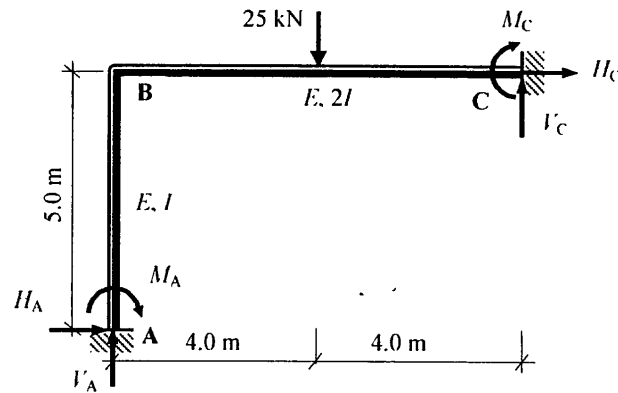
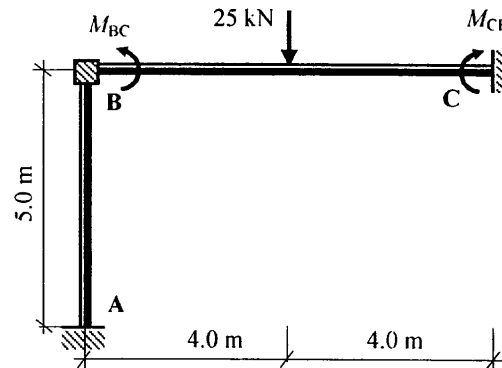


## 5.2.3 Solutions: Moment Distribution – No-Sway Rigid-Jointed Frames

**Solution****Topic: Moment Distribution – No-Sway Rigid-Jointed Frames****Problem Number: 5.5****Page No. 1****Fixed-end Moments:**

Member BC

$$M_{BC} = -\frac{PL}{8} = -\frac{25 \times 8}{8} = -25.0 \text{ kNm}$$

$$M_{CB} = +\frac{PL}{8} = +\frac{25 \times 8}{8} = +25.0 \text{ kNm}$$

**Distribution Factors : Joint B**

$$k_{BA} = \left(\frac{I}{5}\right) = 0.2I$$

$$k_{\text{total}} = 0.45I$$

$$DF_{BA} = \frac{k_{BA}}{k_{\text{Total}}} = \frac{0.2}{0.45} = 0.44$$

$$k_{BC} = \left(\frac{2I}{8}\right) = 0.25I$$

$$DF_{BC} = \frac{k_{BC}}{k_{\text{Total}}} = \frac{0.25}{0.45} = 0.56$$

In this case, since there is only one internal joint, only one balancing operation and one carry-over will be required during the distribution of the moments.

### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

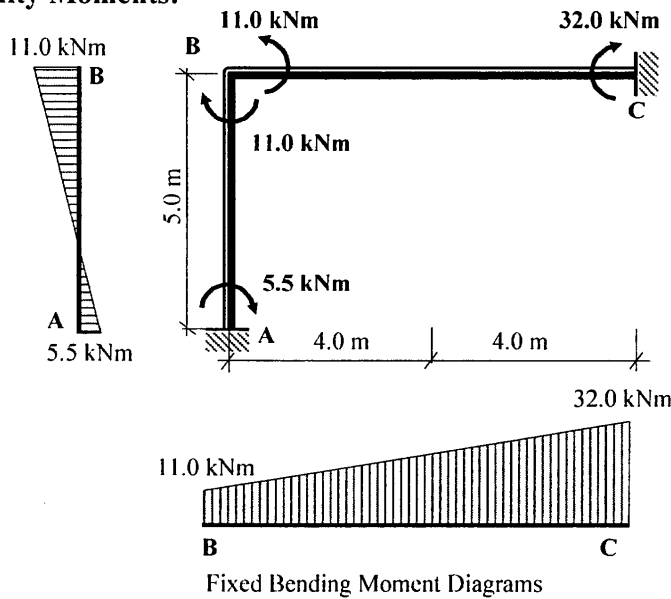
Problem Number: 5.5

Page No. 2

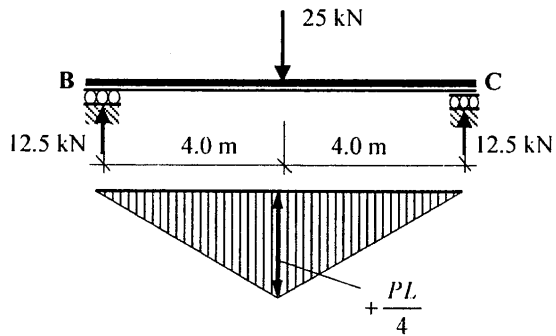
Moment Distribution Table:

Joint	A	B		C
	AB	BA	BC	CB
Distribution Factors	0	0.44	0.56	0
Fixed-end Moments			- 25.0	+ 25.0
Balance		+ 11.0	+ 14.0	
Carry-over	+ 5.5			+ 7.0
Total	+ 5.5	+ 11.0	- 11.0	+ 32.0

Continuity Moments:



Free bending moment:



Member BC:

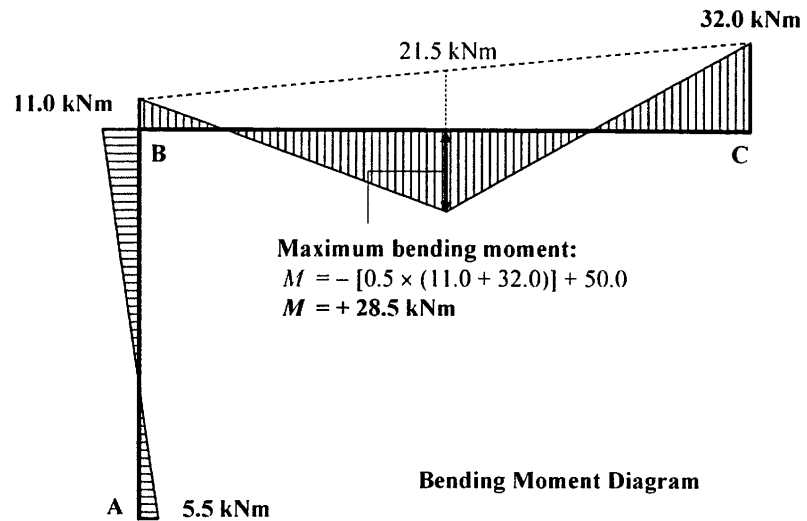
$$M_{\text{free}} = \frac{PL}{4} = \frac{25 \times 8}{4} = 50.0 \text{ kNm}$$

### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

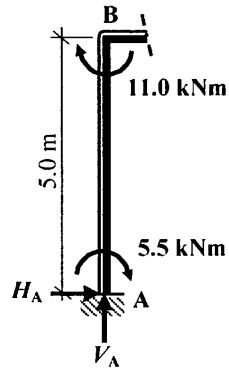
Problem Number: 5.5

Page No. 3



Maximum bending moment:  
 $M = -[0.5 \times (11.0 + 32.0)] + 50.0$   
 $M = +28.5 \text{ kNm}$

Consider Member AB:



$$+ve \curvearrowright \Sigma M_B = 0$$

$$+ 5.5 + 11.0 - (H_A \times 5.0) = 0 \quad \therefore H_A = +3.3 \text{ kN} \rightarrow$$

For the complete frame:

$$+ve \rightarrow \Sigma F_x = 0$$

$$3.3 + H_C = 0 \quad \therefore H_C = -3.3 \text{ kN} \leftarrow$$

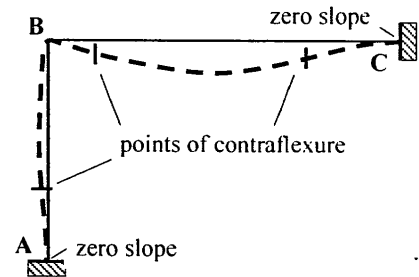
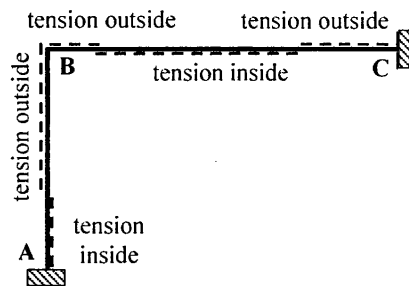
$$+ve \curvearrowright \Sigma M_A = 0$$

$$+ 5.5 + (25.0 \times 4.0) - (3.3 \times 5.0) + 32.0 - (V_C \times 8.0) = 0$$

$$\therefore V_C = +15.13 \text{ kN} \uparrow$$

$$+ve \uparrow \Sigma F_y = 0$$

$$V_A - 25.0 + 15.13 = 0 \quad \therefore V_A = +9.87 \text{ kN} \uparrow$$



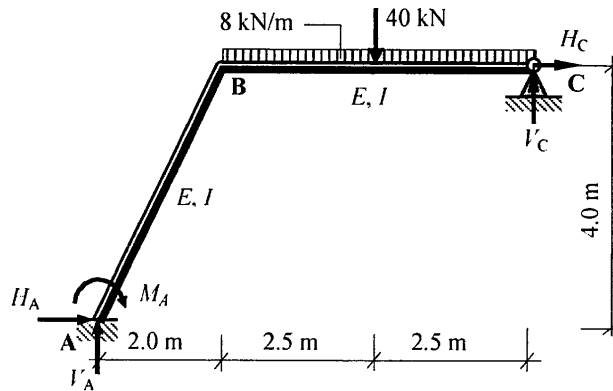
**Deflected Shape**

### Solution

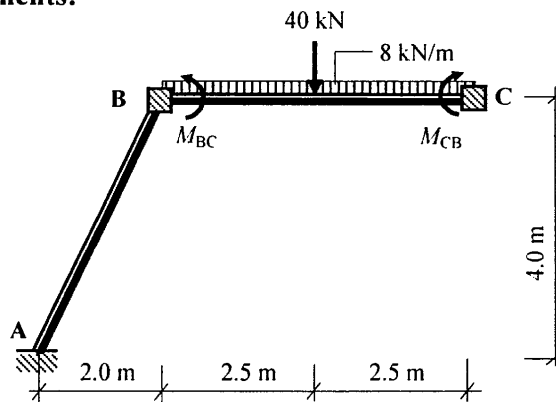
Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.6

Page No. 1



Fixed-end Moments:



$$\text{Length of member AB} = \sqrt{(2.0^2 + 4.0^2)} = 4.472 \text{ m}$$

Member BC\*

$$M_{BC} = -\frac{PL}{8} - \frac{wL^2}{12} = -\frac{40.0 \times 5}{8} - \frac{8.0 \times 5^2}{12} = -41.67 \text{ kNm}$$

$$M_{CB} = +\frac{PL}{8} + \frac{wL^2}{12} = +\frac{40.0 \times 5}{8} + \frac{8.0 \times 5^2}{12} = +41.67 \text{ kNm}$$

\* Since support C is pinned, the fixed-end moments are  $(M_{BC} - 0.5M_{CB})$  at B and zero at C.

$$(M_{BC} - 0.5M_{CB}) = [-41.67 - (0.5 \times 41.67)] = -62.51 \text{ kNm.}$$

### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.6

Page No. 2

#### Distribution Factors : Joint B

$$k_{BA} = \left( \frac{I}{4.472} \right) = 0.22I$$

$$k_{total} = 0.37I$$

$$DF_{BA} = \frac{k_{BA}}{k_{Total}} = \frac{0.22}{0.37} = 0.59$$

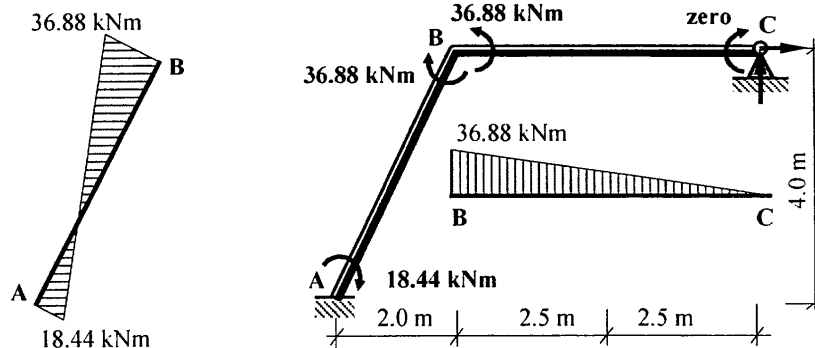
$$k_{BC} = \frac{3}{4} \times \left( \frac{I}{5} \right) = 0.15I$$

$$DF_{BC} = \frac{k_{BC}}{k_{Total}} = \frac{0.15}{0.37} = 0.41$$

#### Moment Distribution Table:

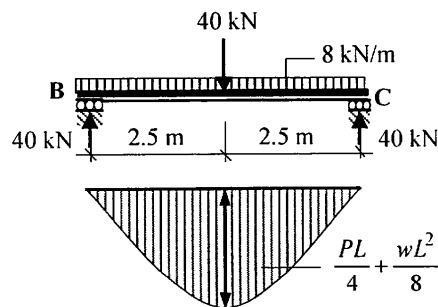
Joint	A	B		C
	AB	BA	BC	CB
Distribution Factors	0	0.59	0.41	1.0
Fixed-end Moments			- 62.51	
Balance		+ 36.88	+ 25.63	
Carry-over	+ 18.44			
Total	+ 18.44	+ 36.88	- 36.88	0

#### Continuity Moments:



Fixed Bending Moment Diagrams

#### Free bending moment:



Member BC:

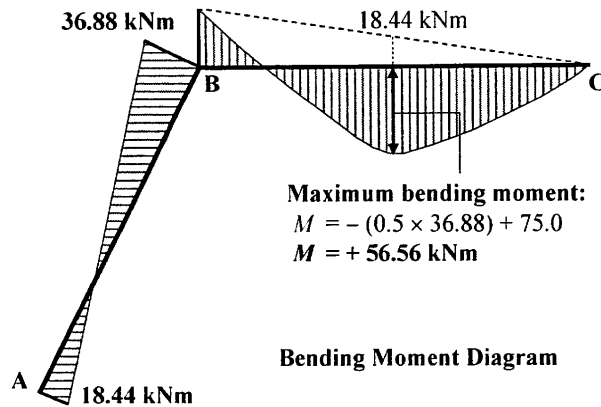
$$\begin{aligned}
 M_{free} &= \frac{PL}{4} + \frac{wL^2}{8} \\
 &= \frac{40 \times 5}{4} + \frac{8.0 \times 5.0^2}{8} \\
 &= 75.0 \text{ kNm}
 \end{aligned}$$

