Nonlinear Adaptive Robust Control
– Theory and Applications

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OUTLINE

- Motivation
- General ARC Design Philosophy and Structure
- Specific Design Issues
- Application Examples
  - Motion and Pressure Control of Electro-Hydraulic Systems
  - Precision Motion Control of Linear Motors
  - Dynamic Friction Compensation
- Conclusions

Obstacles for Control Algorithms Design

- Inherent *Nonlinearities*
  - Friction forces; Nonlinear process dynamics; …
- Modeling *Uncertainties*
  - Unknown but reproducible or slowly changing terms

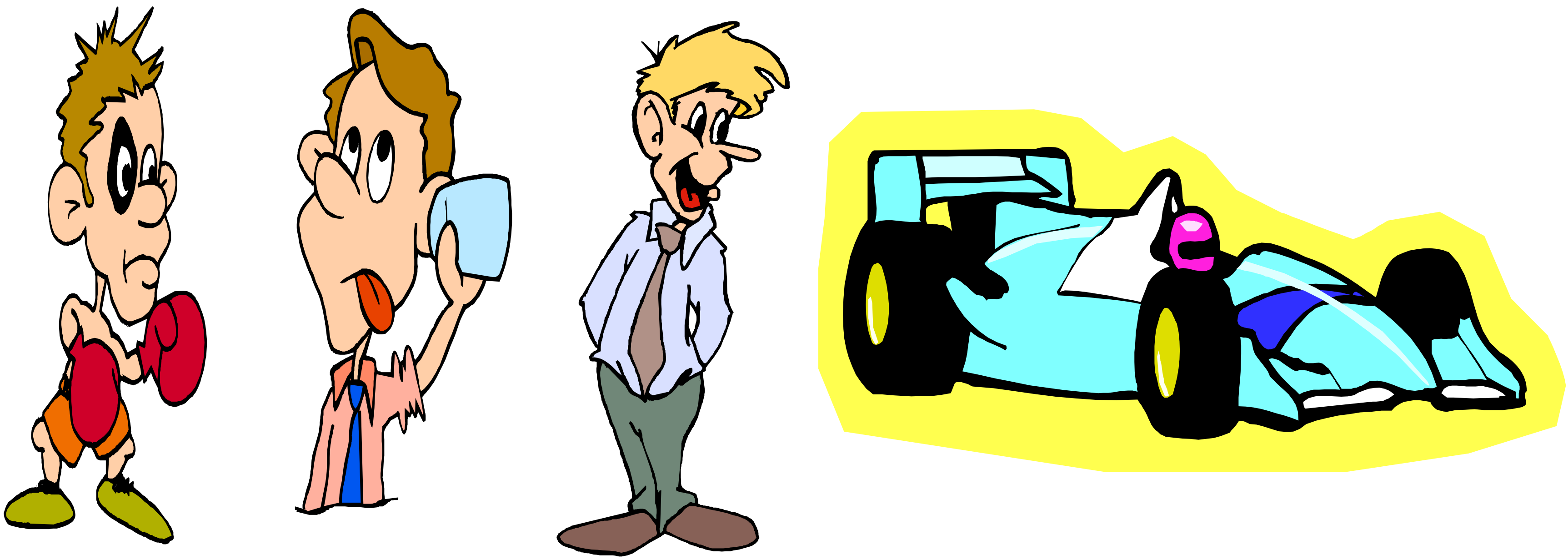
    (e.g., unknown parameters, repeatable run out, …)
  - Non-reproducible terms

    (e.g., random external shock disturbances, non-repeatable run out)

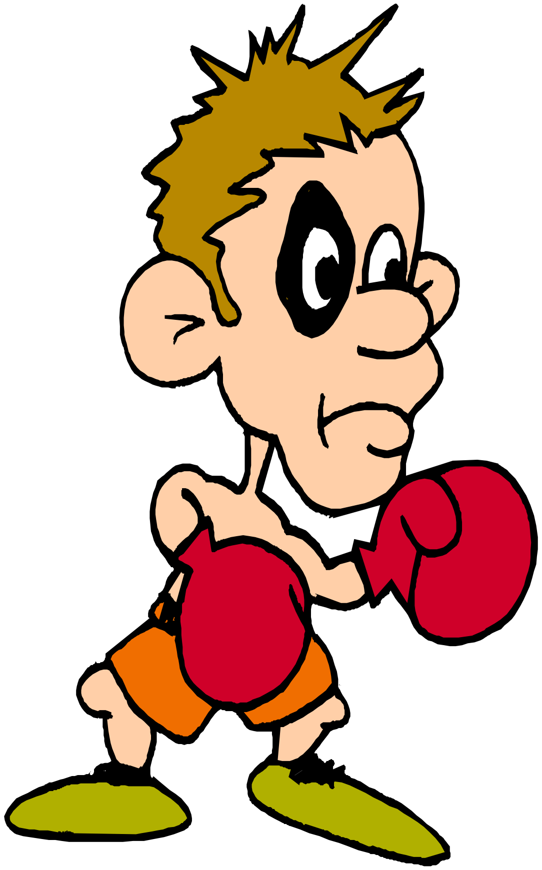
Strategies For Performance Improvement

- Nonlinear Physical Model Based Analysis and Synthesis
  - Deal with the inherent physical nonlinearities directly
- Fast Robust Feedback for Maximum Attenuation of Various Uncertainty Effect
  - Effective to both repeatable and non-repeatable uncertainties
- Controlled Learning for Uncertainty Reduction
  - Effective handling of repeatable uncertainties

Different Types of Race Car Drivers



Mr. Robust Control (DRC)



A Boxer

Fast Instantaneous Reaction !

Mr. Adaptive Control (AC)

A Thinker

Good Learning Ability

but

Not so Fast Instantaneous Reaction !

Mr. Adaptive Robust Control

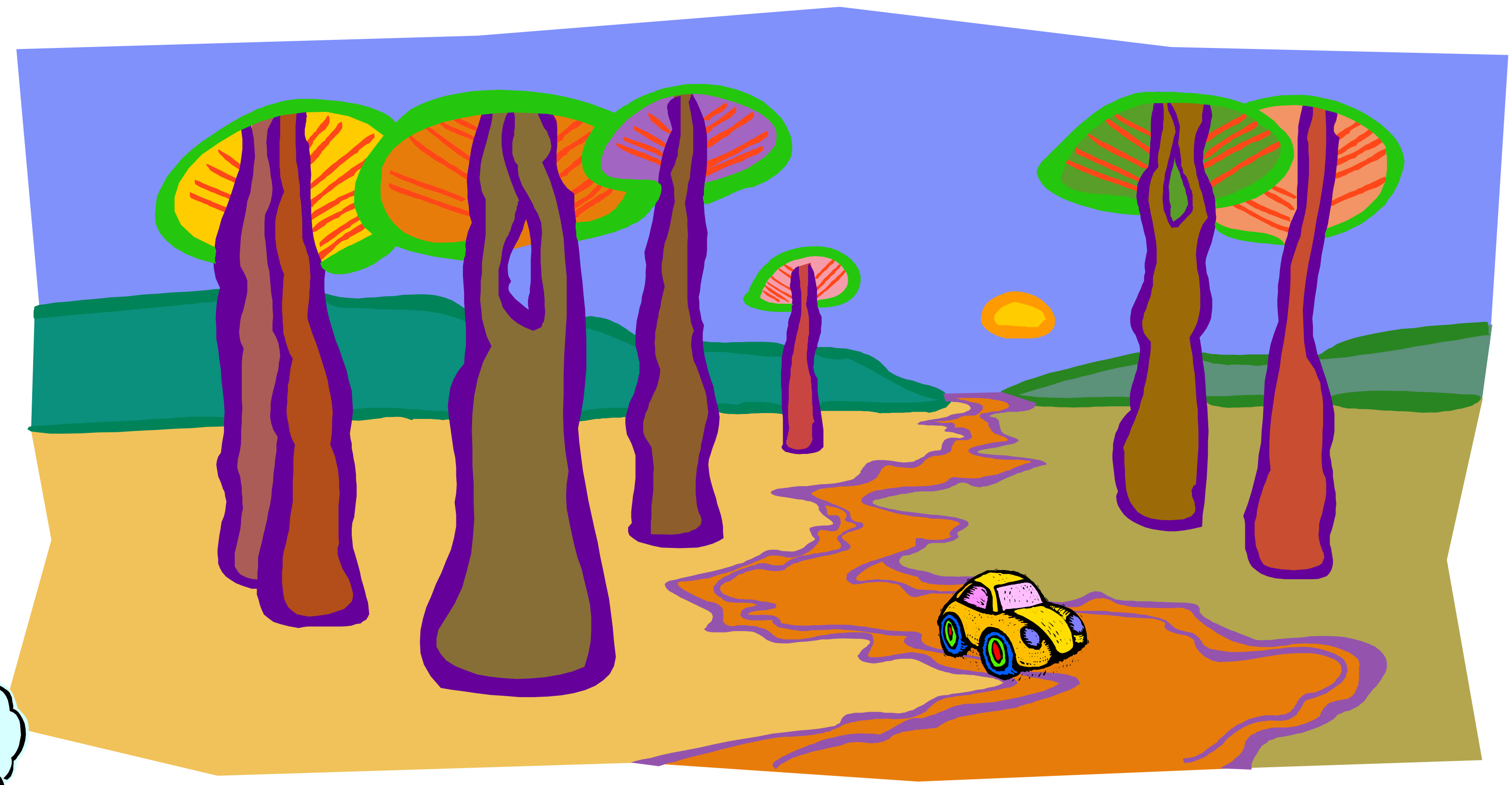
A Thinker with a good body

Good Learning Ability

and

Fast Instantaneous Reaction !

Drive to Yellow Stone National Park



What is the order of arrivals of three drivers ?

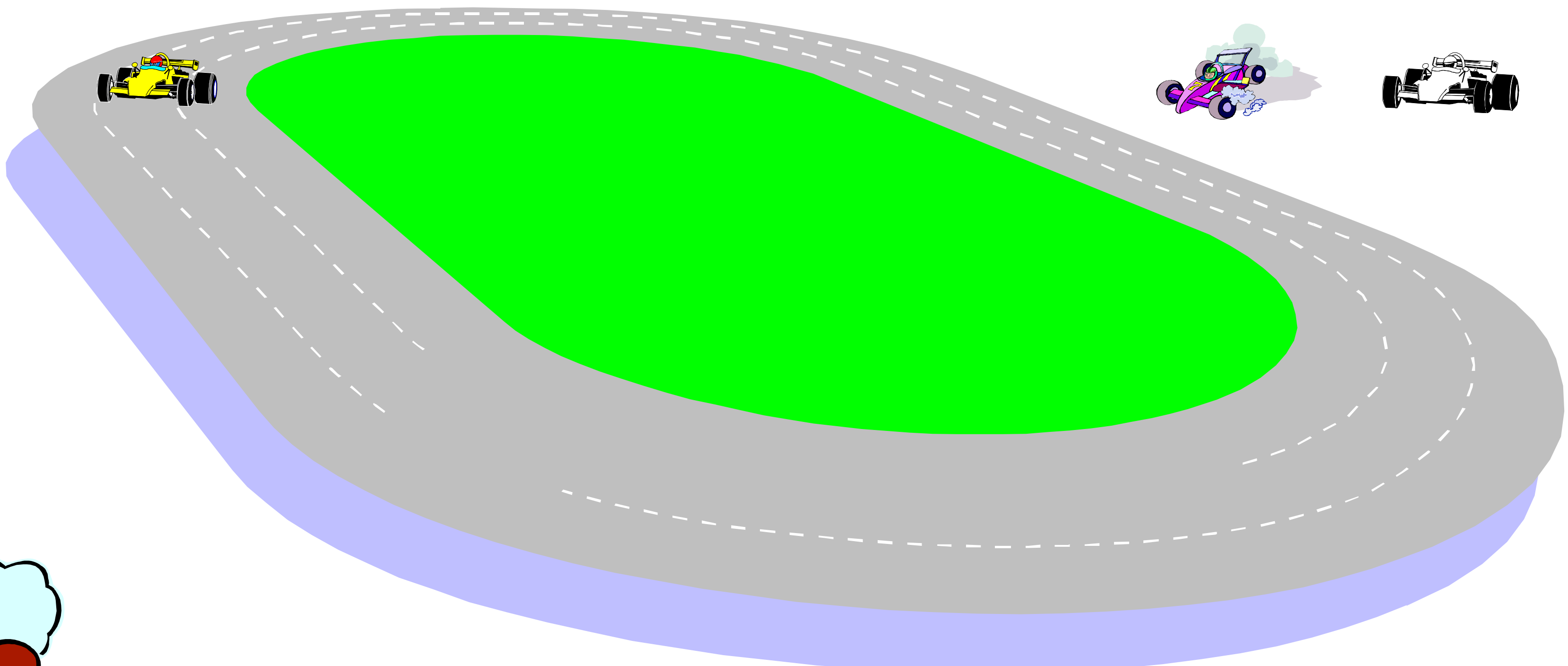
Drive to Yellow Stone National Park

Random Road Profile

Order of arrivals of three drivers:

1. ARC	or	1. ARC/DRC
2. DRC		
3. AC		3. AC

The INDY 500



What is the winning order of three drivers in individual practices ?

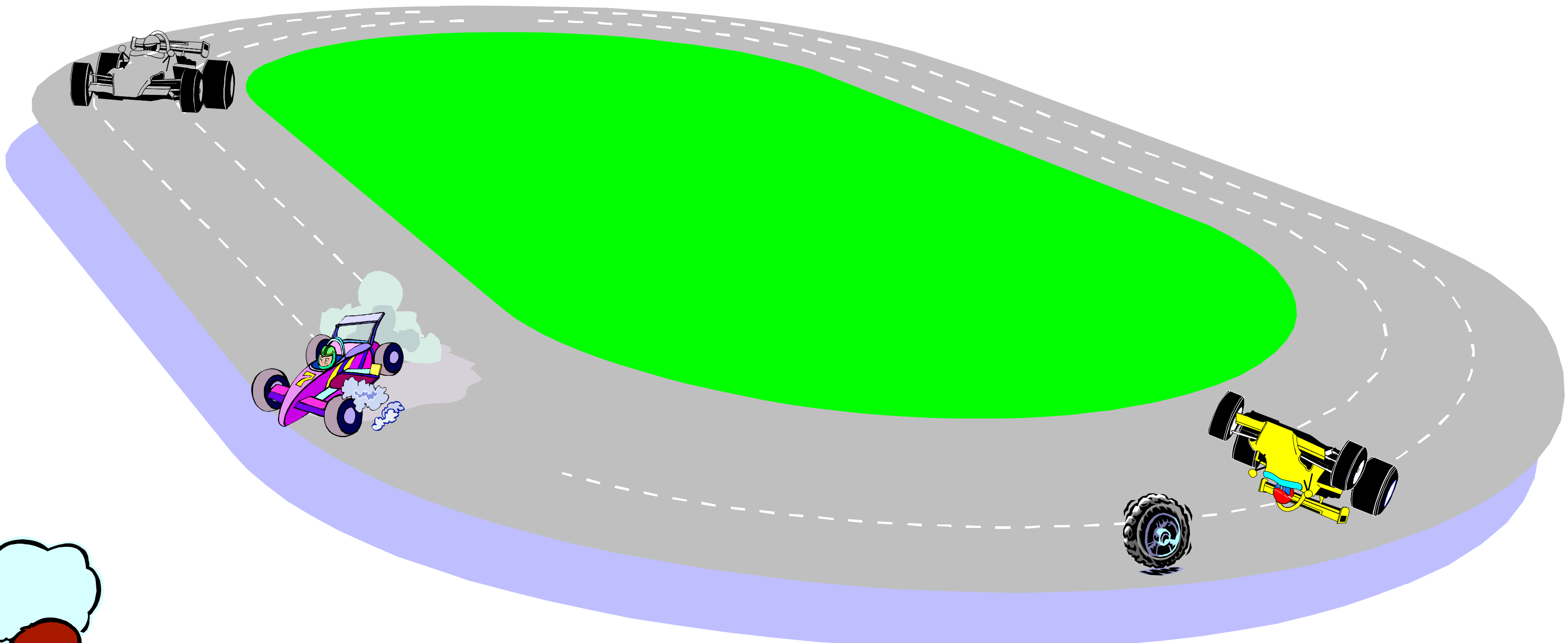
The INDY 500

Repeatable Road Profile

Winning order of drivers:

1. ARC
2. AC
3. DRC

The INDY 500



What is the winning order of three drivers in actual competition ?

The INDY 500

Semi-Repeatable Driving Condition

Winning order of drivers:

1. ARC

2. AC or **2. DRC**

3. DRC **3. AC**

DETERMINISTIC ROBUST CONTROL (DRC)

- **Sliding Mode Control (SMC)**

  (Utkin'77, Young, Slotine, Sastry, Hedrick, Zinober, Zak, …)

  - Matching Condition
  - Control Chattering Due to Discontinuous Control Law
  - Smoothing Techniques (Slotine'85)
    - Asymptotic Tracking is lost and a trade off exists between the actuator requirements and the achievable tracking accuracy.

- **Lyapunov Function Based Min-Max Methods**

  (Leitmann, Corless and Leitmann'81, Barmish, Chen, ...)

  - Unmatched Uncertainties (Qu'93, Freeman and Kokotovic'93,…)

ADAPTIVE CONTROL OF LINEAR SYSTEMS

- **Stable Adaptive Control** (late 1950-1980) (Astrom, Narendra,...)
- **Robust Adaptive Control** (1980s)

  To achieve robustness to small disturbances, time-varying parameters and unmodeled dynamics (Rohrs, et al, 1985, Egardt, 1979)

  - Use Appropriate Reference Input

    Persistent Excitation (PE) Conditions (Boyd and Sastry, Annaswamy,…)

  - Robustify Adaptation Law

    (i) Dead zone; (ii) σ -modification (Ioannou and Kokotovic, 83);

    (iii) ε -modification; (iv) Discontinuous Projection (Sastry, Goodwin,…)

- **Recent Trends**
  - Improve Transient Performance (Zhang and Bitmead, 90)

    Using VSC (Fu,92, Narendra and Boskovic,87), (Datta,93)

  - Relax Assumptions

    Minimum Phase; Relative Degree; Sign of high-frequency gain; Order.

ADAPTIVE CONTROL OF NONLINEAR SYSTEMS

- **Adaptive Control of Robot Manipulators** (1985-present)
  (Slotine and Li'88, Sadegh and Horowitz'90, Spong'89…)
- **Adaptive Control of Feedback Linearizable Systems**
  --Certainty-Equivalence Based (1987-early 90s)
  - Nonlinearity-Constrained Schemes (Sastry and Isidori'89, …)
  - Uncertainty-Constrained Schemes --extended matching condition
- **Systematic Design Method--Backstepping** (1990-present)
  Parametric-strict Feedback Form (Kanellakopoulos, Kokotovic, and Morse'91)
  - Without Overparametrization (Krstic, et al'92)
  - Improved Transient Performance (Kanellakopoulos, et al'93)
- **Robust Adaptive Control of Nonlinear Systems**
  Robot (Reed and Ioannou'89, …); **Backstepping** (Polycarpou and Ioannou'93, Pan and Basar'96, Freeman, et al'96, Marino and Tomei'98, …)
  - **Robust stability; Achievable performances in terms of L_∞ norm are not so transparent**

NONLINEAR ADAPTIVE CONTROL (AC)

- Reduce/Eliminate uncertainties through parameter adaptation to achieve *asymptotic tracking*.

Limitations:

- Need certain invariant properties (e.g., parameters being unknown but *constant*).
- Possible *instability* in the presence of even small measurement noises and disturbances

DETERMINISTIC ROBUST CONTROL (DRC)

- Attenuate the effect of model uncertainties through robust feedback.

