Indirect Adaptive Robust Control of SISO Nonlinear Systems in Semi Strict Feedback Forms

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OUTLINE

- Practical Motivations
- Theoretical Problem Formulation
- Indirect Adaptive Robust Control (IARC)
  - Tracking Error Dynamics based Z-swapping Identifier
  - Actual System Dynamics based X-swapping Identifier
- Experimental Results
  - Precision Motion Control of Linear Motor Drive Systems
  - Precision Motion Control of Electro-Hydraulic Systems
- Conclusions

Smart Machines and Systems

- Performance Oriented
  - Precision manufacturing demands machines with positioning accuracy down to *sub-micrometers* (for quality) at *higher acceleration/speed* (for productivity)
  - ...
- Intelligent and User Friendly
  - Being able to deal with *various working conditions*
  - *Machine health monitoring* and self-fault detections
  - *Prognostics* capability for on-demand services
  - …

Performance Oriented Control Issues

- Inherent Process *Nonlinearities*
  - Stringent performance requirements demand *explicit compensation* of process nonlinearities

    (e.g., HDD pivot bearing friction modeling and compensation)
  - …
- Modeling *Uncertainties*
  - Simply attenuating modeling uncertainties via linear high-gain robust feedback is not sufficient
  - Needs to *reduce model uncertainties through on-line adaptation or learning*

Accurate Parameter Estimation

- Provide Improved Control Performance
  - Can be used to achieve a better model compensation to reduce the effect of model uncertainties
  - Enable on-line tuning of robust feedback gains to maximize the attenuation level of high-gain robust feedback

- Essential for Adding Intelligent Features
  - Features like prognostics, machine health monitoring, fault detection, etc, can be added when one knows the time history of certain critical parameters
  - Non-Conservative Task Planning
  - …

Theoretical Problem Formulations

- System

$$\dot{x}_1 = b_1 x_2 + \varphi_1(x_1, t)^T \theta + \Delta_1(\overline{x}_n, t)$$

$$\cdots$$

$$\dot{x}_n = b_n u + \varphi_n(\overline{x}_n, t)^T \theta + \Delta_n(\overline{x}_n, t)$$

where

$\theta_b = [\theta^T, b_1, \ldots b_n]^T$: unknown parameters and gains

$\Delta_i(\overline{x}_n, t)$: uncertain nonlinearities/disturbances

Theoretical Problem Formulations

- System

$$\dot{x}_1 = b_1 x_2 + \boxed{\varphi_1(x_1, t)}^T \theta + \Delta_1(\bar{x}_n, t)$$

$$\cdots$$

Nonlinearities

$$\dot{x}_n = b_n u + \boxed{\varphi_n(\bar{x}_n, t)}^T \theta + \Delta_n(\bar{x}_n, t)$$

where

$\theta_b = [\theta^T, b_1, \ldots b_n]^T$: unknown parameters and gains

$\Delta_i(\bar{x}_n, t)$: uncertain nonlinearities/disturbances

Theoretical Problem Formulations

- System

Parametric uncertainties

$$\dot{x}_1 = b_1 x_2 + \varphi_1(x_1, t)^T \theta + \Delta_1(\overline{x}_n, t)$$

$$\cdots$$

$$\dot{x}_n = b_n u + \varphi_n(\overline{x}_n, t)^T \theta + \Delta_n(\overline{x}_n, t)$$

where

$\theta_b = [\theta^T, b_1, \ldots b_n]^T$: unknown parameters and gains

$\Delta_i(\overline{x}_n, t)$: uncertain nonlinearities/disturbances

Theoretical Problem Formulations

- System

Nonlinearity uncertainties

$$\dot{x}_1 = b_1 x_2 + \varphi_1(x_1, t)^T \theta + \Delta_1(\overline{x}_n, t)$$

$$\cdots$$

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Theoretical Problem Formulations

- System

$$\dot{x}_1 = b_1 x_2 + \varphi_1(x_1, t)^T \theta + \Delta_1(\overline{x}_n, t)$$

$$\cdots$$

$$\dot{x}_n = b_n u + \varphi_n(\overline{x}_n, t)^T \theta + \Delta_n(\overline{x}_n, t)$$

Unmatched Uncertainties

where

$\theta_b = [\theta^T, b_1, \ldots b_n]^T$: unknown parameters and gains

$\Delta_i(\overline{x}_n, t)$: uncertain nonlinearities/disturbances

Theoretical Problem Formulations

- System

$$\dot{x}_1 = b_1 x_2 + \varphi_1(x_1, t)^T \theta + \Delta_1(\overline{x}_n, t)$$

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where

$\theta_b = [\theta^T, b_1, \ldots b_n]^T$: unknown parameters and gains

$\Delta_i(\overline{x}_n, t)$: uncertain nonlinearities/disturbances

- Assumptions
  - A1: $\theta_b \in \Omega_{\theta_b}$, A known bounded convex set
  - A2: $|\Delta_i(\overline{x}_n, t)| \le \delta_i(\overline{x}_i) d_i(t), \ \forall i$ Semi-strict assumption

Control Design Objectives

- Primary Objectives
  - In general, output x_1 tracks any feasible trajectory $x_{1d}(t)$ with a guaranteed transient performance and final tracking accuracy with all signals in the system being bounded
  - Asymptotic output tracking in the presence of parametric uncertainties only, i.e.,

$$z_1 = x_1 - x_{1d} \rightarrow 0 \quad \text{as} \quad t \rightarrow \infty \quad \text{when} \quad \Delta_i = 0, \forall i,$$

- Secondary Objective
  - Accurate on-line estimation of unknown parameters, i.e., $\hat{\theta}_b(t) \rightarrow \theta_b \quad \text{as} \quad t \rightarrow \infty$

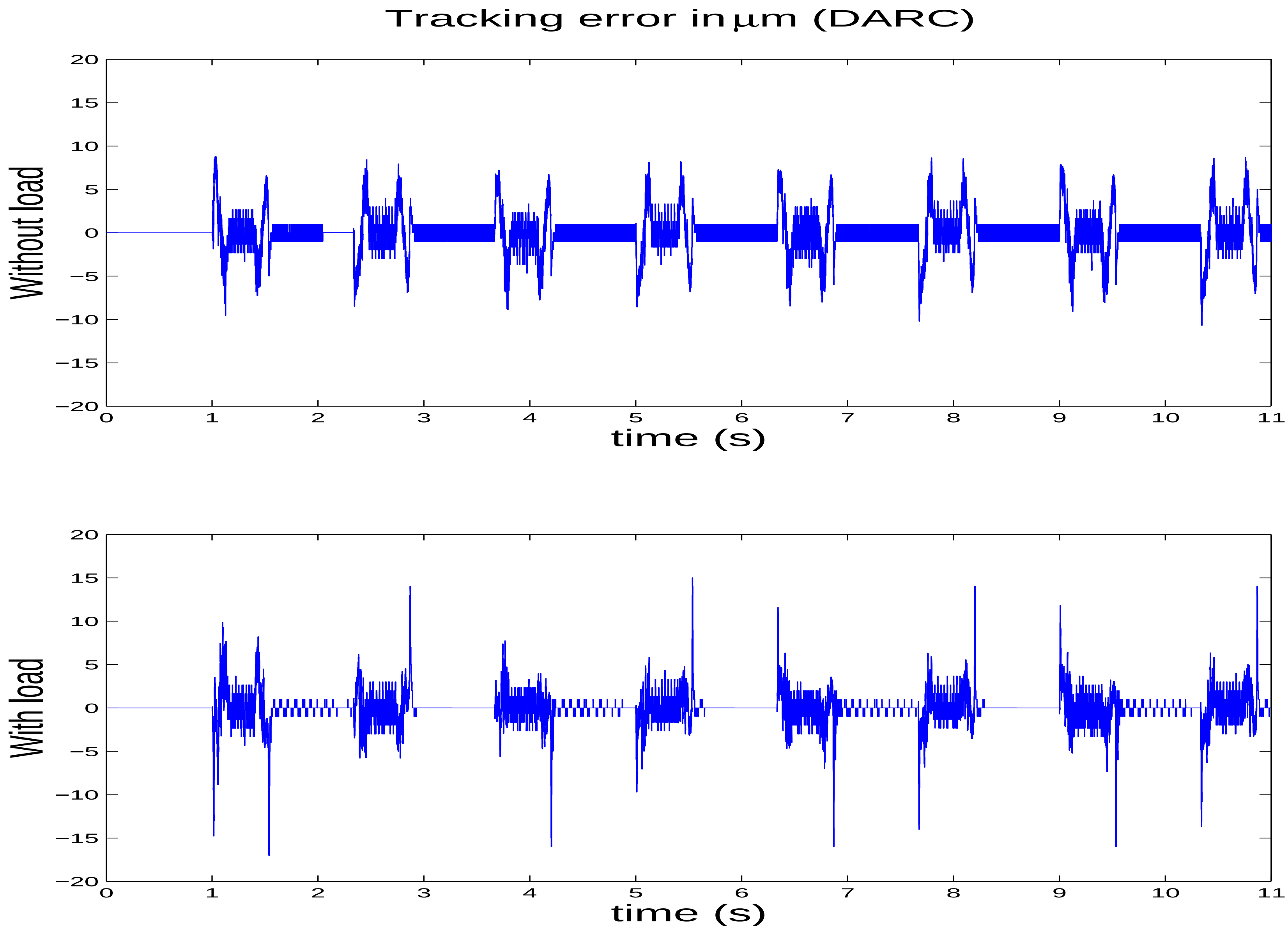
Literature Survey

- Backstepping Adaptive Designs
  (KKK's book'95, …)
  - Consider parametric uncertainty only (i.e., $\Delta_i = 0, \ \forall i$)
- Robust Adaptive Backstepping Designs
  (Polycarpou and Ioannou'93, Pan and Basar'96, Freeman, et al'96, Marino and Tomei'98, …)
  - Robust stability; Achievable performances in terms of L_∞ norm are not so transparent
  - Accurate parameter estimation is not emphasized
- Direct Adaptive Robust Control (DARC)
  (Yao and Tomizuka'94, 01, Yao'97, …)
  - Excellent Output Tracking Performance
  - Verified through Several Applications

