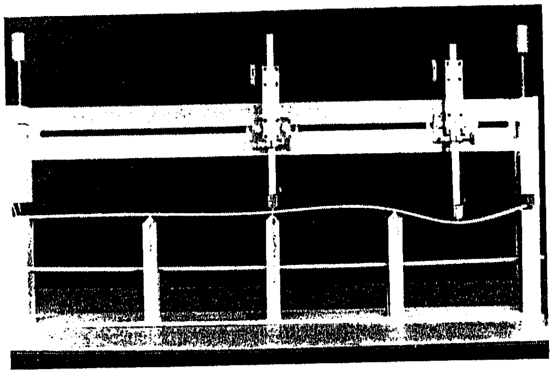


Figure 2-56

After we have acquired some understanding of the deflected shapes of beams under different loadings and support conditions, it will be possible for us to sketch the deflected shape of a beam with greater confidence. For example, we can envision a positive moment or a negative moment in a certain portion of a beam by pointing to the respective concave curvature or convex curvature in the deflected beam. Similarly, we can expect that at a point of inflection corresponding to the transition from concave to convex curvature, the moment equals zero since it can neither be positive nor negative.

The deflected shape of a rigid frame can also be sketched by using the same principle. However, greater caution must be exercised in determining the curvatures of the members meeting at a joint since the free-body diagram of an isolated rigid joint must be in equilibrium. Two examples of the deflected shapes of statically determinate rigid frames are shown in Figs. 2-57 and 2-58.

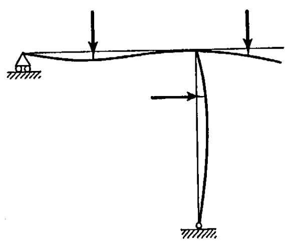


Figure 2-57