Limitations:

- limited final tracking accuracy (tracking errors can only be reduced by increasing feedback gains)

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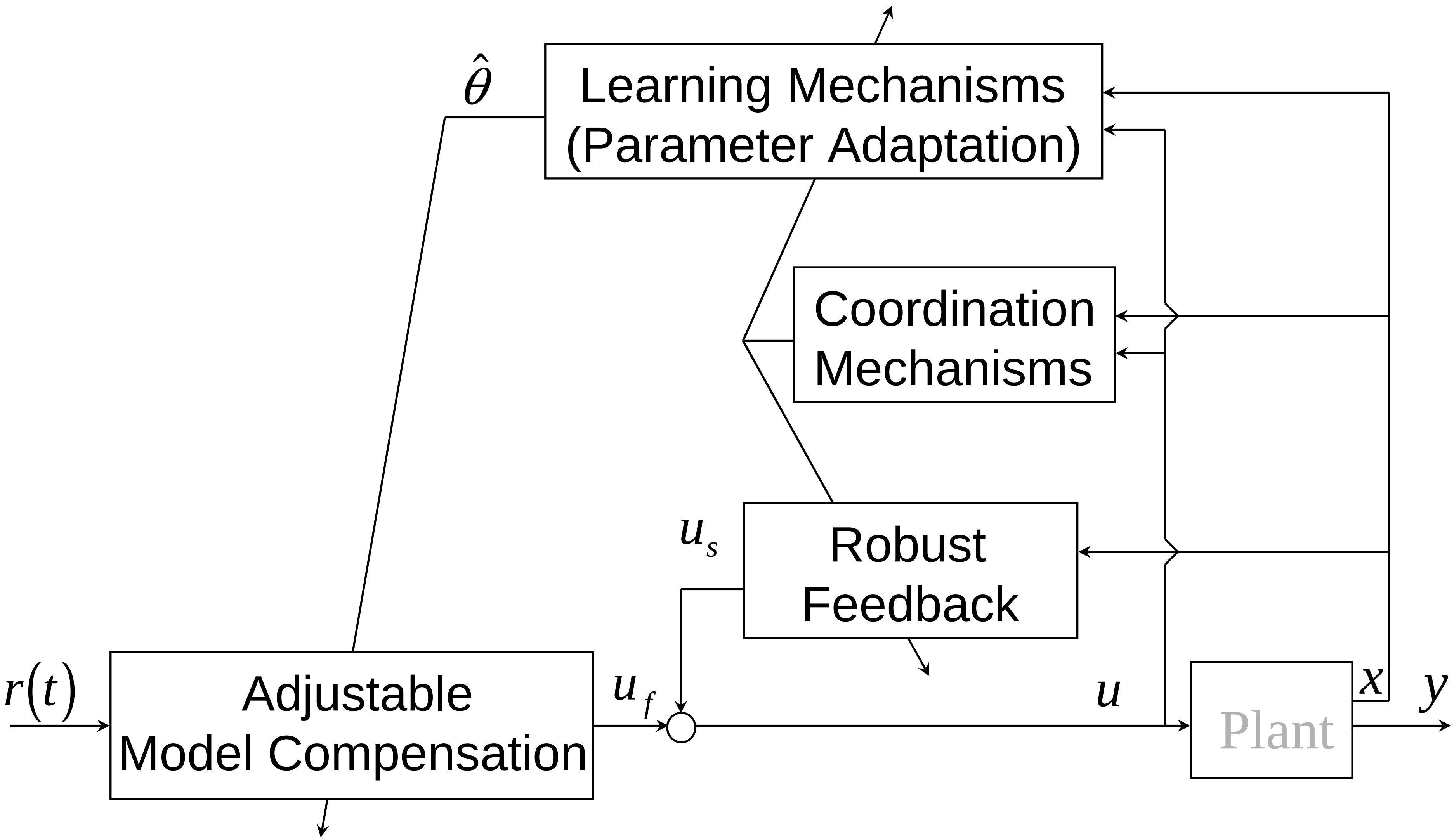
ADAPTIVE ROBUST CONTROL (ARC)

- **Problem Formulation**
  - Practical situation that the system is subjected to both repeatable and non-repeatable uncertainties
- **Means Used To Achieve High Performance**
  - **Fast Robust Filter Structures** to attenuate the effect of model uncertainties as much as possible
  - **Learning Mechanisms** (e.g., parameter adaptation) to reduce repeatable model uncertainties
- **General Design Philosophy**
  - Robust performance provided by robust feedback should not be lost when introducing learning mechanisms; Learning mechanisms are introduced only when their destabilizing effects can be controlled



General Structure of ARC Controllers

FIRST-ORDER UNCERTAIN SYSTEM

$$\dot{x} = f(x,t) + u, \qquad f = \varphi^T(x,t)\theta + \Delta(x,t)$$

θ: unknown parameters

Δ: uncertain nonlinearities

Assumptions

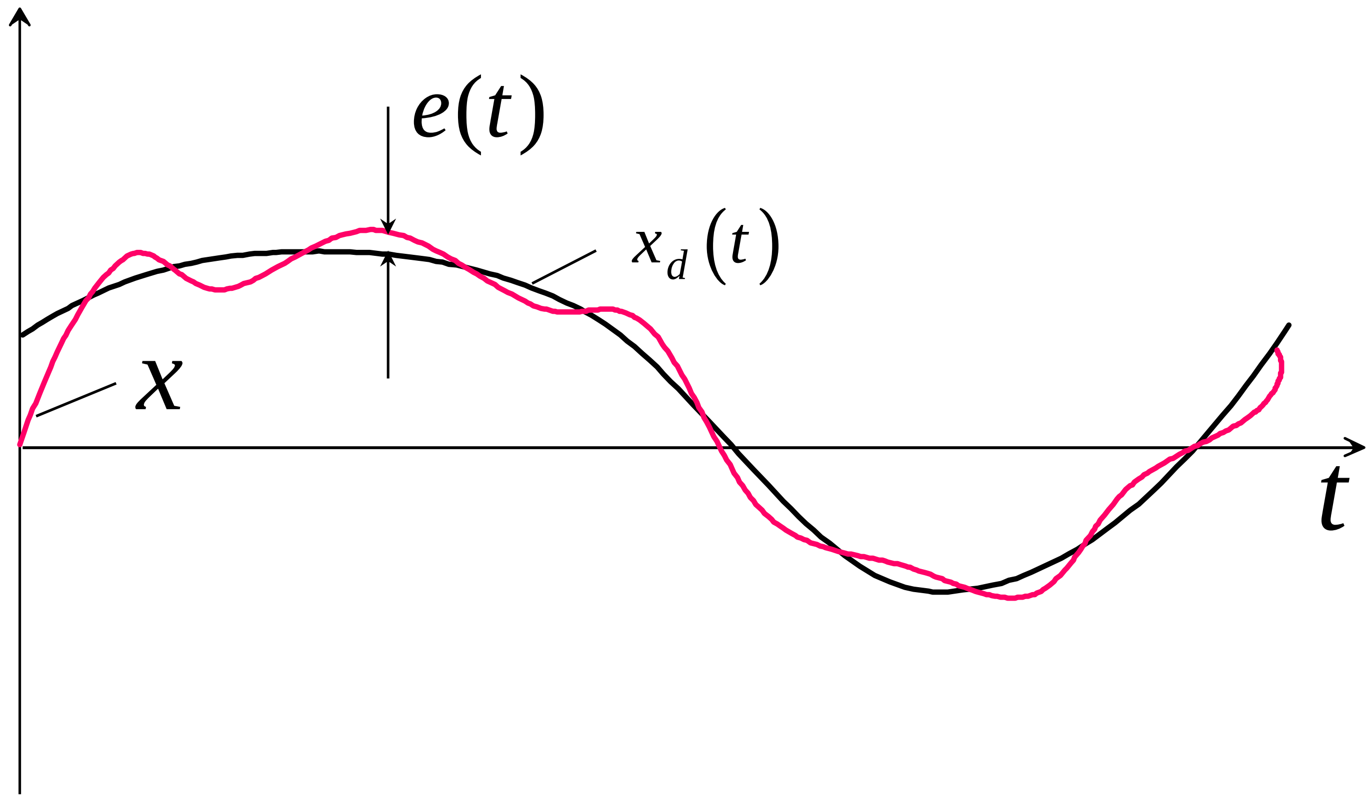
$$\theta \in \Omega_\theta = (\theta_{\min}, \quad \theta_{\max})$$

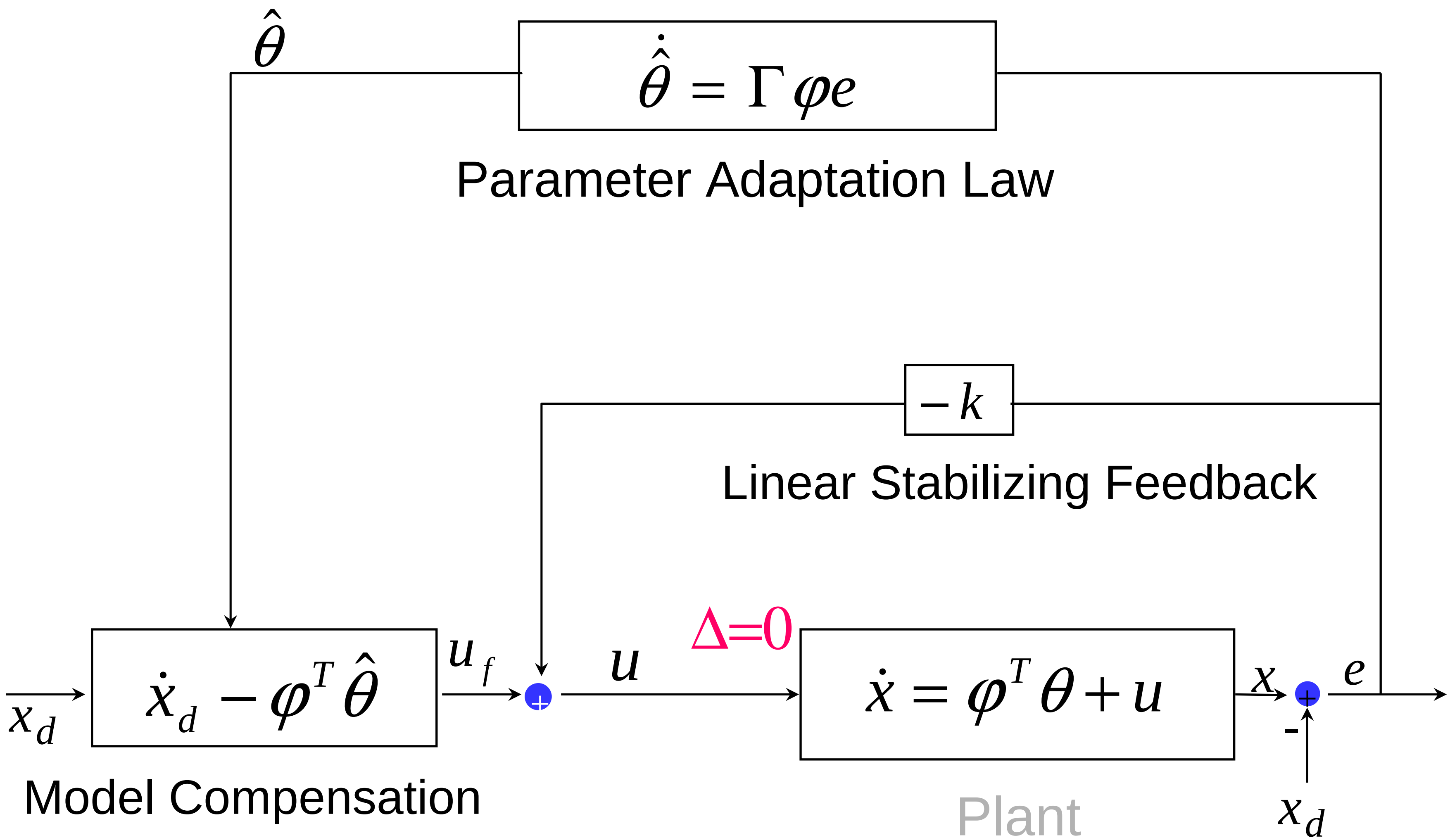
$$|\Delta| \leq \delta(x, t)$$

where Ω_θ and $\delta(x, t)$ are known

OBJECTIVE

For any desired output trajectory $x_d(t)$, design a bounded control input $u(t)$ such that the tracking error, $e = x - x_d$, is as small as possible





Adaptive Control of a First-order Uncertain System

ADAPTIVE CONTROL METHOD

- Assume that there is no uncertain nonlinearity

$$\Delta = 0$$

- On-line parameter adaptation
- Error dynamics

$$\dot{e} + ke = -\varphi^T \tilde{\theta}$$

- Lyapunov function

$$V_a = \frac{1}{2} e^2 + \frac{1}{2\gamma} \tilde{\theta}^2$$

$$\dot{V}_a = e\left[-\varphi^T \tilde{\theta} - ke\right] + \frac{1}{\gamma} \tilde{\theta} \dot{\hat{\theta}} = -ke^2$$

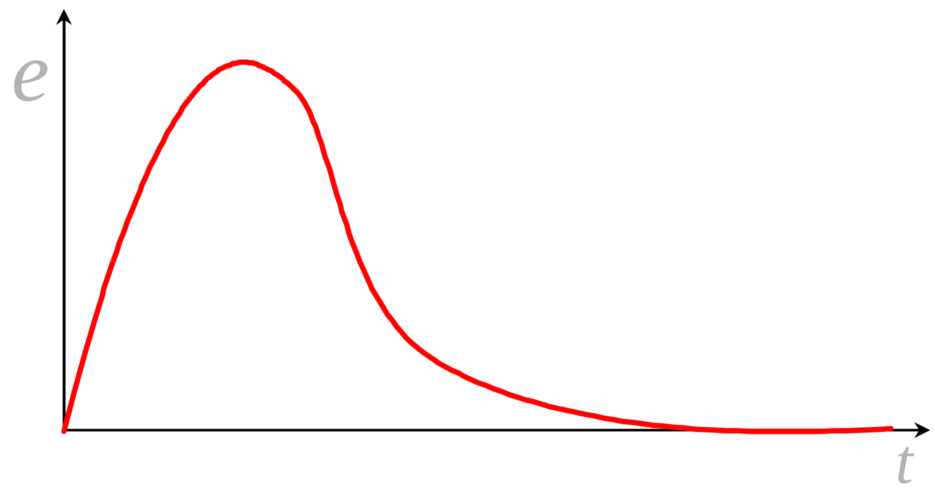
- RESULTS

$$e \to 0 \quad \text{and} \quad \varphi^T \tilde{\theta} \to 0 \quad \text{as} \quad t \to \infty$$

i.e., Zero final tracking error for any feedback gain since the parametric uncertainty is eliminated.

LIMITATIONS OF ADAPTIVE CONTROL

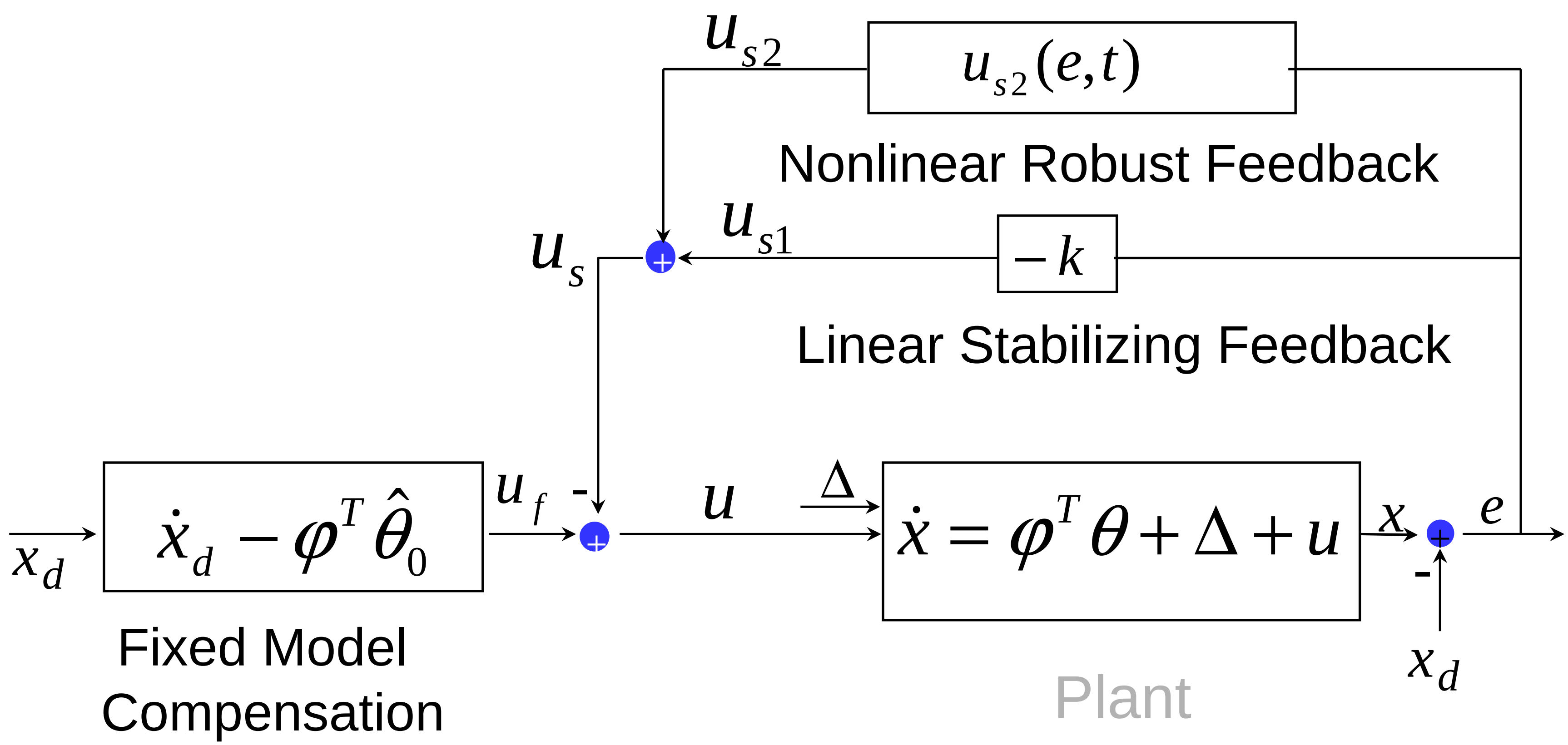
- Transient performance is ***unknown***



- Uncertain nonlinearities are not considered

  When $\Delta \neq 0$, what is the performance ?

- May be ***unstable*** when $\Delta \neq 0$



Deterministic Robust Control of a First-order System

DETERMINISTIC ROBUST CONTROL

- Error dynamics

$$\dot{e} + ke = -\varphi^T \tilde{\theta}_0 + \Delta + u_{s2}$$

- Choose the robust control u_{s2} such that

$$e\left(u_{s2} - \varphi^T \tilde{\theta}_0 + \Delta\right) \le \varepsilon \quad \text{and} \quad \mathrm{eu}_{s2} \le 0$$

where ε is a design parameter.

