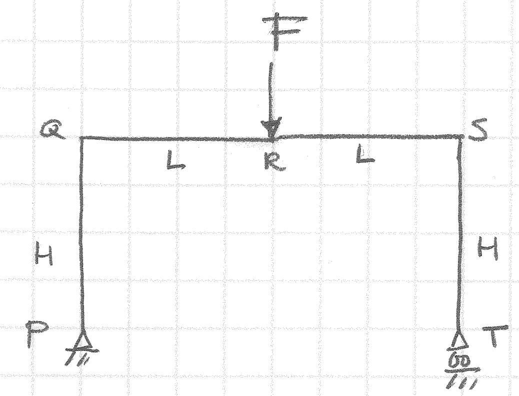
## Example:




EI : columns PQ & ST : $EI_c$

beam QRS : $EI_b$

Sketch deflected shape.

Find $v_R$

$\theta_P$

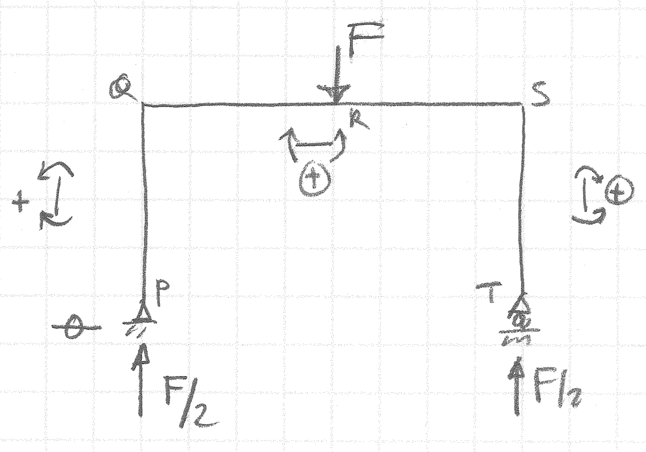
$\theta_Q$ , $u_Q$

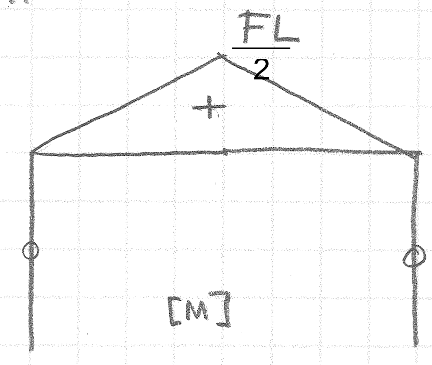
$\theta_S$ , $u_S$

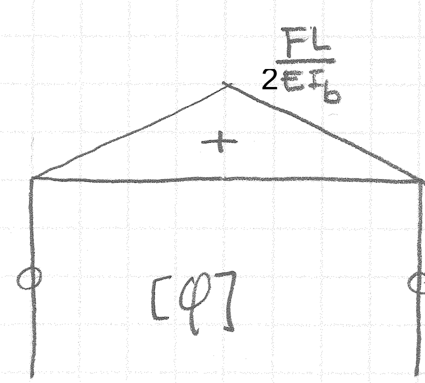
$\theta_T$ & $u_T$

While the use of "moment-area theorems" is rather straightforward once one recognizes the symmetric nature of the loading and structure, and hence the response, let's find the displacements using "geometric influence factor" ("virtual force") method.

First, let's get the curvature diagram.

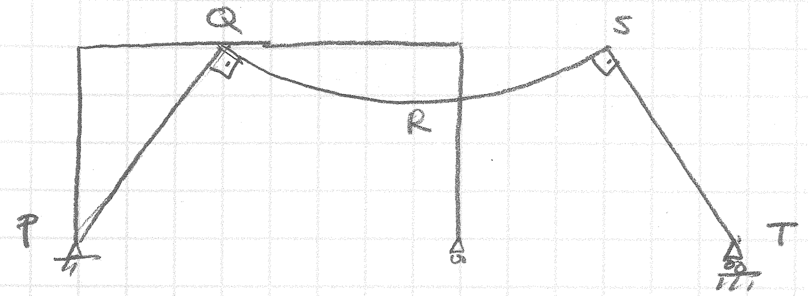



seeing the curvature diagram perhaps it is now easier to visualize the deflected shape?