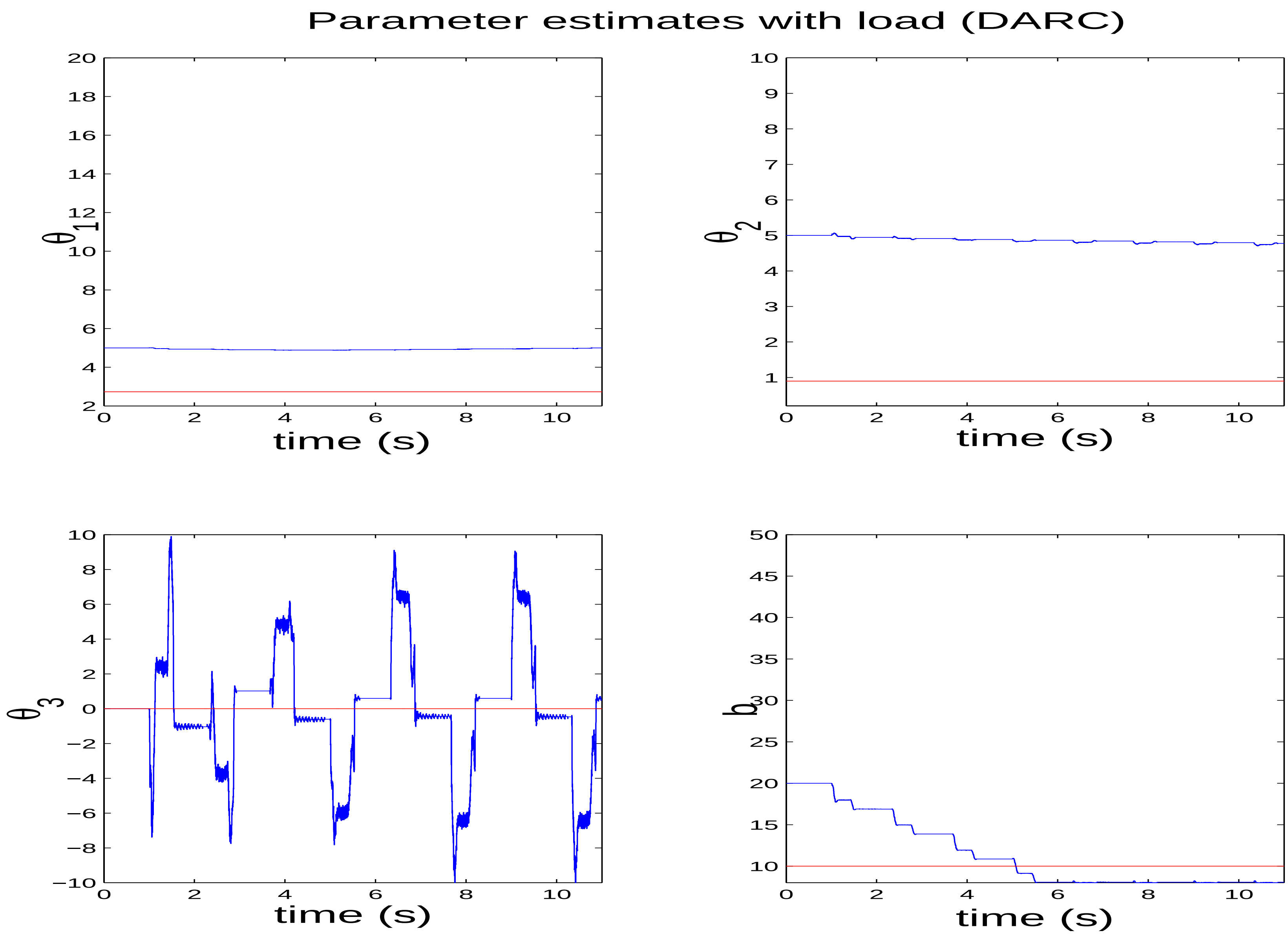
High Speed Linear Motor Driven Precision Positioning Stage

Tracking Errors for Typical Industrial Motion

(Point-to-Point with Velocity of 1m/sec and Acceleration of 12m/sec^2)



Parameter Estimations of DARC (Loaded)



Practical Issues of DARC Design

- Individual Parameter Estimates Seldom Converge
  - The design of control law and parameter estimation law are synthesized jointly through an "energy" function with reducing output tracking error being the sole objective
  - Gradient type estimation algorithm only with certain actual tracking error as driving signal
  - Small actual tracking error in implementation prone to be corrupted by neglected factors such as sampling delay and noises
  - Explicit monitoring of signal excitation level not possible as otherwise one loses the integral type fast dynamic compensation capability of parameter adaptation process

Indirect Adaptive Robust Control

- Total Separation of Parameter Estimation from Robust Control Law Design
  - Theoretical boundedness of parameter estimates and their derivatives achieved through the use of a rate-limited projection type adaptation law structure with preset adaptation rate limits, as opposed to the traditional way of using complicated mathematical derivations such as the normalization and/or nonlinear damping in the modular backstepping adaptive designs that may not work for systems with disturbances and uncertain nonlinearities
- Use Actual System Dynamics to Construct Reliable Parameter Estimation Model

Projection Type Adaptation Law with Rate Limits

- Adaptation Law Structure

$$\dot{\hat{\theta}}_b = sat_{\dot{\theta}_M}\left(\Pr oj_{\hat{\theta}_b}\left(\Gamma(t)\tau\right)\right), \quad \hat{\theta}_b(0) \in \Omega_{\theta_b}$$

where

$$sat_{\dot{\theta}_M}(\zeta) = s_0\zeta, \qquad s_0 = \begin{cases} 1, & \|\zeta\| \le \dot{\theta}_M \\ \dfrac{\dot{\theta}_M}{\|\zeta\|}, & \|\zeta\| > \dot{\theta}_M \end{cases}$$

← A preset rate limit,
A designer choice !

$$\Pr oj_{\hat{\theta}_b}(\xi) = \begin{cases} \xi & \text{if} \quad \hat{\theta}_{\mathrm{b}} \in {}^{\circ}\Omega_{\theta_b} \quad \text{or} \quad n_{\hat{\theta}_b}^T \xi \le 0 \\ \left(I - \Gamma \dfrac{n_{\hat{\theta}_b} n_{\hat{\theta}_b}^T}{n_{\hat{\theta}_b}^T \Gamma n_{\hat{\theta}_b}}\right)\xi & \hat{\theta}_{\mathrm{b}} \in \partial\Omega_{\theta_b} \quad \text{and} \quad n_{\hat{\theta}_b}^T \xi > 0 \end{cases}$$

Benefits of Rate Limited Adaptation Law Structure

- **P1: Parameter Estimates always within Bounded Set**

$$\hat{\theta}_b(t) \in \bar{\Omega}_{\theta_b}, \quad \forall t$$

- **P2: Parameter Adaptation Rate always within Preset Rate**

$$\left\| \dot{\hat{\theta}}_b(t) \right\| \leq \dot{\theta}_M, \quad \forall t$$

- **P3: Ideal Performance of Parameter Adaptation Preserved**

$$\tilde{\theta}_b^T \left(\Gamma^{-1} \mathrm{Proj}_{\hat{\theta}_b}(\Gamma \tau) - \tau \right) \leq 0, \quad \forall \tau$$

Note:

» P1 and P2 enable the use of the same ARC design technique as in direct approach to construct an ARC control law that achieves a guaranteed transient and final tracking accuracy, independent of specific estimation function τ to be used

Adaptive Robust Control Law

- **Control Functions**

$$\alpha_i\left(\bar{x}_i, \hat{\theta}, \bar{\hat{b}}_i, t\right) = \alpha_{ia} + \alpha_{is}$$

$$\alpha_{ia} = \frac{1}{\hat{b}_i}\left[\sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l}\hat{b}_l x_{l+1} + \frac{\partial \alpha_{i-1}}{\partial t} - \phi_i^T \hat{\theta} - \hat{b}_{i-1} z_{i-1}\right]$$

$$\alpha_{is} = \alpha_{is1} + \alpha_{is2}, \quad \alpha_{is1} = -\frac{1}{\hat{b}_i} k_i z_i, \quad \alpha_{is2} = -k_{is2} z_i$$

