

CE 361 Introduction to Transportation Engineering  
Homework 3 (HW 3) Solutions

Out: Mon. 13 September 2004  
Due: Mon. 20 September 2004

### HIGHWAY DESIGN FOR PERFORMANCE

1. (20 points) **Capacity and Level of Service.** Problem 3.7 in FTE. There are 20 access points in the one-mile-long “neckdown” segment.

(a) Because  $ATS < 40.0$ ,  $LOS = E$ . (b) A one-mile segment with 9-ft lanes and no shoulders:  $f_G = 1.0$ ,  $E_T = 1.7$ ,  $ER = 1.0$ ,  $f_{HV} = 0.947$ ,  $v_p = 342$ . If 45 mph speed limit is achieved,  $BFFS = 45.0$ ,  $f_{LS} = 6.4$ ,  $f_A = 5.0$ , and (3.6)  $FFS = 45 - 6.4 - 5.0 = 33.6$  mph. (3.5)  $ATS = 27$  mph (LOS D). Because flow rate does not exceed segment capacity, LOS E is not exceeded. For PTSF,  $f_G = 1.00$ ,  $E_T = 1.1$ ,  $ER = 1.0$ ,  $f_{HV} = 0.992$ , (3.2)  $v_p = 326$ , (3.7)  $BPTSF = 24.8$ ,  $f_{dnp} = 23.75$ , (3.8)  $PTSF = 24.8 + 23.75 = 48.5$  (LOS B). LOS is E with or without the neckdown, but ATS drops from 39.4 to 24.8 mph, according to the HCM method.

2. **Analysis of Freeway Lane Blockage using Queueing Diagram.** Capacity of 1800 vph per lane. 3 inbound lanes. Flow rate is 4000 vph. One lane blocked, reducing inbound capacity to 3200 vph. A 3-lane backup (queue) results, as drivers try to merge into two lanes. The queue builds for 15 minutes, at which time the queue reaches the last offramp upstream from the blockage. At this offramp, no driver is willing to join a queue that extends past the offramp. Instead, enough drivers exit, so that the queue on the freeway never extends past the offramp. Thirty minutes after the incident blocked a lane, the lane is reopened to traffic and the full capacity is restored.

A. (15 points) Queueing diagram on next page. Note that AC and DC are parallel because of the exits to the offramp upstream from the blockage.

B. (5 points) Maximum length of queue = max length of vertical line from AC to DC =  $1000 - 800 = 200$  at  $t = 15$  minutes. Also,  $1800 - 1600 = 200$  at  $t = 30$ . So queue length is 200 vehs from  $t = 15$  to  $t = 30$ .

C. (5 points) Maximum time in queue = max length of horizontal line from AC to DC. The 1000<sup>th</sup> vehicle arrives at  $t = 15$ . The equation of DC2 is  $D = 800 + \frac{3200}{60}t$ ; set that equal to 1000 and solve for  $t$ .  $t =$

$$\frac{1000 - 800}{3200/60} = 3.75 \text{ minutes. The last vehicle to wait this long leaves the queue at } t = 30 \text{ minutes and}$$

entered the queue at  $30 - 3.75 = 26.25$  minutes. The max time in queue applies to all vehicles that arrived between  $t = 15$  and  $t = 26.25$ .

D. (5 points) Queue dissipation. Starting at  $t = 30$ ,  $AC3 = 1800 + \frac{4000}{60}t$  and  $DC3 = 1600 + \frac{5400}{60}t$ . Set

$$AC3 = DC3 \text{ and solve for } t. t = \frac{(1800 - 1600) * 60}{5400 - 4000} = \frac{12,000}{1400} = 8.57 \text{ minutes after } t = 30, \text{ or } t = 38.6$$