- Example

$$u_{s2} = -\frac{1}{4\varepsilon}\left[\|\theta_{\max} - \theta_{\min}\|^2 \|\varphi\|^2 + \delta^2\right]e$$

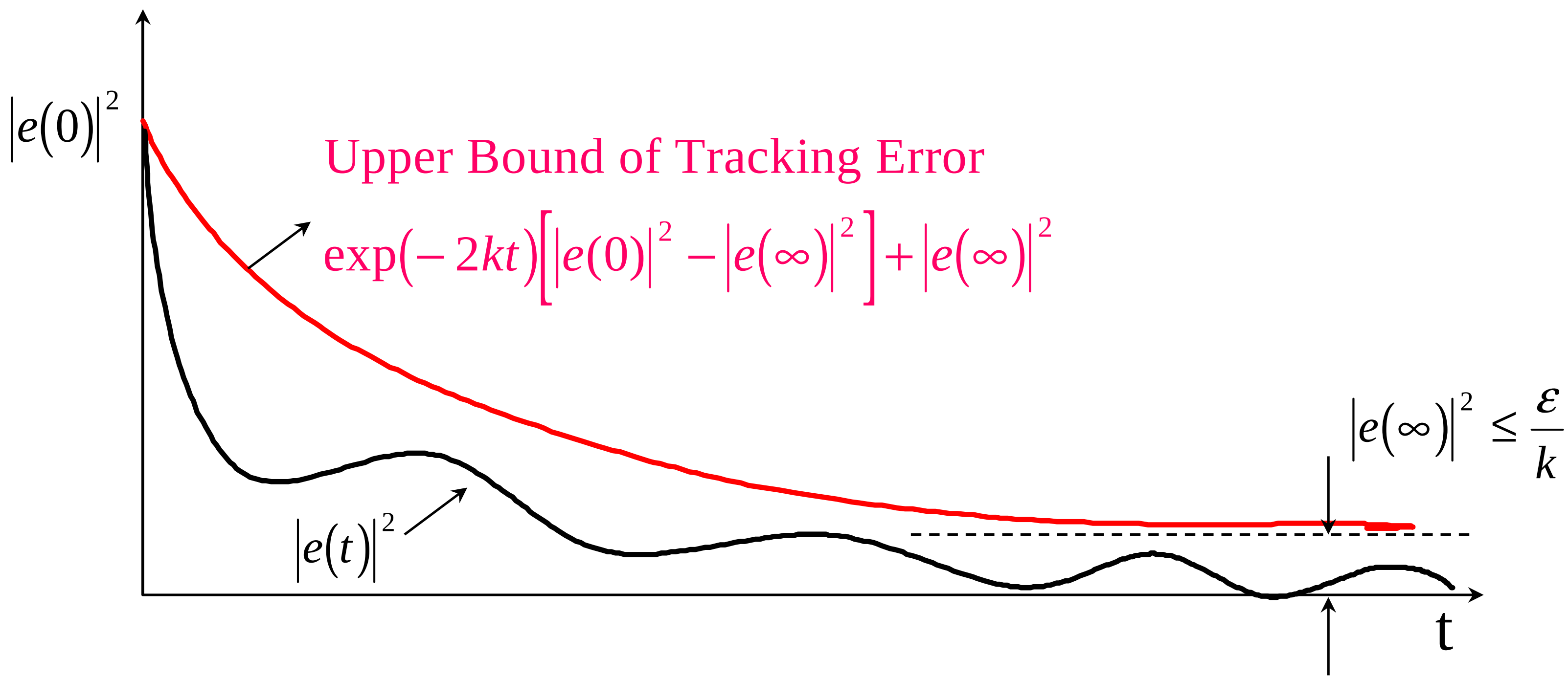
- Stability analysis by a Lyapunov function $V_s = \dfrac{1}{2}e^2$

$$\dot{V}_s \le -2kV_s + \varepsilon$$

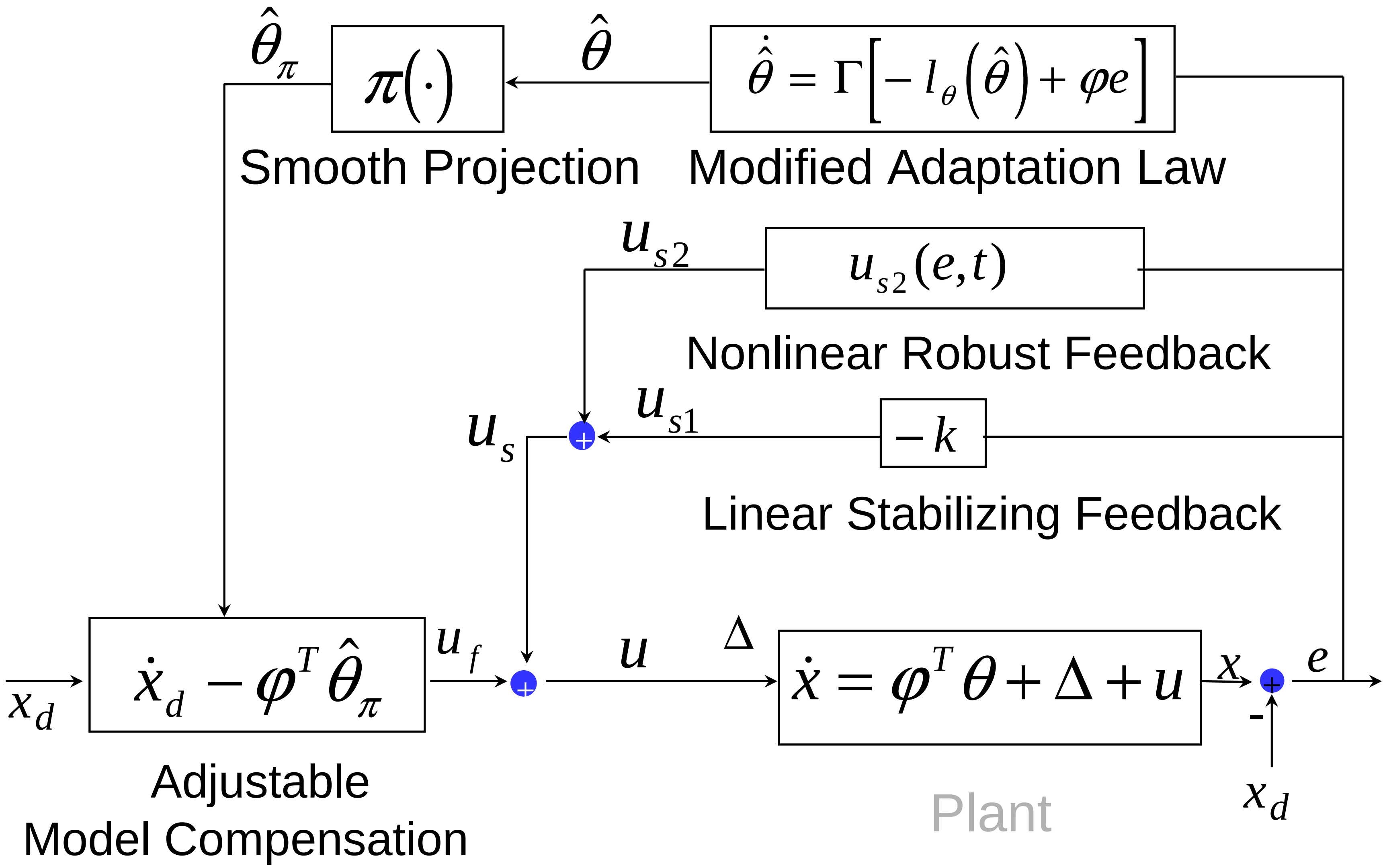
$$V_s \le \exp(-2kt)V_s(0) + \frac{\varepsilon}{2k}\left[1 - \exp(-2kt)\right]$$

DETERMINISTIC ROBUST CONTROL

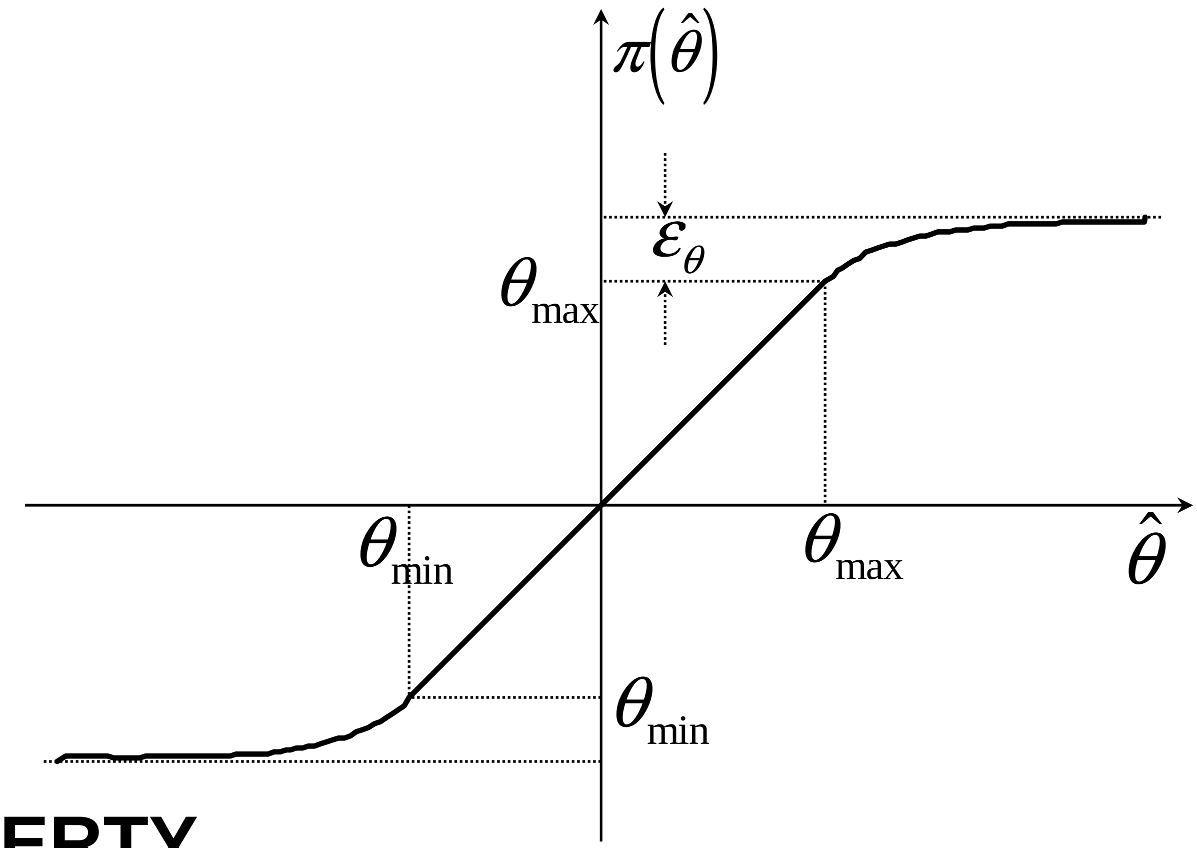
- Guaranteed transient: ***exponential convergence***
- Guaranteed final tracking accuracy:



ARC DESIGN via SMOOTH PROJECTION



SMOOTH PROJECTION



PROPERTY

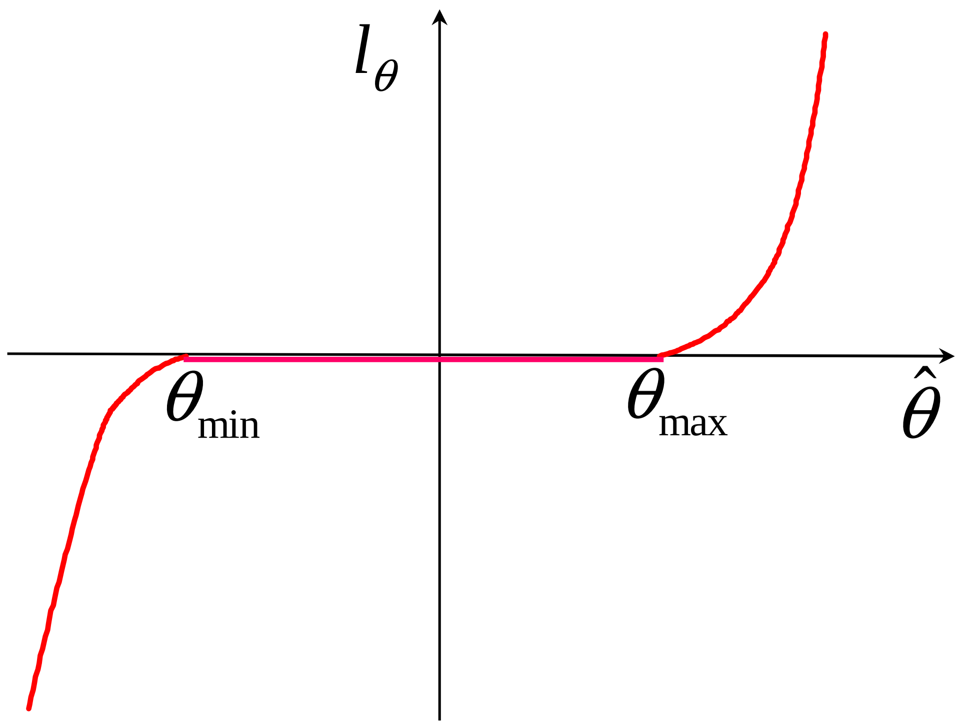
$V_\theta(\tilde{\theta})$ is a positive definite function where (Teel'93)

$$V_\theta(\tilde{\theta}) = \frac{1}{\gamma}\int_0^{\tilde{\theta}} (\pi(\nu + \theta) - \theta) d\nu, \qquad \gamma \geq 0$$

MODIFICATION OF ADAPTATION LAW

l_θ acts as a nonlinear damping and satisfies

$$
\begin{array}{llll}
i. & l_\theta\left(\hat{\theta}\right) = 0 & if & \hat{\theta} \in \Omega_\theta \\
ii. & \tilde{\theta}_\pi^T l_\theta\left(\hat{\theta}\right) \geq 0 & if & \hat{\theta} \notin \Omega_\theta
\end{array}
$$



ARC DESIGN via SMOOTH PROJECTION

- Error dynamics

$$\dot{e} + ke = -\varphi^T \tilde{\theta}_\pi + \Delta + u_{s2}$$

- Choose the robust control u_{s2} such that

i. $e\left(u_{s2} - \varphi^T \tilde{\theta}_\pi + \Delta\right) \le \varepsilon$

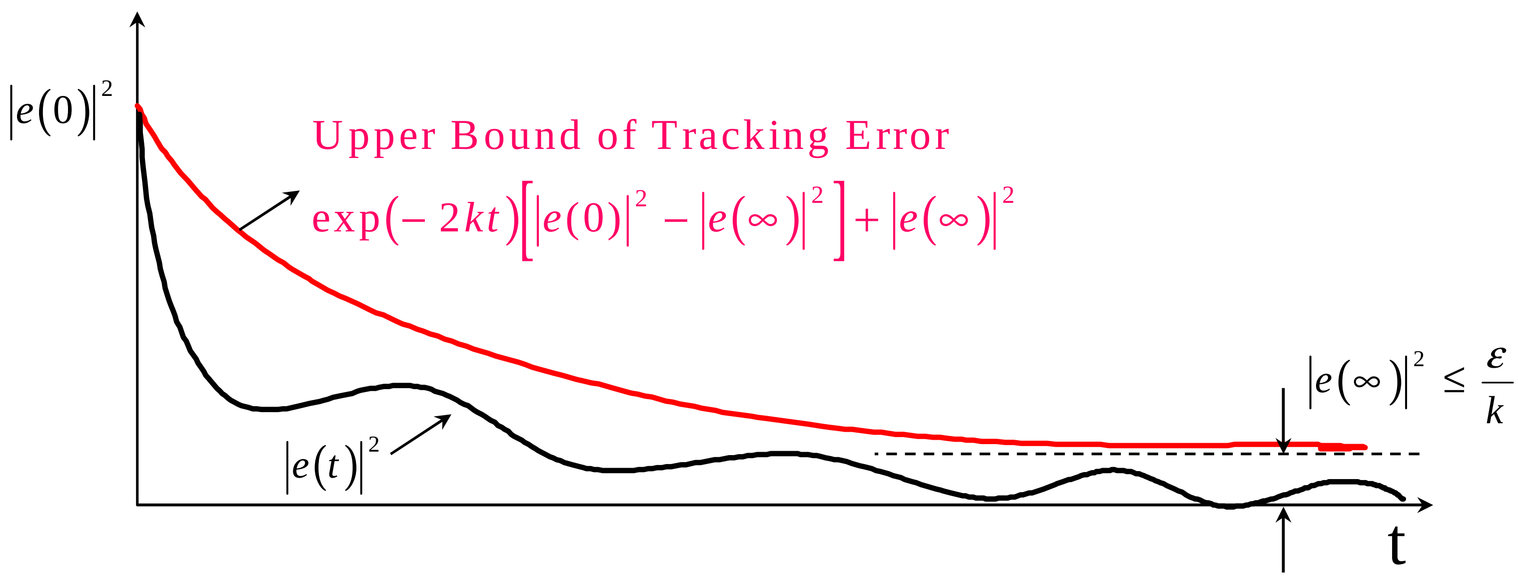
ii. $eu_{s2} \le 0$

- Example

$$u_{s2} = -\frac{1}{4\varepsilon}\left[\left\|\theta_{\max} - \theta_{\min} + \varepsilon_\theta\right\|^2 \left\|\varphi\right\|^2 + \delta^2\right] e$$

PERFORMANCE OF ARC

- In general, achieves the same performance as DRC
  - **Guaranteed transient**: exponential convergence
  - **Guaranteed final tracking accuracy**



- In addition, achieves **asymptotic tracking** in the presence of parametric uncertainties as AC

PROOF of ARC PERFORMANCE

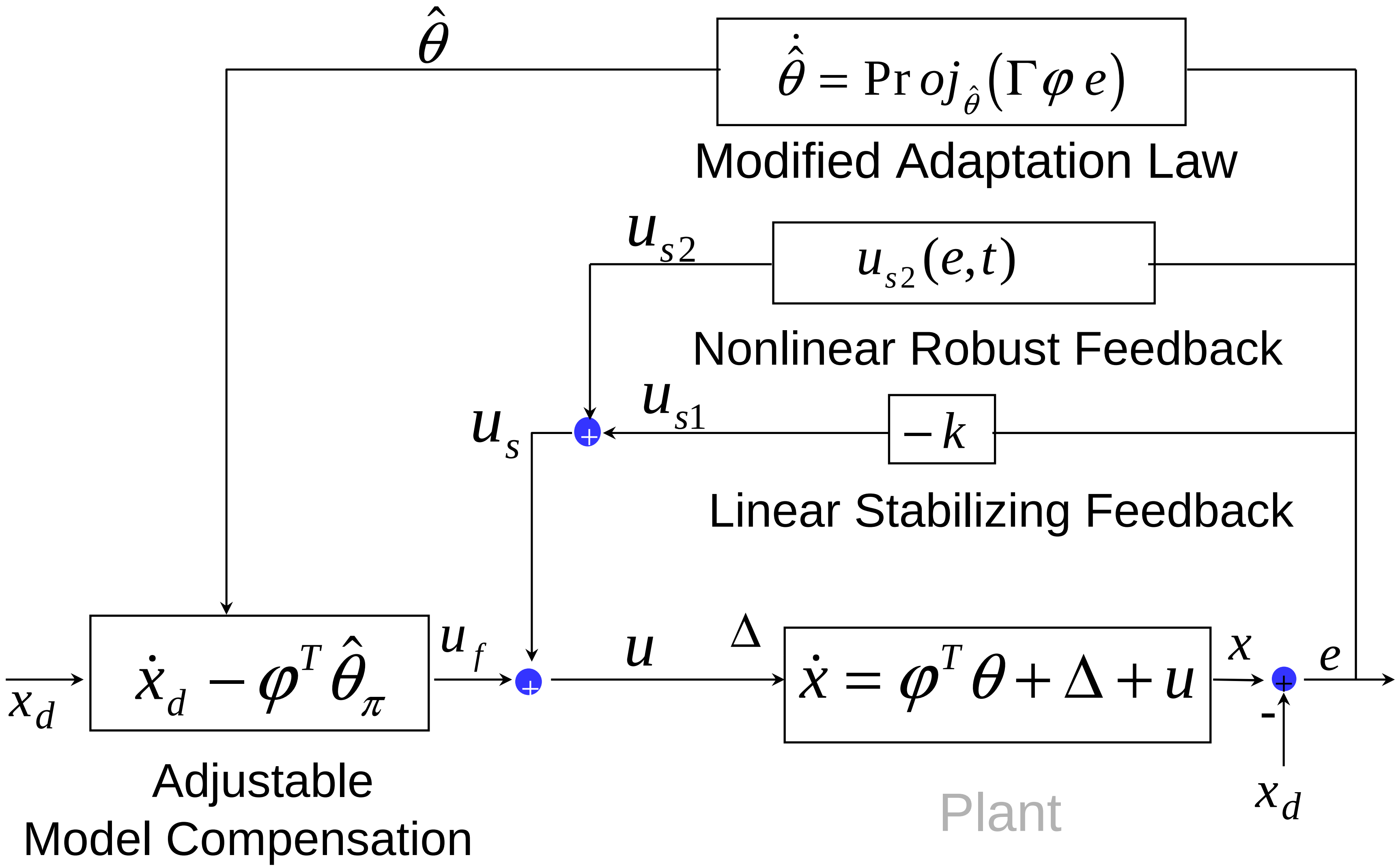
- In general, use the same Lyapunov function V_s as in DRC to recover the results of DRC

$$\begin{aligned}\dot{V}_s &= -ke^2 + e\left(-\varphi^T \tilde{\theta}_\pi + \Delta + u_{s2}\right) \\ &\leq -2kV_s + \varepsilon\end{aligned}$$

- When $\Delta = 0,$ use a new p.d. function $V_t = V_s + V_\theta$ to obtain asymptotic tracking

$$\begin{aligned}\dot{V}_t &= -ke^2 + eu_{s2} - e\varphi^T \tilde{\theta}_\pi + \frac{1}{\gamma}\tilde{\theta}_\pi \dot{\hat{\theta}} \\ &= -ke^2 + eu_{s2} - \tilde{\theta}_\pi l_\theta\left(\hat{\theta}\right) \\ &\leq -ke^2\end{aligned}$$

ARC DESIGN via DISCONTINUOUS PROJECTION



ARC DESIGN via DISCONTINUOUS PROJECTION

- Parameter adaptation with projection

$$\dot{\hat{\theta}} = \mathrm{Pr}\,oj_{\hat{\theta}}\left(\Gamma \varphi e\right)$$