- **Robust Performance Conditions**

$$z_i\left[\hat{b}_i \alpha_{is2} + \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l}\tilde{b}_l x_{l+1} - \phi_i^T \tilde{\theta} - \tilde{b}_{i-1} z_{i-1} - \tilde{b}_i \alpha_{ia}\right.$$
$$\left. - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{b}_l}\dot{\hat{b}}_l - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} + \breve{\Delta}_i\right] \le \varepsilon_{ci} + \varepsilon_{di} \breve{d}_i^{\,2}$$

where

$$z_i = x_i - \alpha_{i-1}$$

$$\phi_i\left(\bar{x}_i, \hat{\theta}, \bar{\hat{b}}_i, t\right) = \varphi_i - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l}\varphi_l, \quad \breve{\Delta}_i = \Delta_i - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l}\Delta_l$$

Theoretical Performance of ARC Law

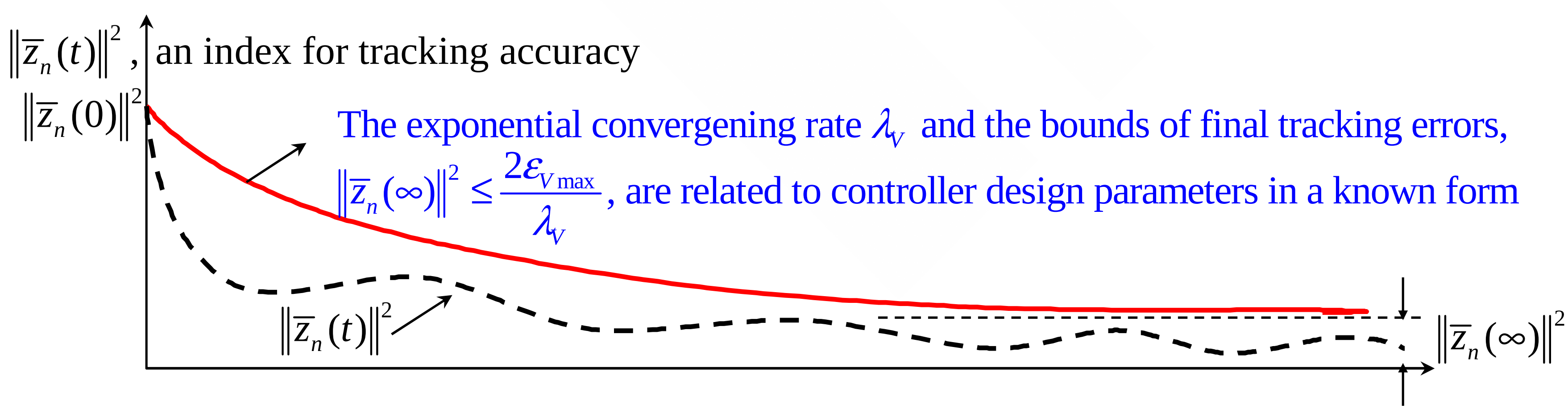
- **Theorem 1**

  With projection type adaptation law with rate limits, in which τ could be any identifier function, the ARC law u= α_n guarantees that, in general, all signals are bounded with tracking errors bounded above by

$$\left\|\bar{z}_n(t)\right\|^2 \leq e^{-\lambda_V t}\left\|\bar{z}_n(0)\right\|^2 + 2\int_0^t e^{-\lambda_V (t-\tau)}\varepsilon_V(\tau)\,d\tau$$

  where $\lambda_V = 2\min\{k_1,\ldots k_n\}$ and $\varepsilon_V = \sum_{i=1}^{n}\varepsilon_{ci} + \varepsilon_{di}\breve{d}_i^2$

- **Practical Significance**
  - **Guaranteed transient and final tracking accuracy, important for applications**



Actual System Dynamics Based Identifier

- **Original System Model in Linear Parametrization Form**

$$\dot{\bar{x}}_n = f_0(\bar{x}_n, u) + F^T(\bar{x}_n, u)\theta_b, \quad \text{assuming} \quad \Delta_i = 0, \forall i$$

where

$$F^T(\bar{x}_n, u) = \begin{bmatrix} \varphi_1{}^T & x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_n{}^T & 0 & \cdots & u \end{bmatrix}$$

- **Construct Filters to Generate Implementable Prediction Error Model**

$$\begin{aligned} \dot{\Omega}^T &= A\Omega^T + F^T(\bar{x}_n, u) \\ \dot{\Omega}_0 &= A(\Omega_0 + \bar{x}_n) - f_0(\bar{x}_n, u) \end{aligned} \qquad A, \text{ any stable matrix}$$

- **Linearly Parametrized Static Prediction Output Model,**

$$\begin{aligned} y &= \bar{x}_n + \Omega_0 \\ \hat{y} &= \Omega^T \hat{\theta}_b \end{aligned} \quad \Rightarrow \quad \boxed{\varepsilon = \hat{y} - y = \Omega^T \tilde{\theta}_b} - \tilde{\varepsilon}, \quad \text{where} \quad \tilde{\varepsilon} = \bar{x}_n + \Omega_0 - \Omega^T \theta_b$$

$$\dot{\tilde{\varepsilon}} = A\tilde{\varepsilon}, \quad \tilde{\varepsilon} \rightarrow 0$$

Estimation Functions

- Gradient Type

$$\tau = -\frac{1}{1+\nu\left\|\Omega\right\|_F^{2}}\Omega\varepsilon, \qquad \nu \ge 0$$

- Least Square Type

$$\tau = -\frac{1}{1+\nu tr\left\{\Omega^T\Gamma\Omega\right\}}\Omega\varepsilon$$

$$\dot{\Gamma} = \begin{cases} \alpha\Gamma - \dfrac{\Gamma\Omega\Omega^T\Gamma}{1+\nu tr\left\{\Omega^T\Gamma\Omega\right\}}, & \text{if} \quad \lambda_{\max}\left(\Gamma(t)\right) \le \rho_M \text{ and } \left\|\Pr oj_{\hat{\theta}_b}(\Gamma\tau)\right\| \le \dot{\theta}_M \\ 0 & \text{otherwise} \end{cases}$$

Theoretical Performance of IARC Law

- **Lemma**

  When the rate limited projection type adaptation law with either the gradient estimator or the least square estimator is used, the following results hold:

$$\varepsilon \in L_2[0,\infty) \cap L_\infty[0,\infty)$$
$$\dot{\hat{\theta}}_b \in L_2[0,\infty) \cap L_\infty[0,\infty)$$

- **Proof**
  - The proof for the above results with projection type adaptation law is well established in the literature. The key difficulty here is to show that, even with any pre-set adaptation rate limit, the above results still hold for the proposed rated limited projection type adaptation law. The technical details are rather involved and given in the paper

Theoretical Performance of IARC Law

- **Theorem 2**

  When the rated-limited projection type adaptation law with either gradient or least-square type identifier function, in the presence of parametric uncertainties only, in addition to the robust performance results in Theorem 1, an improved tracking performance, asymptotic output tracking (i.e., $\bar{z}_n \to 0$), is also acheived.