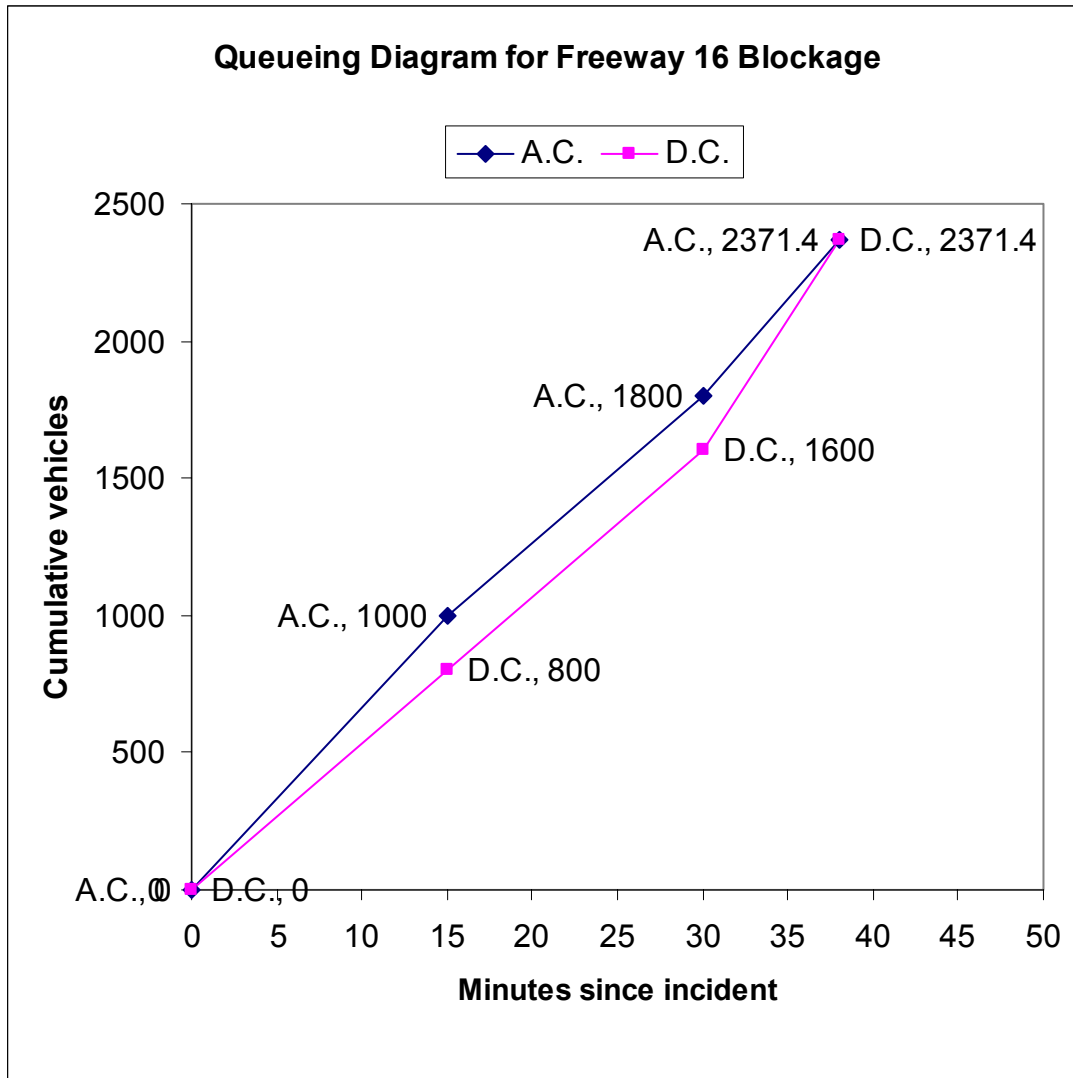
minutes.

E. (5 points) Total delay to all vehicles time in the queue = area between ACs and DCs,  $0 \leq t \leq 38.6$ .

$$\text{Area 1 for } 0 \leq t \leq 15 = \frac{1}{2}(15)(1000 - 800) = 1500 \text{ veh-min. Area 2 for } 15 \leq t \leq 30 = 200 * 15 = 3000$$

$$\text{veh-min. Area 3 for } 30 \leq t \leq 38.6 = \frac{1}{2}(38.6 - 30.0)(1800 - 1600) = 860 \text{ veh-min. Area 1 + Area 2 +}$$

Area 3 = 5360 veh-min = 89.33 veh-hr. (Do not include the delay incurred by drivers who left the freeway at the offramp and had to use surface streets.)