## Solution

**Topic: Moment Distribution – No-Sway Rigid-Jointed Frames**

**Problem Number: 5.6**

**Page No. 3**

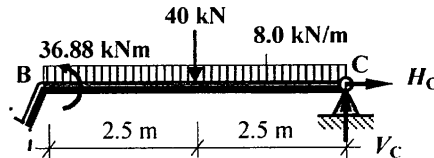


Maximum bending moment:

$$M = -(0.5 \times 36.88) + 75.0$$

$$M = + 56.56 \text{ kNm}$$

Consider Member BC:



$$+ve \curvearrowright \Sigma M_B = 0$$

$$- 36.88 + (40.0 \times 2.5) + (8.0 \times 5.0 \times 2.5) - (V_C \times 5.0) = 0 \quad \therefore V_C = + 32.62 \text{ kN} \uparrow$$

For the complete frame:

$$+ve \curvearrowright \Sigma M_A = 0$$

$$+ 18.44 + (40.0 \times 4.5) + (8.0 \times 5.0 \times 4.5) - (32.62 \times 7.0) + (H_C \times 4.0) = 0$$

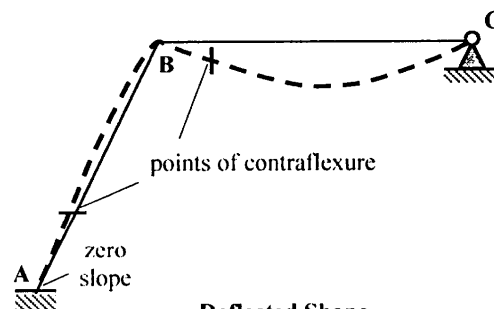
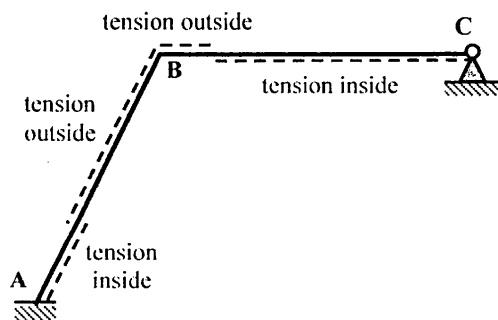
$$\therefore H_C = - 37.53 \text{ kN} \leftarrow$$

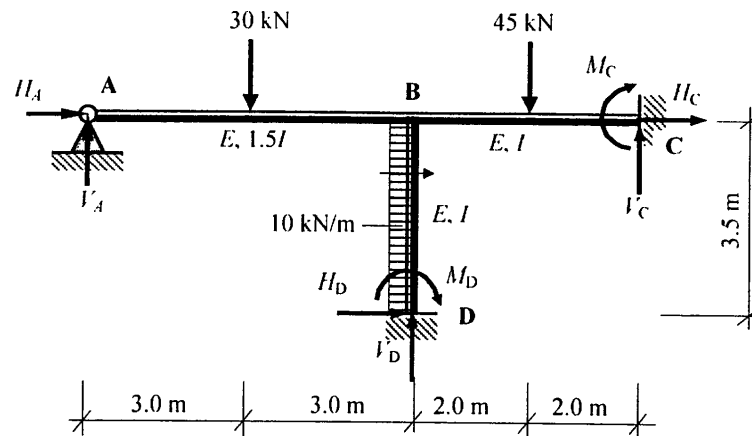
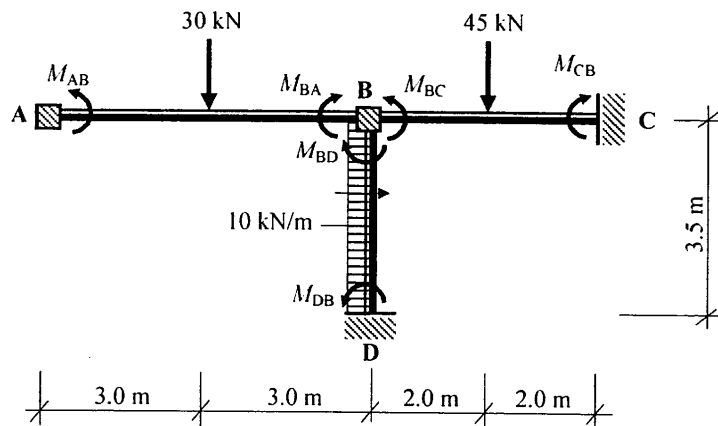
$$+ve \uparrow \Sigma F_y = 0$$

$$V_A - 40.0 - (8.0 \times 5.0) + 32.62 = 0 \quad \therefore V_A = + 47.38 \text{ kN} \uparrow$$

$$+ve \rightarrow \Sigma F_x = 0$$

$$H_A - 37.53 = 0 \quad \therefore H_A = + 37.53 \text{ kN} \rightarrow$$



**Solution****Topic: Moment Distribution – No-Sway Rigid-Jointed Frames****Problem Number: 5.7****Page No. 1****Fixed-end Moments:**

Member AB\*

$$M_{AB} = -\frac{PL}{8} = -\frac{30.0 \times 6}{8} = -22.5 \text{ kNm}$$

$$M_{BA} = +\frac{PL}{8} = +\frac{30.0 \times 6}{8} = +22.5 \text{ kNm}$$

\* Since support A is pinned, the fixed-end moments are zero at A and  $(M_{BA} - 0.5M_{AB})$  at B.

$$(M_{BA} - 0.5M_{AB}) = [22.5 + (0.5 \times 22.5)] = +33.75 \text{ kNm.}$$

**Solution****Topic: Moment Distribution – No-Sway Rigid-Jointed Frames****Problem Number: 5.7****Page No. 2**

Member BC

$$M_{BC} = -\frac{PL}{8} = -\frac{45.0 \times 4}{8} = -22.5 \text{ kNm}$$

$$M_{CB} = +\frac{PL}{8} = +\frac{45.0 \times 4}{8} = +22.5 \text{ kNm}$$

Member BD

$$M_{BD} = +\frac{wL^2}{12} = +\frac{10.0 \times 3.5^2}{12} = +10.21 \text{ kNm}$$

$$M_{DB} = -\frac{wL^2}{12} = -\frac{10.0 \times 3.5^2}{12} = -10.21 \text{ kNm}$$

**Distribution Factors : Joint B**

$$k_{BA} = \frac{3}{4} \times \left( \frac{1.5I}{6.0} \right) = 0.19I$$

$$k_{BC} = \left( \frac{I}{4} \right) = 0.25I$$

$$k_{BD} = \left( \frac{I}{3.5} \right) = 0.29I$$

$$k_{\text{total}} = 0.73I$$

$$DF_{BA} = \frac{k_{BA}}{k_{\text{Total}}} = \frac{0.19}{0.73} = 0.26$$

$$DF_{BC} = \frac{k_{BC}}{k_{\text{Total}}} = \frac{0.25}{0.73} = 0.34$$

$$DF_{BD} = \frac{k_{BD}}{k_{\text{Total}}} = \frac{0.29}{0.73} = 0.40$$

**Moment Distribution Table:**

Joint	A	B			C	D
	AB	BA	BD	BC	CB	DB
<b>Distribution Factors</b>	<b>1.0</b>	<b>0.26</b>	<b>0.40</b>	<b>0.34</b>	<b>0</b>	<b>0</b>
<b>Fixed-end Moments</b>		+ 33.75	+ 10.21	- 22.5	+ 22.5	- 10.21
<b>Balance</b>		- 5.58	- 8.58	- 7.3		
<b>Carry-over</b>					- 3.7	- 4.29
<b>Total</b>	<b>0</b>	<b>+ 28.17</b>	<b>+ 1.63</b>	<b>- 29.8</b>	<b>+ 18.8</b>	<b>- 14.5</b>

Note: the sum of the moments at joint B = zero



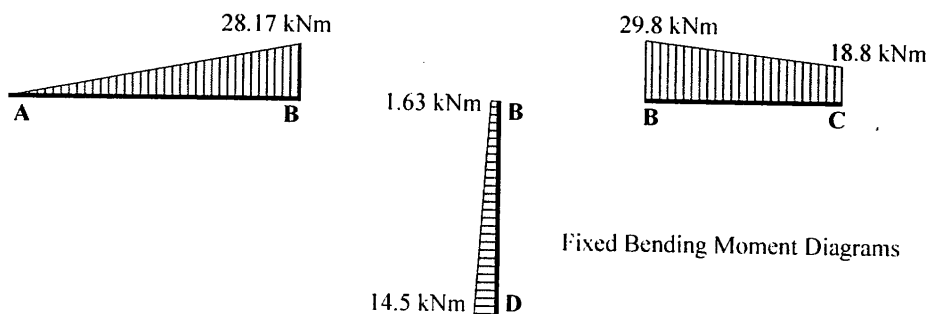
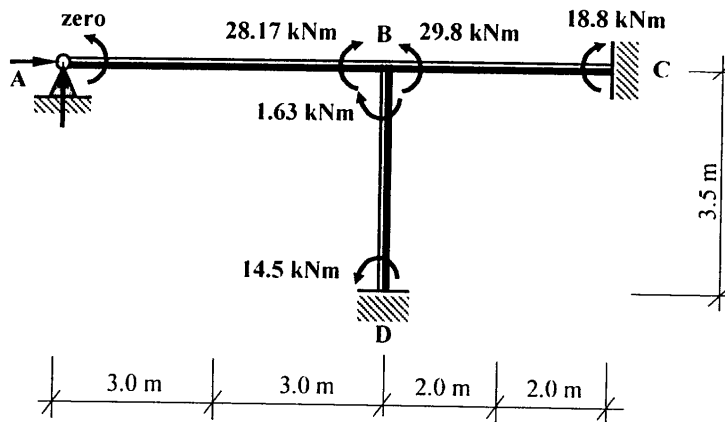
### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

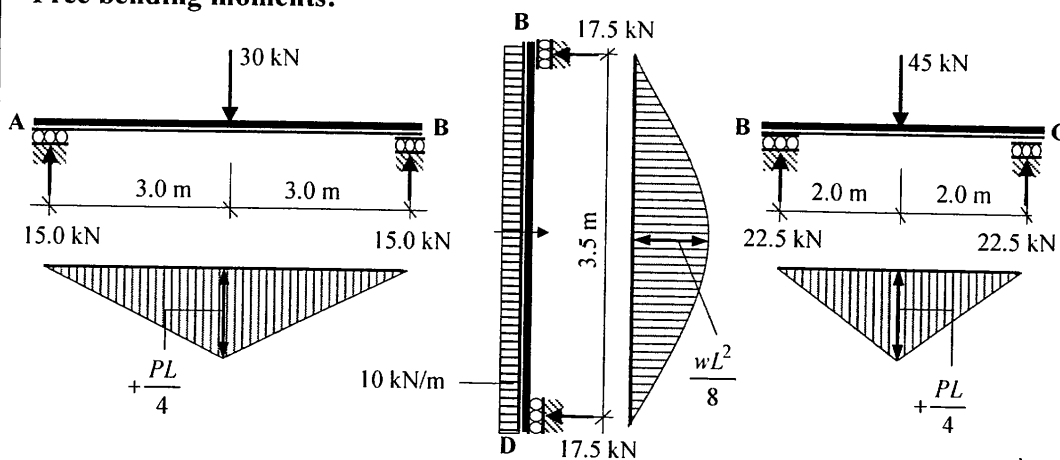
Problem Number: 5.7

Page No. 3

Continuity Moments:



Free bending moments:



Free Bending Moment Diagrams

### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.7

Page No. 4

Member AB:

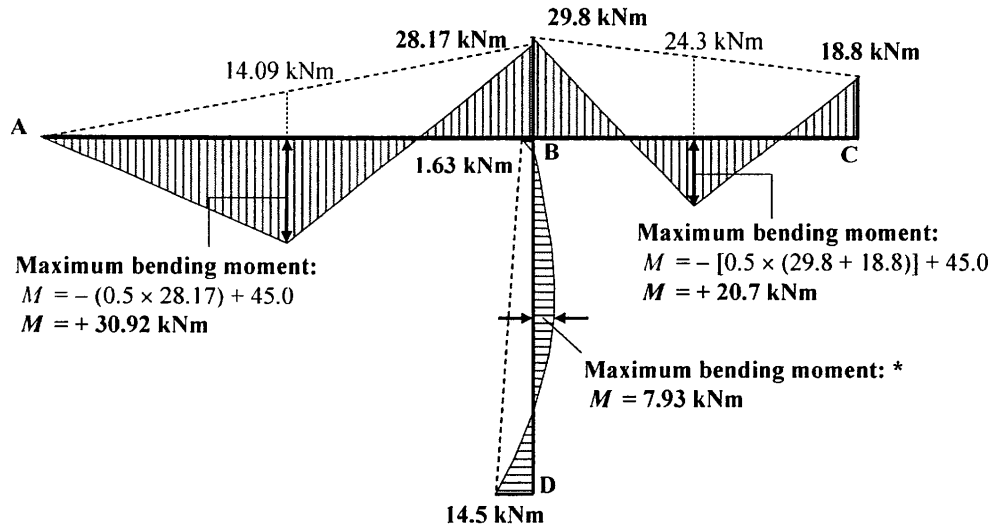
$$M_{\text{free}} = + \frac{PL}{4} = \frac{30.0 \times 6}{4} = 45.0 \text{ kNm}$$

Member BD:

$$M_{\text{free}} = + \frac{wL^2}{8} = \frac{10.0 \times 3.5^2}{8} = 15.31 \text{ kNm}$$