- **Sketch of Proof**

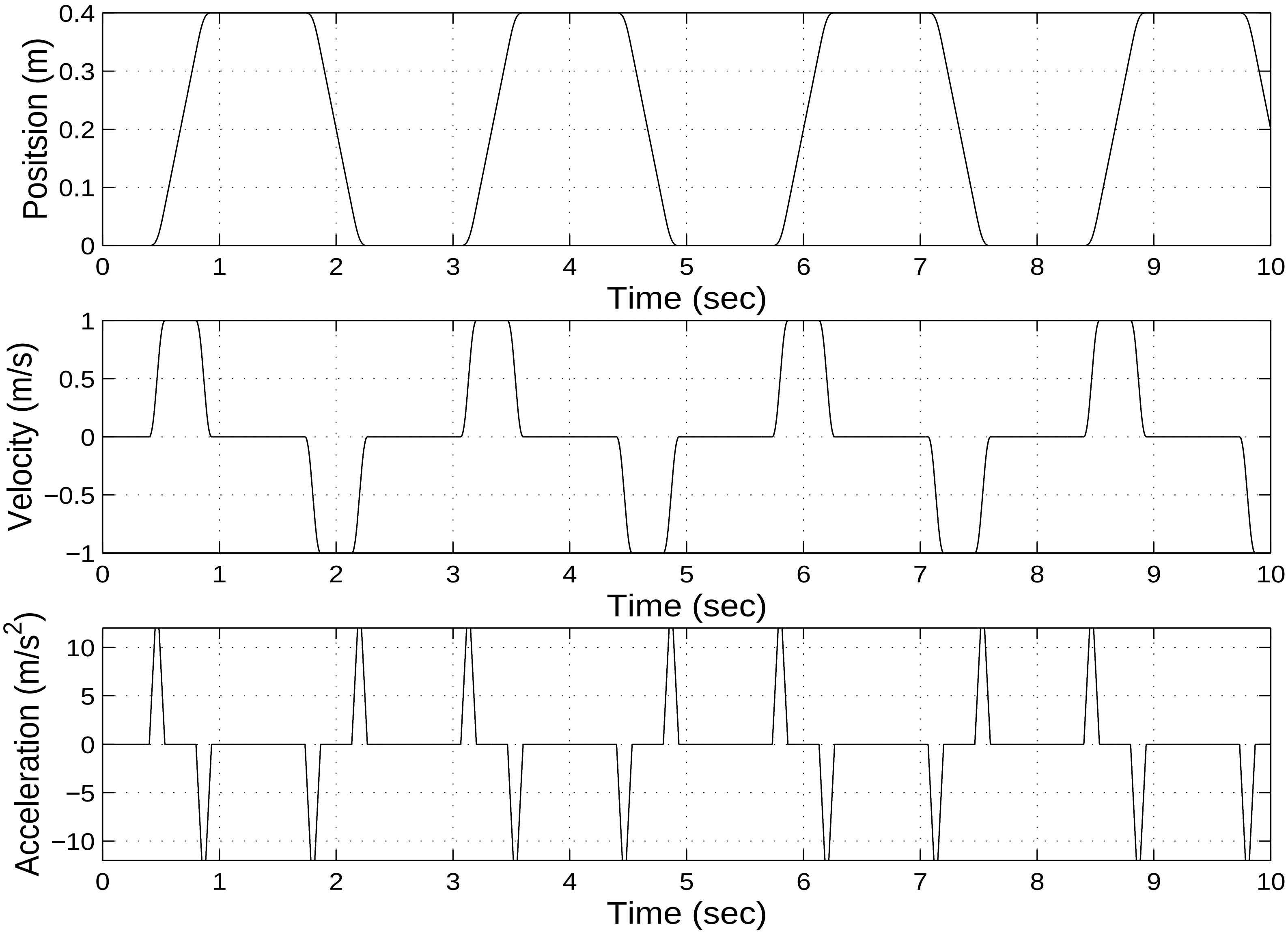
  – The proof follows a similar procedure as in the modular adaptive backstepping designs, which uses some Nonlinear Swapping Lemmas and the results of the previous lemma to show that $\bar{z}_n \in L_2[0,\infty)$. Thus, as $\dot{\bar{z}}_n \in L_\infty[0,\infty)$, the theorem can be proven by using Barbalat's lemma. The details are rather technical and given in the paper.

Note:

» **The above results indicate that the ideal performance results of adaptive designs are preserved.**

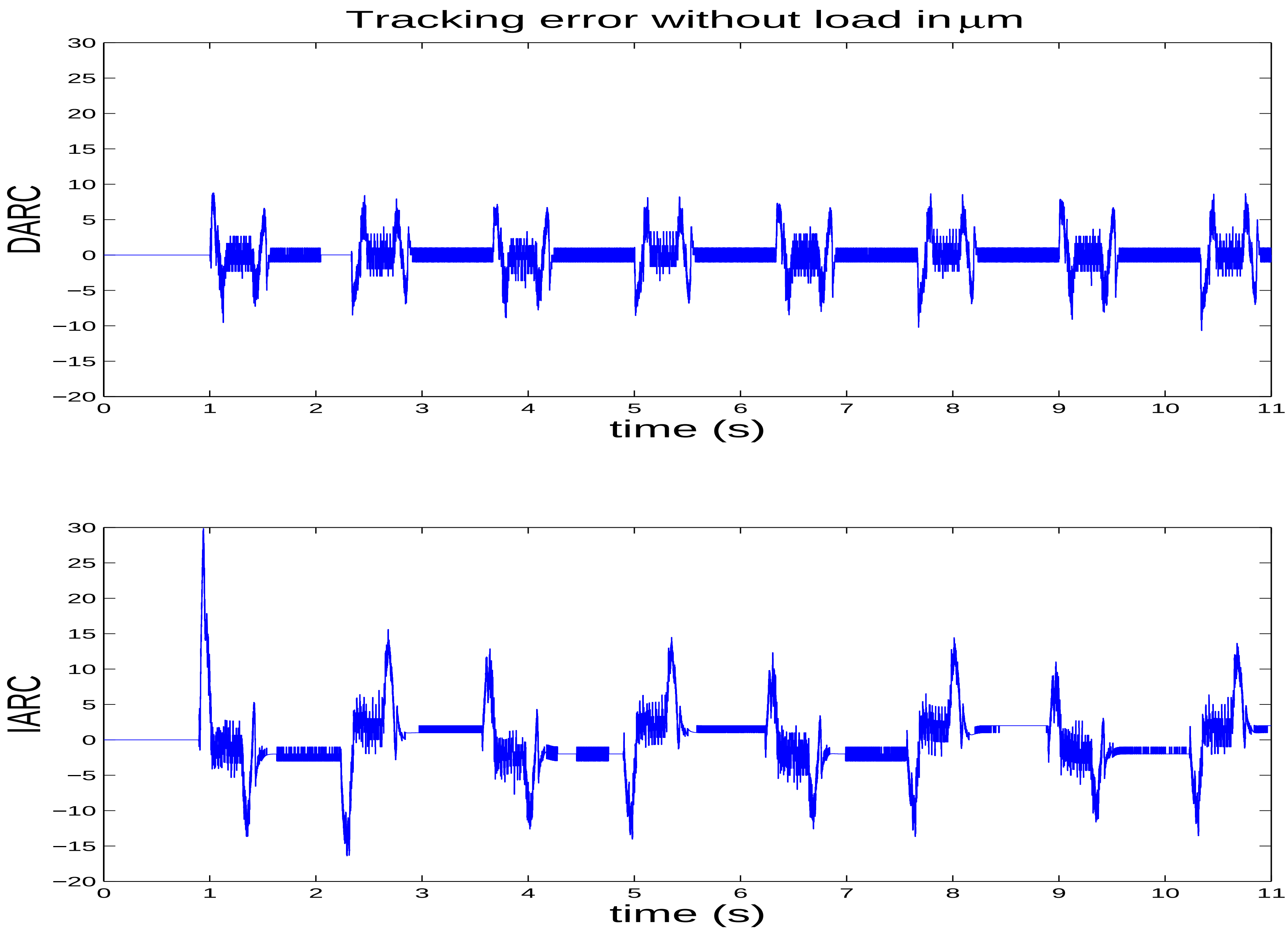
Precision Control of Linear Motors

Point-to-Point Motion Trajectory



Tracking Errors for Typical Industrial Motion

(Point-to-Point with Velocity of 1m/sec and Acceleration of 12m/sec^2)



