Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.9

Fixed-end Moments:

Member AB

\[ M_{AB} = - \frac{wl^2}{12} = - \frac{12.0 \times 4^2}{12} = - 16.0 \text{ kNm} \]

\[ M_{BA} = + \frac{wl^2}{12} = + \frac{12.0 \times 4^2}{12} = + 16.0 \text{ kNm} \]
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Member CD*

\[ M_{CD} = -\frac{PL}{8} = -\frac{36.0 \times 4}{8} = -18.0 \text{ kNm} \]

\[ M_{DC} = +\frac{PL}{8} = +\frac{36.0 \times 4}{8} = +18.0 \text{ kNm} \]

* Since support D is pinned, the fixed-end moments are \((M_{CD} - 0.5M_{DC})\) at C and zero at D.

\((M_{CD} - 0.5M_{DC}) = [-18.0 - (0.5 \times 18.0)] = -27.0 \text{ kNm}.\)

Distribution Factors: Joint B

\[ k_{BA} = \left( \frac{1}{4.0} \right) = 0.25I \quad k_{total} = 0.51I \]

\[ DF_{BA} = \frac{k_{BA}}{k_{Total}} = \frac{0.25}{0.5} = 0.5 \]

\[ k_{BC} = \left( \frac{1}{4.0} \right) = 0.25I \]

\[ DF_{BC} = \frac{k_{BC}}{k_{Total}} = \frac{0.25}{0.5} = 0.5 \]

Distribution Factors: Joint C

\[ k_{CB} = \left( \frac{1}{4.0} \right) = 0.25I \quad k_{total} = 0.44I \]

\[ DF_{CB} = \frac{k_{CB}}{k_{Total}} = \frac{0.25}{0.44} = 0.57 \]

\[ k_{CD} = \frac{3}{4} \left( \frac{1}{4.0} \right) = 0.19I \]

\[ DF_{CD} = \frac{k_{CD}}{k_{Total}} = \frac{0.19}{0.44} = 0.43 \]

Moment Distribution Table:

<table>
<thead>
<tr>
<th>Joint</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tbody>
<tr>
<td></td>
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<td>BA</td>
<td>BC</td>
<td>CB</td>
</tr>
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<td>Distribution Factors</td>
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<td>0.5</td>
<td>0.57</td>
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<tr>
<td>Fixed-end Moments</td>
<td>-16.0</td>
<td>+16.0</td>
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<td>-27.0</td>
</tr>
<tr>
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<td>-8.0</td>
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<td>Total</td>
<td>-22.35</td>
<td>+3.31</td>
<td>-3.31</td>
<td>+12.72</td>
</tr>
</tbody>
</table>
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Continuity Moments:

Fixed Bending Moment Diagrams

Free bending moments:

Free Bending Moment Diagrams

Member AB: \( M_{\text{free}} = \frac{(12.0 \times 4^2)}{8} = 24.0 \text{ kNm} \)
Member CD: \( M_{\text{free}} = \frac{(36.0 \times 4)}{4} = 36.0 \text{ kNm} \)
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Maximum bending moment:
\[ M = 12.16 \text{ kNm} \]

Bending Moment Diagram

- The maximum value along the length of member AB can be found by identifying the point of zero shear as follows:

\[ \Sigma M_B = 0 \]
\[ -22.35 - (12.0 \times 4.0 \times 2.0) + 3.31 + (V_A \times 4.0) = 0 \]
\[ x = (28.76/12.0) = 2.4 \text{ m} \]

\[ M_{\text{maximum}} = (0.5 \times 2.4 \times 28.76) - 22.35 = 12.16 \text{ kNm} \]
Consider Member CD:

\[ +\text{ve} \sum M_C = 0 \]
\[ -12.72 + (36.0 \times 2.0) - (V_D \times 4.0) = 0 \]

\[ \therefore V_D = +14.82 \text{ kN} \uparrow \]

For the complete frame:

\[ +\text{ve} \sum F_y = 0 \]
\[ 28.76 - (12.0 \times 4.0) - 36.0 + 14.82 + V_C = 0 \]

\[ \therefore V_C = +40.42 \text{ kN} \uparrow \]

\[ +\text{ve} \sum M_A = 0 \]
\[ -22.35 + (12.0 \times 4.0 \times 2.0) + (36.0 \times 6.0) - (40.42 \times 4.0) - (14.82 \times 8.0) - (H_D \times 4.0) = 0 \]

\[ \therefore H_D = +2.35 \text{ kN} \rightarrow \]

\[ +\text{ve} \quad \sum F_x = 0 \]
\[ +H_A + H_D = 0 \]

\[ \therefore H_A = -2.35 \text{ kN} \leftarrow \]