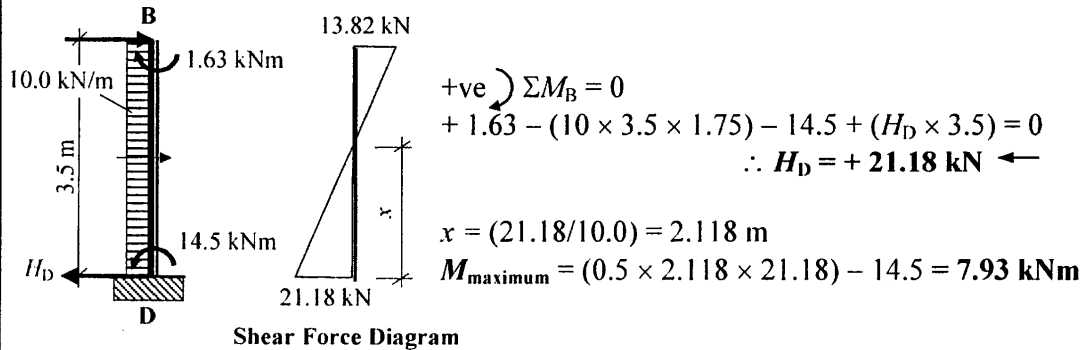
Member BC:

$$M_{\text{free}} = + \frac{PL}{4} = \frac{45.0 \times 4}{4} = 45.0 \text{ kNm}$$



Bending Moment Diagram

\* The maximum value along the length of member DB can be found by identifying the point of zero shear as follows:



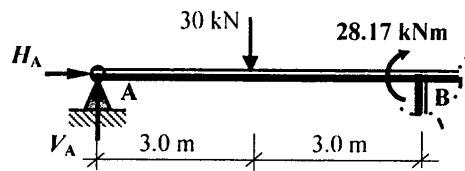
## Solution

**Topic: Moment Distribution – No-Sway Rigid-Jointed Frames**

**Problem Number: 5.7**

**Page No. 4**

Consider Member AB:

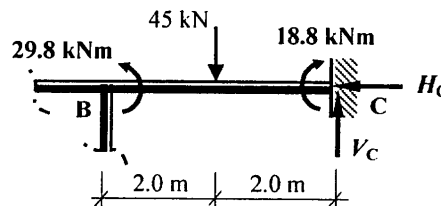


$$+ve \curvearrowright \Sigma M_B = 0$$

$$+ 28.17 - (30.0 \times 3.0) + (V_A \times 6.0) = 0$$

$$\therefore V_A = + 10.31 \text{ kN} \quad \uparrow$$

Consider Member BC:



$$+ve \curvearrowright \Sigma M_B = 0$$

$$- 29.8 + (45.0 \times 2.0) + 18.8 - (V_C \times 4.0) = 0$$

$$\therefore V_C = + 19.75 \text{ kN} \quad \uparrow$$

For the complete frame:

$$+ve \uparrow \Sigma F_y = 0$$

$$10.31 - 30.0 - 45.0 + 19.75 + V_D = 0$$

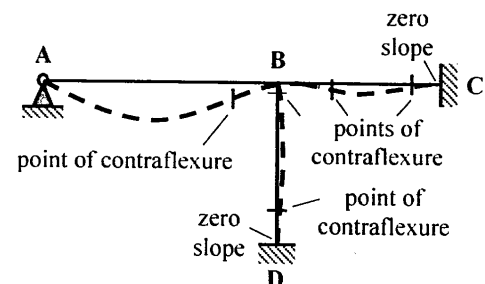
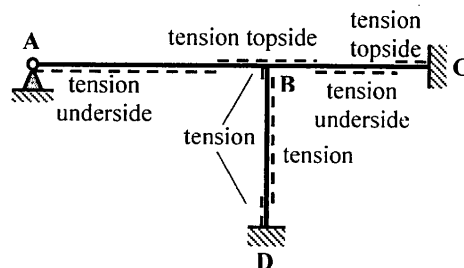
$$\therefore V_D = + 44.94 \text{ kN} \quad \uparrow$$

**There is insufficient information from the moment distribution analysis to determine the values of  $H_A$  and  $H_C$  separately; i.e.**

$$+ve \rightarrow \Sigma F_x = 0$$

$$(10.0 \times 3.5) + H_A + H_D + H_C = 0$$

$$\therefore H_A + H_C = (35.0 - 21.18) = 13.82 \text{ kN}$$

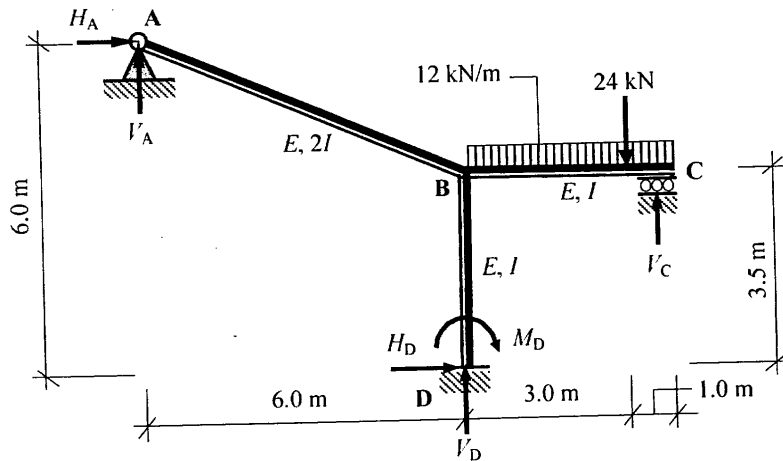


**Deflected Shape**

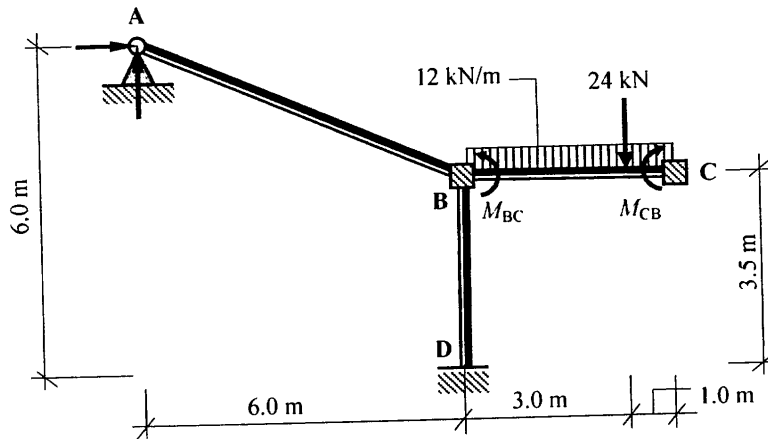
## Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames  
 Problem Number: 5.8

Page No. 1



Fixed-end Moments:



Member BC\*

$$M_{BC} = -\frac{Pab^2}{L^2} - \frac{wL^2}{12} = -\frac{24.0 \times 3 \times 1^2}{4^2} - \frac{12.0 \times 4^2}{12} = -20.5 \text{ kNm}$$

$$M_{CB} = +\frac{Pa^2b}{L^2} + \frac{wL^2}{12} = +\frac{24.0 \times 3^2 \times 1}{4^2} + \frac{12.0 \times 4^2}{12} = +29.5 \text{ kNm}$$

\* Since support C is a roller, the fixed-end moments are  $(M_{BC} - 0.5M_{CB})$  at B and zero at C.

$$(M_{BC} - 0.5M_{CB}) = [-20.5 - (0.5 \times 29.5)] = -35.25 \text{ kNm.}$$

### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.8

Page No. 2

Length of member AB =  $\sqrt{(6.0^2 + 2.5^2)} = 6.5 \text{ m}$

**Distribution Factors : Joint B**

$$\left. \begin{aligned}
 k_{BA} &= \frac{3}{4} \times \left( \frac{2I}{6.5} \right) = 0.23I \\
 k_{BC} &= \frac{3}{4} \times \left( \frac{I}{4.0} \right) = 0.19I \\
 k_{BD} &= \left( \frac{I}{3.5} \right) = 0.29I
 \end{aligned} \right\} k_{\text{total}} = 0.71I$$

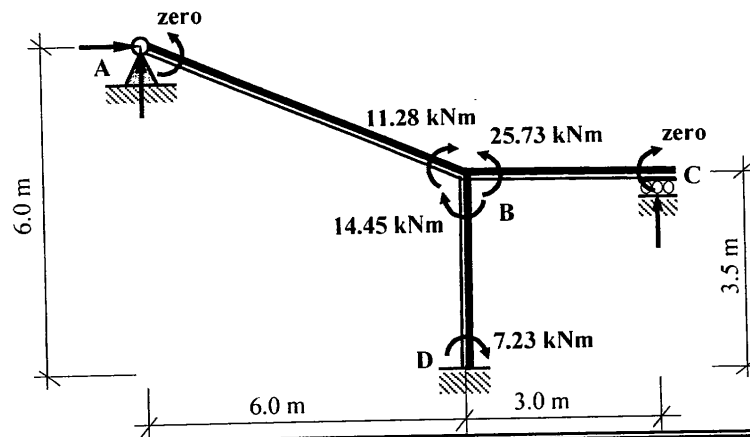
$$\begin{aligned}
 DF_{BA} &= \frac{k_{BA}}{k_{\text{Total}}} = \frac{0.23}{0.71} = 0.32 \\
 DF_{BC} &= \frac{k_{BC}}{k_{\text{Total}}} = \frac{0.19}{0.71} = 0.27 \\
 DF_{BD} &= \frac{k_{BD}}{k_{\text{Total}}} = \frac{0.29}{0.71} = 0.41
 \end{aligned}$$

**Moment Distribution Table:**

Joint	A	B			C	D
	AB	BA	BD	BC	CB	DB
Distribution Factors	1.0	0.32	0.41	0.27	1.0	0
Fixed-end Moments				-35.25		
Balance		+11.28	+14.45	+9.52		
Carry-over						+7.23
Total	0	+11.28	+14.45	-25.73	0	+7.23

Note: the sum of the moments at joint B = zero

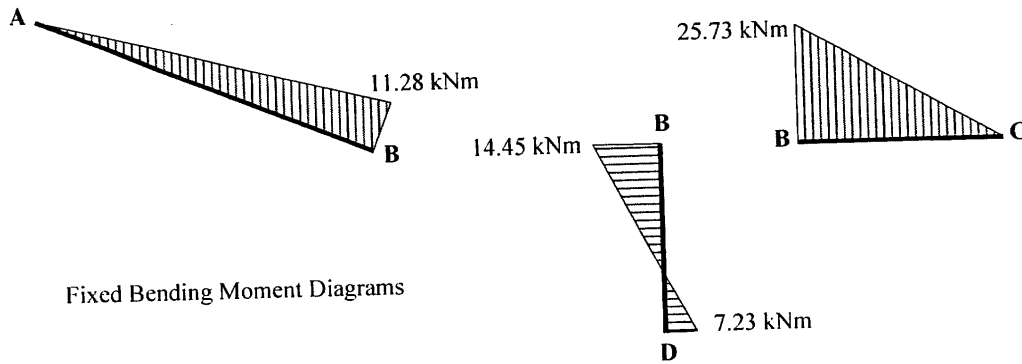
**Continuity Moments:**



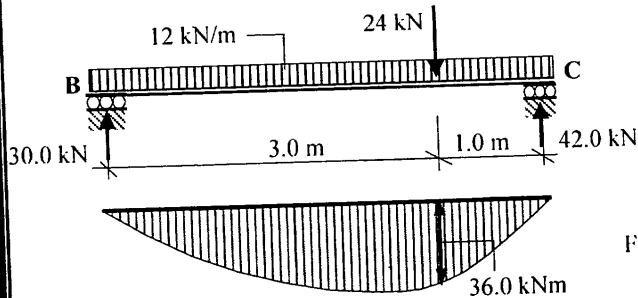
### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames  
 Problem Number: 5.8

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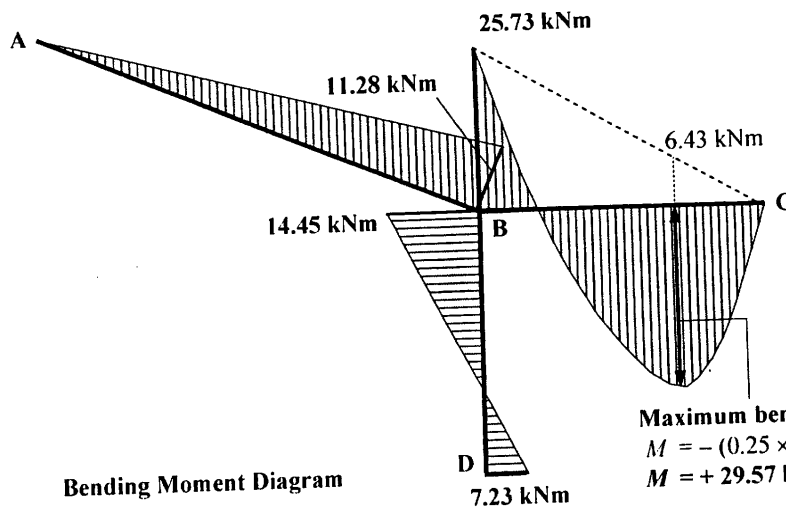


Free bending moments:



Note:  
 In this problem, the point of zero shear in member BC occurs under the point load.