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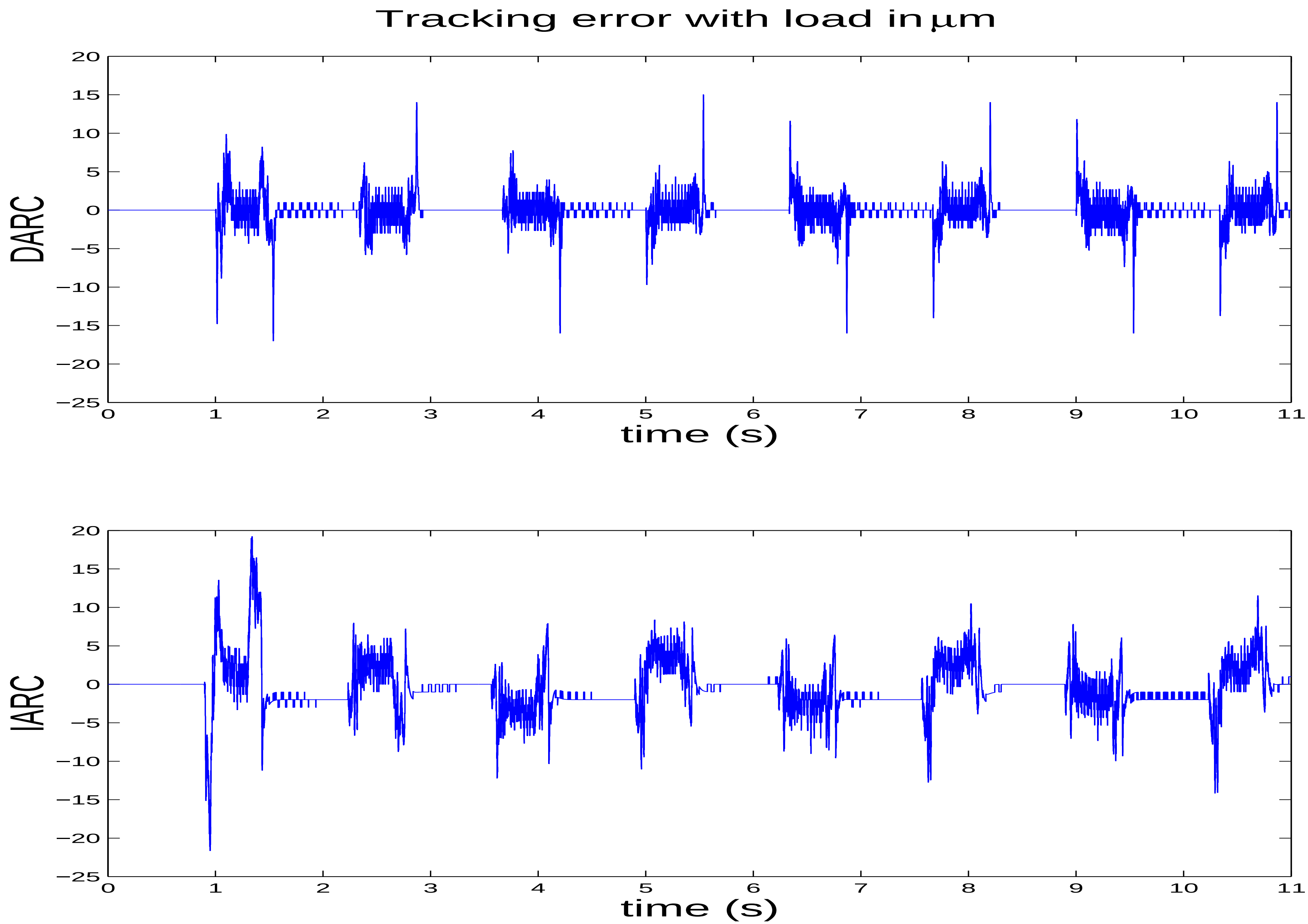
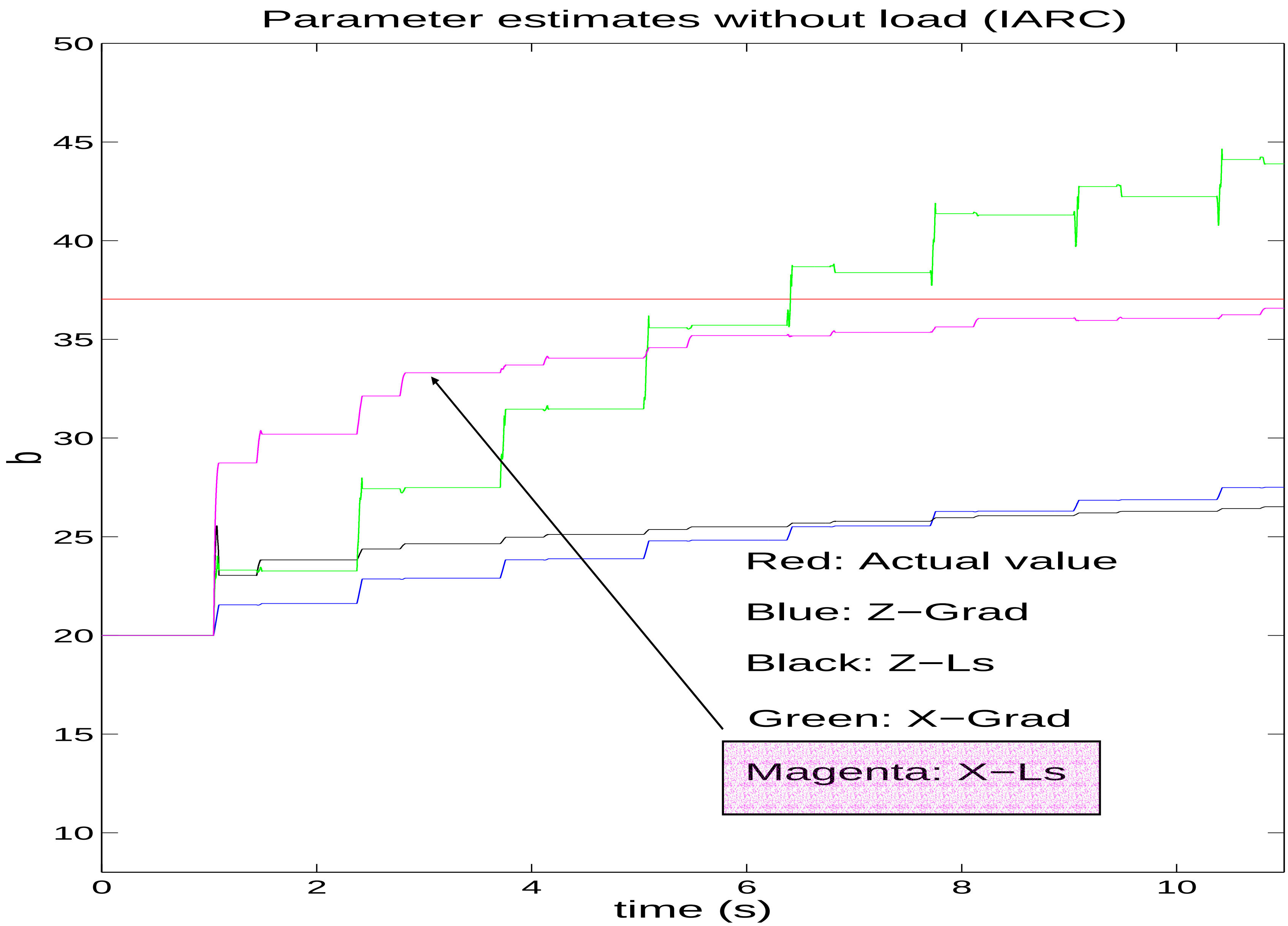


Parameter Estimations of IARC (No Load)



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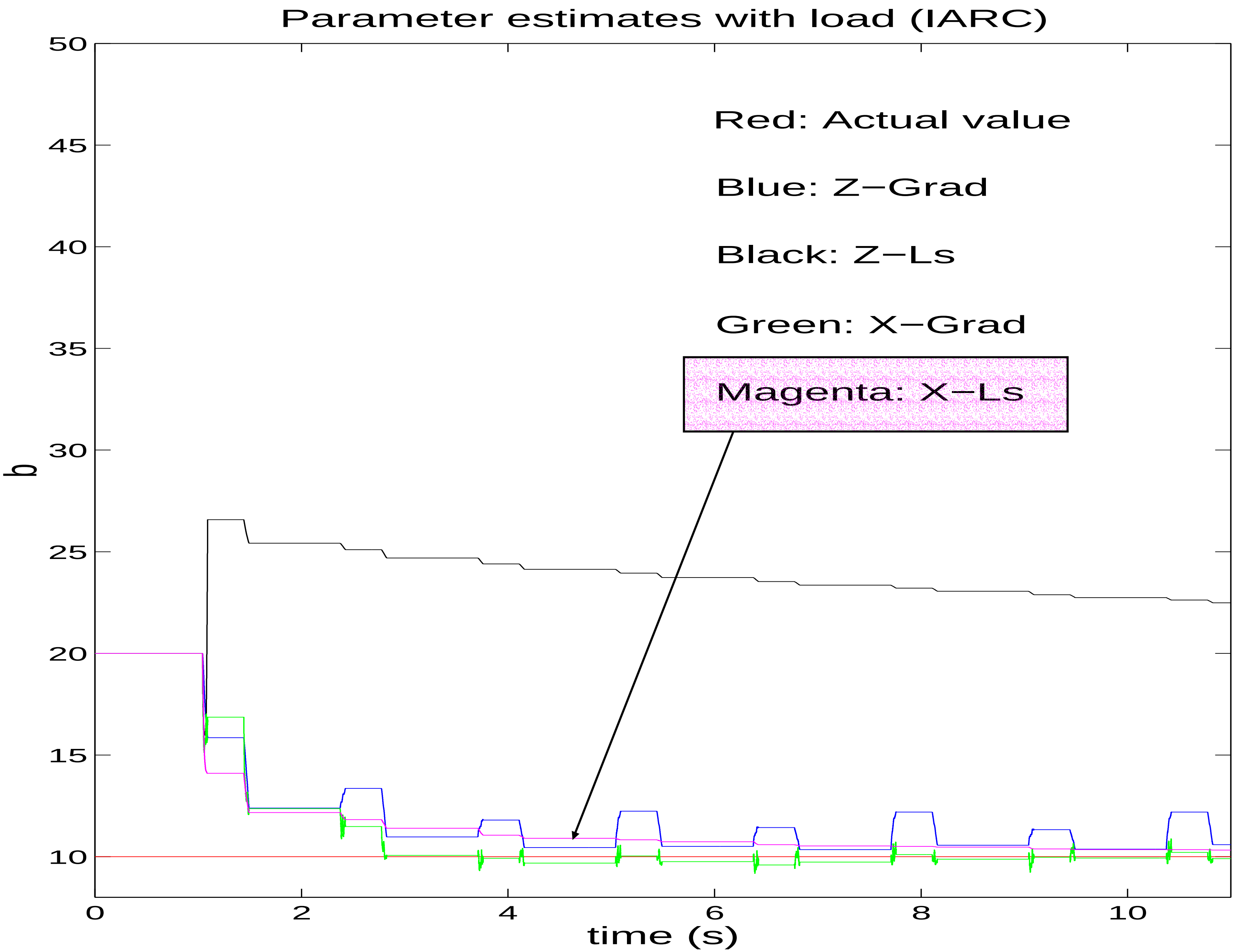

Desirable Features of IARC Design

- Accurate Parameter Estimates
  - **The design of control law and parameter estimation law are totally separated – no more one stone two birds problem.**
  - **The parameter estimation law allows for estimation algorithms with faster parameter convergence properties.**
  - **Driving signal is based on actual system dynamics, not tracking error dynamics, which is less sensitive to neglected factors.**
  - **Explicit monitoring of signal excitation level can be used to improve parameter convergences.**

