5.2.3 Solutions: Moment Distribution – No-Sway Rigid-Jointed Frames

Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames
Problem Number: 5.5

Fixed-end Moments:

Member BC
\[ M_{BC} = - \frac{PL}{8} = - \frac{25 \times 8}{8} = -25.0 \text{ kNm} \]
\[ M_{CB} = + \frac{PL}{8} = + \frac{25 \times 8}{8} = +25.0 \text{ kNm} \]

Distribution Factors: Joint B

\[ k_{BA} = \left( \frac{I}{5} \right) = 0.2I \]
\[ k_{BC} = \left( \frac{2I}{8} \right) = 0.25I \]
\[ k_{total} = 0.45I \]

\[ DF_{BA} = \frac{k_{BA}}{k_{Total}} = \frac{0.2}{0.45} = 0.44 \]
\[ DF_{BC} = \frac{k_{BC}}{k_{Total}} = \frac{0.25}{0.45} = 0.56 \]

In this case, since there is only one internal joint, only one balancing operation and one carry-over will be required during the distribution of the moments.
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Moment Distribution Table:

<table>
<thead>
<tr>
<th>Joint</th>
<th>A</th>
<th></th>
<th></th>
<th>B</th>
<th></th>
<th></th>
<th>C</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AB</td>
<td>BA</td>
<td>BC</td>
<td></td>
<td></td>
<td></td>
<td>CB</td>
<td></td>
</tr>
<tr>
<td>Distribution Factors</td>
<td>0</td>
<td>0.44</td>
<td>0.56</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed-end Moments</td>
<td></td>
<td>-25.0</td>
<td>+25.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td></td>
<td>+11.0</td>
<td>+14.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carry-over</td>
<td>+5.5</td>
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<td></td>
<td>+7.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>+5.5</td>
<td>+11.0</td>
<td>-11.0</td>
<td>+32.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Continuity Moments:

Fixed Bending Moment Diagrams

Free bending moment:

Member BC:

\[ M_{free} = \frac{PL}{4} = \frac{25 \times 8}{4} = 50.0 \text{ kNm} \]
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Maximum bending moment:
\[ M = -[0.5 \times (11.0 + 32.0)] + 50.0 \]
\[ M = +28.5 \text{ kNm} \]

Bending Moment Diagram

Consider Member AB:

\[ +ve \sum M_B = 0 \]
\[ +5.5 + 11.0 - (H_A \times 5.0) = 0 \]
\[ \therefore H_A = +3.3 \text{ kN} \]

For the complete frame:

\[ +ve \sum F_x = 0 \]
\[ 3.3 + H_C = 0 \]
\[ \therefore H_C = -3.3 \text{ kN} \]

\[ +ve \sum M_A = 0 \]
\[ +5.5 + (25.0 \times 4.0) - (3.3 \times 5.0) + 32.0 - (V_C \times 8.0) = 0 \]
\[ \therefore V_C = +15.13 \text{ kN} \]

\[ +ve \sum F_y = 0 \]
\[ V_A - 25.0 + 15.13 = 0 \]
\[ \therefore V_A = +9.87 \text{ kN} \]

Deflected Shape