Slope-Deflection Method Examples

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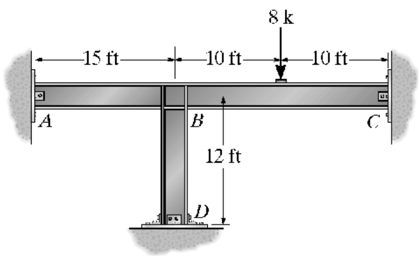
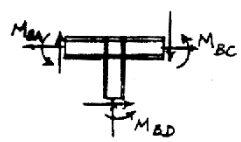
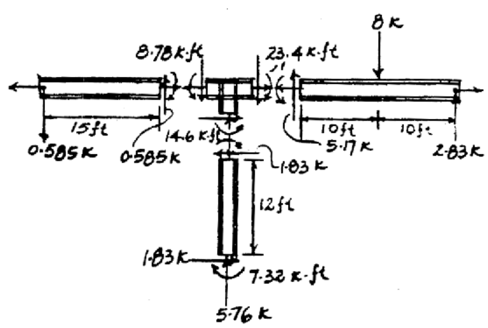
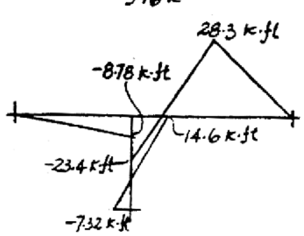
Example 1

Determine the moments at *B* and *D*, then draw the moment diagram. Assume *A* and *C* are pinned and *B* and *D* are fixed connected. *EI* is constant.









$$(FEM)_{BA} = 0$$

$$(FEM)_{BC} = \frac{-3(8)(20)}{16} = -30\ \text{k·ft}$$

$$(FEM)_{BD} = (FEM)_{DB} = 0$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{BA} = 3E\left(\frac{I}{15}\right)(\theta_B - 0) + 0$$

$$M_{BA} = 0.2EI\theta_B \qquad (1)$$

$$M_{BC} = 3E\left(\frac{I}{20}\right)(\theta_B - 0) - 30$$

$$M_{BC} = 0.15EI\theta_B - 30 \qquad (2)$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{BD} = 2E\left(\frac{I}{12}\right)(2\theta_B + 0 - 0) + 0$$

$$M_{BD} = 0.3333EI\theta_B \qquad (3)$$

$$M_{DB} = 2E\left(\frac{I}{12}\right)(2(0) + \theta_B - 0) + 0$$

$$M_{DB} = 0.1667EI\theta_B \qquad (4)$$

Equilibrium

$$M_{BA} + M_{BC} + M_{BD} = 0 \qquad (5)$$

Solving Eqs. 1 – 5 :

$$\theta_B = \frac{43.90}{EI}$$

$M_{BA} = 8.78$ k·ft **Ans**

$M_{BC} = -23.4$ k·ft **Ans**

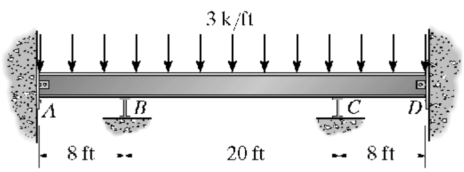
$M_{BD} = 14.6$ k·ft **Ans**

$M_{DB} = 7.32$ k·ft **Ans**

Example 2

Determine the moments at *B* and *C*. Assume *B* and *C* are rollers and *A* and *D* are pinned. *EI* is constant.