Conclusions

- A Theoretical Framework for IARC is established
  - **Adopts a more practical and cleaner interface in separating the design of control law and parameter estimation law than traditional indirect adaptive control designs – the use of rate-limited projection type adaptation law structure with the adaptation rate-limit being pre-set by practicing engineers for a more controlled adaptation process**
  - **Theoretically guarantees transient and final tracking accuracy for both parameter variations and disturbances, while preserving the ideal asymptotic output tracking performance of adaptive designs.**
- Experimental Results verify both the excellent output tracking performance as well as the improved parameter estimations of IARC

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Mathematical Model

$$\dot{x}_1 = x_2$$

$$M\dot{x}_2 = u - Bx_2 - F_{fn} - F_r + F_d$$

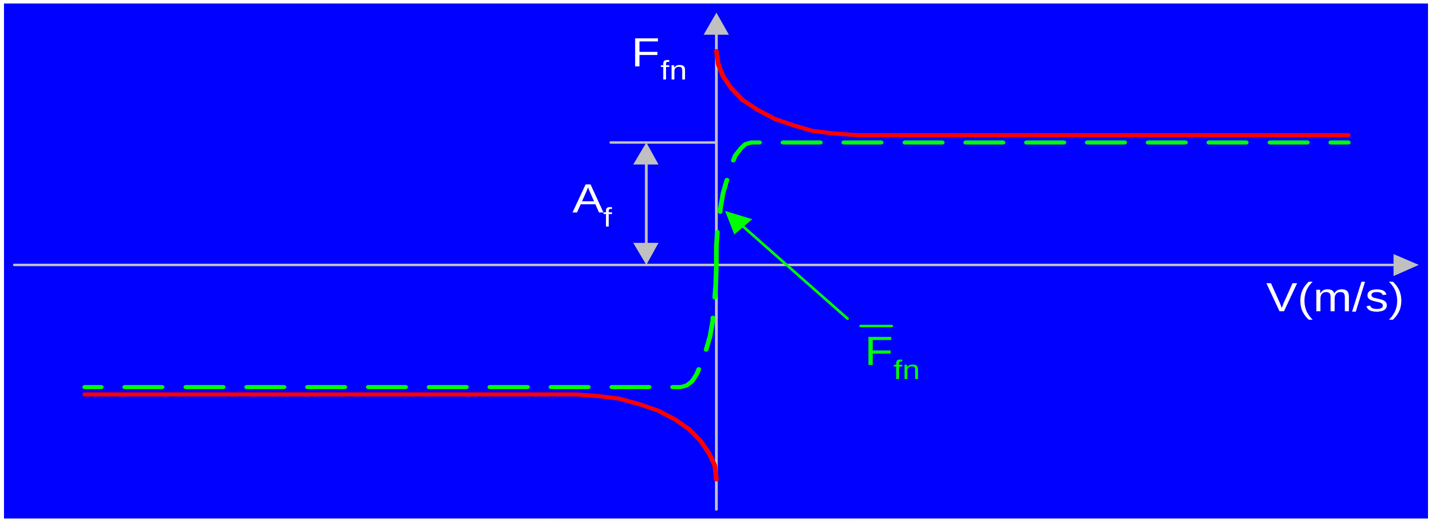
$$y = x_1$$

where

x_1	: position	M	: mass of load
x_2	: velocity	u	: input voltage
y	: output	B	: viscous friction const
F_{fn}	: nonlinear friction	F_r	: force ripple
F_d	: lumped disturbance		

Model of Friction Force

- F_{fn} is discontinuous at zero velocity



- A continuous friction model $\overline{F}_{fn}$ is used to approximate F_{fn}

$$\overline{F}_{fn}(x_2) = A_f S_f(x_2)$$

ARC Controller Design Model

$\Rightarrow$ $$M\dot{x}_2 = u - Bx_2 - A_f S_f(x_2) + d$$

where

$$d = (\overline{F}_{fn} - F_{fn}) - F_r + \Delta$$

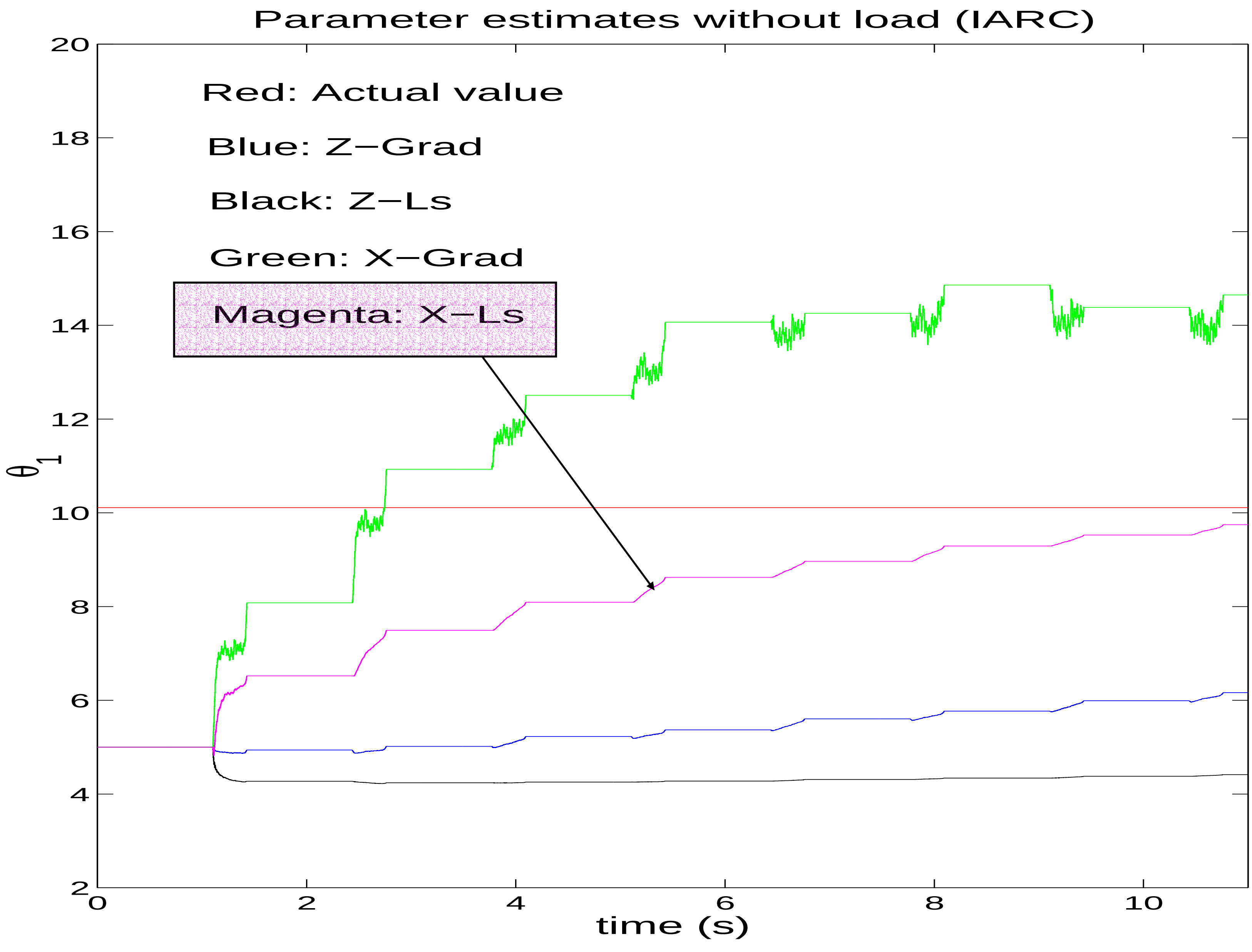
$\Rightarrow$ $$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= bu + \varphi^T\theta + \Delta \\ y &= x_1 \end{aligned}$$