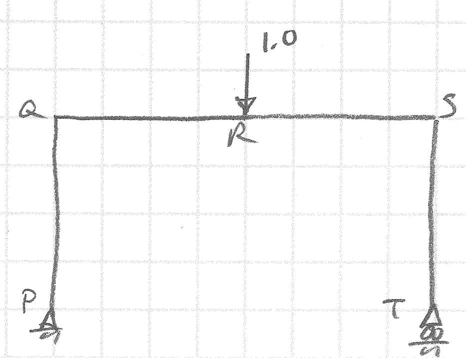
*



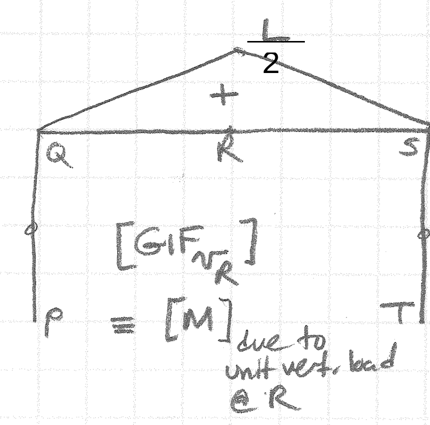

deflected shape

to find $v_R$, vertical deflection @ R, we will need $GIF_{v_R}$.

$GIF_{v_R}$ diagram is equivalent to the bending moment diagram due to application of a unit vertical load applied @ R.




$\Rightarrow$




(**)

(*) from page 1

$$v_R = \int_{\text{structure}} \varphi \cdot GIF_{v_R} \cdot dl = \int_P^Q \cancel{\varphi}^{\,0} \cdot GIF_{v_R}\, dl + \int_Q^S \varphi \cdot GIF_{v_R}\, dl + \int_S^T \cancel{\varphi}^{\,0} \cdot GIF_{v_R}\, dl$$

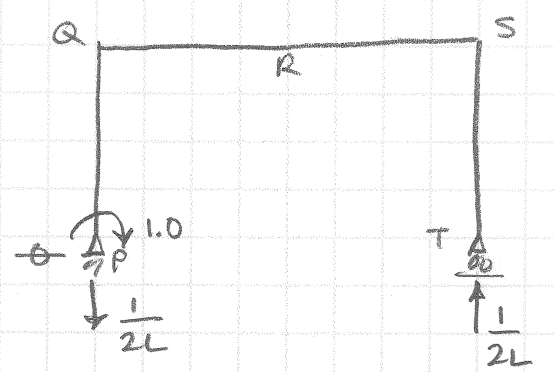
(**)

$$= 0 + \frac{1}{3} \cdot \left(\frac{FL}{2EI_b}\right)\left(\frac{L}{2}\right)(2L) + 0$$

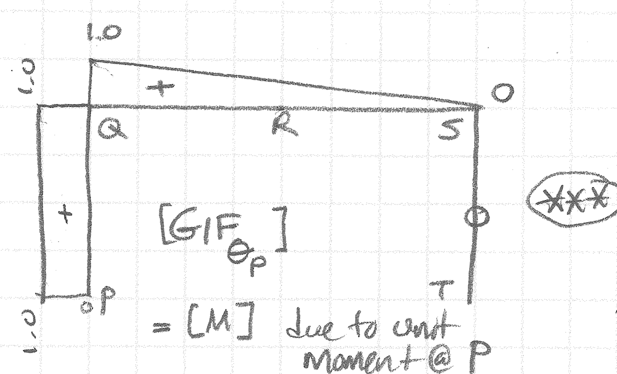
$$v_R = \frac{1}{6}\,\frac{FL^3}{EI_b} \downarrow$$