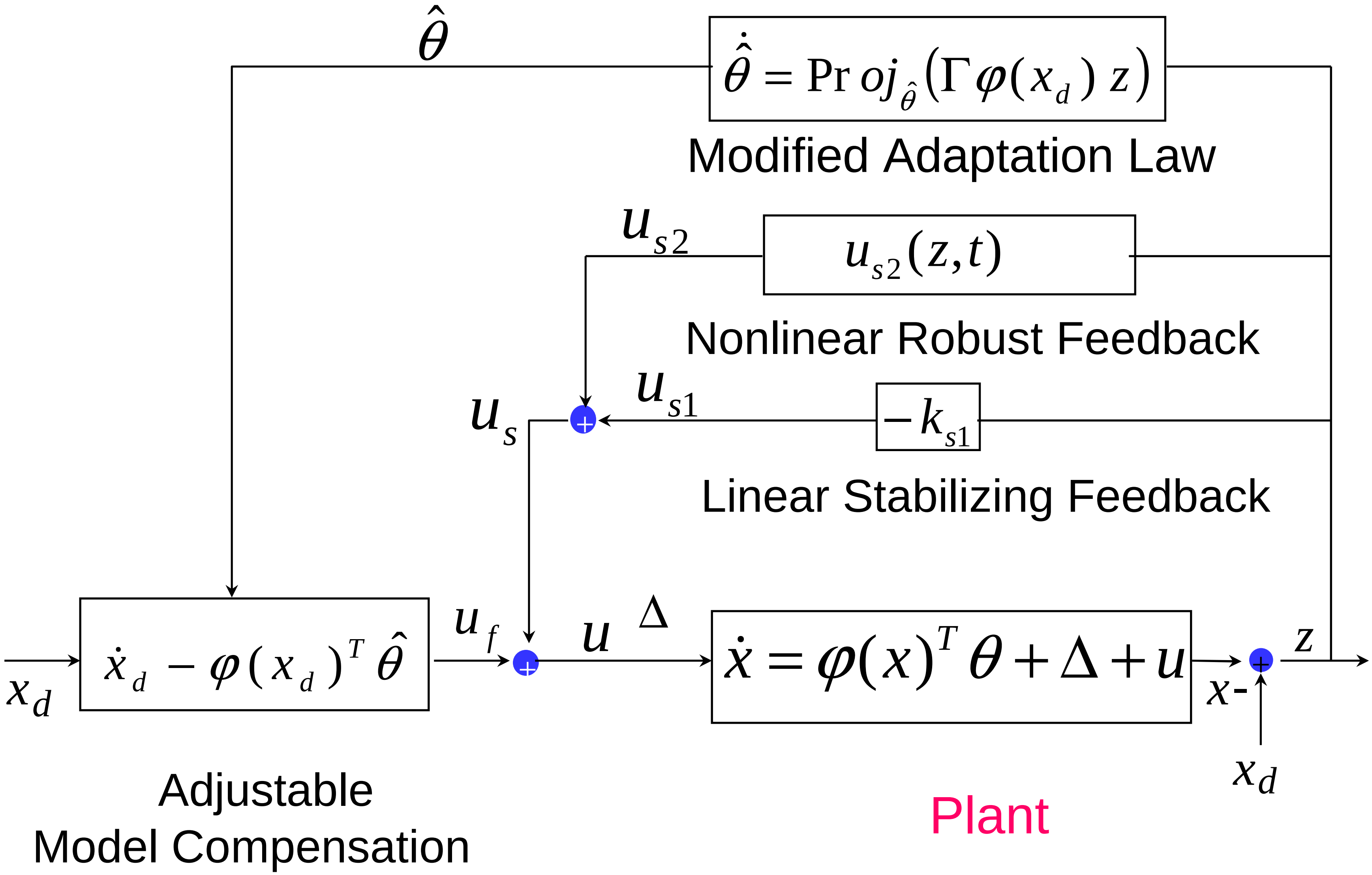
$$\mathrm{Pr}\,oj_{\hat{\theta}_i}\left(\bullet_i\right) = \begin{cases} 0 & \text{if} \quad \hat{\theta}_{\mathrm{i}} = \hat{\theta}_{\mathrm{imax}} \quad \text{and} \quad \bullet_i > 0 \\ 0 & \text{if} \quad \hat{\theta}_{\mathrm{i}} = \hat{\theta}_{\mathrm{imin}} \quad \text{and} \quad \bullet_i < 0 \\ \bullet_i & \quad \text{otherwise} \end{cases}$$

- Properties

P1. $\hat{\theta} \in \overline{\Omega}_\theta = \left\{ \hat{\theta} : \quad \theta_{\mathrm{min}} \le \hat{\theta} \le \theta_{\mathrm{max}} \right\}$

P2. $\tilde{\theta}\left(\Gamma^{-1}\,\mathrm{Pr}\,oj_{\hat{\theta}}\left(\Gamma \bullet\right) - \bullet\right) \le 0, \qquad \forall \bullet$

DESIRED COMPENSATION ARC DESIGN



SIMULATION OF A FIRST-ORDER SYSTEM

- The Plant

$$\dot{x} = \theta \, sin(\pi x) + \Delta + u$$

$$\varphi = sin(\pi x) \qquad \Delta = (-1)^{round(t)}$$

$$\theta = 18 \qquad \Omega_\theta = (0, \; 20)$$

- The Controller Parameters

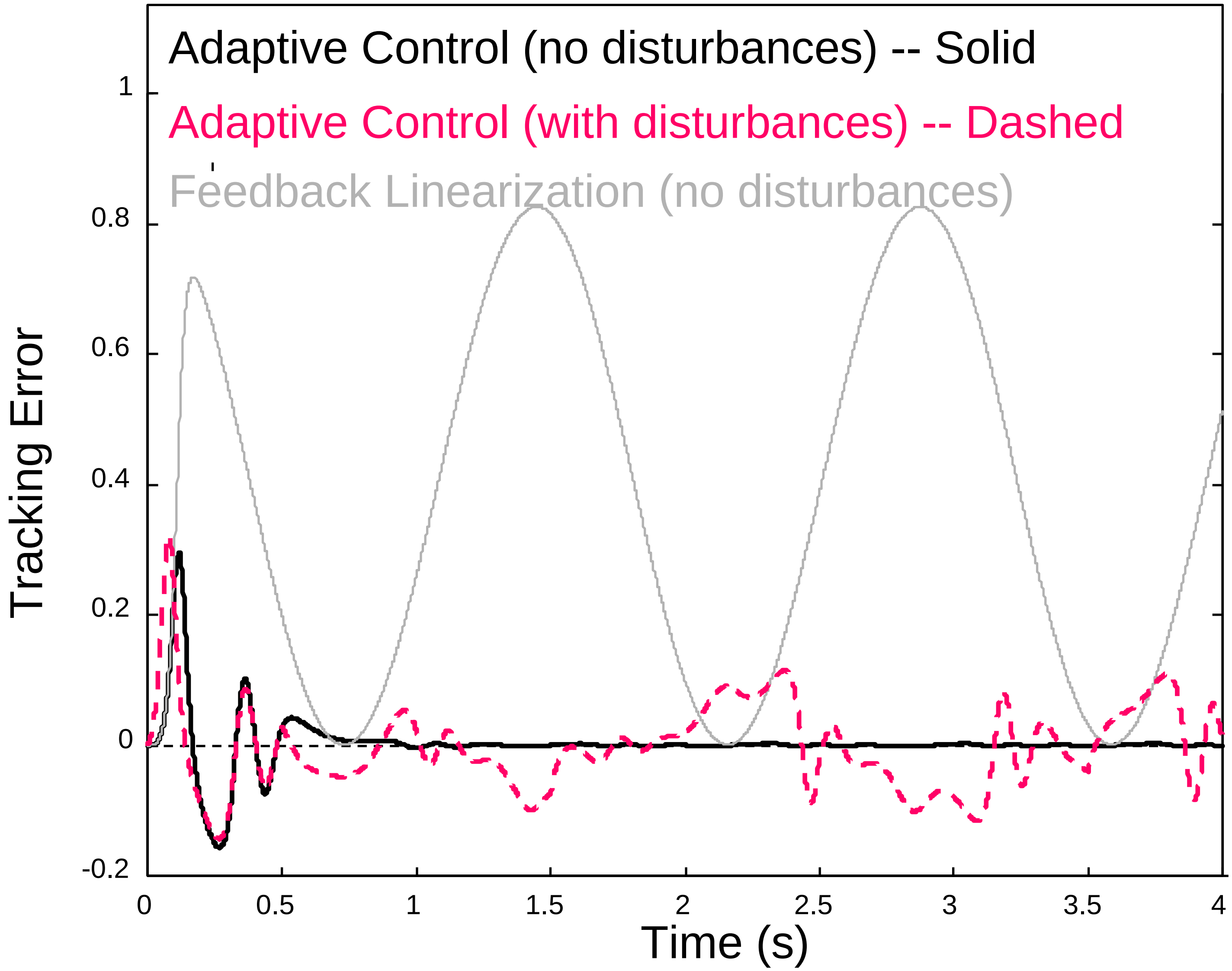
$$\delta = 1 \qquad \hat{\theta}_0 = 2 \qquad \mathrm{Dt} = 1\mathrm{ms}$$

$$\mathrm{k} = 10 \qquad \varepsilon_0 = 0.3 \qquad \varepsilon_1 = 0.001$$

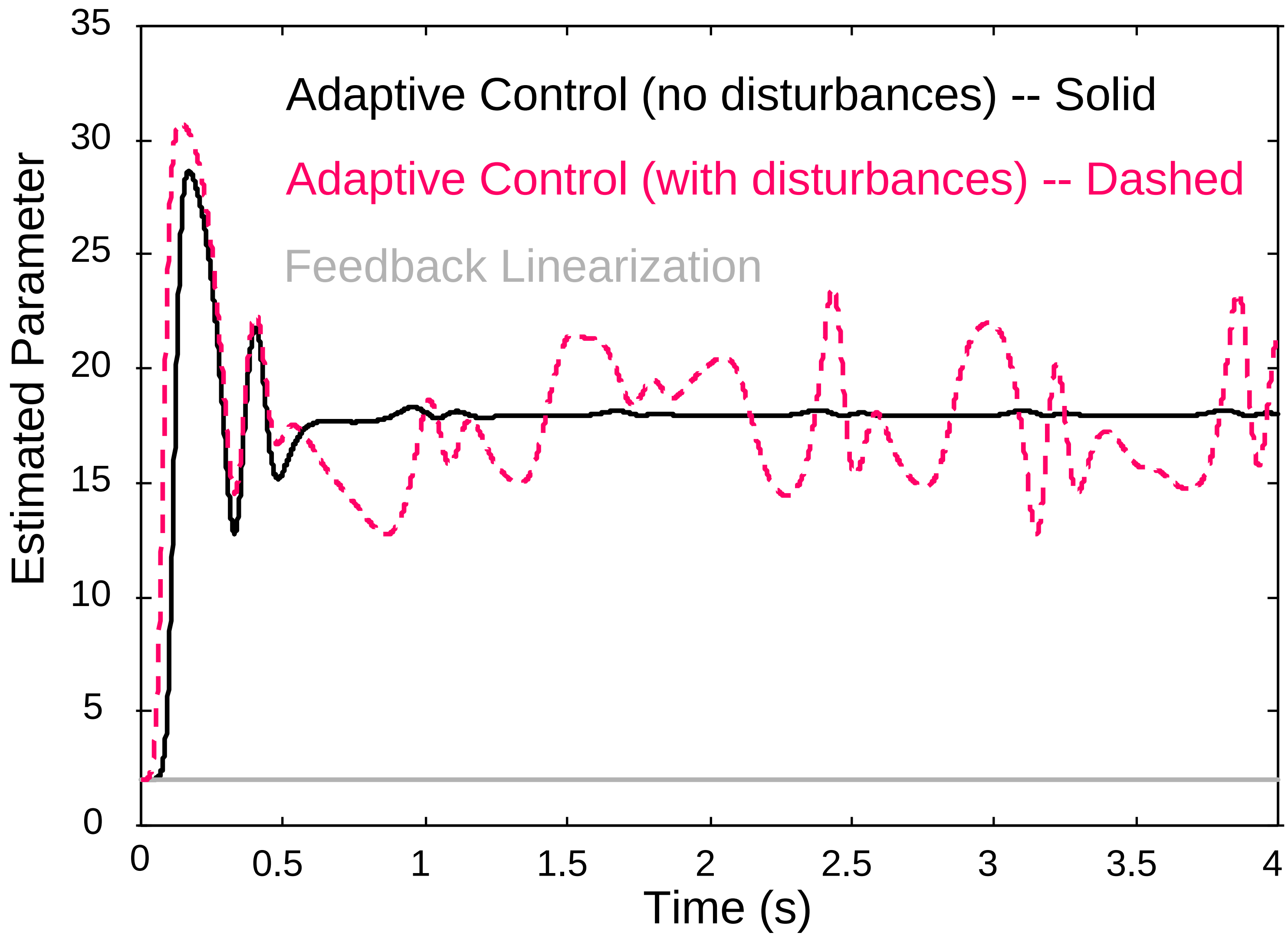
$$\gamma = 2000$$

- The Desired Trajectory

$$x_d = 0.5(1 - \cos(1.4\pi \, \mathrm{t}))$$



Adaptive Control of a First-order Uncertain System



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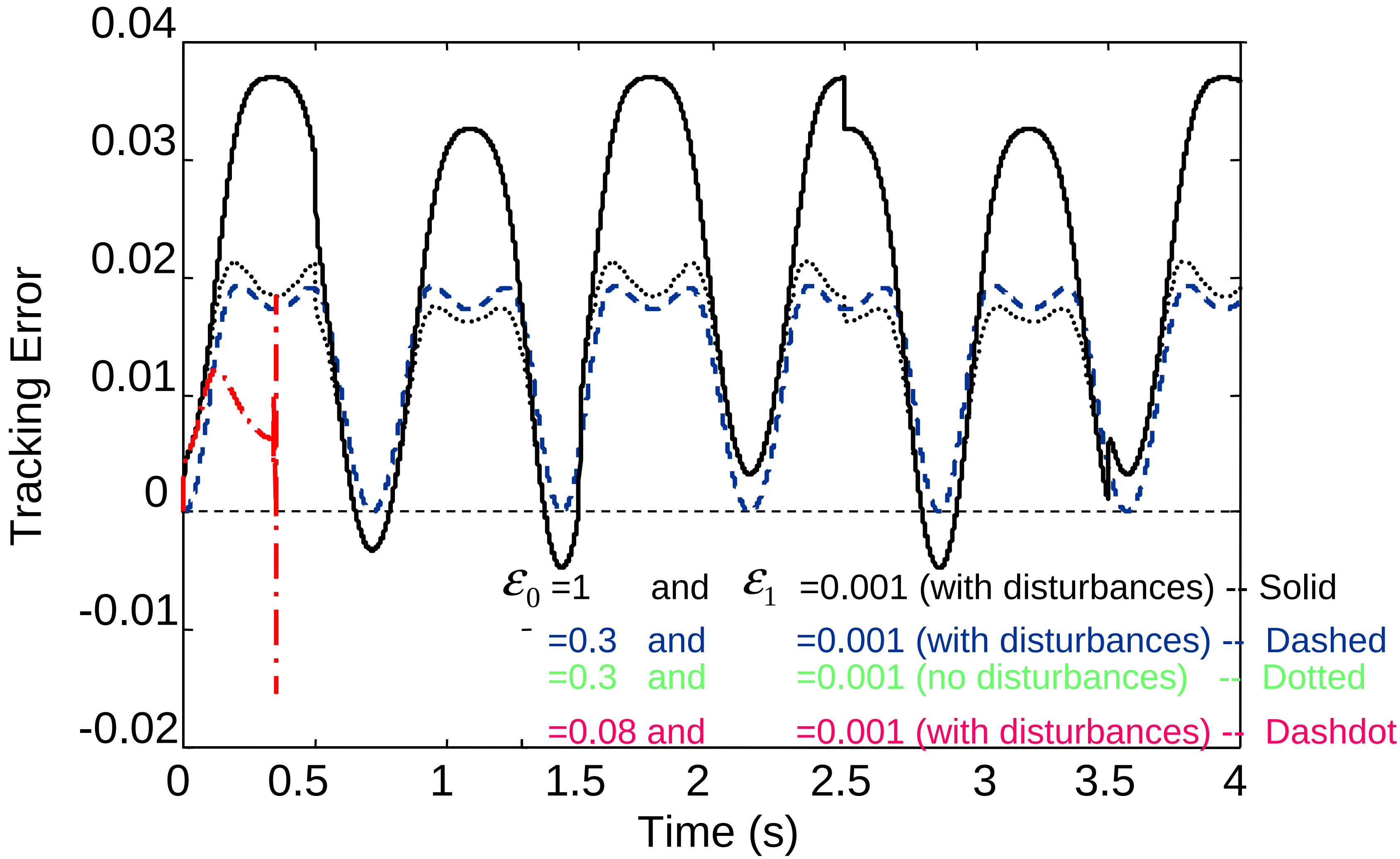
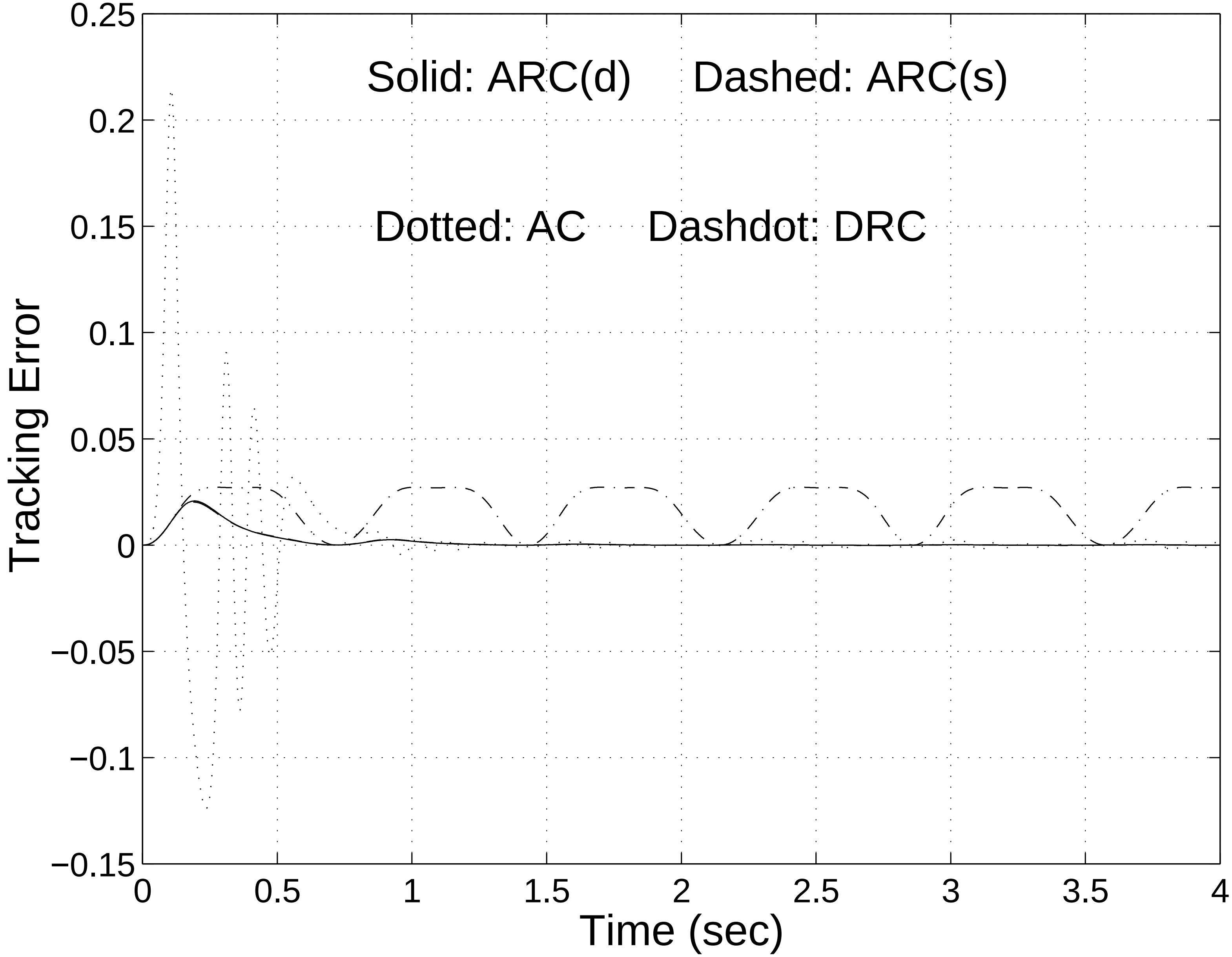
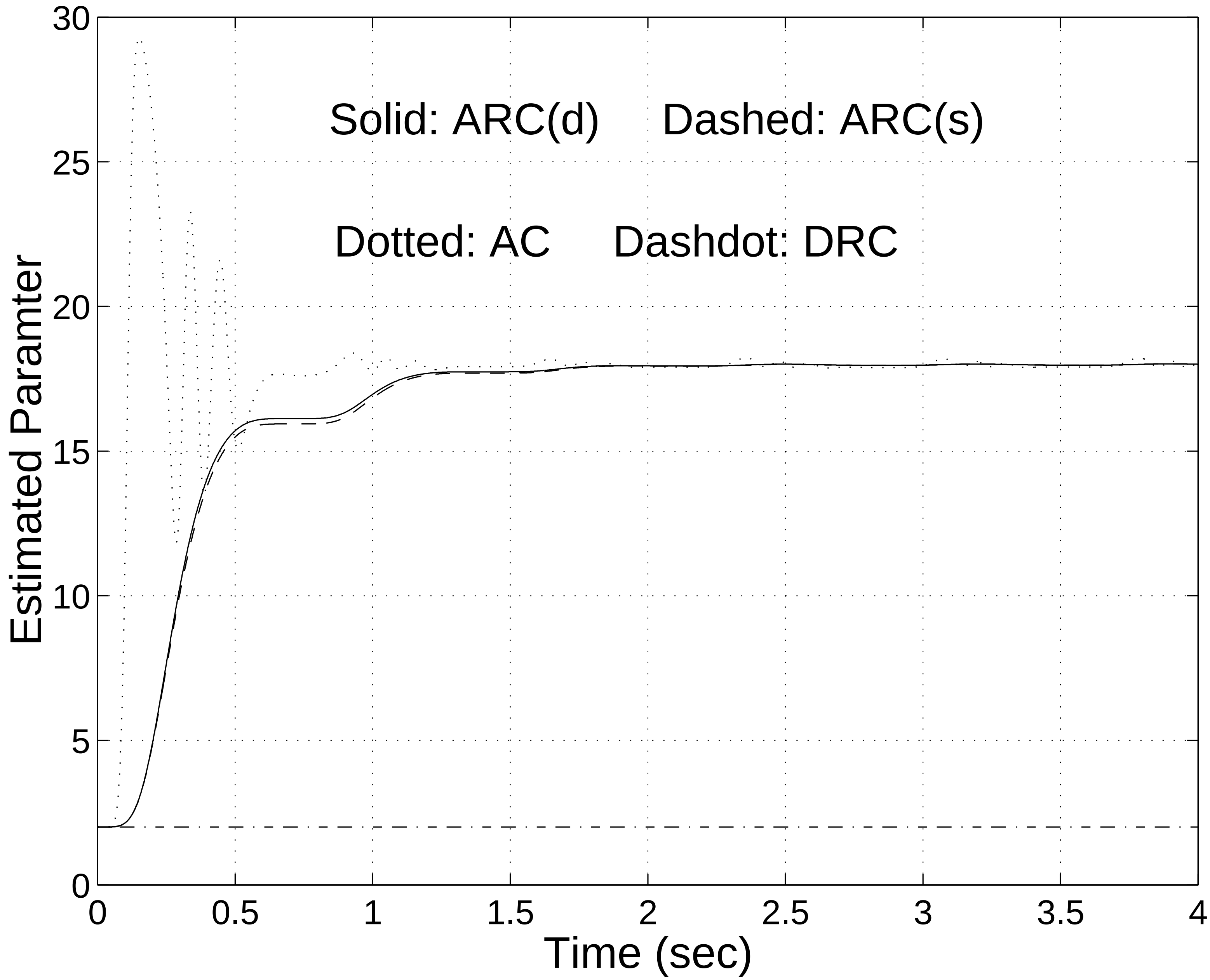