In semi strict feedback from

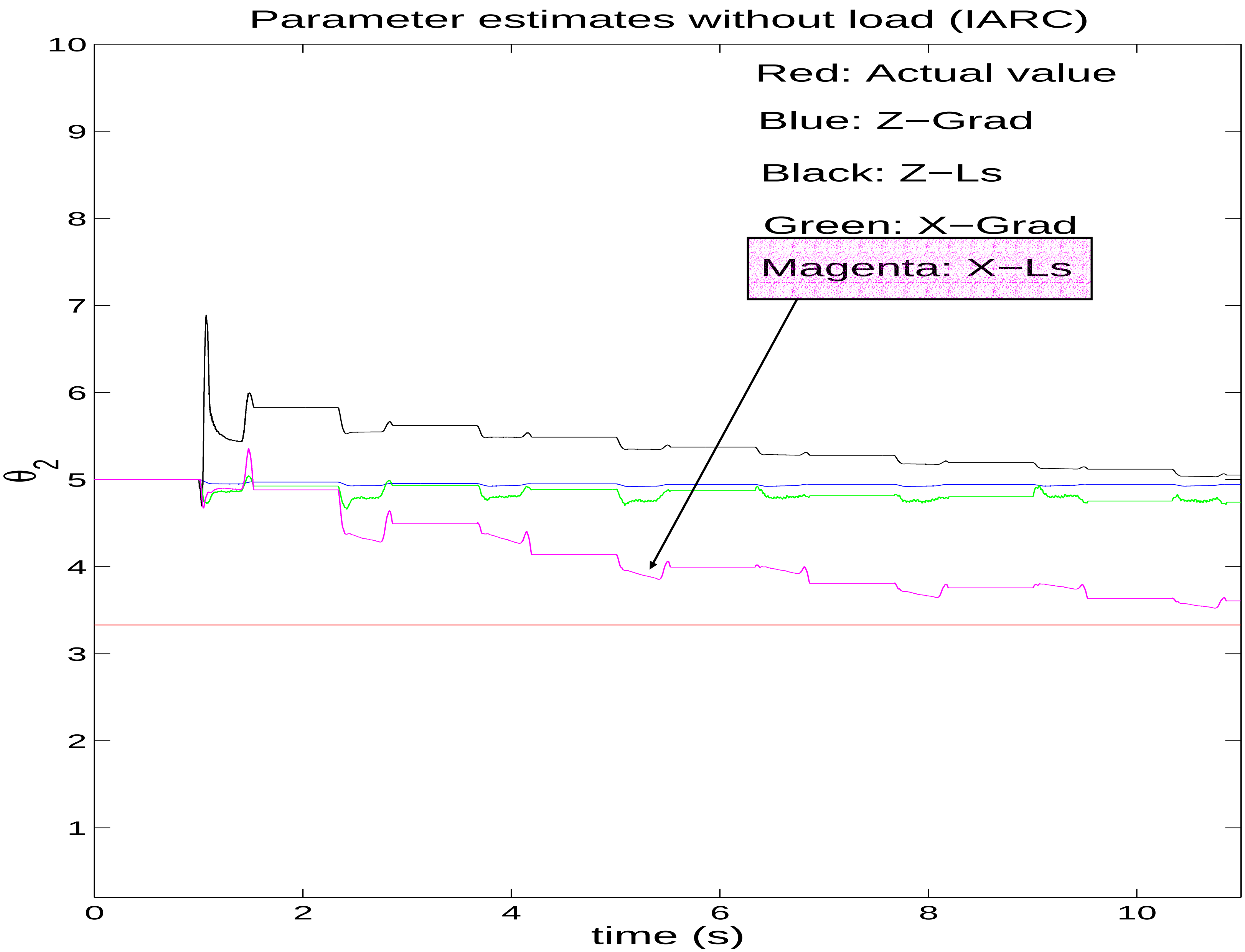
where

$$b = 1/M, \quad \theta = \begin{bmatrix} B/M & A_f/M & d_n/M \end{bmatrix}, \quad \Delta = \tfrac{1}{M}\tilde{d} = \tfrac{1}{M}[d - d_n]$$

Parameter Estimations of IARC (No Load)



Parameter Estimations of IARC (No Load)



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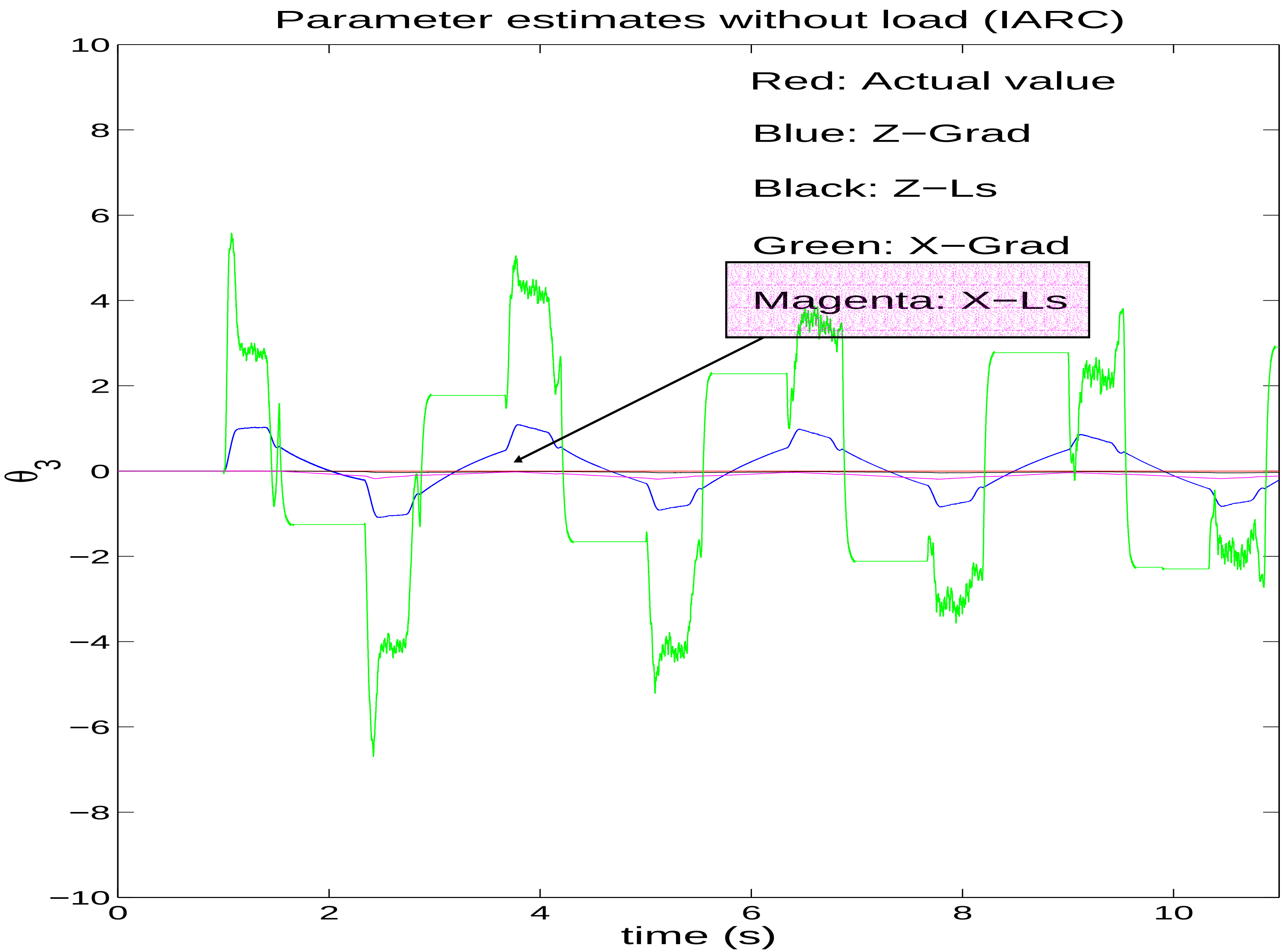

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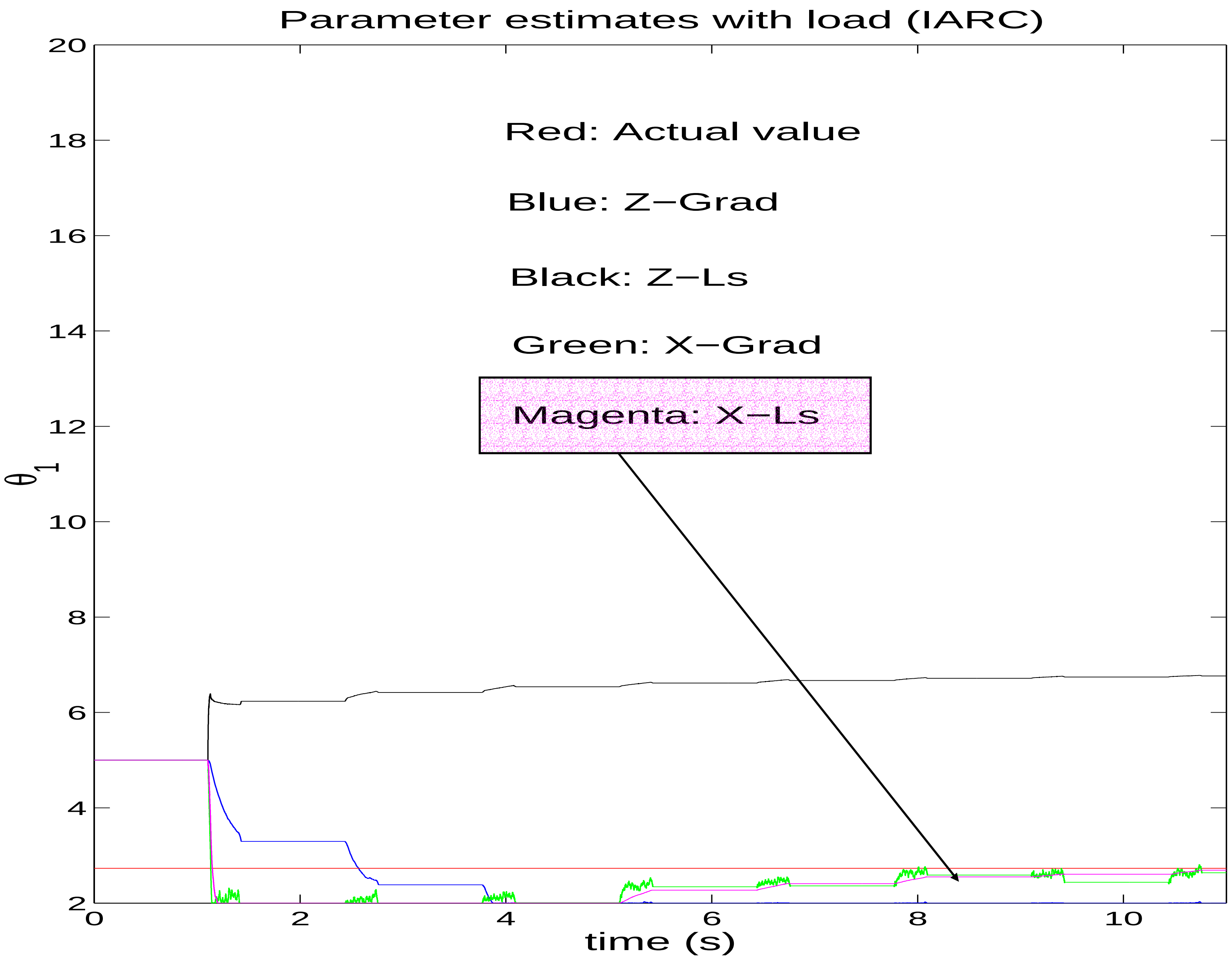