Member BC:  $M_{free} = + [(42.0 \times 1.0) - (12.0 \times 1.0 \times 0.5)] = + 36.0 \text{ kNm}$



Maximum bending moment:  
 $M = - (0.25 \times 25.73) + 36.0$   
 $M = + 29.57 \text{ kNm}$

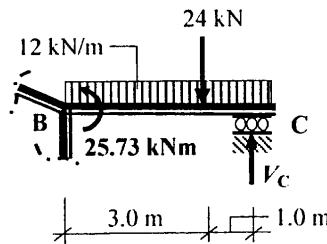
**Solution**

**Topic: Moment Distribution – No-Sway Rigid-Jointed Frames**

**Problem Number: 5.8**

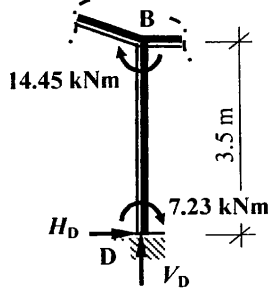
**Page No. 4**

Consider Member BC:



$$\begin{aligned}
 &+ve \curvearrowright \Sigma M_B = 0 \\
 &- 25.73 + (12.0 \times 4.0 \times 2.0) + (24.0 \times 3.0) - (V_C \times 4.0) = 0 \quad \therefore V_C = + 35.57 \text{ kN} \quad \uparrow
 \end{aligned}$$

Consider Member BD:



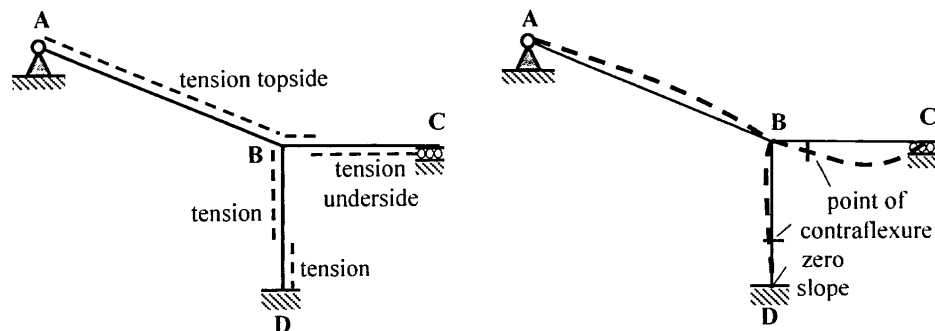
$$\begin{aligned}
 &+ve \curvearrowright \Sigma M_B = 0 \\
 &+ 14.45 + 7.23 - (H_D \times 3.5) = 0 \quad \therefore H_D = + 6.19 \text{ kN} \quad \rightarrow
 \end{aligned}$$

For the complete frame:

$$\begin{aligned}
 &+ve \rightarrow \Sigma F_x = 0 \\
 &+ H_A + H_D = 0 \quad \therefore H_A = - 6.19 \text{ kN} \quad \leftarrow
 \end{aligned}$$

$$\begin{aligned}
 &+ve \curvearrowright \Sigma M_A = 0 \\
 &+ 7.23 + (12.0 \times 4.0 \times 8.0) + (24.0 \times 9.0) - (35.57 \times 10.0) - (6.19 \times 6.0) - (V_D \times 6.0) = 0 \\
 &\therefore V_D = + 35.73 \text{ kN} \quad \uparrow
 \end{aligned}$$

$$\begin{aligned}
 &+ve \uparrow \Sigma F_y = 0 \\
 &35.73 - (12.0 \times 4.0) - 24.0 + 35.57 + V_A = 0 \quad \therefore V_A = + 0.7 \text{ kN} \quad \uparrow
 \end{aligned}$$



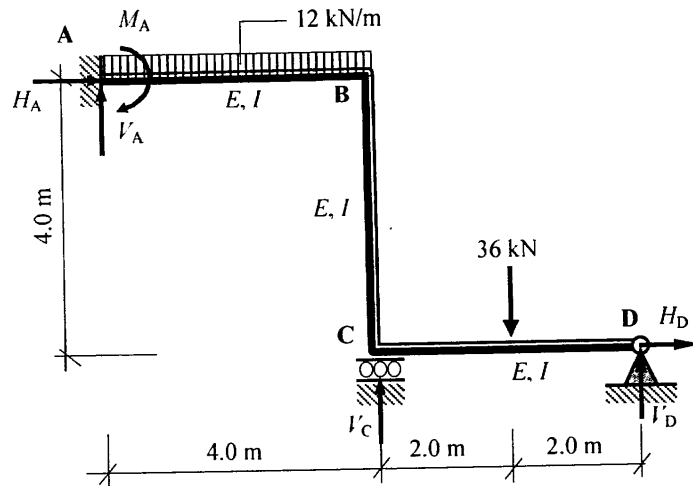
**Deflected Shape**

### Solution

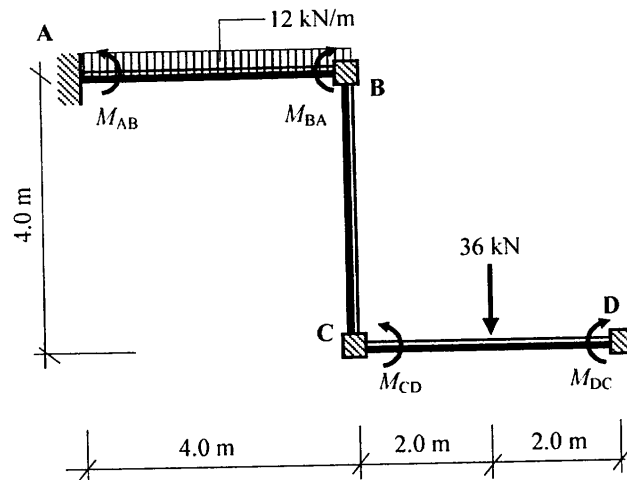
Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.9

Page No. 1



Fixed-end Moments:



Member AB

$$M_{AB} = -\frac{wL^2}{12} = -\frac{12.0 \times 4^2}{12} = -16.0 \text{ kNm}$$

$$M_{BA} = +\frac{wL^2}{12} = +\frac{12.0 \times 4^2}{12} = +16.0 \text{ kNm}$$



**Solution****Topic: Moment Distribution – No-Sway Rigid-Jointed Frames****Problem Number: 5.9****Page No. 2**

Member CD\*

$$M_{CD} = -\frac{PL}{8} = -\frac{36.0 \times 4}{8} = -18.0 \text{ kNm}$$

$$M_{DC} = +\frac{PL}{8} = +\frac{36.0 \times 4}{8} = +18.0 \text{ kNm}$$

\* Since support D is pinned, the fixed-end moments are  $(M_{CD} - 0.5M_{DC})$  at C and zero at D.

$$(M_{CD} - 0.5M_{DC}) = [-18.0 - (0.5 \times 18.0)] = -27.0 \text{ kNm.}$$

**Distribution Factors : Joint B**

$$k_{BA} = \left(\frac{I}{4.0}\right) = 0.25I$$

$$k_{\text{total}} = 0.51I$$

$$DF_{BA} = \frac{k_{BA}}{k_{\text{Total}}} = \frac{0.25}{0.5} = 0.5$$

$$k_{BC} = \left(\frac{I}{4.0}\right) = 0.25I$$

$$DF_{BC} = \frac{k_{BC}}{k_{\text{Total}}} = \frac{0.25}{0.5} = 0.5$$

**Distribution Factors : Joint C**

$$k_{CB} = \left(\frac{I}{4.0}\right) = 0.25I$$

$$k_{\text{total}} = 0.44I$$

$$DF_{CB} = \frac{k_{CB}}{k_{\text{Total}}} = \frac{0.25}{0.44} = 0.57$$

$$k_{CD} = \frac{3}{4} \times \left(\frac{I}{4.0}\right) = 0.19I$$

$$DF_{CD} = \frac{k_{CD}}{k_{\text{Total}}} = \frac{0.19}{0.44} = 0.43$$

**Moment Distribution Table:**

Joint	A	B		C		D
	AB	BA	BC	CB	CD	DC
<b>Distribution Factors</b>	<b>0</b>	<b>0.5</b>	<b>0.5</b>	<b>0.57</b>	<b>0.43</b>	<b>1.0</b>
<b>Fixed-end Moments</b>	-16.0	+16.0			-27.0	
<b>Balance</b>		-8.0	-8.0	+15.39	+11.61	
<b>Carry-over</b>	-4.0		+7.7	-4.0		
<b>Balance</b>		-3.85	-3.85	+2.28	+1.72	
<b>Carry-over</b>	-1.79		+1.14	-1.93		
<b>Balance</b>		-0.57	-0.57	+1.1	+0.83	
<b>Carry-over</b>	-0.29		+0.55	-0.29		
<b>Balance</b>		-0.27	-0.27	+0.17	+0.12	
<b>Carry-over</b>	-0.13					
<b>Total</b>	-22.35	+3.31	-3.31	+12.72	-12.72	

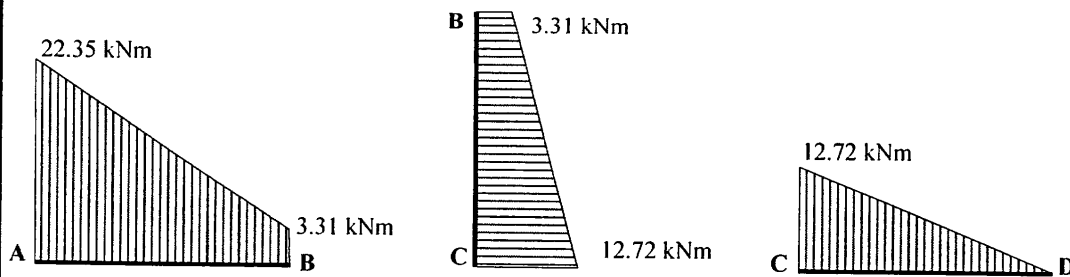
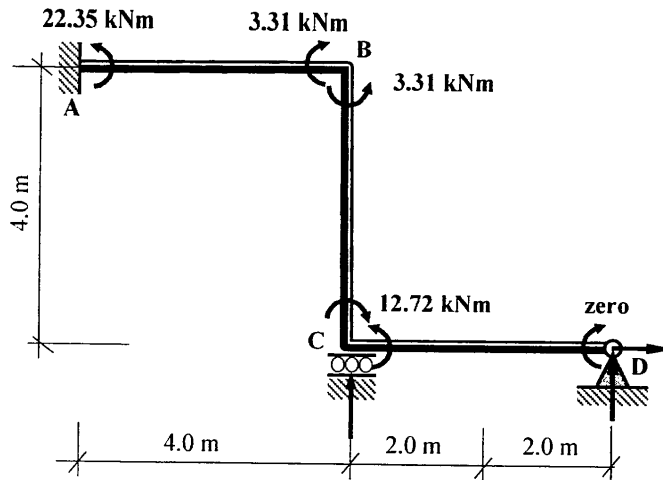
### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.9

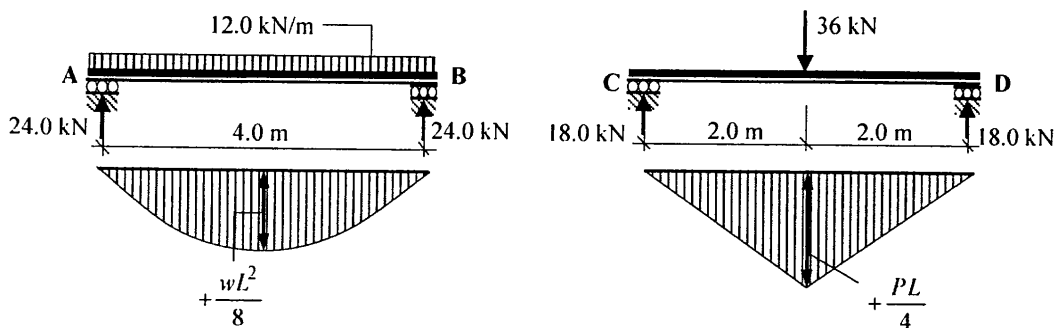
Page No. 3

#### Continuity Moments:



Fixed Bending Moment Diagrams

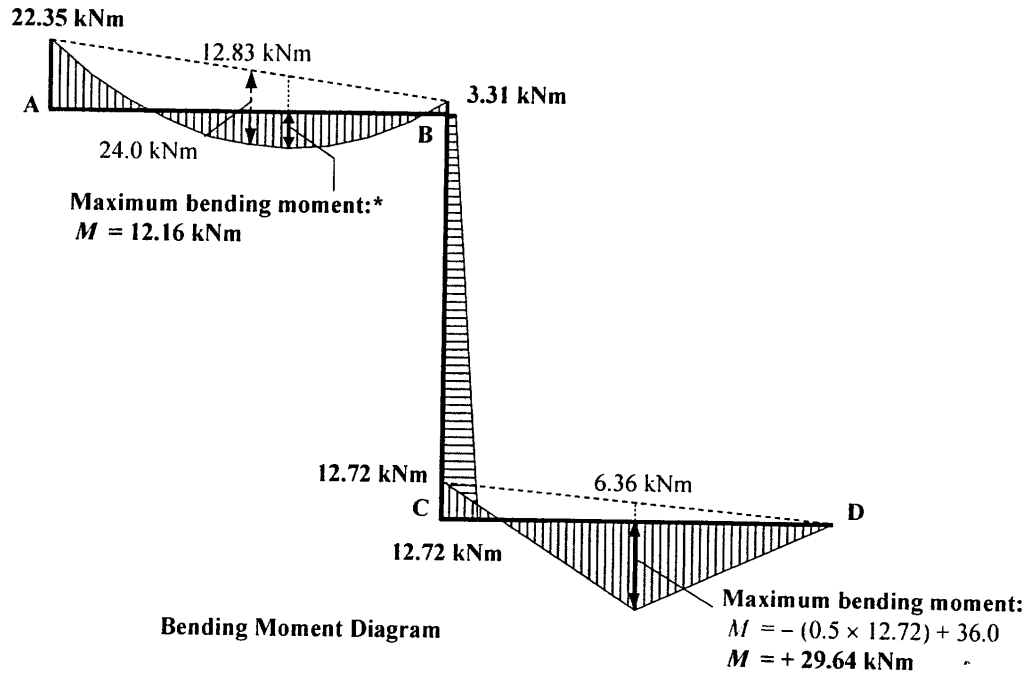
#### Free bending moments:



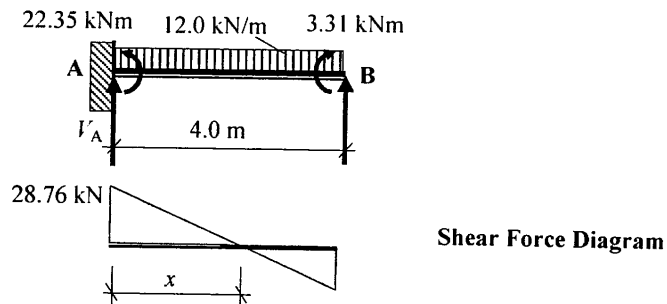
Free Bending Moment Diagrams

Member AB:  $M_{\text{free}} = (12.0 \times 4^2)/8 = 24.0 \text{ kNm}$

Member CD:  $M_{\text{free}} = (36.0 \times 4)/4 = 36.0 \text{ kNm}$

**Solution****Topic: Moment Distribution – No-Sway Rigid-Jointed Frames****Problem Number: 5.9****Page No. 4**

- The maximum value along the length of member AB can be found by identifying the point of zero shear as follows:



$$+ve \curvearrowright \Sigma M_B = 0$$

$$-22.35 - (12.0 \times 4.0 \times 2.0) + 3.31 + (V_A \times 4.0) = 0 \quad \therefore V_A = +28.76 \text{ kN} \uparrow$$

$$x = (28.76/12.0) = 2.4 \text{ m}$$

$$M_{\text{maximum}} = (0.5 \times 2.4 \times 28.76) - 22.35 = 12.16 \text{ kNm}$$