3. **Analyzing a Stable Queue.** Offramp traffic has a constant cycle, so that 960 vph can enter the surface streets from the ramp. Normally, 400 vph use the ramp during the morning peak period.

A. (10 points) AC1 = 4000 vph, AC2 = 3200 vph. Vehicles diverted to the offramp = (4000-

$$3200) * \frac{15 \text{ min}}{60 \text{ min/hr}} = 200 \text{ vehs.}$$

Arrival rate on the offramp before was 400 vph and 400+800+1200 vph after the freeway blockage.  $\mu = 960$  vph, so queue is stable only before the freeway blockage.

B. (10 points) Type of queueing system (x/y/z) is M/D/1. Random traffic arrivals/pre-timed or fixed traffic signal cycle/one signal for all "customer" vehicles.

C. For the cases in which a stable queue exists, answer the following questions.

i. (5 points) Average queue length. (3.15) with  $\rho = \frac{400}{600} = 0.42$ ;  $\bar{Q} = \frac{(0.42)^2}{2(1 - 0.42)} = 0.15$  vehs.

ii. (5 points) Average time spent waiting in a queue. (3.16)  $\bar{W} = \frac{0.42}{2\left(\frac{960}{3600}\right)(1 - 0.42)} = 1.36$  sec.

4. (15 points) **Poisson Calculations at River Bridge Toll Facility.** Problem 3.24 (a-c) in FTE.

- A. Arrival rate,  $\lambda = 55 \text{ veh}/20 \text{ min} * (1 \text{ min}/60 \text{ sec}) = 0.0458 \text{ veh}/\text{sec}$   
 Time period,  $t = 30 \text{ sec}$

$$\text{From } \Pr(n) = \frac{(\lambda \cdot t)^n e^{-\lambda \cdot t}}{n!} = \frac{(0.0458 \cdot 30)^0 e^{-0.0458 \cdot 30}}{0!} = 0.3477$$

Number of Vehicles Arriving, n	Probability of n vehicles arriving, Pr(n)	Total Probability Pr (n ≤ N)
0	0.3477	0.3477
1	0.3477	0.6953
2	0.1738	0.8691

$$\Pr(n \geq 2) = 1.00 - 0.3477 - 0.3477 = 0.2512.$$

- B. For M/M/1 queueing model and  $\mu = 450 \text{ vph}$   
 Arrival rate,  $\lambda = (55 + 133) / 40 \text{ min} * (60 \text{ min} / 1 \text{ hr}) = 282 \text{ veh}/\text{hr}$

$$\bar{Q} = \frac{\rho^2}{(1-\rho)} = \frac{\left(\frac{\lambda}{\mu}\right)^2}{1 - \frac{\lambda}{\mu}} = \frac{\left(\frac{282}{450}\right)^2}{1 - \frac{282}{450}} = \mathbf{0.41 \text{ veh}}$$

$$\bar{W} = \frac{\left(\frac{\lambda}{\mu}\right)}{\mu - \lambda} = \frac{\left(\frac{282}{450}\right)}{450 - 282} = 0.0037 \text{ hr}/\text{veh} * 3600 \text{ s} / \text{hr} = \mathbf{13.43 \text{ s}/\text{veh}}$$

$$\bar{t} = \frac{1}{\mu - \lambda} = \frac{1}{450 - 282} = 0.0060 \text{ hr}/\text{veh} * 3600 \text{ s}/\text{hr} = \mathbf{21.43 \text{ s}/\text{veh}}$$

- C. Consecutive hourly time periods when  $\lambda > \mu$  are from 7:40 to 9:00AM. See queueing diagram below.