Note that in the solution, for spans *AB* and *CD* the short-hand slope-deflection formula along with pinned-fixed FEMs are used.



$$(\text{FEM})_{BA} = \frac{3(8)^2}{8} = 24 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = \frac{-3(20)^2}{12} = -100 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = 100 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CD} = -24 \text{ k}\cdot\text{ft}$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) + 24$$

$$M_{BA} = \frac{3EI\theta_B}{8} + 24 \qquad (1)$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BC} = 2E\left(\frac{I}{20}\right)(2\theta_B + \theta_C - 0) - 100$$

$$M_{BC} = 0.2EI\theta_B + 0.1EI\theta_C - 100 \qquad (2)$$

$$M_{CB} = 2E\left(\frac{I}{20}\right)(2\theta_C + \theta_B - 0) + 100$$

$$M_{CB} = 0.2EI\theta_C + 0.1EI\theta_B + 100 \qquad (3)$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

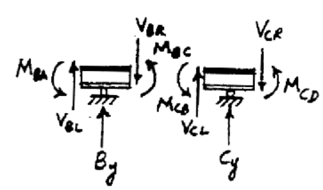
$$M_{CD} = 3E\left(\frac{I}{8}\right)(\theta_C - 0) - 24$$

$$M_{CD} = \frac{3EI\theta_C}{8} - 24 \qquad (4)$$

Equilibrium

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

$$M_{CB} + M_{CD} = 0 \qquad (6)$$



Solving Eqs. 1 - 6 :

$$\theta_B = \frac{160}{EI} \qquad \theta_C - \frac{160}{EI}$$

$M_{BA} = 84.0$ k·ft **Ans**

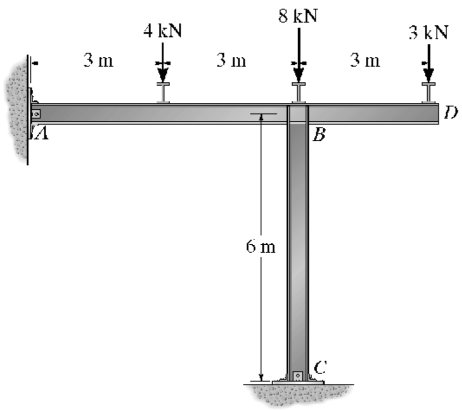
$M_{BC} = -84.0$ k·ft **Ans**

$M_{CB} = 84.0$ k·ft **Ans**

$M_{CD} = -84.0$ k·ft **Ans**

Example 3

Determine the moments at the ends of each member of the frame. The support at *A* and *C* and joint *B* are fixed connected. *EI* is constant.



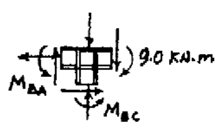
$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = \frac{2EI}{6}(0 + \theta_B) - \frac{(4)(6)}{8}$$

$$M_{BA} = \frac{2EI}{6}(2\theta_B) + \frac{4(6)}{8}$$

$$M_{BC} = \frac{2EI}{6}(2\theta_B)$$

$$M_{CB} = \frac{2EI}{6}(\theta_B)$$



Equilibrium

$$M_{BA} + M_{BC} - 9 = 0$$

$$\frac{2EI}{6}(2\theta_B) + \frac{4(6)}{8} + \frac{2EI}{6}(2\theta_B) - 9 = 0$$

$$\theta_B = \frac{4.5}{EI}$$

Thus.

$$M_{AB} = \frac{2EI}{6}\left(0 + \frac{4.5}{EI}\right) - \frac{4(6)}{8} = -1.50 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

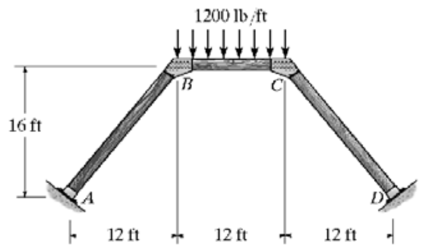
$$M_{CB} = \frac{2EI}{6}\left(\frac{4.5}{EI}\right) = 1.50 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$M_{BA} = \frac{2EI}{6}\left(2\frac{4.5}{EI}\right) - \frac{4(6)}{8} = 6.00 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$M_{BC} = \frac{2EI}{6}\left(2\left(\frac{4.5}{EI}\right)\right) = 3.00 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

Example 4

Determine the moments at each joint and support of the battered-column frame. The joints are rigid. The supports are fixed connected. *EI* is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{20}(0 + \theta_B) + 0$$