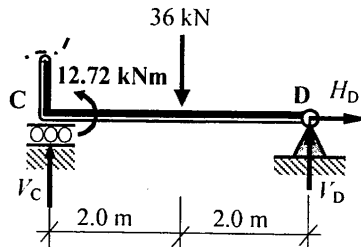
### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.9

Page No. 4

Consider Member CD:



$$\begin{aligned}
 &+ve \curvearrowright \Sigma M_C = 0 \\
 &- 12.72 + (36.0 \times 2.0) - (V_D \times 4.0) = 0 \qquad \therefore V_D = + 14.82 \text{ kN} \uparrow
 \end{aligned}$$

For the complete frame:

$$\begin{aligned}
 &+ve \uparrow \Sigma F_y = 0 \\
 &28.76 - (12.0 \times 4.0) - 36.0 + 14.82 + V_C = 0 \qquad \therefore V_C = + 40.42 \text{ kN} \uparrow
 \end{aligned}$$

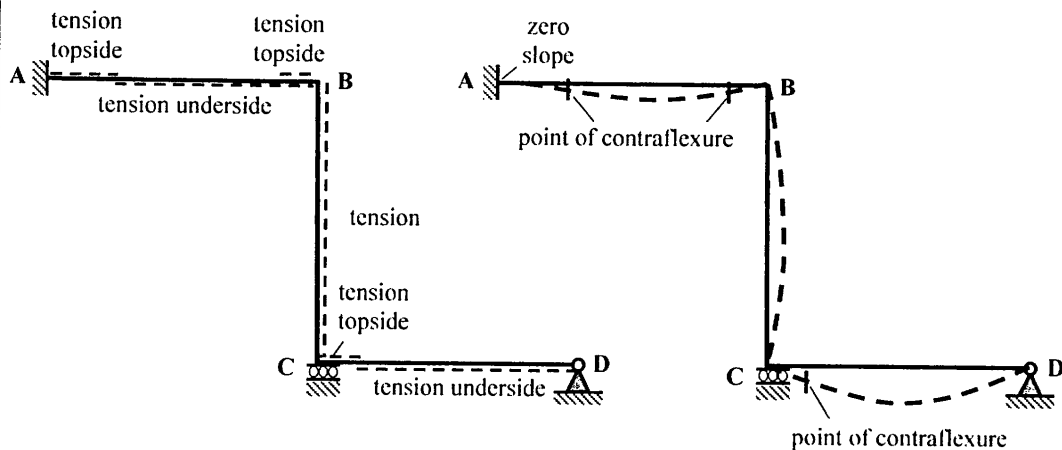
$$\begin{aligned}
 &+ve \curvearrowright \Sigma M_A = 0 \\
 &- 22.35 + (12.0 \times 4.0 \times 2.0) + (36.0 \times 6.0) - (40.42 \times 4.0) - (14.82 \times 8.0) - (H_D \times 4.0) = 0
 \end{aligned}$$

$$\therefore H_D = + 2.35 \text{ kN} \rightarrow$$

$$+ve \rightarrow \Sigma F_x = 0$$

$$+ H_A + H_D = 0$$

$$\therefore H_A = - 2.35 \text{ kN} \leftarrow$$



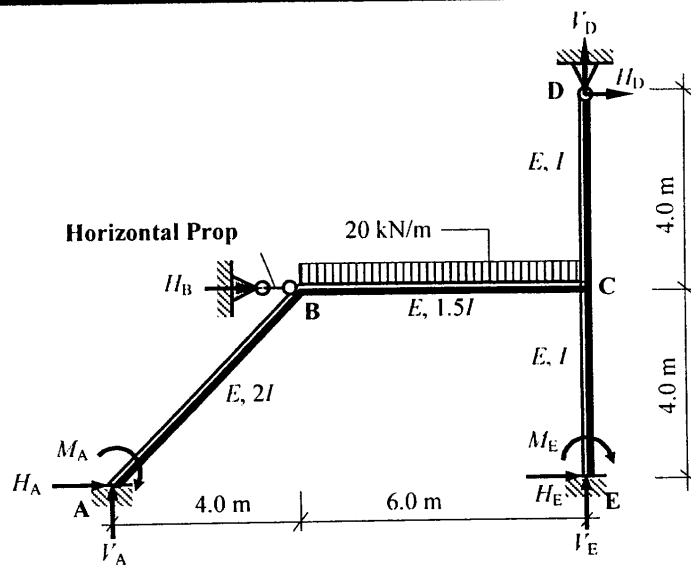
Deflected Shape

## Solution

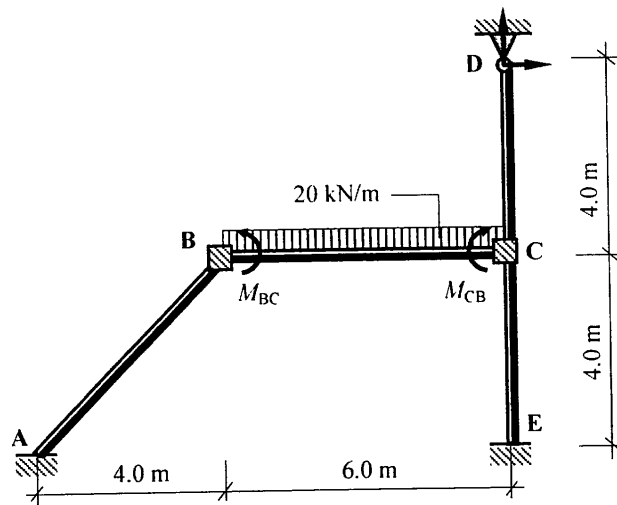
Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.10

Page No. 1



Fixed-end Moments:



Member BC

$$M_{BC} = -\frac{wL^2}{12} = -\frac{20.0 \times 6^2}{12} = -60.0 \text{ kNm}$$

$$M_{CB} = +\frac{wL^2}{12} = +\frac{20.0 \times 6^2}{12} = +60.0 \text{ kNm}$$

**Solution****Topic: Moment Distribution – No-Sway Rigid-Jointed Frames****Problem Number: 5.10****Page No. 2**

$$\text{Length of member AB} = \sqrt{(4.0^2 + 4.0^2)} = 5.657 \text{ m}$$

**Distribution Factors : Joint B**

$$k_{BA} = \left( \frac{2.0I}{5.657} \right) = 0.35I$$

$$DF_{BA} = \frac{k_{BA}}{k_{\text{Total}}} = \frac{0.35}{0.6} = 0.58$$

$$k_{\text{total}} = 0.6I$$

$$k_{BC} = \left( \frac{1.5I}{6.0} \right) = 0.25I$$

$$DF_{BC} = \frac{k_{BC}}{k_{\text{Total}}} = \frac{0.25}{0.6} = 0.42$$

**Distribution Factors : Joint C**

$$k_{CB} = \left( \frac{1.5I}{6.0} \right) = 0.25I$$

$$DF_{CB} = \frac{k_{CB}}{k_{\text{Total}}} = \frac{0.25}{0.69} = 0.36$$

$$k_{CD} = \frac{3}{4} \times \left( \frac{I}{4.0} \right) = 0.19I$$

$$k_{\text{total}} = 0.69I$$

$$DF_{CD} = \frac{k_{CD}}{k_{\text{Total}}} = \frac{0.19}{0.69} = 0.28$$

$$k_{CE} = \left( \frac{I}{4.0} \right) = 0.25I$$

$$DF_{CE} = \frac{k_{CE}}{k_{\text{Total}}} = \frac{0.25}{0.69} = 0.36$$

**Moment Distribution Table:**

Joint	A		B		C			E	D
	AB		BA	BC	CB	CD	CE	EC	DC
<b>Distribution Factors</b>	0		0.58	0.42	0.36	0.28	0.36	0	1.0
<b>Fixed-end Moments</b>				- 60.0	+ 60.0				
<b>Balance</b>			+ 34.8	+ 25.2	- 21.6	- 16.8	- 21.6		
<b>Carry-over</b>	+ 17.4			- 10.8	+ 12.6			- 10.8	
<b>Balance</b>			+ 6.26	+ 4.54	- 4.54	- 3.52	- 4.54		
<b>Carry-over</b>	+ 3.13			- 2.27	+ 2.27			- 2.27	
<b>Balance</b>			+ 1.32	+ 0.95	- 0.82	- 0.63	- 0.82		
<b>Carry-over</b>	+ 0.66			- 0.41	+ 0.48			- 0.41	
<b>Balance</b>			+ 0.24	+ 0.17	- 0.17	- 0.14	- 0.17		
<b>Carry-over</b>	+ 0.12							- 0.09	
<b>Total</b>	+ 21.3		+ 42.6	- 42.6	+ 48.2	- 21.1	- 27.1	- 13.6	0

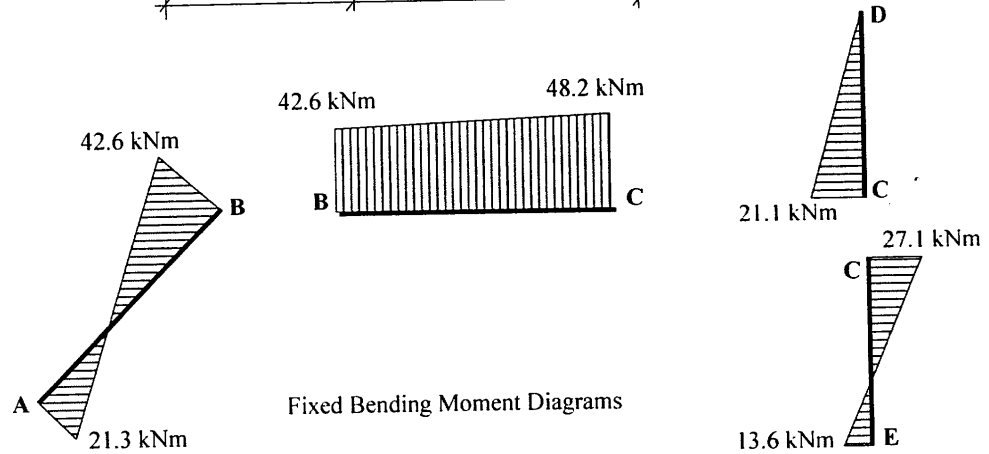
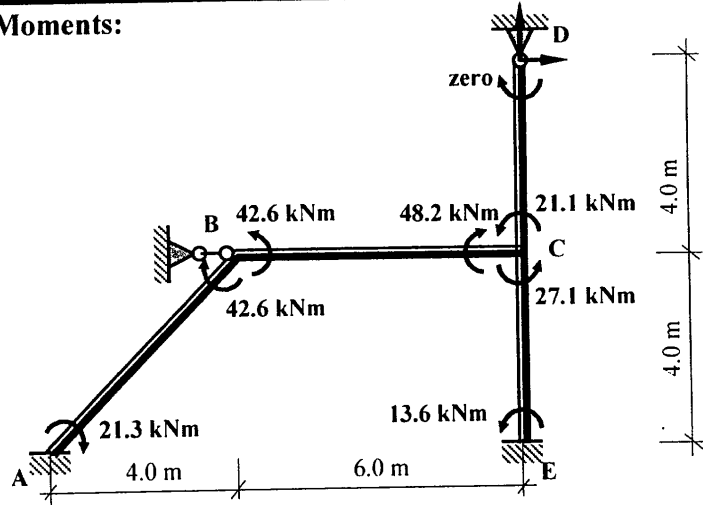
### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

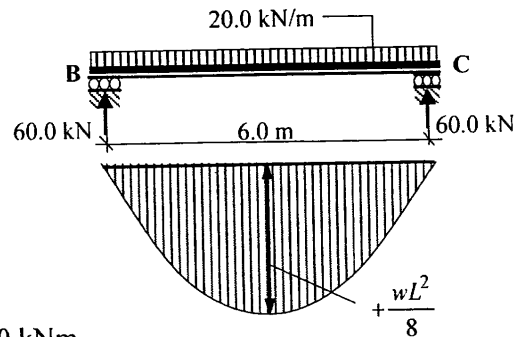
Problem Number: 5.10

Page No. 3

Continuity Moments:



Free bending moments:



Member BC:  $M_{\text{free}} = (20.0 \times 6^2)/8 = 90.0 \text{ kNm}$

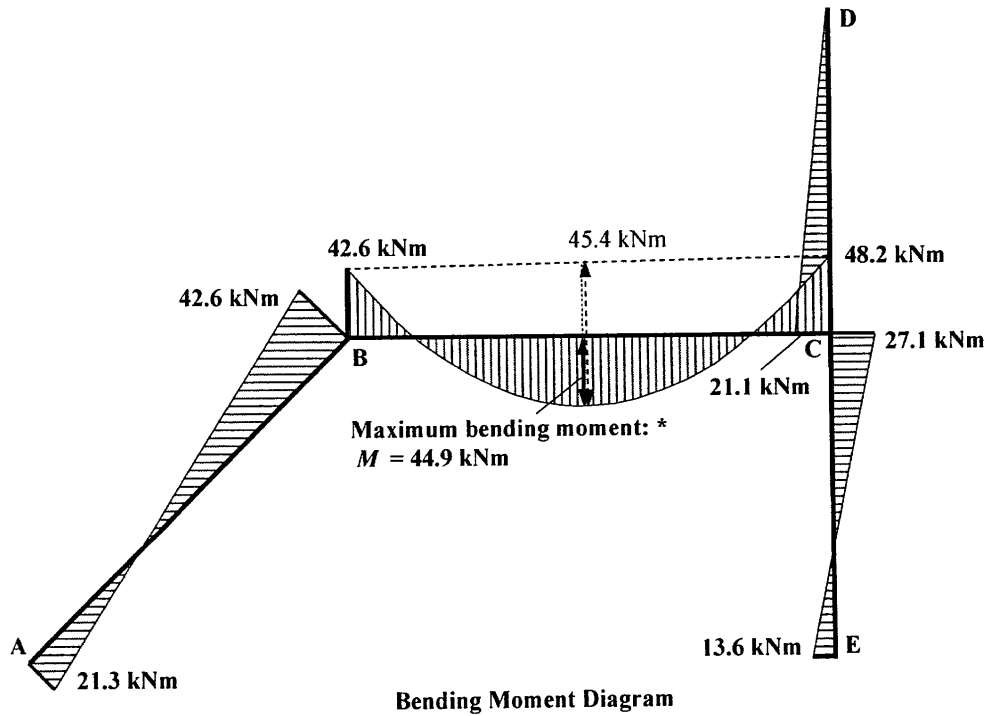
Free Bending Moment Diagram

## Solution

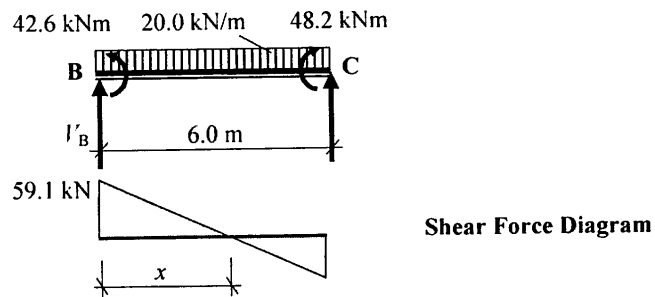
Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.10

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\* The maximum value along the length of member BC can be found by identifying the point of zero shear as follows:



$$+ve \curvearrowright \Sigma M_C = 0$$

$$-42.6 - (20.0 \times 6.0 \times 3.0) + 48.2 + (V_B \times 6.0) = 0$$

$$\therefore V_B = +59.1 \text{ kN} \uparrow$$

$$x = (59.1/20.0) = 2.96 \text{ m}$$

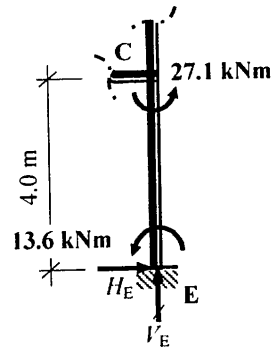
$$M_{\text{maximum}} = (0.5 \times 2.96 \times 59.1) - 42.6 = 44.9 \text{ kNm}$$



## Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames  
 Problem Number: 5.10

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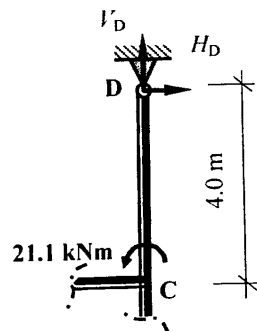


Consider Member CE:

$$+ve \curvearrowright \Sigma M_C = 0$$

$$- 27.1 - 13.6 - (H_E \times 4.0) = 0$$

$$\therefore H_E = -10.18 \text{ kN} \leftarrow$$

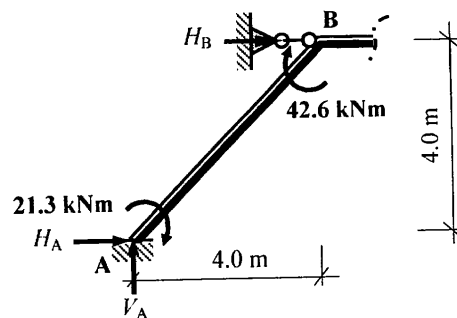


Consider Member CD:

$$+ve \curvearrowright \Sigma M_C = 0$$

$$- 21.1 + (H_D \times 4.0) = 0$$

$$\therefore H_D = +5.28 \text{ kN} \rightarrow$$



Consider Member AB:

$$+ve \curvearrowright \Sigma M_B = 0$$

$$+ 42.6 + 21.3 - (H_A \times 4.0) + (V_A \times 4.0) = 0$$

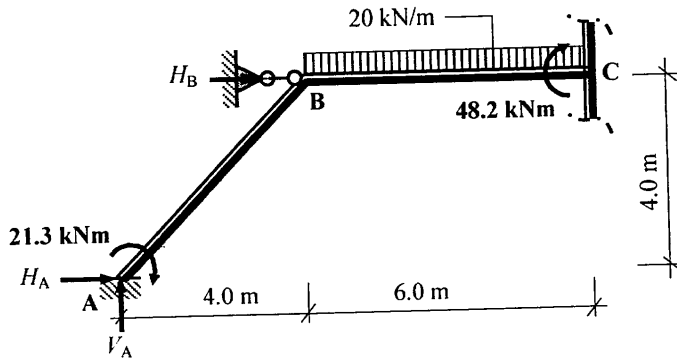
$$\therefore H_A = V_A + 15.98$$

### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.10

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Consider a section at C:

$$+ve \curvearrowright \Sigma M_C = 0$$

$$+ 48.2 - (20.0 \times 6.0 \times 3.0) + 21.3 - (H_A \times 4.0) + (V_A \times 10.0) = 0$$

$$\therefore H_A = 2.5V_A - 72.63$$

$$\therefore V_A + 15.98 = 2.5V_A - 72.63$$

$$\therefore V_A = 59.1 \text{ kN} \quad \uparrow$$

$$\therefore H_A = 75.1 \text{ kN} \quad \rightarrow$$

For the complete frame:

$$+ve \rightarrow \Sigma F_x = 0$$

$$+ 75.1 + 5.28 - 10.18 + H_B = 0$$

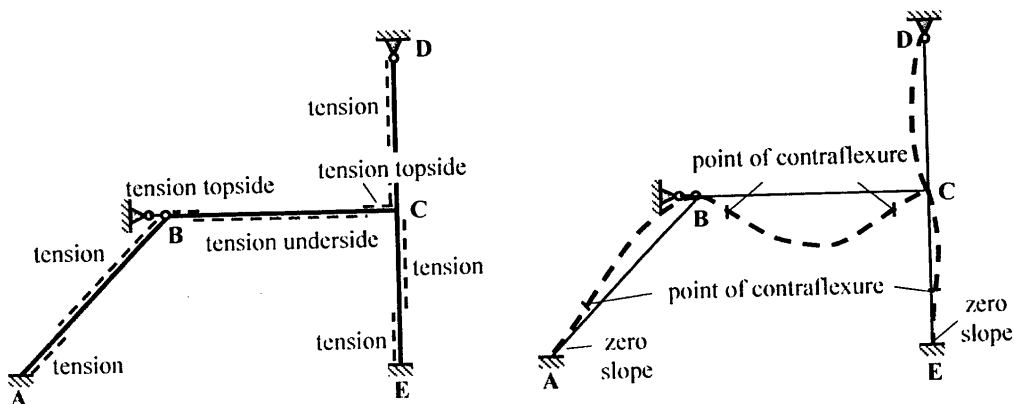
$$\therefore H_B = + 70.2 \text{ kN} \quad \leftarrow$$

There is insufficient information from the moment distribution analysis to determine the values of  $V_D$  and  $V_E$  separately; i.e.

$$+ve \uparrow \Sigma F_y = 0$$

$$-(20.0 \times 6.0) + 59.1 + V_D + V_E = 0$$

$$\therefore V_D + V_E = + (120.0 - 59.1) = + 60.9 \text{ kN}$$



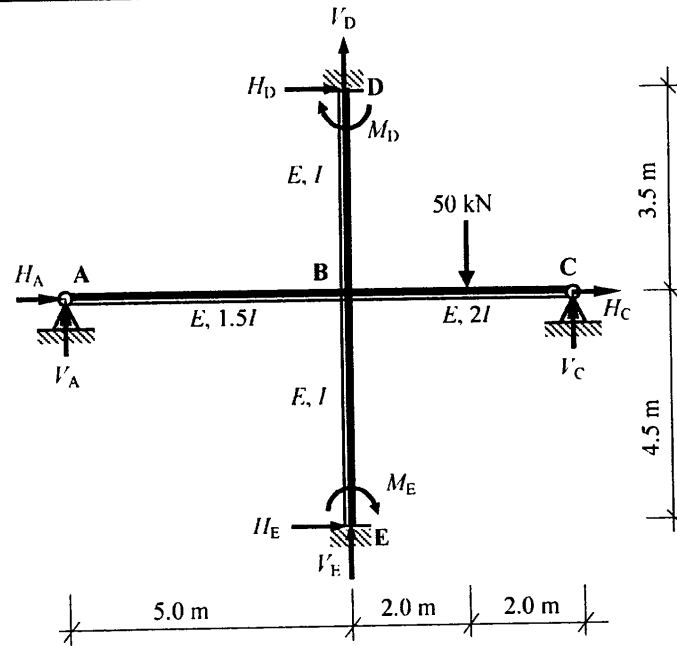
Deflected Shape

### Solution

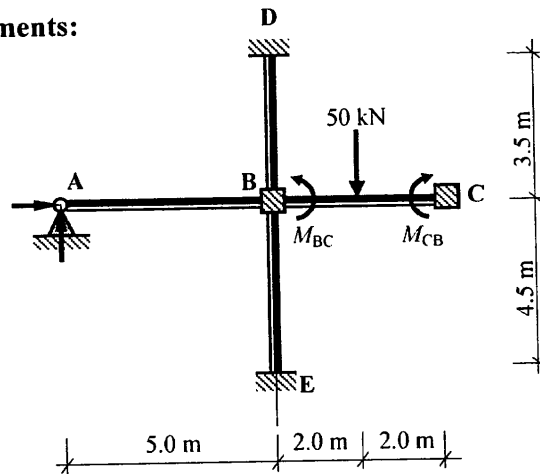
Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.11

Page No. 1



Fixed-end Moments:



Member BC \*

$$M_{BC} = -\frac{PL}{8} = -\frac{50.0 \times 4}{8} = -25.0 \text{ kNm}$$

$$M_{CB} = +\frac{PL}{8} = +\frac{50.0 \times 4}{8} = +25.0 \text{ kNm}$$

\* Since support C is pinned, the fixed-end moments are  $(M_{BC} - 0.5M_{CB})$  at B and zero at C.

$$(M_{BC} - 0.5M_{CB}) = [-25.0 - (0.5 \times 25.0)] = -37.5 \text{ kNm.}$$

### Solution

Topic: Moment Distribution – No-Sway Rigid-Jointed Frames  
 Problem Number: 5.11

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#### Distribution Factors : Joint B

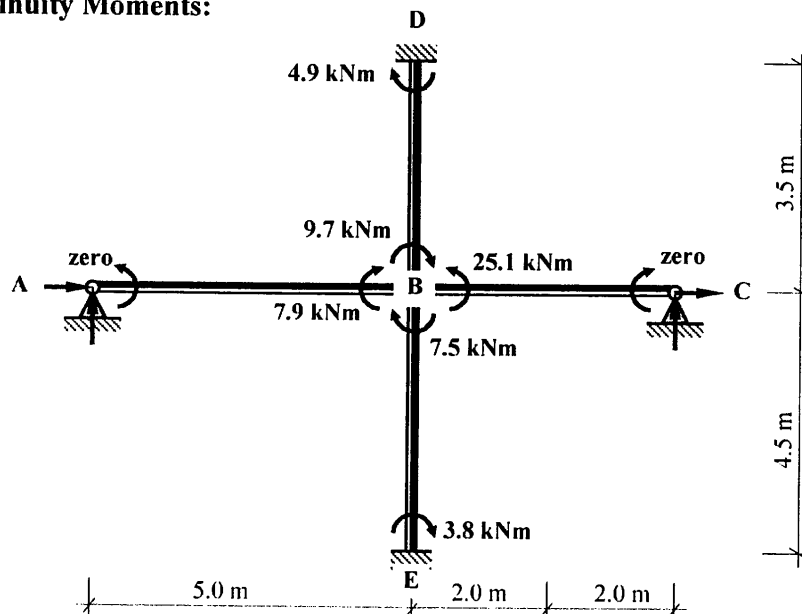
$$\left. \begin{aligned}
 k_{BA} &= \frac{3}{4} \times \left( \frac{1.5I}{5.0} \right) = 0.23I \\
 k_{BC} &= \frac{3}{4} \times \left( \frac{2I}{4.0} \right) = 0.38I \\
 k_{BD} &= \left( \frac{I}{3.5} \right) = 0.29I \\
 k_{BE} &= \left( \frac{I}{4.5} \right) = 0.22I
 \end{aligned} \right\} k_{\text{total}} = 1.12I$$

$$\begin{aligned}
 DF_{BA} &= \frac{k_{BA}}{k_{\text{Total}}} = \frac{0.23}{1.12} = 0.21 \\
 DF_{BC} &= \frac{k_{BC}}{k_{\text{Total}}} = \frac{0.38}{1.12} = 0.33 \\
 DF_{BD} &= \frac{k_{BD}}{k_{\text{Total}}} = \frac{0.29}{1.12} = 0.26 \\
 DF_{BE} &= \frac{k_{BE}}{k_{\text{Total}}} = \frac{0.22}{1.12} = 0.20
 \end{aligned}$$

#### Moment Distribution Table:

Joint	A	D		B			E	C	
	AB	DB		BD	BA	BC	BE	EB	CB
<b>Distribution Factors</b>	1.0	0		0.26	0.21	0.33	0.2	0	1.0
<b>Fixed-end Moments</b>						-37.5			
<b>Balance</b>				+9.7	+7.9	+12.4	+7.5		
<b>Carry-over</b>		+4.9						+3.8	
<b>Total</b>	0	+4.9		+9.7	+7.9	-25.1	+7.5	+3.8	0