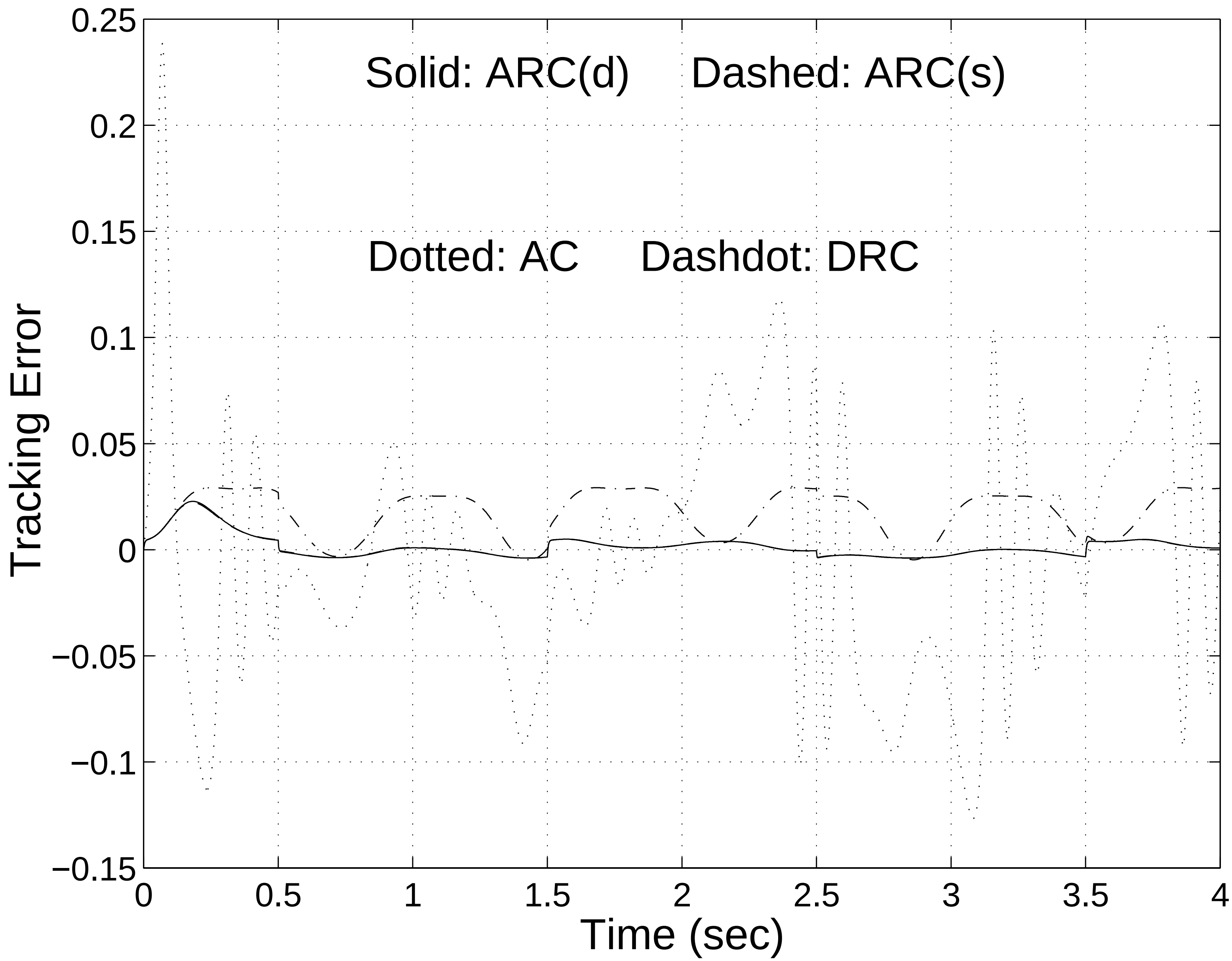
Deterministic Robust Control of a First-Order System



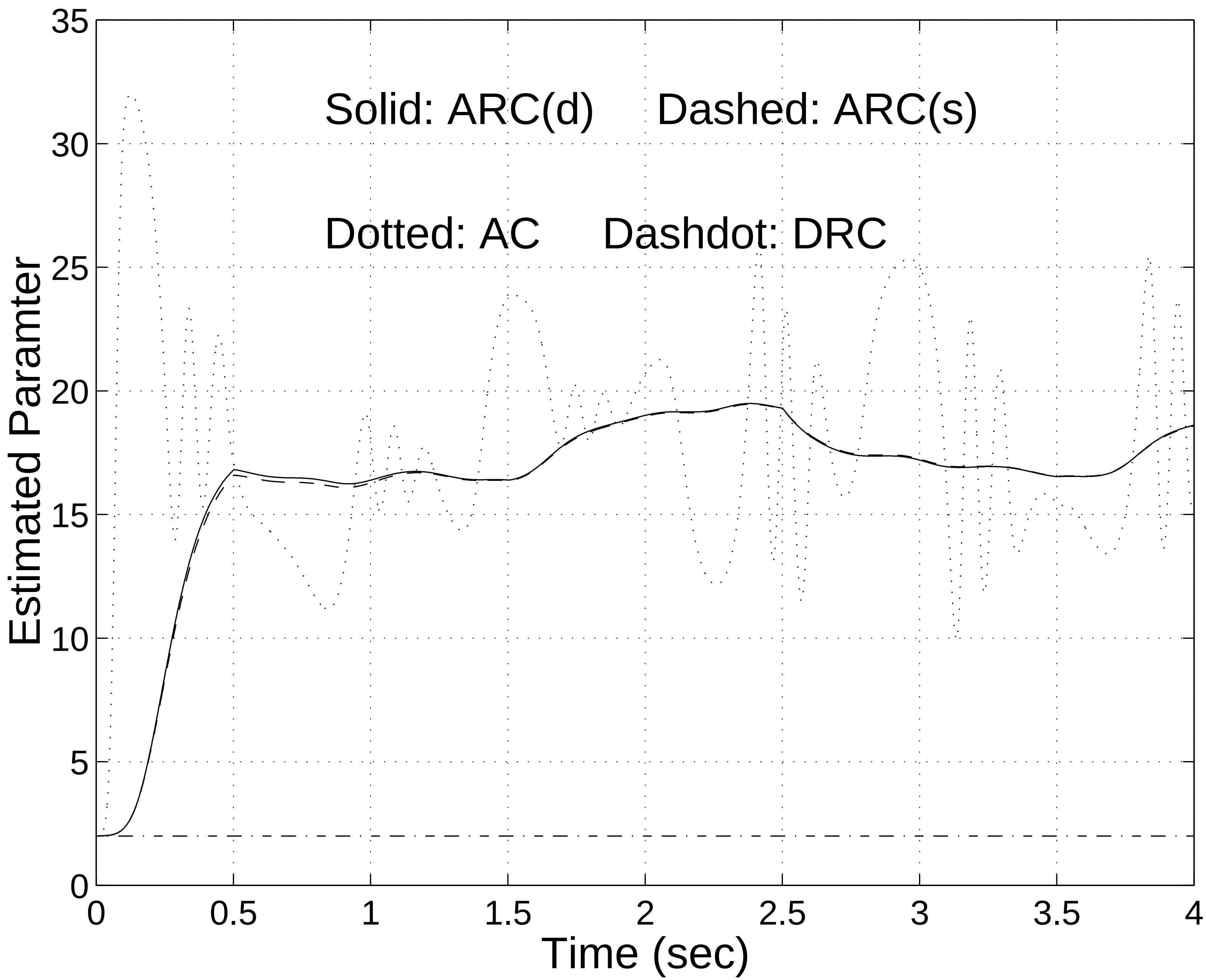
Tracking Errors In the Presence of Parametric Uncertainty Only



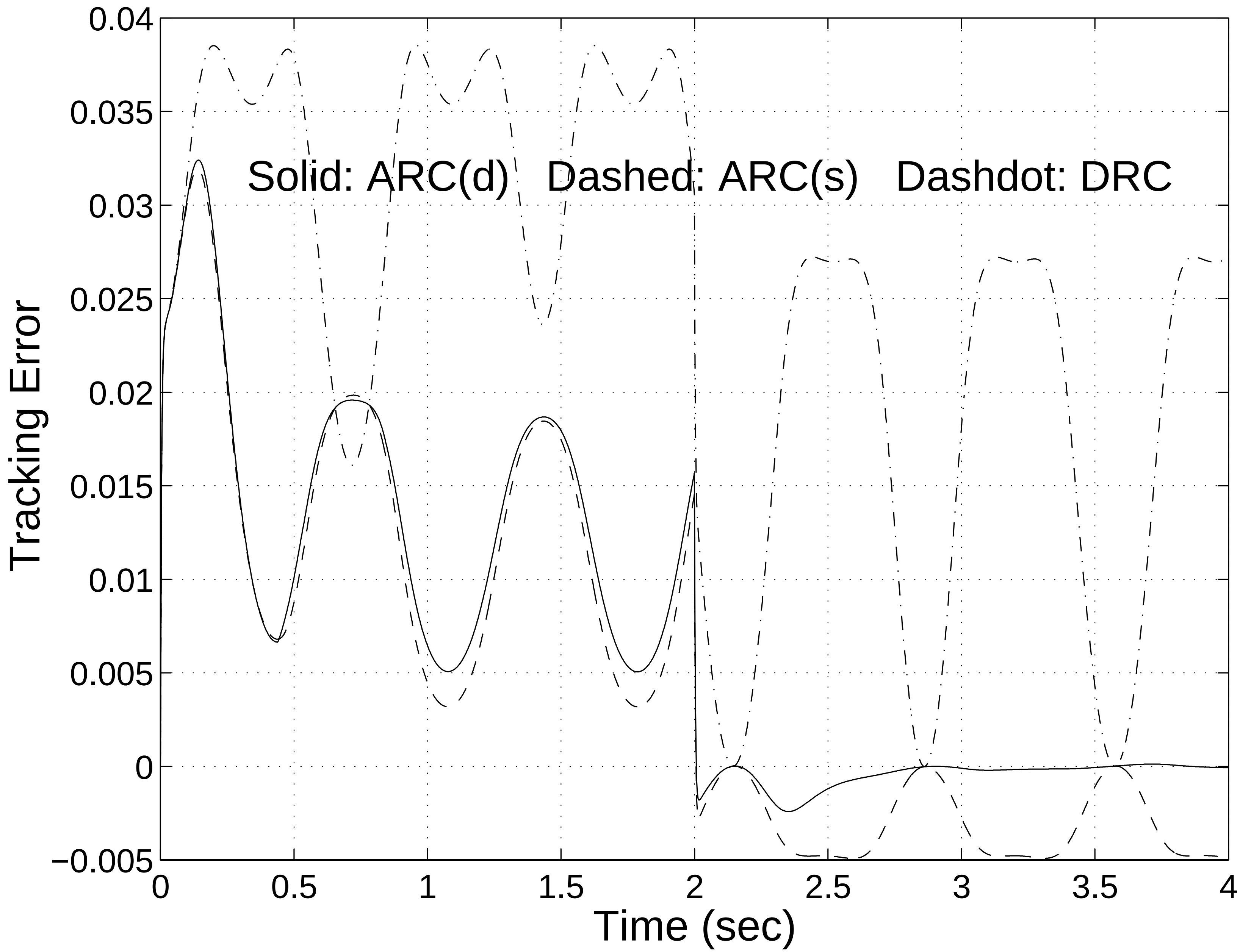
Estimates In the Presence of Parametric Uncertainty Only



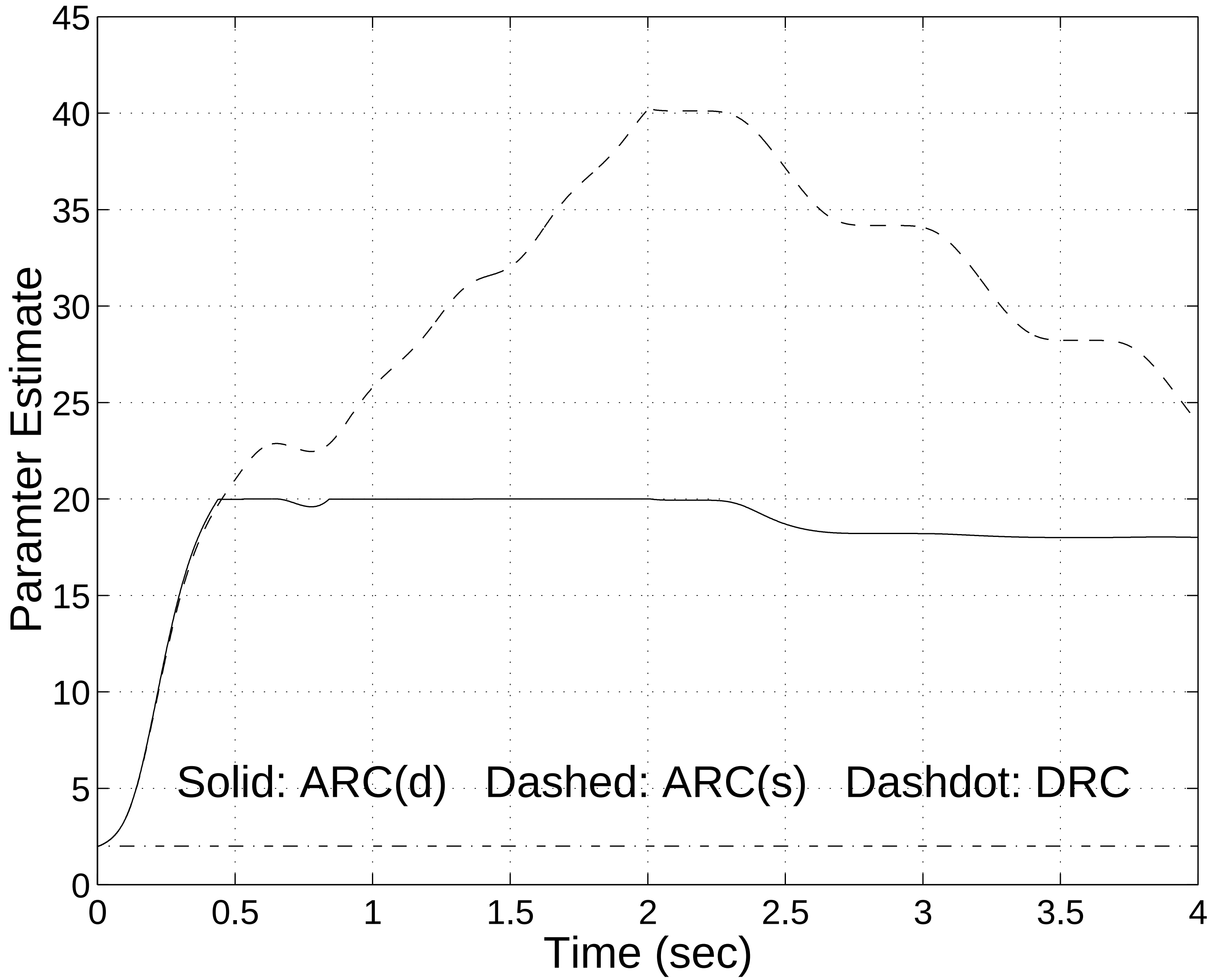
In the Presence of Parametric Uncertainty and Small Disturbances



In the Presence of Parametric Uncertainty and Small Disturbances



In the Presence of Parametric Uncertainty and Large Disturbances



In the Presence of Parametric Uncertainty and Large Disturbances

ADAPTIVE ROBUST CONTROL (ARC)

- **Departs From Robust Adaptive Control (RAC)**
  - In terms of fundamental view point, puts more emphasis on the underline robust control law design for a guaranteed robust output tracking performance in general.
  - In terms of achievable performance, guarantees a prescribed transient performance and tracking accuracy.
  - In terms of design approaches and proof, uses two Lyapunov functions; one the same as DRC and the other as RAC.

- **Departs From Deterministic Robust Control (DRC)**
  - Achieves asymptotic tracking in the presence of parametric uncertainties without using discontinuous control law or infinite gain feedback—overcome the design conservativeness.