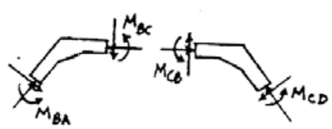
$$M_{BA} = \frac{2EI}{20}(2\theta_B + 0) + 0$$

$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C) - \frac{1.2(12)^2}{12}$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B) + \frac{1.2(12)^2}{12}$$

$$M_{CD} = \frac{2EI}{20}(2\theta_C + 0) + 0$$

$$M_{DC} = \frac{2EI}{20}(0 + \theta_C) + 0$$



Equilibrium

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{20}(2\theta_D + \frac{2EI}{12}(2\theta_B + \theta_C) - 14.4 = 0$$

$$0.5333\theta_D + 0.1667\theta_C = \frac{14.4}{EI} \qquad (1)$$

$$M_{CB} + M_{CD} = 0$$

$$\frac{2EI}{12}(2\theta_C - \theta_B) + 14.4 + \frac{2}{20}(2\theta_C) = 0$$

$$0.5333\theta_C + 0.1667\theta_B = \frac{-14.4}{EI} \qquad (2)$$

Solving Eqs. 1-2:

$$\theta_B = \frac{39.27}{EI}$$

$M_{AB} = 3.93\ \text{k·ft}$ **Ans**

$M_{BA} = 7.85\ \text{k·ft}$ **Ans**

$M_{BC} = -7.85\ \text{k·ft}$ **Ans**

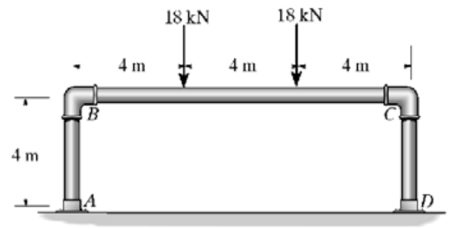
$M_{CB} = 7.85\ \text{k·ft}$ **Ans**

$M_{CD} = -7.85\ \text{k·ft}$ **Ans**

$M_{DC} = -3.93\ \text{k·ft}$ **Ans**

Example 5

The frame is made from pipe that is rigid connected. Determine the moments developed at each of the joints and supports under the given loading. *EI* is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{4}(0 + \theta_B) + 0$$

$$M_{BA} = \frac{2EI}{4}(2\theta_B + 0) + 0$$

$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C) - \frac{2(18)(12)}{9}$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B) + \frac{2(18)(12)}{9}$$

$$M_{CD} = \frac{2EI}{4}(2\theta_C + 0) + 0$$

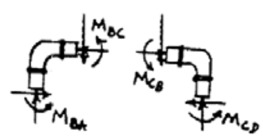
$$M_{DC} = \frac{2EI}{4}(0 + \theta_C) + 0$$

Equilibrium

$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{4}(2\theta_B) + \frac{2EI}{12}(2\theta_B + \theta_C) - 48 = 0$$

$$1.333\theta_B + 0.1667\theta_C = \frac{48}{EI} \qquad (1)$$



$$M_{CB} + M_{CD} = 0$$

$$\frac{2EI}{12}(2\theta_C + \theta_B) + 48 + \frac{2EI}{4}(2\theta_C) = 0$$

$$1.333\theta_C + 0.1667\theta_B = -\frac{48}{EI} \qquad (2)$$

Solving Eqs. (1) and (2) :

$$\theta_B = -\theta_C = \frac{41.143}{EI}$$

$M_{AB} = 20.6$ kN·m **Ans**

$M_{BA} = 41.1$ kN·m **Ans**

$M_{BC} = -41.1$ kN·m **Ans**

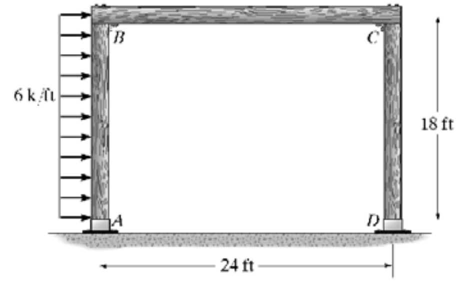
$M_{CB} = 41.1$ kN·m **Ans**

$M_{CD} = -41.1$ kN·m **Ans**

$M_{DC} = -20.6$ kN·m **Ans**

Example 6

Determine the moments at each joint and support. The connections at *B* and *C* are rigid-fixed. The supports at *A* and *D* are fixed. *EI* is constant.