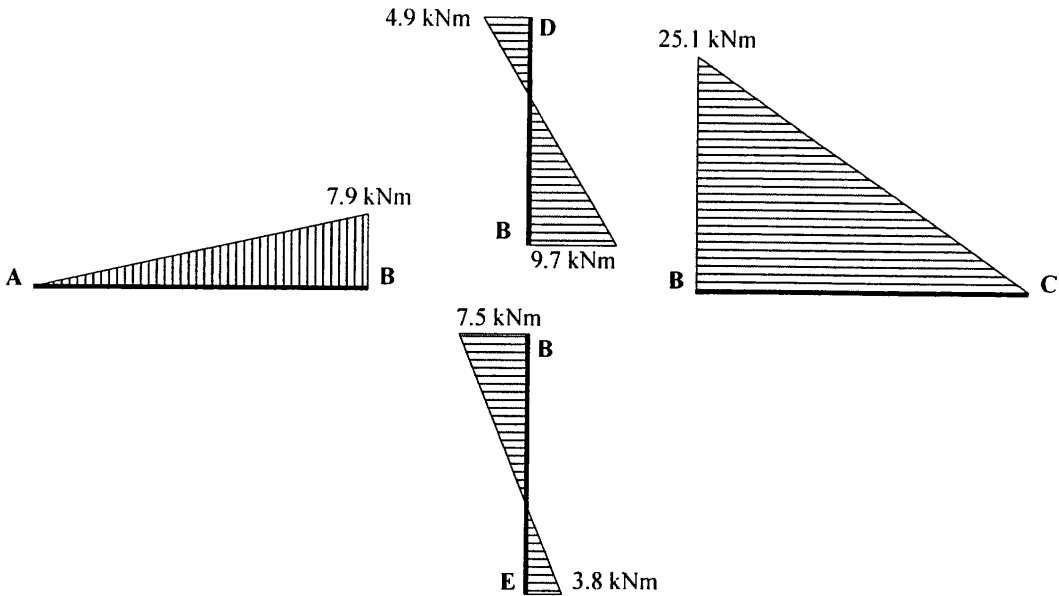
#### Continuity Moments:



**Solution**

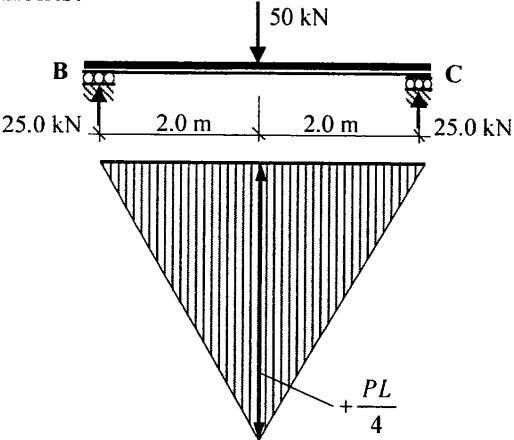
**Topic: Moment Distribution – No-Sway Rigid-Jointed Frames**  
**Problem Number: 5.11**

**Page No. 3**



Fixed Bending Moment Diagrams

**Free bending moments:**



Free Bending Moment Diagram

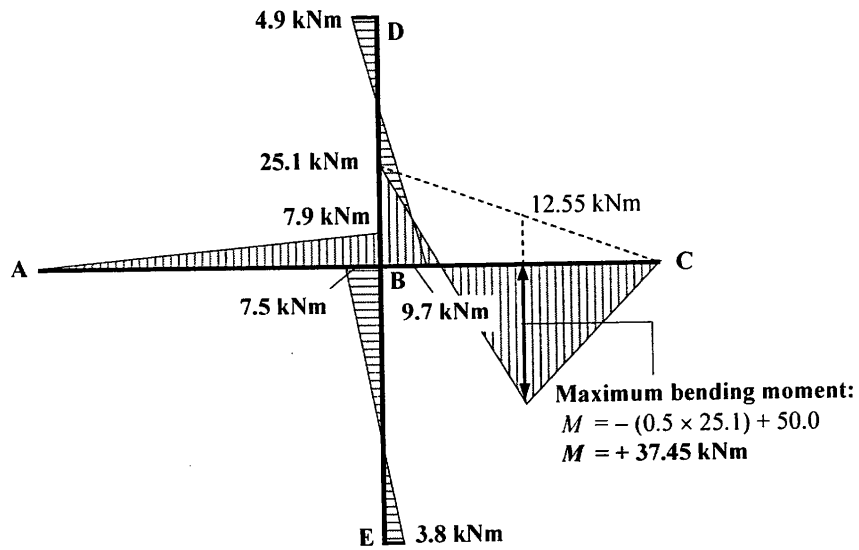
Member BC:  $M_{\text{free}} = (50.0 \times 4)/4 = 50.0 \text{ kNm}$

### Solution

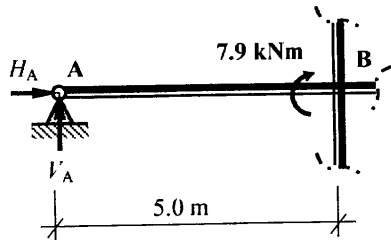
Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.11

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Bending Moment Diagram

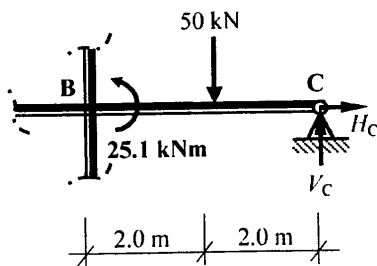


Consider Member AB:

$$+ve \curvearrowright \Sigma M_B = 0$$

$$+ 7.9 + (V_A \times 5.0) = 0$$

$$\therefore V_A = +1.58 \text{ kN} \quad \uparrow$$



Consider Member BC:

$$+ve \curvearrowright \Sigma M_B = 0$$

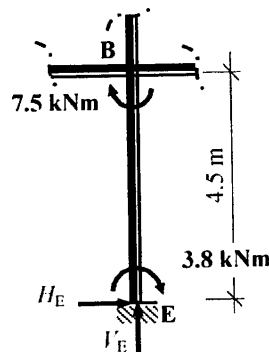
$$- 25.1 + (50.0 \times 2.0) - (V_C \times 4.0) = 0$$

$$\therefore V_C = +18.73 \text{ kN} \quad \uparrow$$

Consider Member BE:

$$+ve \curvearrowright \Sigma M_B = 0$$

$$+ 7.5 + 3.8 - (H_E \times 4.5) = 0$$



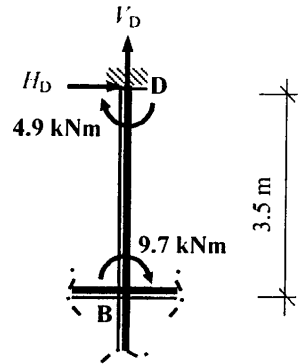
$$\therefore H_E = +2.51 \text{ kN} \quad \rightarrow$$

**Solution**

**Topic: Moment Distribution – No-Sway Rigid-Jointed Frames**

**Problem Number: 5.11**

**Page No. 4**



Consider Member BD:

$$+ve \curvearrowright \Sigma M_B = 0$$

$$+ 9.7 + 4.9 + (H_D \times 3.5) = 0$$

$$\therefore H_D = -4.17 \text{ kN} \leftarrow$$

There is insufficient information from the moment distribution analysis to determine the values of  $H_A$ ,  $H_C$ ,  $V_D$  and  $V_E$  separately; i.e.

$$+ve \rightarrow \Sigma F_x = 0$$

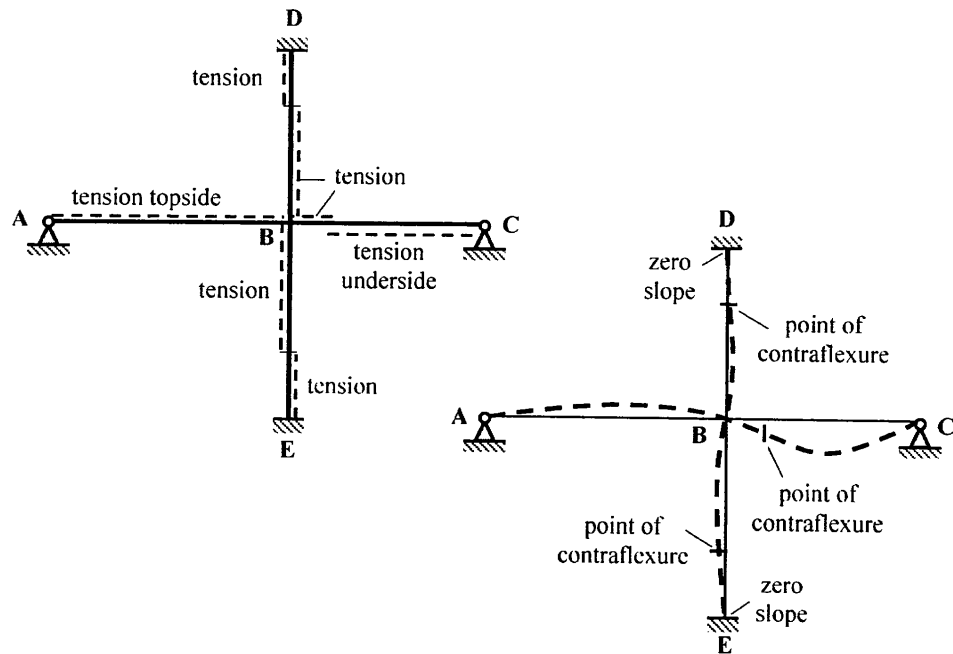
$$H_A + H_C - 4.17 + 2.51 = 0$$

$$\therefore H_A + H_C = +1.66 \text{ kN}$$

$$+ve \uparrow \Sigma F_y = 0$$

$$-50.0 + 1.58 + 18.73 + V_D + V_E = 0$$

$$\therefore V_D + V_E = +29.69 \text{ kN}$$



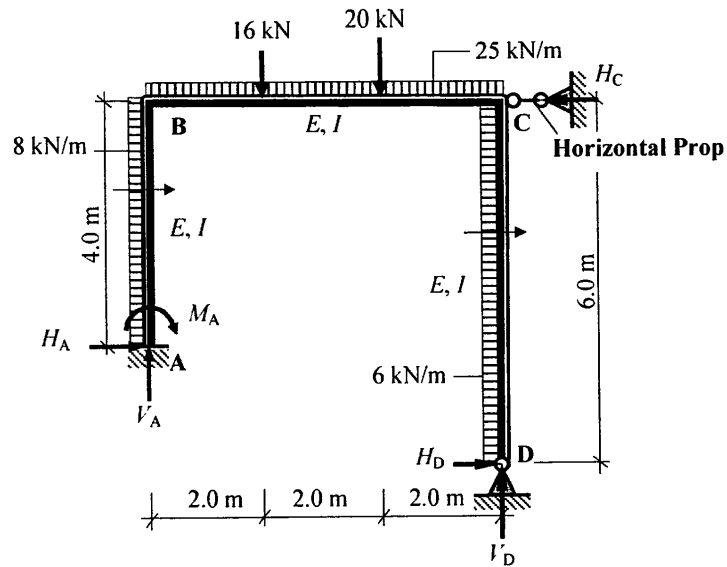
**Deflected Shape**

### Solution

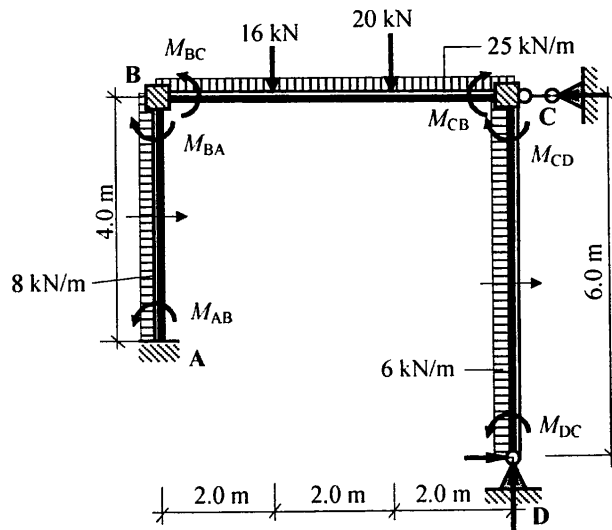
Topic: Moment Distribution – No-Sway Rigid-Jointed Frames

Problem Number: 5.12

Page No. 1



Fixed-end Moments:



Member AB

$$M_{AB} = -\frac{wL^2}{12} = -\frac{8.0 \times 4^2}{12} = -10.67 \text{ kNm}$$

$$M_{BA} = +\frac{wL^2}{12} = +\frac{8.0 \times 4^2}{12} = +10.67 \text{ kNm}$$



**Solution****Topic: Moment Distribution – No-Sway Rigid-Jointed Frames****Problem Number: 5.12****Page No. 2**

Member BC

$$M_{BC} = -\frac{wL^2}{12} - \frac{P_1ab^2}{L^2} - \frac{P_2ab^2}{L^2}$$

$$= -\left[\left(\frac{25.0 \times 6^2}{12}\right) + \left(\frac{16.0 \times 2.0 \times 4.0^2}{6^2}\right) + \left(\frac{20.0 \times 4.0 \times 2.0^2}{6^2}\right)\right] = -98.1 \text{ kNm}$$

$$M_{CB} = +\frac{wL^2}{12} + \frac{P_1a^2b}{L^2} + \frac{P_2a^2b}{L^2}$$

$$= +\left[\left(\frac{25.0 \times 6^2}{12}\right) + \left(\frac{16.0 \times 2.0^2 \times 4.0}{6^2}\right) + \left(\frac{20.0 \times 4.0^2 \times 2.0}{6^2}\right)\right] = +99.9 \text{ kNm}$$

Member CD \*

$$M_{CD} = +\frac{wL^2}{12} = +\frac{6.0 \times 6^2}{12} = +18.0 \text{ kNm}$$

$$M_{DC} = -\frac{wL^2}{12} = -\frac{6.0 \times 6^2}{12} = -18.0 \text{ kNm}$$

\* Since support D is pinned, the fixed-end moments are  $(M_{CD} - 0.5M_{DC})$  at C and zero at D.