$\therefore$ because the result is positive, the vertical displacement at R is in the same direction of the unit vertical load.

for $\theta_P$ : apply unit moment @ P




$\Rightarrow$

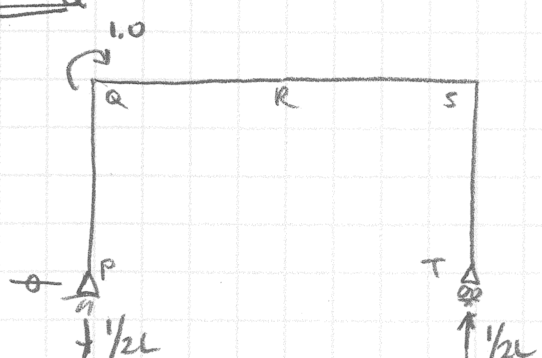



$$\theta_P = \int_{\text{structure}} \overset{*}{\varphi} \cdot \overset{***}{GIF_{\theta_P}}\, dl = \int_P^Q \overset{0}{\cancel{\varphi}} \cdot GIF_{\theta_P}\, dl + \int_Q^S \varphi \cdot GIF_{\theta_P}\, dl + \int_S^T \overset{0}{\cancel{\varphi}} \cdot \overset{0}{\cancel{GIF_{\theta_P}}}\, dl$$

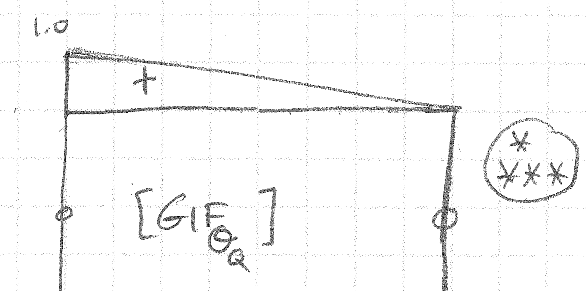
$$= 0 + \frac{1}{4}\left(\frac{FL}{2EI_b}\right)(1.0)(2L) + 0$$

$$\theta_P = \frac{FL^2}{4EI_b} \curvearrowright \text{(CW)}$$

for $\theta_Q$ :



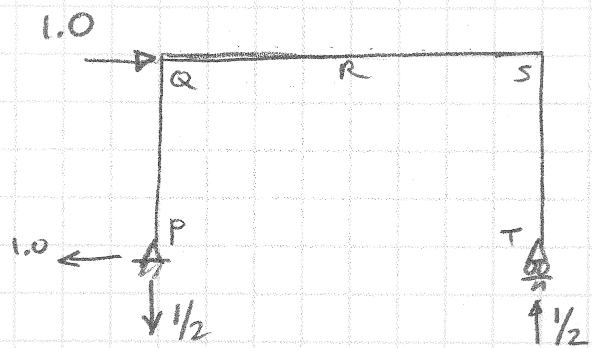

$\Rightarrow$



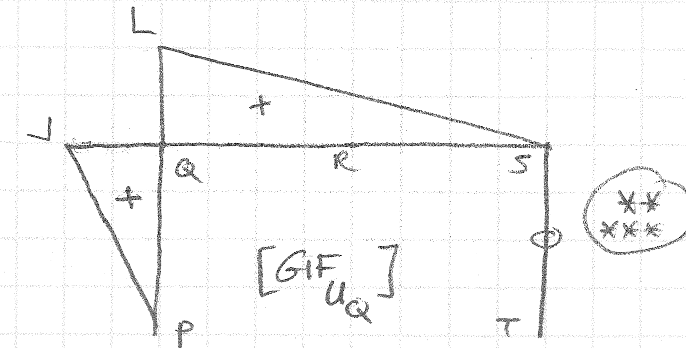

$$\theta_Q = \int_{\text{str.}} \overset{*}{\varphi} \cdot \underset{*\,***}{GIF_{\theta_Q}}\, dl \qquad 0 + \frac{1}{4}\left(\frac{FL}{2EI_b}\right)(1.0)(2L) + 0 = \frac{FL^2}{4EI_b} \curvearrowright \text{(CW)}$$

note that $\theta_Q = \theta_P$ is an expected result because beam PQ has no curvature and must remain straight.