$$(FEM)_{AB} = \frac{-6(18)^2}{12} = -162 \text{ k}\cdot\text{ft}$$

$$(FEM)_{BA} = 162 \text{ k}\cdot\text{ft}$$

$$(FEM)_{BC} = (FEM)_{CB} = 0$$

$$(FEM)_{CD} = (FEM)_{DC} = 0$$

$$\psi_{AB} = \psi_{DC}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2E\left(\frac{I}{18}\right)(2(0) + \theta_B - 3\,\psi_{AB}) - 162$$

$$M_{AB} = 0.1111EI\theta_B - 0.3333EI\psi_{AB} - 162 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{18}\right)(2\theta_B + 0 - 3\psi_{AB}) + 162$$

$$M_{BA} = 0.2222EI\theta_B - 0.333EI\psi_{AB} + 162 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{24}\right)(2\theta_B + \theta_C - 3(0)) + 0$$

$$M_{BC} = 0.1667EI\theta_B + 0.08333EI\theta_C \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{24}\right)(2\theta_C + \theta_B - 3(0)) + 0$$

$$M_{CB} = 0.1667EI\theta_B + 0.08333EI\theta_C \qquad (4)$$

$$M_{CD} = 2E\left(\frac{I}{18}\right)(2\theta_C + 0 - 3\psi_{AB}) + 0$$

$$M_{CD} = 0.2222EI\theta_C - 03333EI\psi_{AB} \qquad (5)$$

$$M_{DC} = 2E\left(\frac{I}{18}\right)(2(0) + \theta_C - 3\psi_{AB}) + 0$$

$$M_{DC} = 0.1111EI\theta_C - 0.3333EI\psi_{AB} \qquad (6)$$

Equilibrium

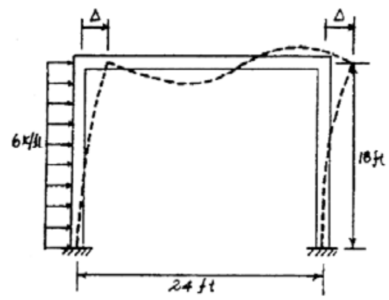
$$M_{BA} + M_{BC} = 0 \qquad (7)$$

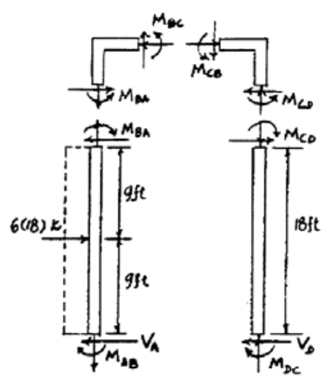
$$M_{CD} + M_{CB} = 0 \qquad (8)$$

$$V_A + V_D - 6(18) = 0$$

$$-\frac{(M_{AB} + M_{BA} - 972)}{18} - \frac{(M_{CD} + M_{DC})}{18} - 108 = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = -972 \qquad (9)$$