$$(M_{CD} - 0.5M_{DC}) = [+18.0 + (0.5 \times 18.0)] = +27.0 \text{ kNm.}$$

**Distribution Factors : Joint B**

$$k_{BA} = \left(\frac{I}{4.0}\right) = 0.25I$$

$$k_{\text{total}} = 0.42I$$

$$DF_{BA} = \frac{k_{BA}}{k_{\text{Total}}} = \frac{0.25}{0.42} = 0.6$$

$$k_{BC} = \left(\frac{I}{6.0}\right) = 0.17I$$

$$DF_{BC} = \frac{k_{BC}}{k_{\text{Total}}} = \frac{0.17}{0.42} = 0.4$$

**Distribution Factors : Joint C**

$$k_{CB} = \left(\frac{I}{6.0}\right) = 0.17I$$

$$k_{\text{total}} = 0.3I$$

$$DF_{CB} = \frac{k_{CB}}{k_{\text{Total}}} = \frac{0.17}{0.3} = 0.57$$

$$k_{CD} = \frac{3}{4} \times \left(\frac{I}{6.0}\right) = 0.13I$$

$$DF_{CD} = \frac{k_{CD}}{k_{\text{Total}}} = \frac{0.13}{0.3} = 0.43$$

### Solution

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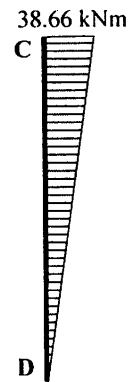
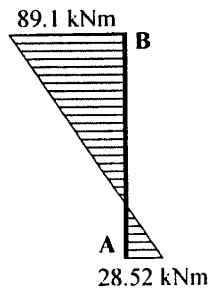
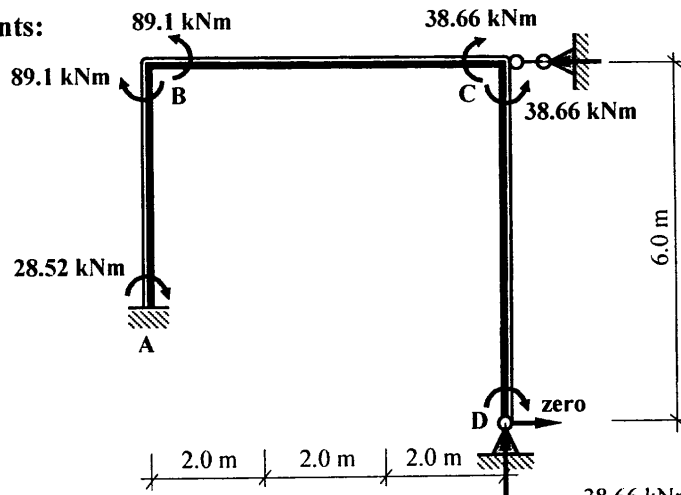
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Moment Distribution Table:

Joint	A		B		C		D
	AB	BA	BC	CB	CD	DC	
Distribution Factors	0	0.6	0.4	0.57	0.43		1.0
Fixed-end Moments	-10.67	+ 10.67	- 98.1	+ 99.9	+ 27.0		
Balance		+ 52.46	+ 34.97	- 72.3	- 54.6		
Carry-over	+ 26.23		- 36.2	+ 17.49			
Balance		+ 21.72	+ 14.48	- 9.97	- 7.52		
Carry-over	+ 10.86		- 4.99	+ 7.24			
Balance		+ 3.0	+ 1.99	- 4.13	- 3.11		
Carry-over	+ 1.5		- 2.07	+ 1.0			
Balance		+ 1.2	+ 0.87	- 0.57	- 0.43		
Carry-over	+ 0.6						
Total	+ 28.52	+ 89.1	- 89.1	+ 38.66	- 38.66		0

Continuity Moments:



Fixed Bending Moment Diagrams

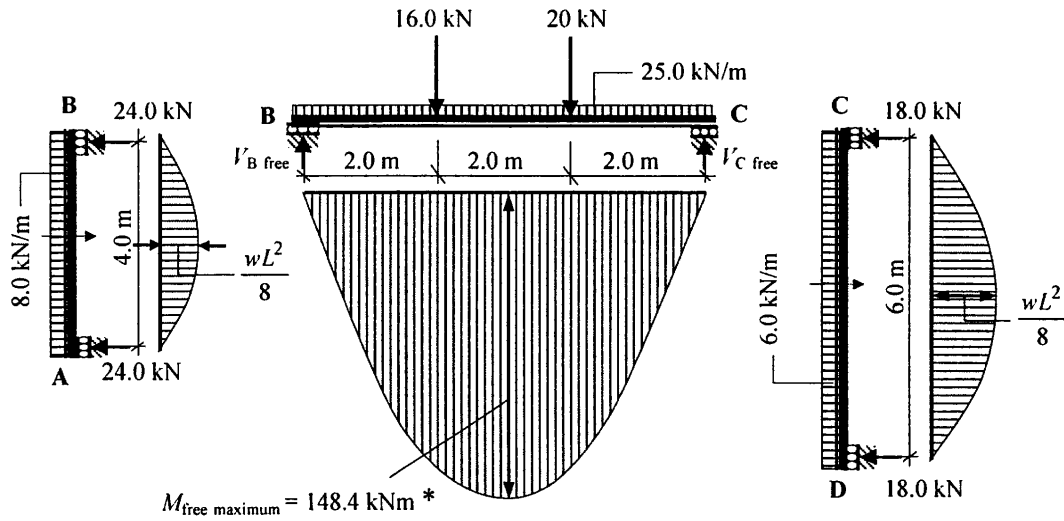
### Solution

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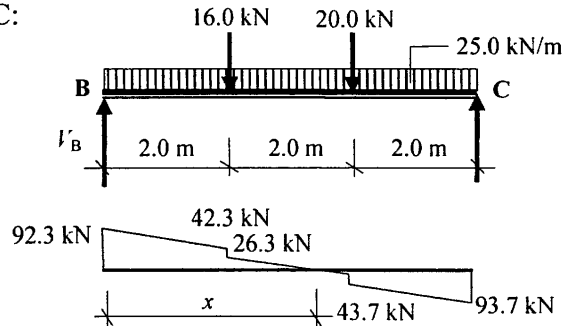
Free bending moments:



Free Bending Moment Diagrams

Member AB:  $M_{\text{free}} = (8.0 \times 4^2)/8 = 16.0 \text{ kNm}$

\* Member BC:



$$+\text{ve } \sum M_C = 0$$

$$-(16.0 \times 4.0) - (20.0 \times 2.0) - (25.0 \times 6.0 \times 3.0) + (V_B \times 6.0) = 0 \quad V_B = +92.3 \text{ kN}$$

Position of zero shear  $x = [2.0 + (26.3 / 25.0)] = 3.05 \text{ m}$

$$M_{\text{maximum free bending moment}} = [0.5 \times (92.3 + 42.3) \times 2.0] + (0.5 \times 1.05 \times 26.3) = 148.4 \text{ kNm}$$

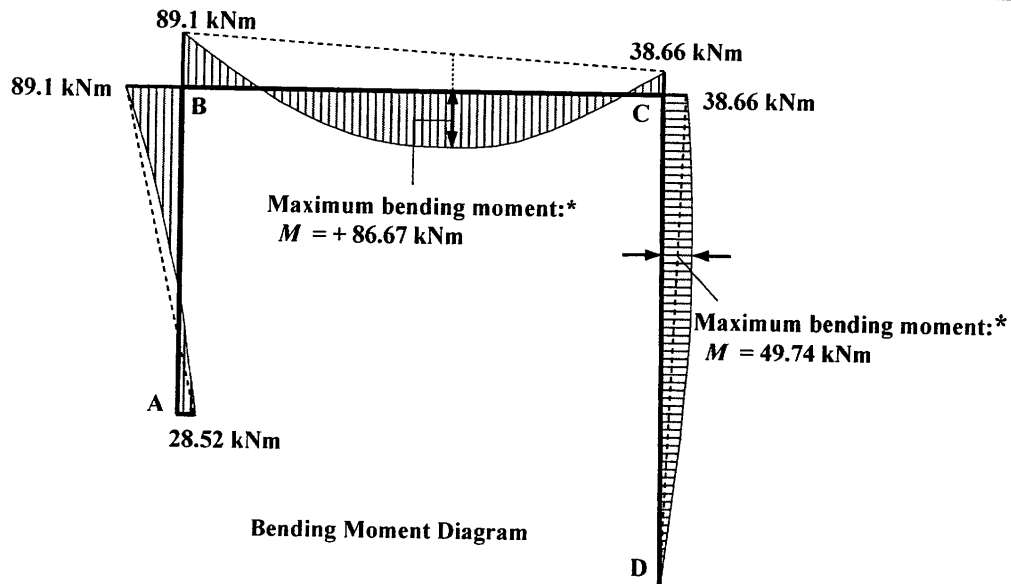
Member DC:  $M_{\text{free}} = (6.0 \times 6^2)/8 = 27.0 \text{ kNm}$

**Solution**

**Topic: Moment Distribution – No-Sway Rigid-Jointed Frames**

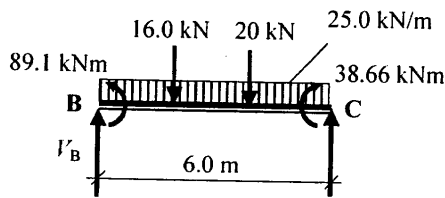
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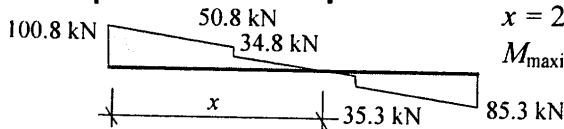


\* The maximum value along the length of members BC and DC can be found by identifying the point of zero shear as follows:

Member BC:



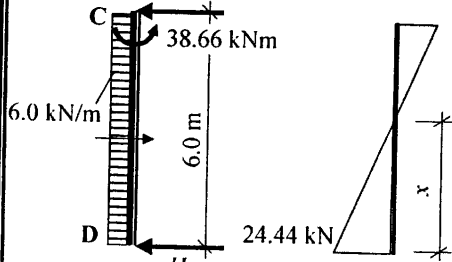
$$\begin{aligned}
 +ve \curvearrowright \Sigma M_C &= 0 \\
 - 89.1 - (16.0 \times 4.0) - (20.0 \times 2.0) \\
 - (25.0 \times 6.0 \times 3.0) + 38.66 + (V_B \times 6.0) &= 0 \\
 V_B &= + 100.8 \text{ kN}
 \end{aligned}$$



$$\begin{aligned}
 x &= 2.0 + (34.8/25.0) = 3.39 \text{ m} \\
 M_{\text{maximum}} &= [0.5 \times (100.8 + 50.8) \times 2.0] \\
 &\quad + (0.5 \times 1.39 \times 34.8) - 89.1 \\
 M_{\text{maximum}} &= 86.67 \text{ kNm}
 \end{aligned}$$

**Shear Force Diagram**

Member CD:



$$\begin{aligned}
 +ve \curvearrowright \Sigma M_C &= 0 \\
 - 38.66 - (6.0 \times 6.0 \times 3.0) + (H_D \times 6.0) &= 0 \\
 H_D &= + 24.44 \text{ kN} \leftarrow
 \end{aligned}$$

$$\begin{aligned}
 x &= (24.4/6.0) = 4.07 \text{ m} \\
 M_{\text{maximum}} &= (0.5 \times 4.07 \times 24.44) = 49.74 \text{ kNm}
 \end{aligned}$$

**Shear Force Diagram**

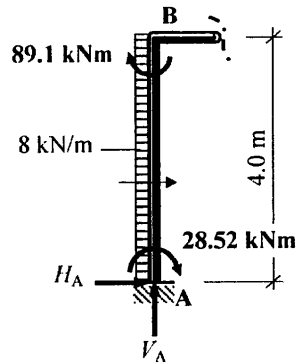
### Solution

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Consider Member AB:



Consider Member AB:

$$+ve \curvearrowright \Sigma M_B = 0$$

$$+ 89.1 + 28.52 - (8.0 \times 4.0 \times 2.0) - (H_A \times 4.0) = 0$$

$$\therefore H_A = + 13.41 \text{ kN} \rightarrow$$

For the complete frame:

$$+ve \rightarrow \Sigma F_x = 0$$

$$13.41 + (8.0 \times 4.0) + (6.0 \times 6.0) - 24.44 - H_C = 0$$

$$\therefore H_C = + 56.97 \text{ kN} \leftarrow$$

$$+ve \curvearrowright \Sigma M_A = 0$$

$$+ 28.52 + (8.0 \times 4.0 \times 2.0) + (25.0 \times 6.0 \times 3.0) + (16.0 \times 2.0) + (20.0 \times 4.0)$$

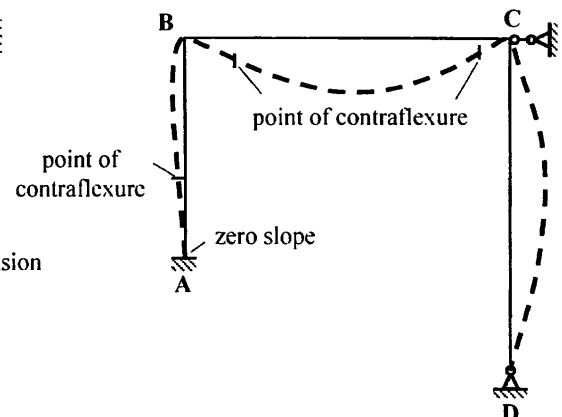
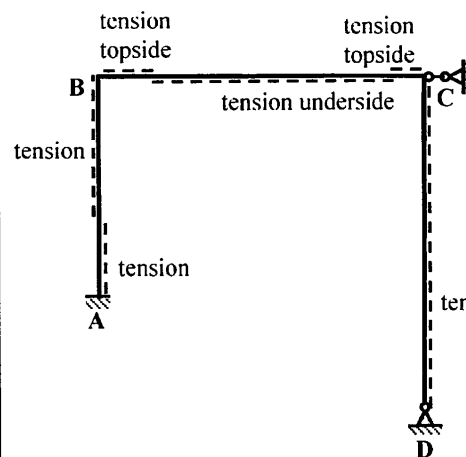
$$- (56.97 \times 4.0) + (6.0 \times 6.0 \times 1.0) + (24.44 \times 2.0) - (V_D \times 6.0) = 0$$

$$\therefore V_D = + 85.25 \text{ kN} \uparrow$$

$$+ve \uparrow \Sigma F_y = 0$$

$$V_A - (25.0 \times 6.0) - 16.0 - 20.0 + 85.25 = 0$$

$$\therefore V_A = + 100.75 \text{ kN} \uparrow$$



Deflected Shape