- **Departs From Other Existing Combined Schemes**

  (Narendra and Boskovic'92, Slotine and Li'88, Chen'92, …)
  - Achieves a guaranteed transient performance and tracking accuracy

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Specific Design Issues

- Means to achieve fast robust feedback
- Learning techniques (e.g., parameter adaptation) to reduce model uncertainties for an improved performance
- Desired compensation to alleviate the effect of measurement noises
- Direct/Indirect and Integrated ARC designs
- Extensions

Maximizing Disturbances Attenuation

- General Means :

  High-gain feedback to raise the closed-loop bandwidth

- Problems:

  May run into *Control Saturation Problem* during large transients caused by sudden changes of large command inputs

- Solutions:

  Separating the achievable closed-loop bandwidths to command inputs and disturbances

Respect System Dynamics

- Design Principle

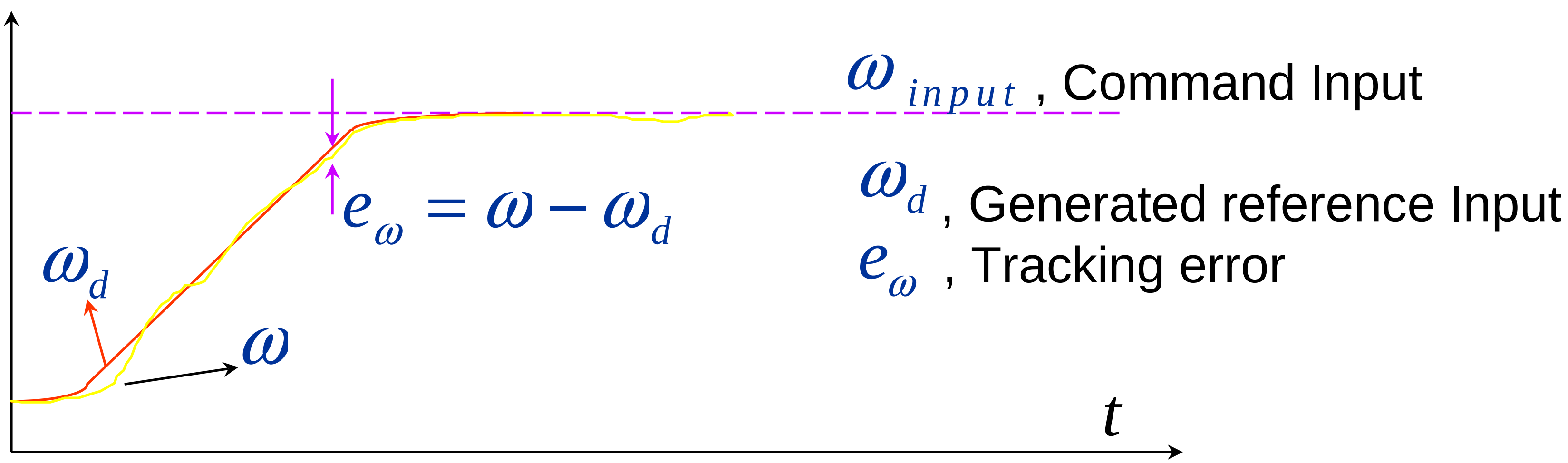
Only ask system to track feasible reference trajectories

- Solutions

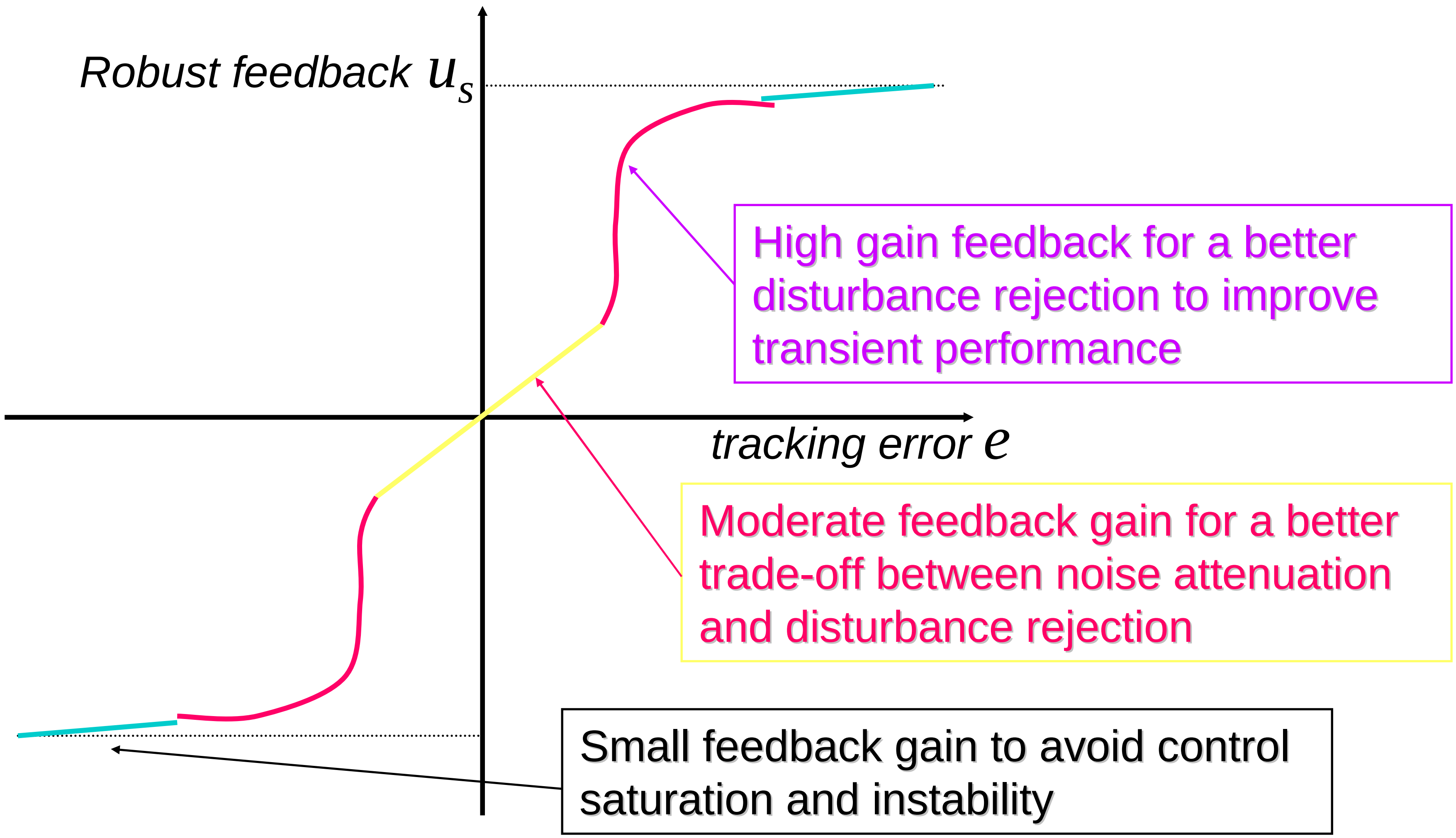
On-line reference trajectory generation

and

Physical model based nonlinear compensation !



Nonlinear Feedback Gains for Improved Transient Performance and Better Trade-off in Meeting Various Needs



Specific Design Issues

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Controlled Learning Process

Use prior knowledge such as physical bounds of parameter variations to achieve a controlled learning process; this helps get rid off the destabilizing effect of on-line learning and enable a fast adaptation loop to be used for a better performance !

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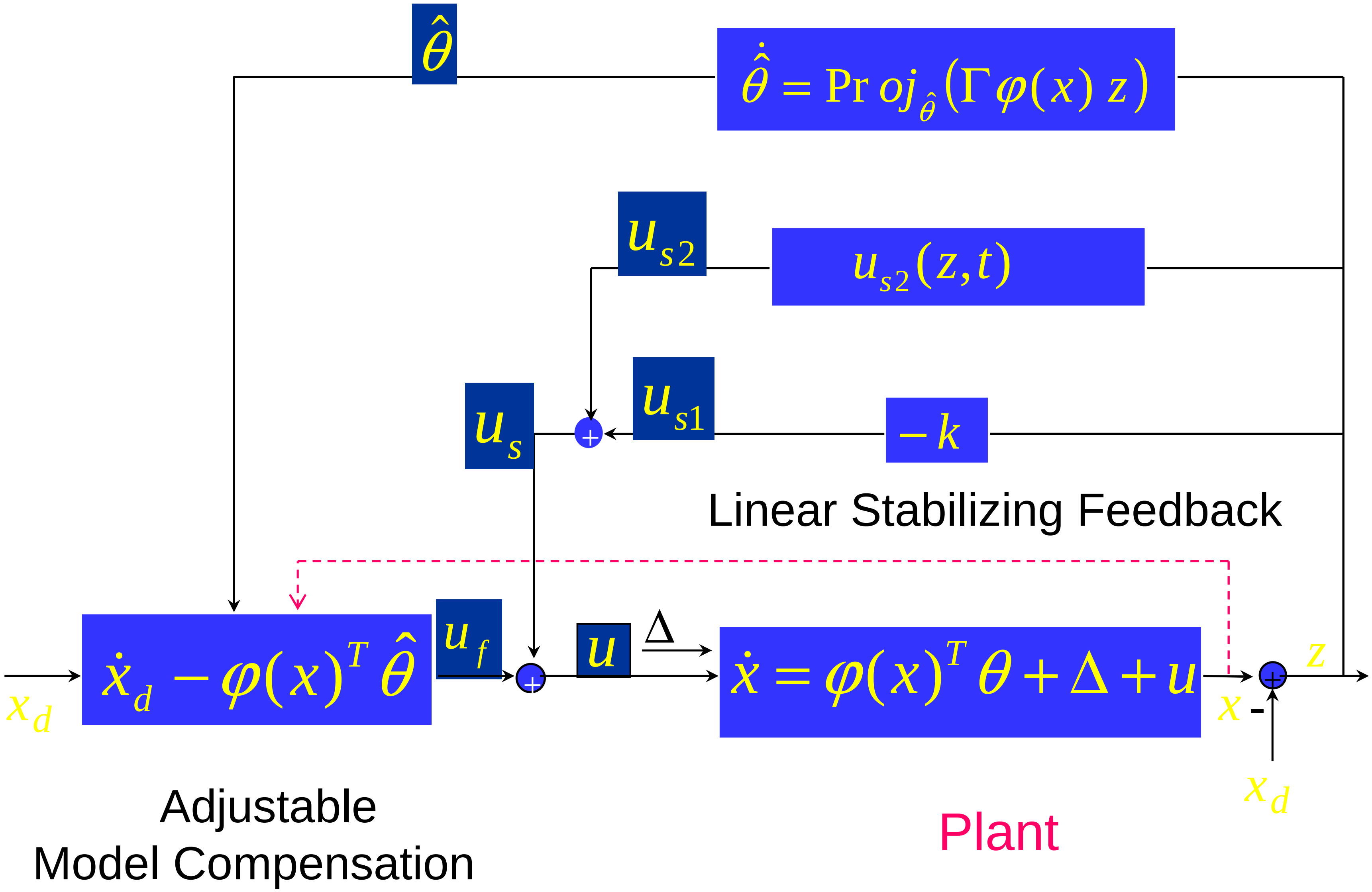
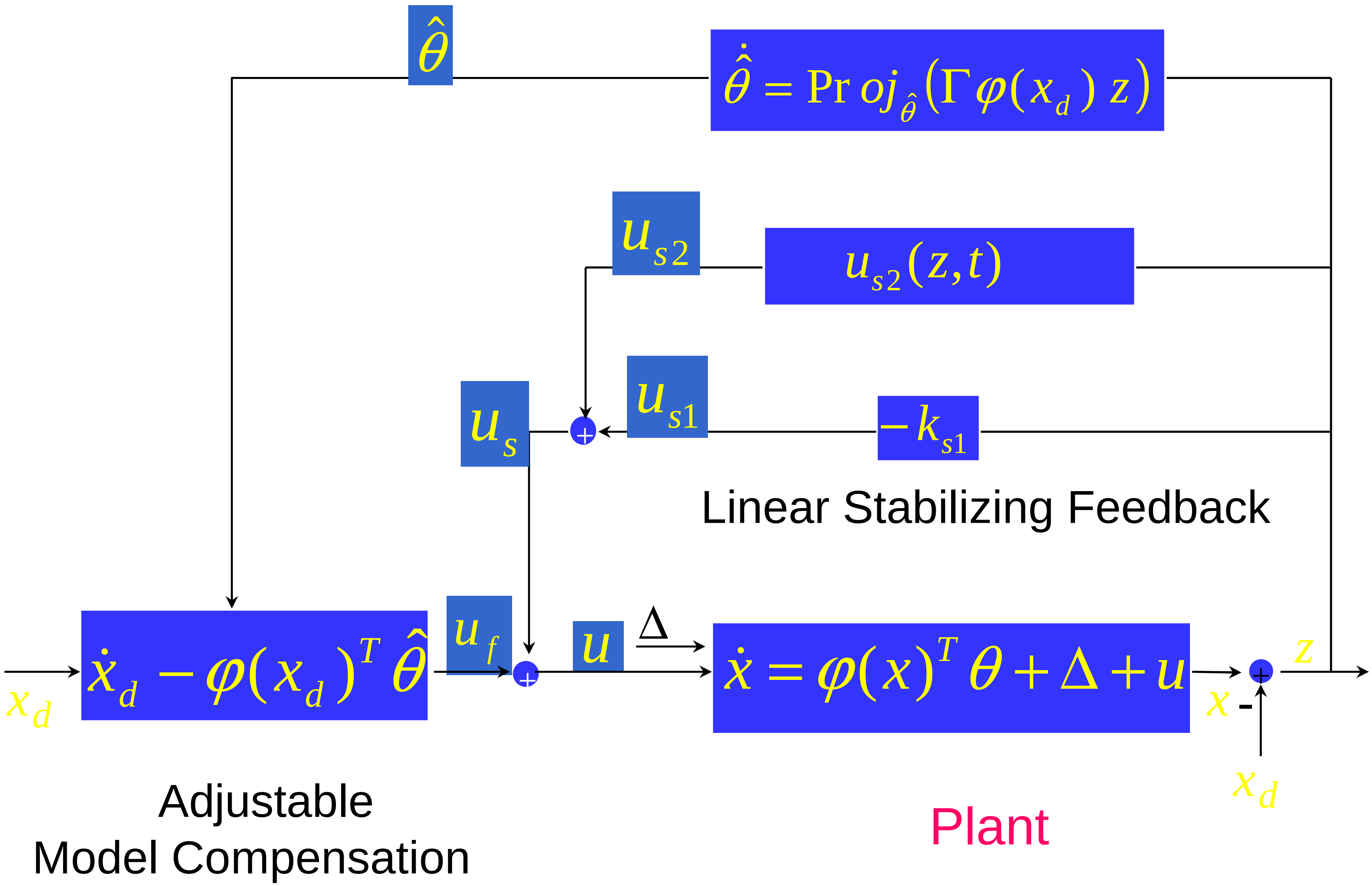
DISCONTINUOUS PROJECTION BASED ARC DESIGN



DESIRED COMPENSATION ARC DESIGN



Desired Compensation ARC Structure

- Reducing the effect of measurement noise
  - ☞ Regressor does not depend on measurements
- Fast adaptation rate in implementation
- An almost total separation of robust control law design and parameter adaptation design; this facilitates controller gain tuning process considerably
- Off-line calculation of regressors

Specific Design Issues

- Means to Achieve a Fast Robust Feedback
- Learning techniques (e.g., parameter adaptation) to reduce model uncertainties for an improved performance
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Specific Design Issues

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MIMO Semi-Strict FEEDBACK FORM I

i-th Subsystem

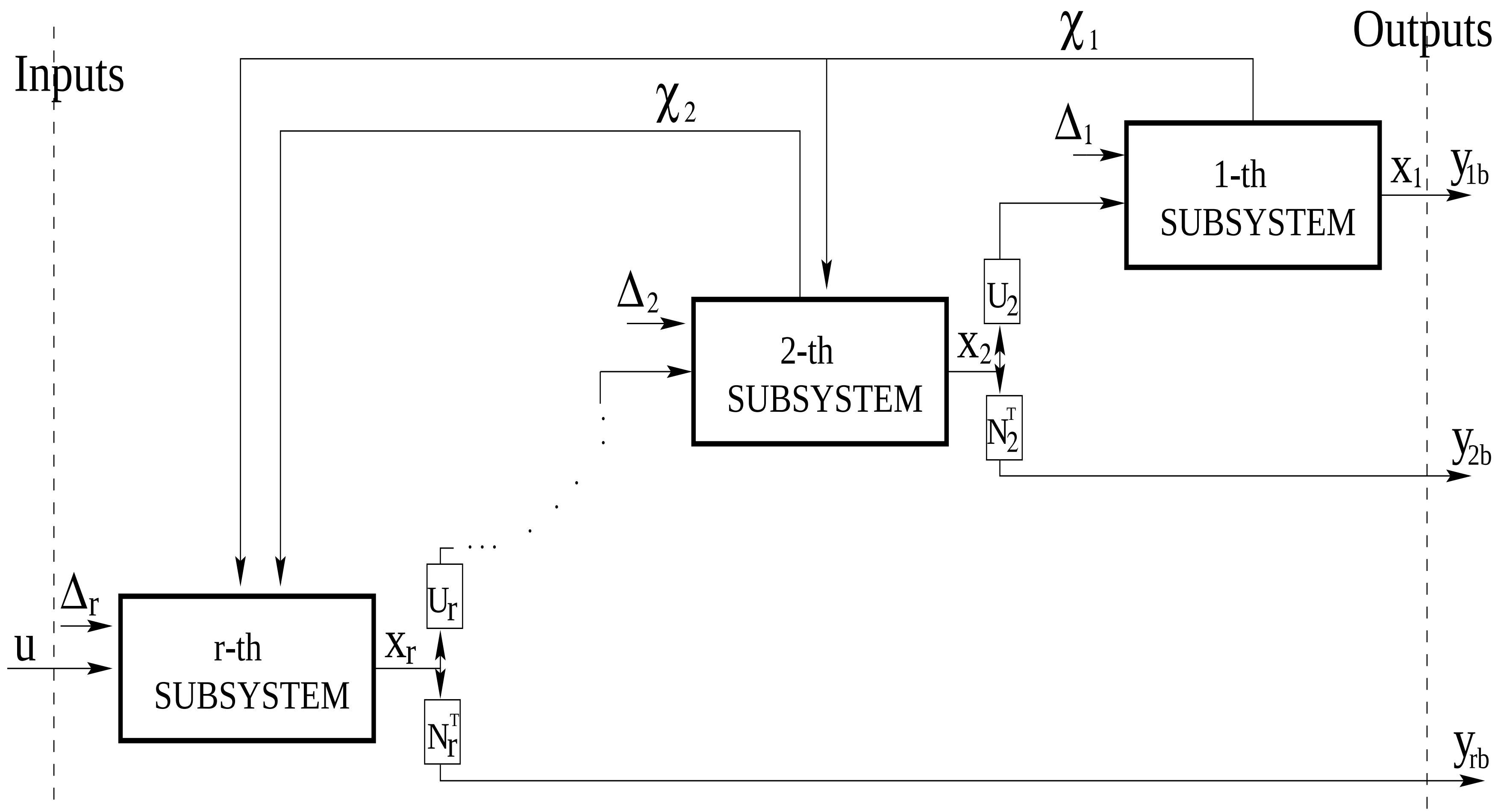
$$\dot{x}_i = f_i^0(\bar{\chi}_i, t) + F_i(\bar{\chi}_i, t)\theta + B_i(\bar{\chi}_i, \theta, t)\bar{x}_{i+1,m_i} + D_i(\bar{\chi}_i, t)\Delta_i$$
$$\dot{\eta}_i = \Phi_i^0(\bar{\chi}_i, t) + \Phi_i^1(\bar{\chi}_i, t)\theta \qquad 1 \le i \le r - 1$$

r-th Subsystem

$$\dot{x}_r = M^{-1}(\bar{\chi}_i, \beta, t)\left[f_r^0(\bar{\chi}_r, t) + F_r(\bar{\chi}_r, t)\theta + F_\beta \beta + B_i(\bar{\chi}_r, \theta, \beta, t)u + D_r \Delta_r \right]$$
$$\dot{\eta}_r = \Phi_r(\bar{\chi}_r, \theta, t)$$

Output

$$y = \bar{y}_r = \left[y_{1b}^T, \ldots, y_{rb}^T \right]^T$$



MIMO Semi-Strict FEEDBACK FORM I

ASSUMPTIONS

- B_i is nonsingular and can be linearly parametrized
- M is an s.p.d. matrix and can be linearly parametrized
- The $\eta_i - subsystem$ is BIBS stable w.r.t. input $\left(\overline{\chi}_{i-1}, x_i\right)$
- There exist known functions $\delta_i(\overline{\chi}_i, t)$ such that