Solving these equations :

$$\theta_B = \frac{265.09}{EI}, \qquad \theta_C = \frac{795.27}{EI}$$

$$\psi_{AB} = \frac{994.09}{EI}$$

$M_{AB} = -464$ k·ft **Ans**

$M_{BA} = -110$ k·ft **Ans**

$M_{BC} = 110$ k·ft **Ans**

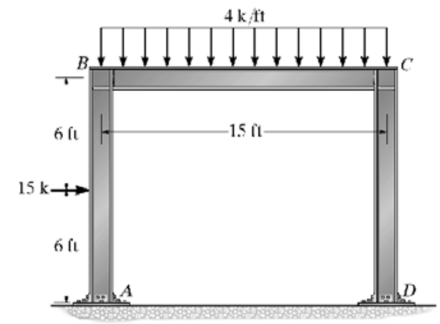
$M_{CB} = 155$ k·ft **Ans**

$M_{CD} = -155$ k·ft **Ans**

$M_{DC} = -243$ k·ft **Ans**

Example 7

Determine the moments at *A*, *B*, *C*, and *D*. Draw the moment diagram. The members are fixed connected at the supports and joints. *EI* is constant.



$$(FEM)_{AB} = \frac{-15(12)}{8} = -22.5\ \text{k}\cdot\text{ft}$$

$$(FEM)_{BA} = 22.5\ \text{k}\cdot\text{ft}$$

$$(FEM)_{BC} = \frac{-4(15)^2}{12} = -75.0\ \text{k}\cdot\text{ft}$$

$$(FEM)_{CB} = 75.0\ \text{k}\cdot\text{ft}$$

$$(FEM)_{CD} = (FEM)_{DC} = 0$$

$$\psi_{AB} = \psi_{DC}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = 2E\left(\frac{I}{12}\right)(2(0) + \theta_B - 3\psi_{AB}) + 0$$

$$M_{AB} = 0.1667EI\theta_B - 0.5EI\psi_{AB} - 22.5 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)(2\theta_B + 0 - 3\psi_{AB}) + 22.5$$

$$M_{BA} = 0.3333EI\theta_B - 0.5EI\psi_{AB} + 22.5 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{15}\right)(2\theta_B + \theta_C - 3(0)) - 75.0$$

$$M_{BC} = 0.2667EI\theta_B + 0.1333EI\theta_C - 75.0 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{15}\right)(2\theta_C + \theta_B - 3(0)) + 75.0$$

$$M_{CB} = 0.2667EI\theta_B + 0.1333EI\theta_C + 75.0 \qquad (4)$$

$$M_{CD} = 2E\left(\frac{I}{12}\right)(2\theta_C + 0 - 3\psi_{AB}) + 0$$

$$M_{CD} = 0.3333EI\theta_C - 0.5EI\psi_{AB} \qquad (5)$$

$$M_{DC} = 2E\left(\frac{I}{12}\right)(2(0) + \theta_C - 3\psi_{AB}) + 0$$

$$M_{DC} = 0.1667EI\theta_C - 0.5EI\psi_{AB} \qquad (6)$$

Equilibrium

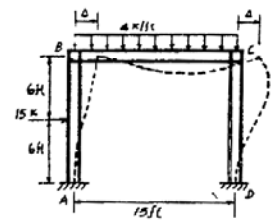
$$M_{BA} + M_{BC} = 0 \qquad (7)$$

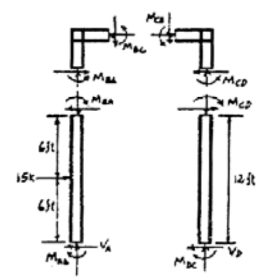
$$M_{CD} + M_{CB} = 0 \qquad (8)$$

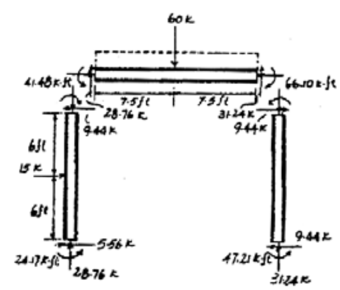
$$V_A + V_D - 15 = 0$$

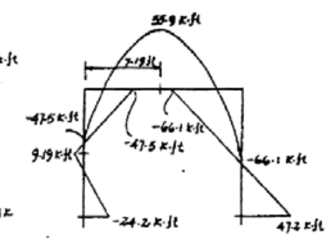
$$-\frac{(M_{AB} + M_{BA} - 90)}{12} - \frac{(M_{CD} + M_{DC})}{12} - 15 = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = -90 \qquad (9)$$