## for $u_Q$

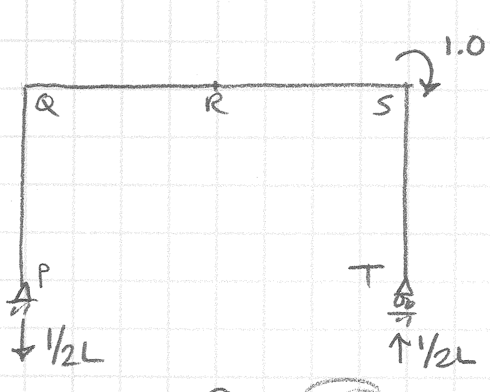



$\Rightarrow$

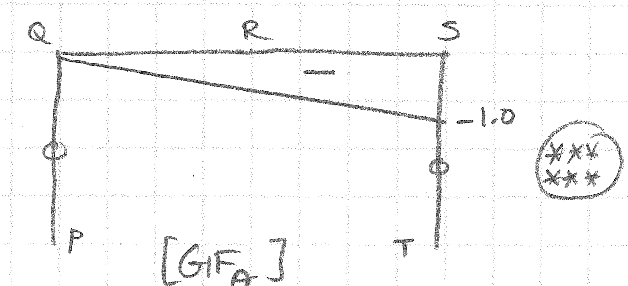



$$u_Q = \int_{str} \overset{*}{\varphi} \cdot GIF_{u_Q}\, d\ell = \frac{1}{4}\left(\frac{FL}{2EI_b}\right)(L)(2L) = \frac{FL^3}{4EI_b} \rightarrow$$

## for $\theta_S$




$\Rightarrow$



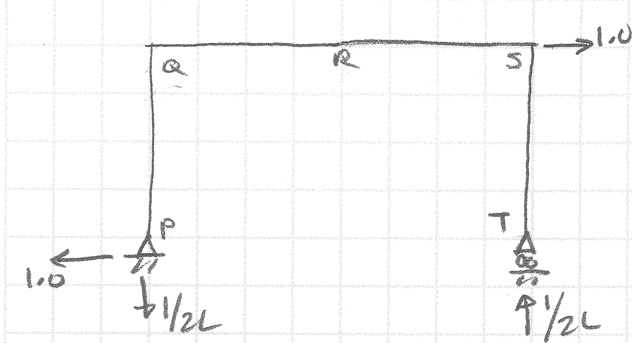

$$\theta_S = \int_{str} \overset{*}{\varphi} \cdot \overset{***}{GIF_{\theta_S}} \cdot d\ell = \cancel{\int_P^Q} 0 + \int_Q^S + \cancel{\int_S^T} 0$$

$$\theta_S = \frac{1}{4}\left(\frac{FL}{2EI_b}\right)(-1.0)(2L) = -\frac{FL^2}{4EI_b}$$

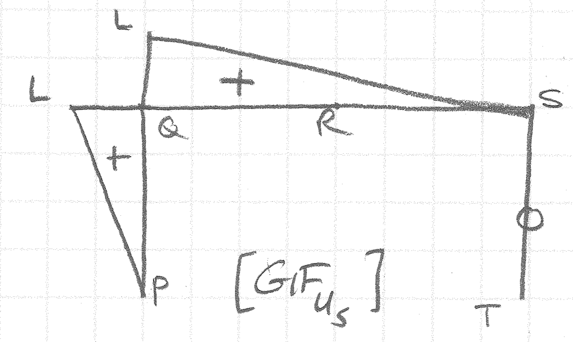
meaning in the opposite of the direction we chose for the unit moment

$$\theta_S = \frac{FL^2}{4EI_b} \circlearrowleft \quad (ccw)$$

for $u_S$ : — what do you ~~expect it to be~~?