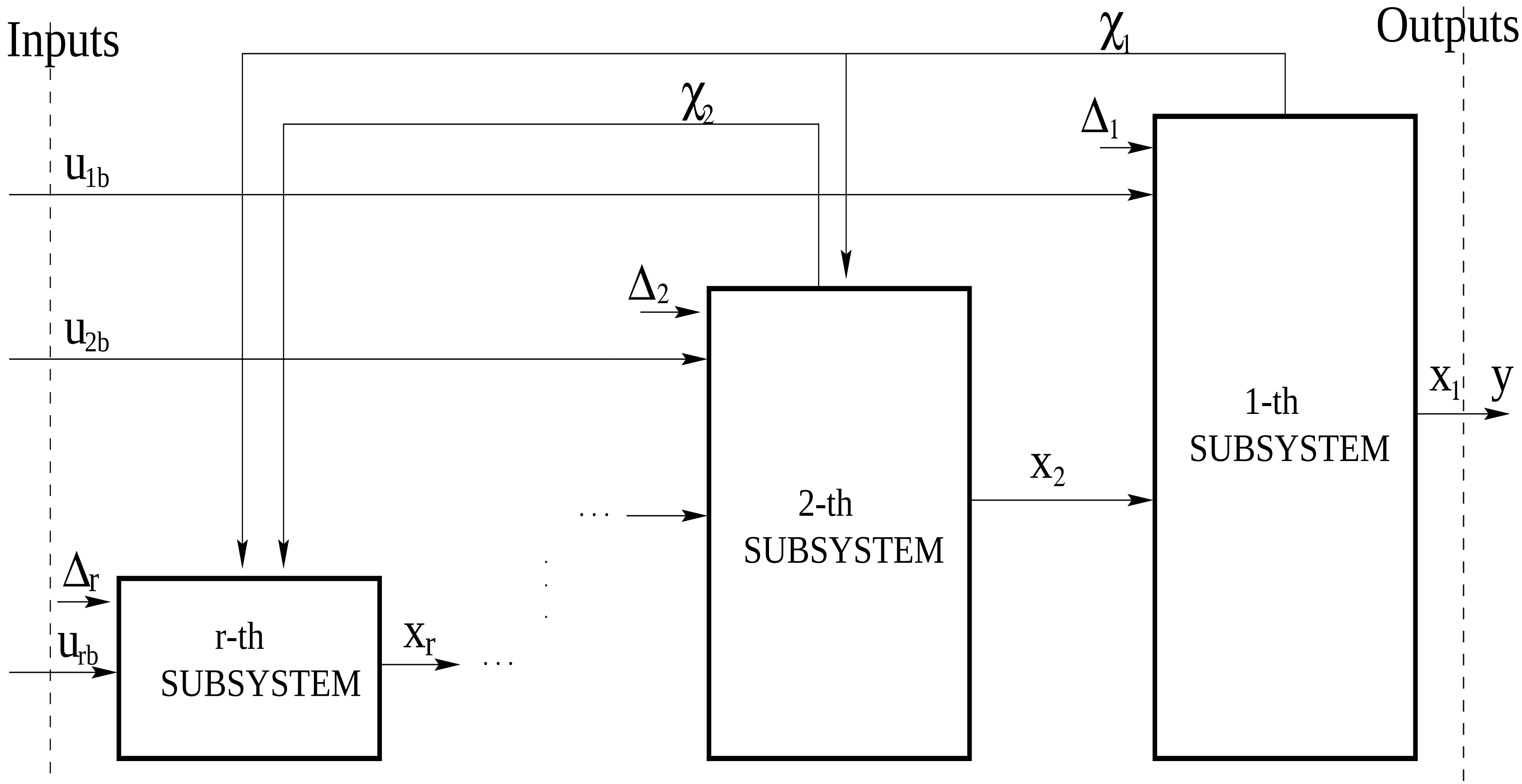
$$\left\| \Delta_i(\chi, \theta, u, t) \right\| \leq \delta_i(\overline{\chi}_i, t), \qquad i = 1, \ldots, r$$

DIFFICULTIES

- **High Relative Degrees**
- **Mismatched Uncertainties**

  Uncertainties do not enter the system in the same channel as control inputs

- **Coupling and Appearance of Parametric Uncertainties in the Input Channels of Each Layer**
- **Last Layer's State Equations *Cannot* be Linearly Parametrized**



MIMO Semi-Strict FEEDBACK FORM II

Extensions

- Output Feedback ARC of Uncertain Linear Systems with Disturbances
  - *Need observers for state estimates*
- Observer Based ARC of a Class of Nonlinear Systems with Dynamic Uncertainties
  - *Partial state feedback*
- Adaptive Robust Control without Knowing Bounds of Parameter Variations
  - *Use fictitious bounds for a controlled learning process*
- Neural Network Adaptive Robust Control
  - *Integrate the universal approximation capability of neural networks into ARC design for general nonlinearities*
- Adaptive Robust Repetitive Control
  - *For periodic unknown disturbances and repetitive tasks*

APPLICATIONS

- **Precision Motion Control of High Speed Machine Tools and Linear Motors**

  Combined the design technique with digital control

- **Control of Electro-Hydraulic Systems**

  Hydraulic Servo-systems; Hydraulic Excavators

- **Trajectory Tracking Control of Robot Manipulators**

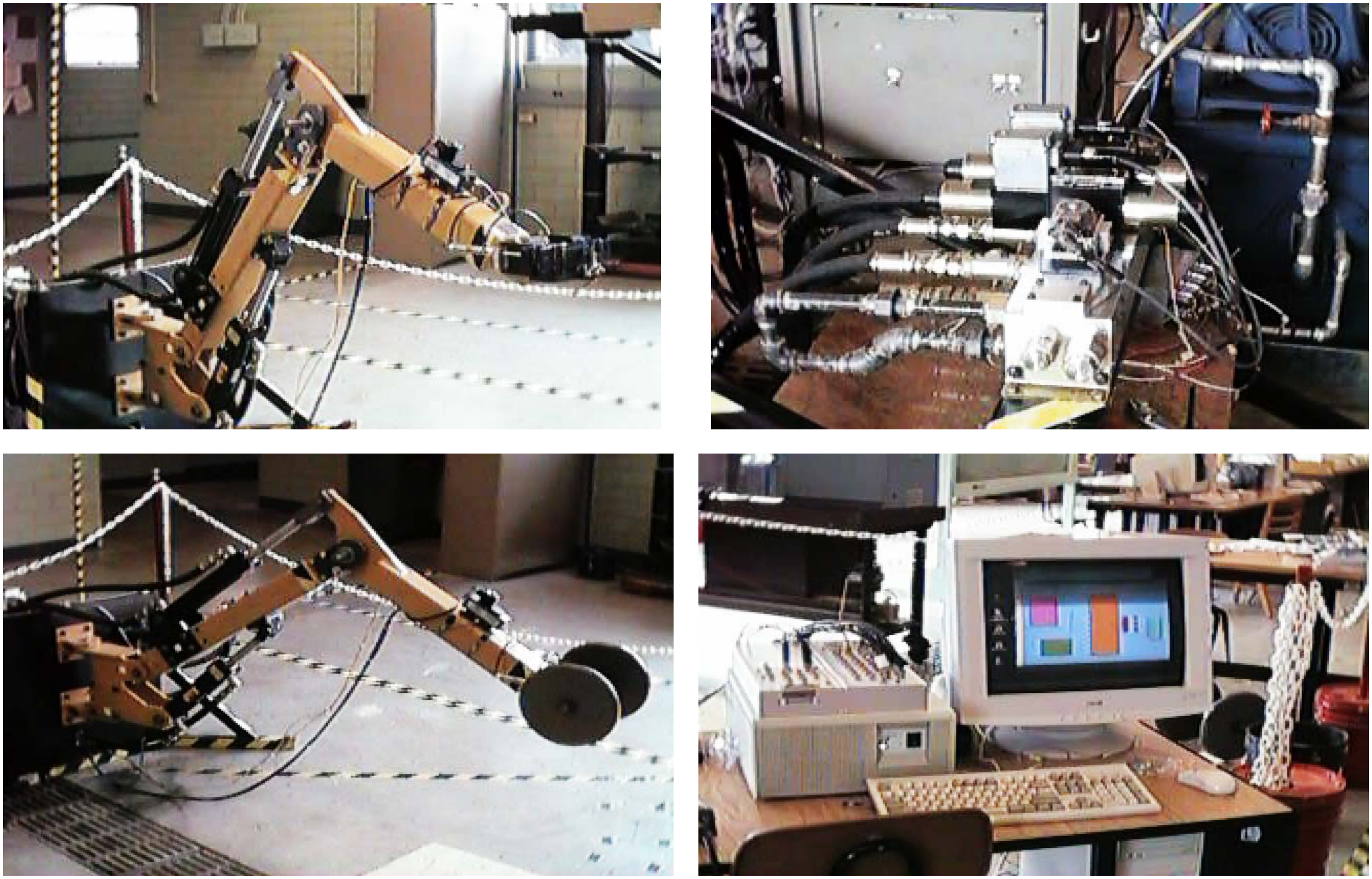
- **Motion and Force Control of Robot Manipulators**

  (a) in contact with a stiff surface with unknown stiffness

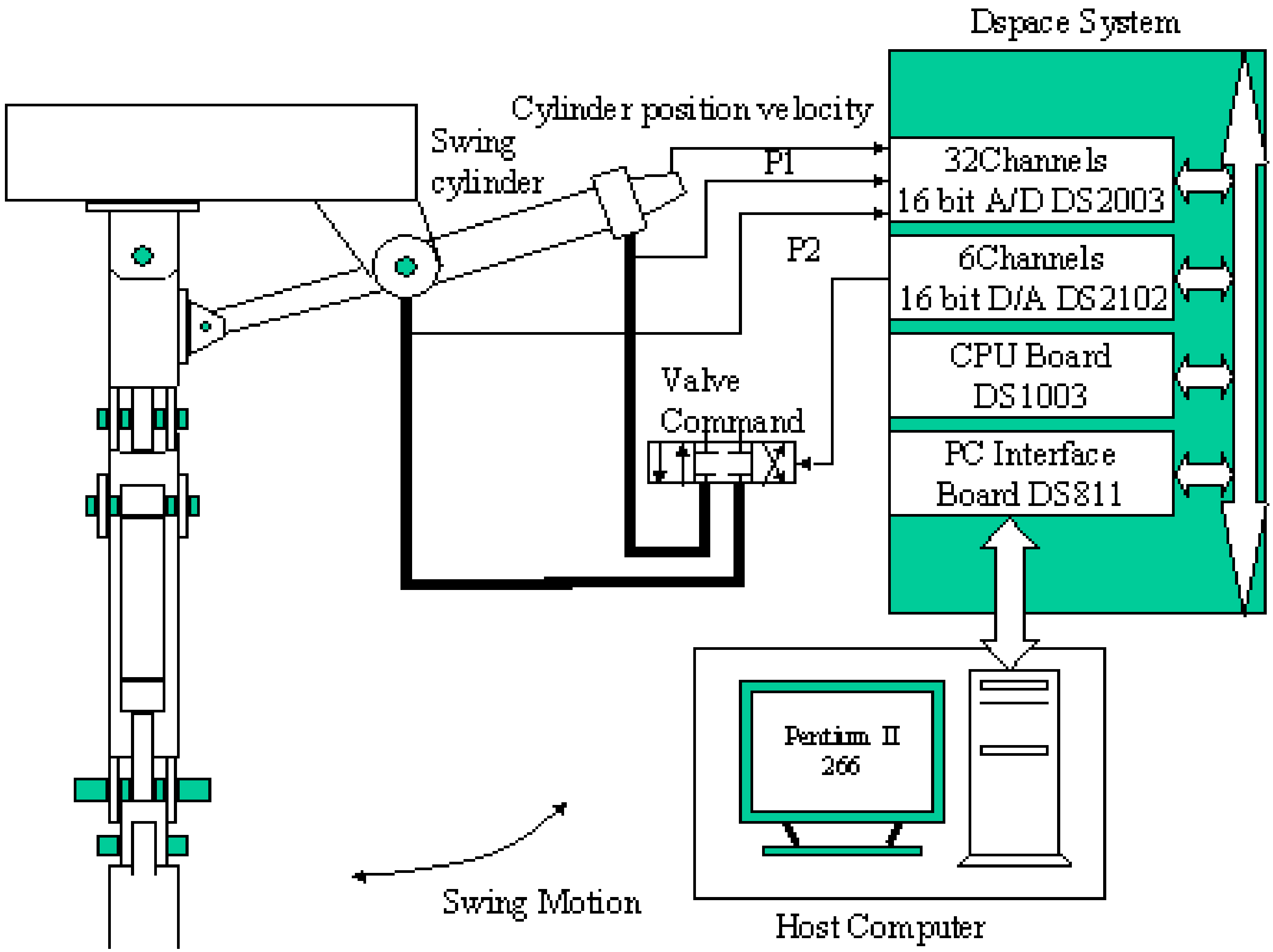
  (b) in contact with a rigid surface

- **Coordinated Control of Multiple Robot Manipulators**

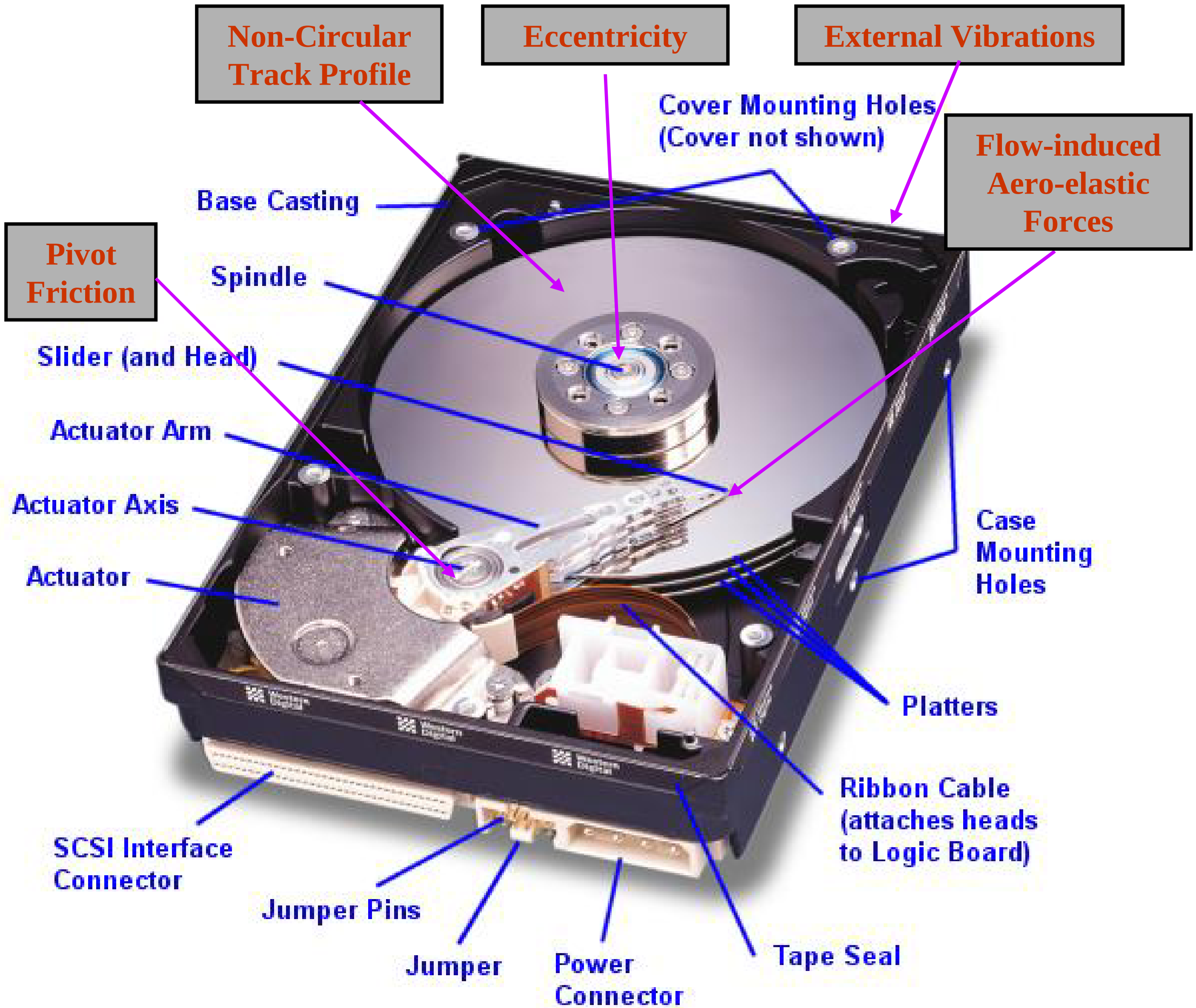
- **Ultra-Precision Control of Piezo-electrical Actuators; Hard-disk Drives; …**



Electro-Hydraulic Experimental Setup



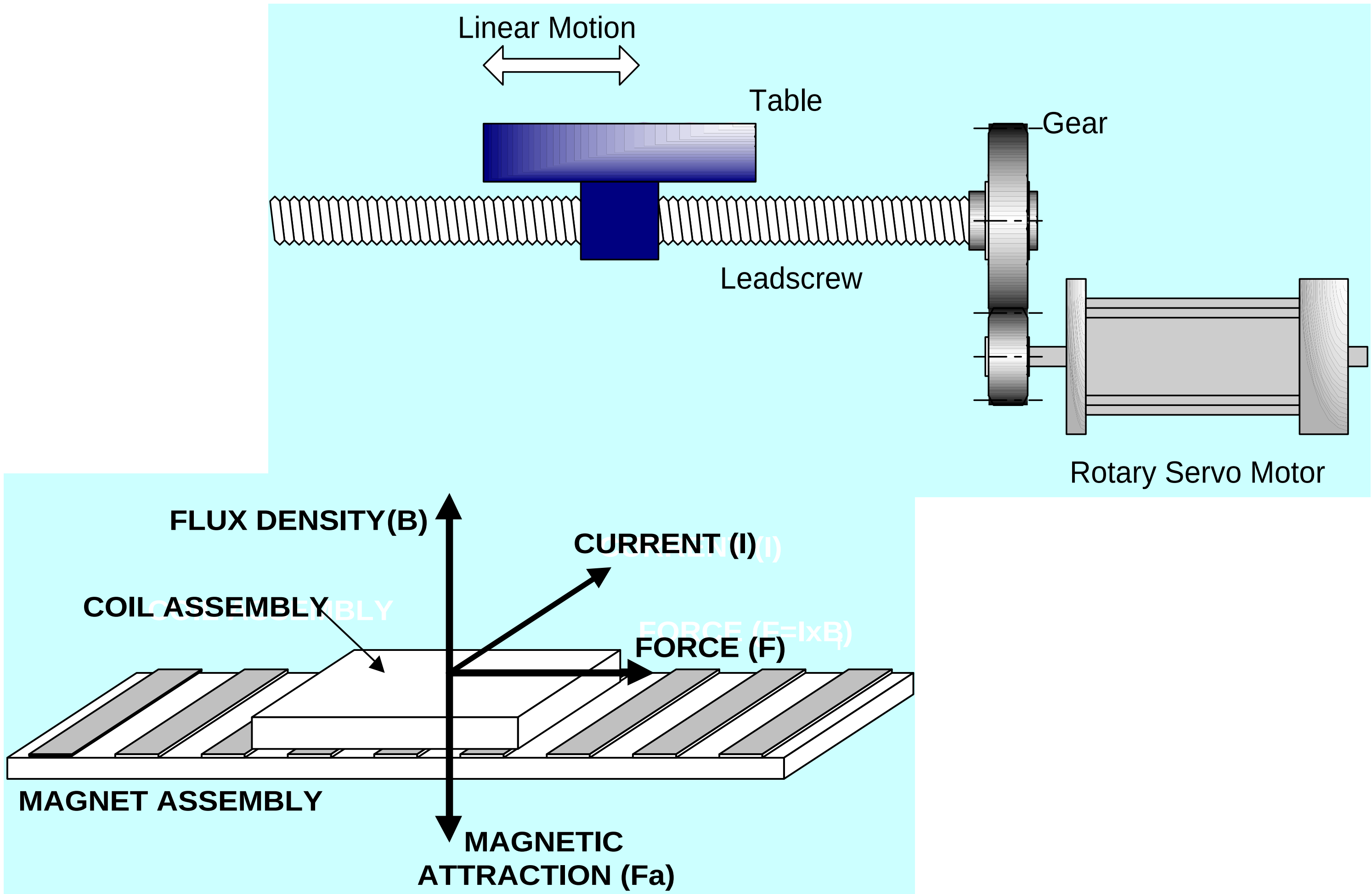
Electro-Hydraulic Experimental Setup



Hard-Disk Drive Experimental Setup

OUTLINE

- Motivation
- General Design Philosophy and Structure
- Specific Design Issues
- Application Examples
  - Motion and Pressure Control of Electro-Hydraulic Systems
  - Precision Motion Control of Linear Motors
  - Dynamic Friction Compensation
- Conclusions



Linear Motor vs. Rotary Motor

Advantages of Linear Motor Drive Systems

- Mechanical simplicity (no mechanical transmission mechanisms), higher reliability, and longer lifetime
- No backlash and less friction, resulting in the potential of having high load positioning accuracy
- No mechanical limitations on achievable acceleration and velocity
- Bandwidth is only limited by encoder resolution, measurement noise, calculation time, and frame stiffness

Difficulties in Control of Linear Motors

- Model Uncertainties

  Parametric uncertainties (e.g., *load inertia*)

  Discontinuous disturbances (e.g., *Coulomb friction*); external disturbances (e.g., *cutting force*)

- Drawback of without mechanical transmissions

  Gear reduction reduces the effect of model uncertainties and external disturbance

- Significant uncertain nonlinearities due to position dependent electro-magnetic *force ripples* (e.g. iron-core linear motors)

- Implementation issues (e.g., *measurement noise*)