Solving these equations :

$$\theta_B = \frac{159.88}{EI}. \qquad \theta_C = \frac{-113.33}{EI}$$

$$\psi_{AB} = \frac{56.64}{EI}$$

$M_{AB} = -24.17$ k·ft $= -24.2$ k·ft **Ans**

$M_{BA} = 47.48$ k·ft $= 47.5$ k·ft **Ans**

$M_{BC} = -47.48$ k·ft $= -47.5$ k·ft **Ans**

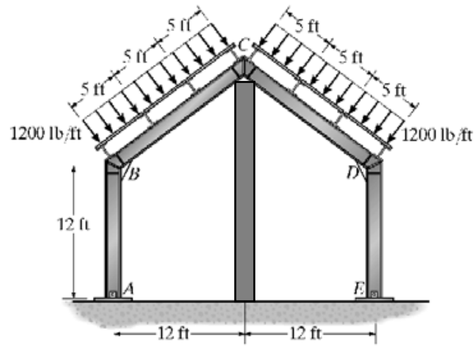
$M_{CB} = 66.01$ k·ft $= 66.0$ k·ft **Ans**

$M_{CD} = -66.01$ k·ft $= -66.0$ k·ft **Ans**

$M_{DC} = -47.21$ k·ft $= -47.2$ k·ft **Ans**

Example 8

Determine the moment at each joint of the gable frame. The roof load is transmitted to each of the purlins over simply supported sections of the roof decking. Assume the supports at *A* and *E* are pins. The joints are fixed connected. The center column, connected to the ridge point *C*, is incompressible. *EI* is constant.
Note that in the solution, in calculating the M_{BA} and M_{DE}, the short-hand/modified slope-deflection formula is used. The center column keeps ridge point *C* from displacing vertically.



$$(FEM)_{BC} = (FEM)_{CD} = \frac{-2(6)(15)}{9} = -20 \text{ k}\cdot\text{ft}$$

$$(FEM)_{CB} = (FEM)_{DC} = 20 \text{ k}\cdot\text{ft}$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{BA} = \frac{3EI}{12}(\theta_B)$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{BC} = \frac{2EI}{15}(2\theta_B + \theta_C) - 20$$

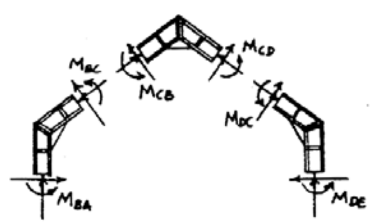
$$M_{CB} = \frac{2EI}{15}(2\theta_C + \theta_B) + 20$$

$$M_{CD} = \frac{2EI}{15}(2\theta_C + \theta_D) - 20$$

$$M_{DC} = \frac{2EI}{15}(2\theta_D + \theta_C) + 20$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (FEM)_N$$

$$M_{DE} = \frac{3EI}{12}(\theta_D)$$



Equilibrium

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$M_{DC} + M_{DE} = 0$$

or

$$\frac{3EI}{12}\theta_B + \frac{2EI}{15}(2\theta_B + \theta_C) - 20 = 0$$

$$0.5167\theta_B + 0.1333\theta_C = \frac{20}{EI}$$

$$\frac{2EI}{15}(2\theta_C + \theta_B) + 20 + \frac{2EI}{15}(2\theta_C + \theta_D) - 20 = 0$$

$$4\theta_C + \theta_B + \theta_D = 0$$

$$\frac{2EI}{15}(2\theta_D + \theta_C) + 20 + \frac{3EI}{12}\theta_D = 0$$

$$0.51667\theta_D + 0.1333\theta_C = -\frac{20}{EI}$$

Solving these equations :