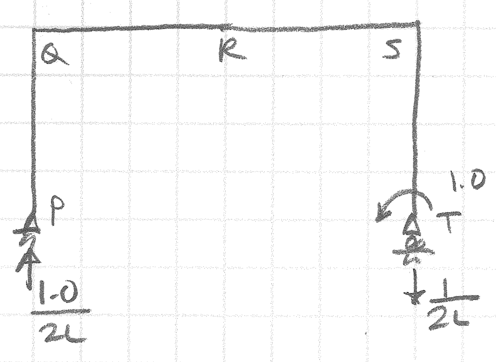



$\Rightarrow$

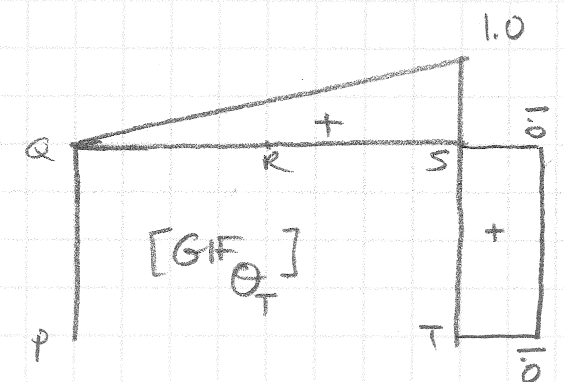



$$u_S = \int \varphi \cdot GIF_{u_S}\, dl = \frac{FL^3}{4EI_b} \longrightarrow \text{as expected?}$$

for $\theta_T$ :



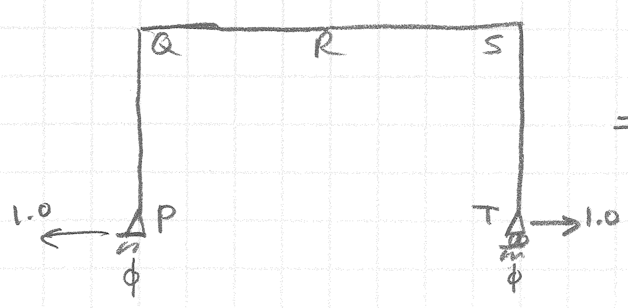

$\Rightarrow$



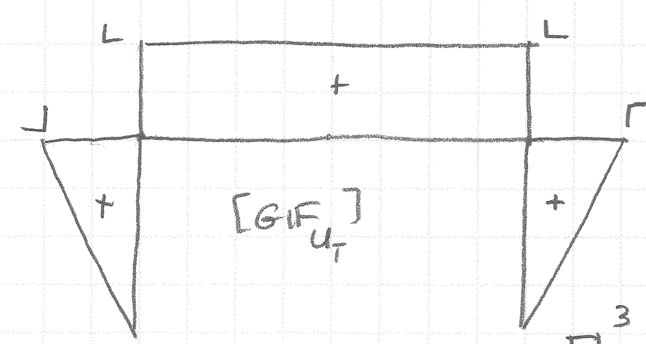

$$\theta_T = \int \varphi \cdot GIF_{\theta_T}\, dl = \frac{FL^2}{4EI_b}$$

$\theta_T =$ ↺ (CCW) reasonable?

for $u_T$ :




$\Rightarrow$




$$u_T = \int \varphi \cdot GIF_{u_T}\, dl = \frac{1}{2} \cdot \left(\frac{FL}{2EI_b}\right) \cdot (L)(L) = \frac{FL^3}{4EI_b}$$

makes sense?