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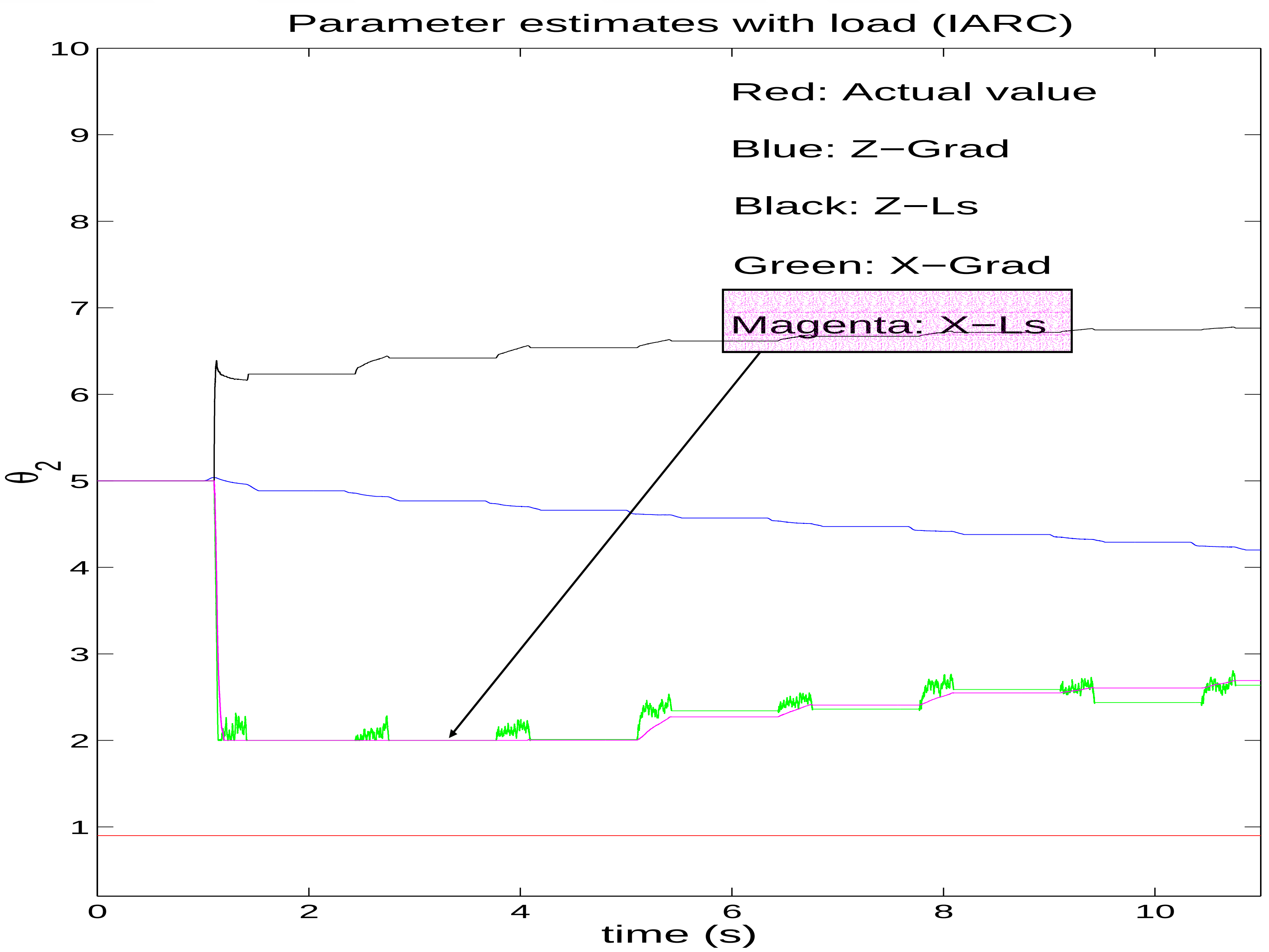

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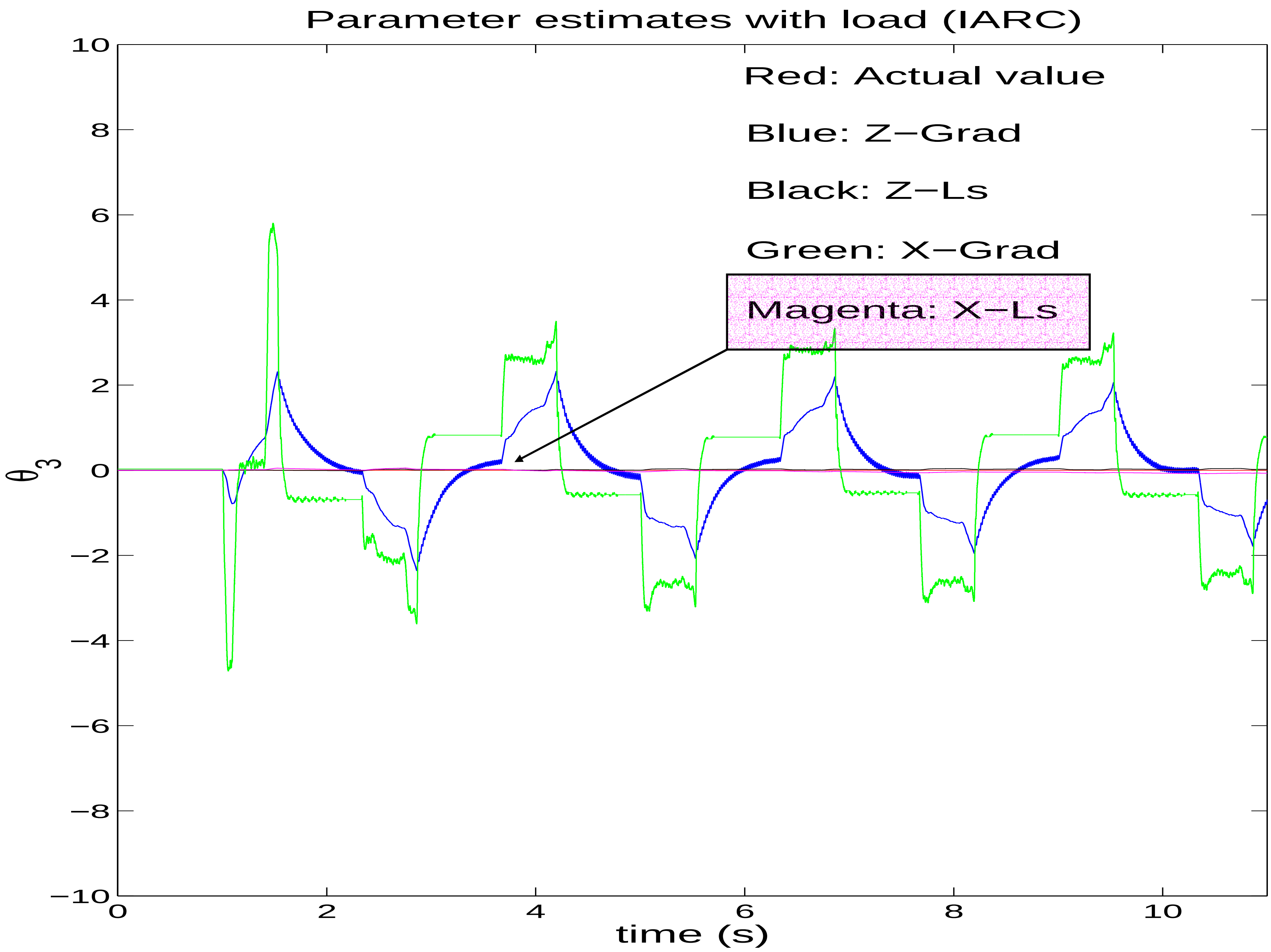

Parameter Estimations of DARC (Unloaded)