$$\theta_C = 0$$
$$\theta_B = -\theta_D = \frac{38.71}{EI}$$

Thus,

$M_{BA} = 9.68$ k·ft **Ans**

$M_{BC} = -9.68$ k·ft **Ans**

$M_{CB} = 25.2$ k·ft **Ans**

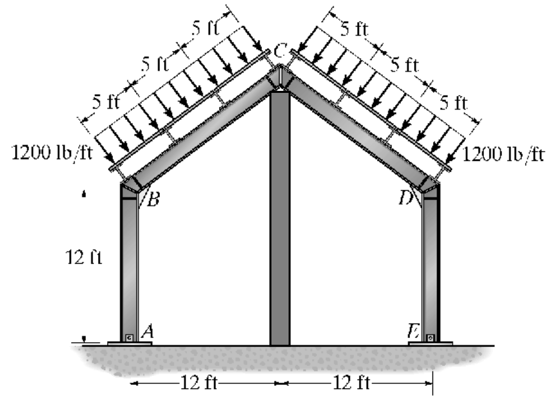
$M_{CD} = -25.2$ k·ft **Ans**

$M_{DC} = 9.68$ k·ft **Ans**

$M_{DE} = -9.68$ k·ft **Ans**

Example 9

Solve example 8 assuming the supports at *A* and *E* are fixed.



$$(FEM)_{BC} = (FEM)_{CD} = \frac{-2(6)(15)}{9} = -20 \text{ k}\cdot\text{ft}$$

$$(FEM)_{CB} = (FEM)_{DC} = 20 \text{ k}\cdot\text{ft}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (FEM)_N$$

$$M_{AB} = \frac{2EI}{12}(\theta_B)$$

$$M_{BA} = \frac{2EI}{12}(2\theta_B)$$

$$M_{BC} = \frac{2EI}{15}(2\theta_B + \theta_C) - 20$$

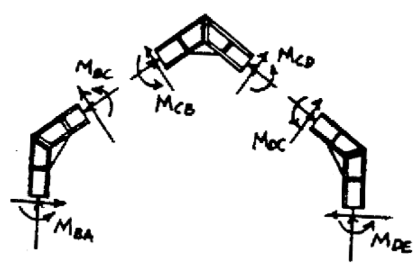
$$M_{CB} = \frac{2EI}{15}(2\theta_C + \theta_B) + 20$$

$$M_{CD} = \frac{2EI}{15}(2\theta_C + \theta_D) - 20$$

$$M_{DC} = \frac{2EI}{15}(2\theta_D + \theta_C) + 20$$

$$M_{DE} = \frac{2EI}{12}(2\theta_D)$$

$$M_{ED} = \frac{2EI}{12}(\theta_D)$$



Equilibrium

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$$M_{DC} + M_{DE} = 0$$

or.

$$\frac{2EI}{12}(2\theta_B) + \frac{2EI}{15}(2\theta_B + \theta_C) - 20 = 0$$

$$0.6\theta_B + 0.1333\theta_C = \frac{20}{EI}$$

$$\frac{2EI}{15}(2\theta_C + \theta_B) + 20 + \frac{2EI}{15}(2\theta_C + \theta_D) - 20 = 0$$

$$0.5333\theta_C + 0.1333\theta_B + 0.1333\theta_D = 0$$

$$\frac{2EI}{15}(2\theta_D + \theta_C) + 20 + \frac{2EI}{12}(2\theta_D) = 0$$

$$0.6\theta_D + 0.1333\theta_C = -\frac{20}{EI}$$

Solving these equations :

$$\theta_C = 0$$

$$\theta_B = -\theta_D = \frac{33.33}{EI}$$

$M_{AB} = 5.56$ k·ft **Ans**

$M_{BA} = 11.1$ k·ft **Ans**

$M_{BC} = -11.1$ k·ft **Ans**

$M_{CB} = 24.4$ k·ft **Ans**

$M_{CD} = -24.4$ k·ft **Ans**

$M_{DC} = 11.1$ k·ft **Ans**

$M_{DE} = -11.1$ k·ft **Ans**

$M_{ED} = -5.56$ k·ft **Ans**