Mathematical Model

$$\dot{x}_1 = x_2$$
$$M\dot{x}_2 = u - Bx_2 - F_{fn}(\dot{q}) - F_r(q) + F_d$$
$$y = x_1$$

where

x_1 : position

x_2 : velocity

y : output

F_{fn} : nonlinear friction

F_d : lumped disturbance

M : mass of load

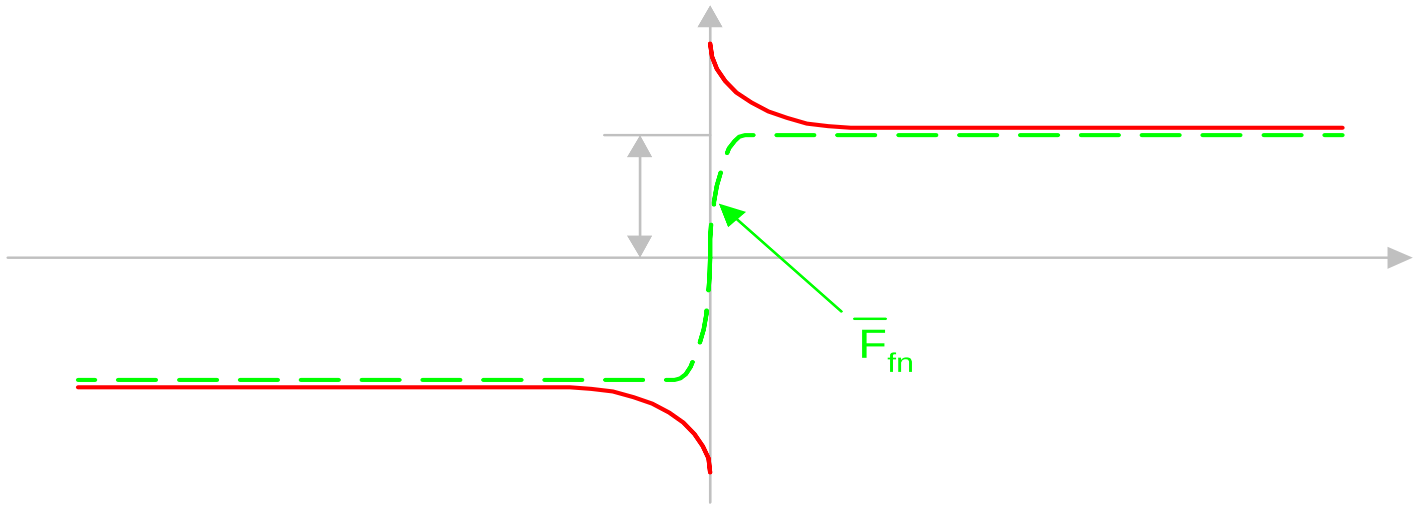
u : input voltage

B : viscous friction const

F_r : force ripple

Model of Friction Force

- F_{fn} is discontinuous at zero velocity

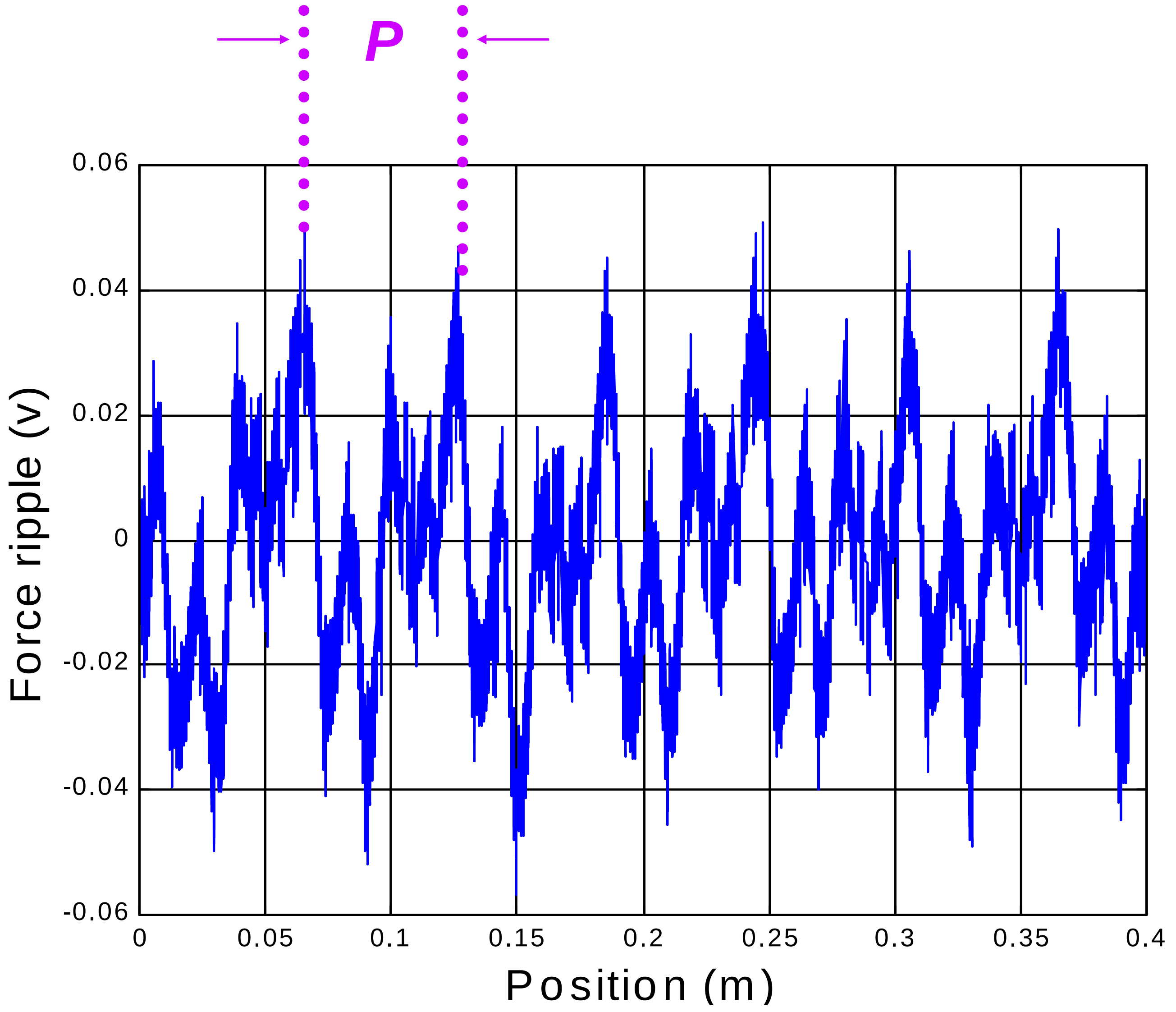


- A continuous friction model $\overline{F}_{fn}$ is used to approximate F_{fn}

$$\overline{F}_{fn}(x_2) = A_f S_f(x_2)$$

Force ripple

- ***Cogging force*** is a magnetic force developed due to the attraction between the permanent magnets and the cores of the coil assembly. It depends only on the relative position of the motor coils with respect to the magnets, and is independent of the motor current
- ***Reluctance force*** is developed due to the variation of self inductance of the windings, which causes a position-dependent force in the direction of motion when current flows through the coils



Measurement of Force Ripple (X-axis)

Model of Ripple Forces

- The permanent magnets of the linear motor are identical and are equally spaced at a pitch of P :

$$F_r(x_1 + P) = F_r(x_1)$$

- It can be approximated quite accurately by the first several harmonics, which is denoted as $\overline{F}_r(x_1)$ and represented by

$$\overline{F_r}(x_1) = A_r^T S_r(x_1)$$

where

$$A_r = [A_{r1s}, A_{r1c}, \cdots, A_{rqs}, A_{rqc}]^T$$

$$S_r(x_1) = [\sin(\frac{2\pi}{P}x_1), \cos(\frac{2\pi}{P}x_1), \cdots, \sin(\frac{2\pi q}{P}x_1), \sin(\frac{2\pi q}{P}x_1)]^T$$

and q is the numbers of harmonics used to approximate $F_r(x_1)$

ARC Controller Design Model

$$\Rightarrow \qquad M\dot{x}_2 = u - Bx_2 - A_f S_f - A_r^T S_r + d$$

where

$$d = (\overline{F}_{fn} - F_{fn}) + (\overline{F}_r - F_r) + \Delta$$

$$\dot{x}_1 = x_2$$

$$\Rightarrow \qquad \theta_1 \dot{x}_2 = u - \theta_2 x_2 - \theta_3 S_f - \theta_{4b}^T S_r(x_1) + \theta_5 + \tilde{d}$$

$$y = x_1$$

where

$$\theta_1 = M, \quad \theta_2 = B, \quad \theta_3 = A_f, \quad \theta_{4b} = A_r$$

$$\theta_5 = d_n, \quad \tilde{d} = d - d_n$$

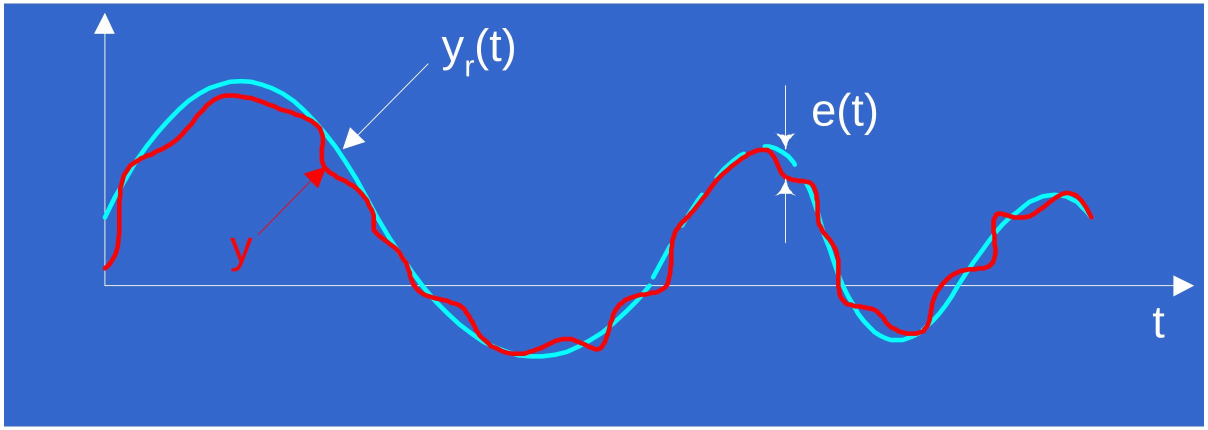
Assumption and Control Objective

- **Assumption:**

$$\theta \in \Omega_\theta = \{\theta : \quad \theta_{\min} < \theta < \theta_{\max}\}$$
$$\tilde{d} \in \Omega_d = \{\tilde{d} : \quad |\tilde{d}| \leq \delta_d(x,t)\}$$

- **Objective:**

Synthesize a control input *u* such that the output *y* track the reference motion trajectory y_r as closely as possible



ARC Controller Design

- Define a switching-function-like quantity as

$$p = \dot{e} + k_1 e = x_2 - x_{2eq}, \qquad x_{2eq} = \dot{y}_d - k_1 e$$

where

$$e = y - y_d(t)$$

- Error dynamics

$$M\dot{p} = u + \varphi^T \theta + \tilde{d}$$

where

$$\varphi^T = [-\dot{x}_{2eq}, -x_2, -S_f(x_2), -S_r(x_1), 1]$$
$$\dot{x}_{2eq} = \ddot{y}_d - k_1 \dot{e}$$

ARC Controller Design (cont'd)

- ARC control law

$$u = u_a + u_s, \qquad u_a = -\varphi^T \hat{\theta}$$

$$u_s = u_{s1} + u_{s2}, \qquad u_{s1} = -k_2 p$$

- Error dynamics

$$M\dot{p} + k_2 p = u_{s2} - \varphi^T \tilde{\theta} + \tilde{d}$$

- Choose robust control u_{s2} such that

$$i \qquad p\left(u_{s2} - \varphi^T \tilde{\theta} + \tilde{d}\right) \le \varepsilon$$

$$ii \qquad p u_{s2} \le 0$$

where ε is a design parameter

- Example

$$u_{s2} = -\frac{1}{4\varepsilon}(\|\theta_{\max} - \theta_{\min}\| \cdot \|\varphi\| + \delta_d)^2 p$$

Performance of ARC

If the adaptation function τ is chosen as $\tau = \varphi p$, then the ARC control law guarantees that

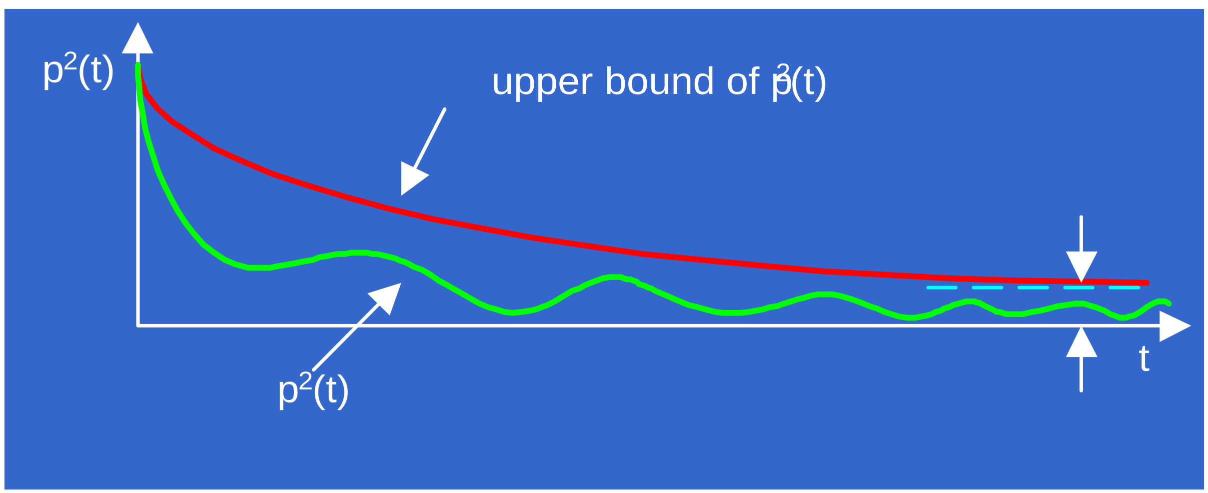
- In general, all signal are bounded. Furthermore, the positive definite function V_s defined by

$$V_s = \frac{1}{2} M p^2$$

is bounded by

$$V_s \le \exp(-\lambda t) V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)]$$

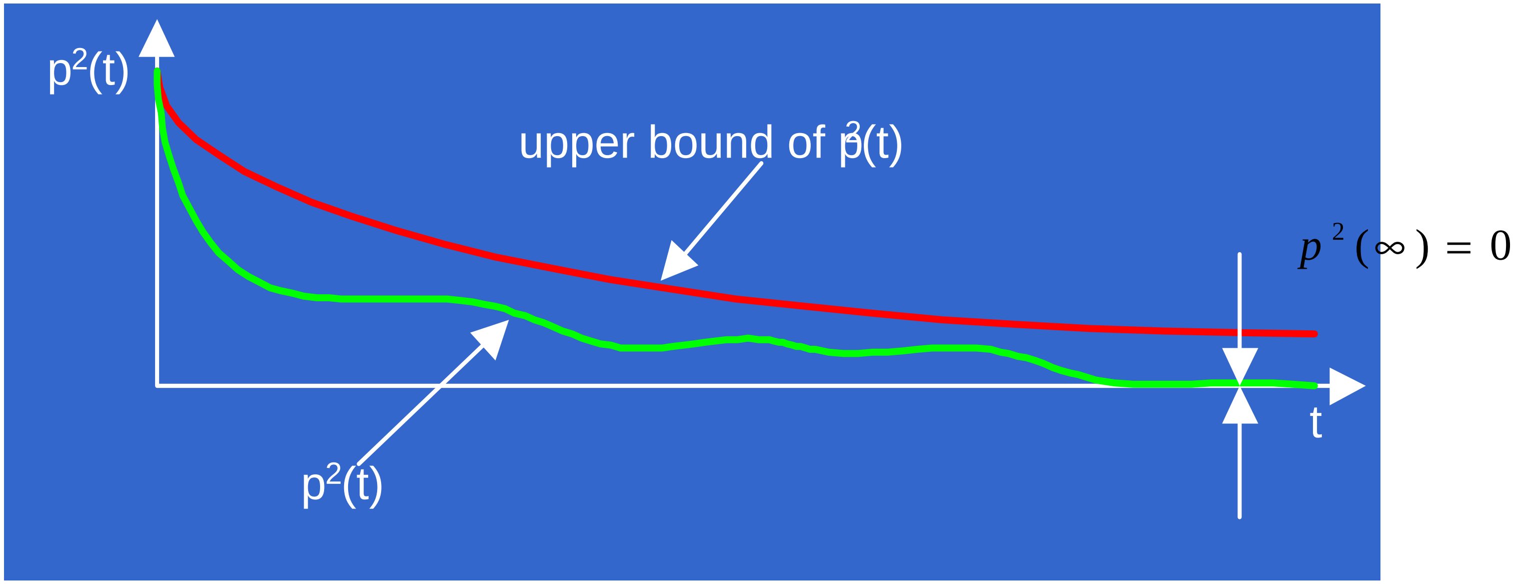
where $\lambda = 2k_2 / \theta_{1\max}$



$$p^2(\infty) \le \frac{2\varepsilon}{\lambda M}$$

Performance of ARC (cont'd)

- In addition, zero final tracking error is achieved in the presence of parametric uncertainties only (i.e., $\tilde{d} = 0, \forall t \geq t_0$)



Implementation Issues

- Regressor φ has to be calculated on-line based on the actual measurement of the velocity x_2
  - The effect of velocity measurement noise may be severe
  - Slow adaptation rate has to be used

- Model compensation u_a depends on the actual feedback of the state
  - Creates certain interactions between the model compensation u_a and the robust control u_s
  - Complicates the controller gain tuning process

Desired Compensation ARC (DCARC)

- DCARC control law

$$u = u_a + u_s, \qquad u_a = -\varphi_d^{\,T} \hat{\theta}$$

$$\tau = \varphi_d\, p$$

where

$$\varphi_d^{\,T} = [-\ddot{y}_d, -\dot{y}_d, -S_f(\dot{y}_d), -S_r(y_d), 1]$$

- Error dynamics

$$M\dot{p} = u_s - \varphi_d^{\,T}\tilde{\theta} + \underbrace{(\theta_1 k_1 - \theta_2)\dot{e} + \theta_3[S_f(\dot{y}_d) - S_f(x_2)] + \theta_{4b}^T[S_r(y_d) - S_r(x_1)]}_{addition\quad terms} + \tilde{d}$$

- Notice that (applying Mean Value Theorem)

$$S_f(\dot{y}_d) - S_f(x_2) = g_f(x_2, t)\dot{e}, \quad S_r(x_1) - S_r(\dot{y}_d) = g_r(x_2, t)e$$

where $g_f(x_2, t)$ and $g_r(x_2, t)$ are certain nonlinear functions

DCARC Controller Design (cont'd)

- Robust control function

$$u_s = u_{s1} + u_{s2}, \qquad u_{s1} = -k_{s1} p$$

where k_{s1} is a nonlinear gain such that the matrix A defined below is p.d.

$$A = \begin{bmatrix} k_{s1} - k_2 - \theta_1 k_1 + \theta_2 + \theta_3 g_f & -\frac{1}{2}(k_1\theta_2 + k_1\theta_3 g_f - \theta_{4b}^T g_r) \\ -\frac{1}{2}(k_1\theta_2 + k_1\theta_3 g_f - \theta_{4b}^T g_r) & \frac{1}{2} M k_1^3 \end{bmatrix}$$

For example

$$k_{s1} \ge k_2 + \theta_1 k_1 - \theta_2 - \theta_3 g_f + \frac{1}{2\theta_1 k_1^3}(\theta_2 k_1 + \theta_3 k_1 g_f + \left|\theta_{4b}^T g_r\right|)^2$$

- Choose u_{s2} such that

$$i \qquad p\left(u_{s2} - \varphi_d{}^T \tilde{\theta} + \tilde{d}\right) \le \varepsilon$$

$$ii \qquad p u_{s2} \le 0$$

Performance of DCARC

If the DCARC law is applied, then

- In general, all signal are bounded. Furthermore, the positive definite function V_s defined by
$$V_s = \frac{1}{2} Mp^2 + \frac{1}{2} Mk_1^2 e^2$$
is bounded by
$$V_s \le \exp(-\lambda t) V_s(0) + \frac{\varepsilon}{\lambda}[1 - \exp(-\lambda t)]$$
where $\lambda = \min\{2k_2/\theta_{1\max}, k_1\}$

- In addition, zero final tracking error (i.e., asymptotic tracking) is achieved in the presence of parametric uncertainties only

Implementation of DCARC Adaptation Law

- Digital implementation of the adaptation law

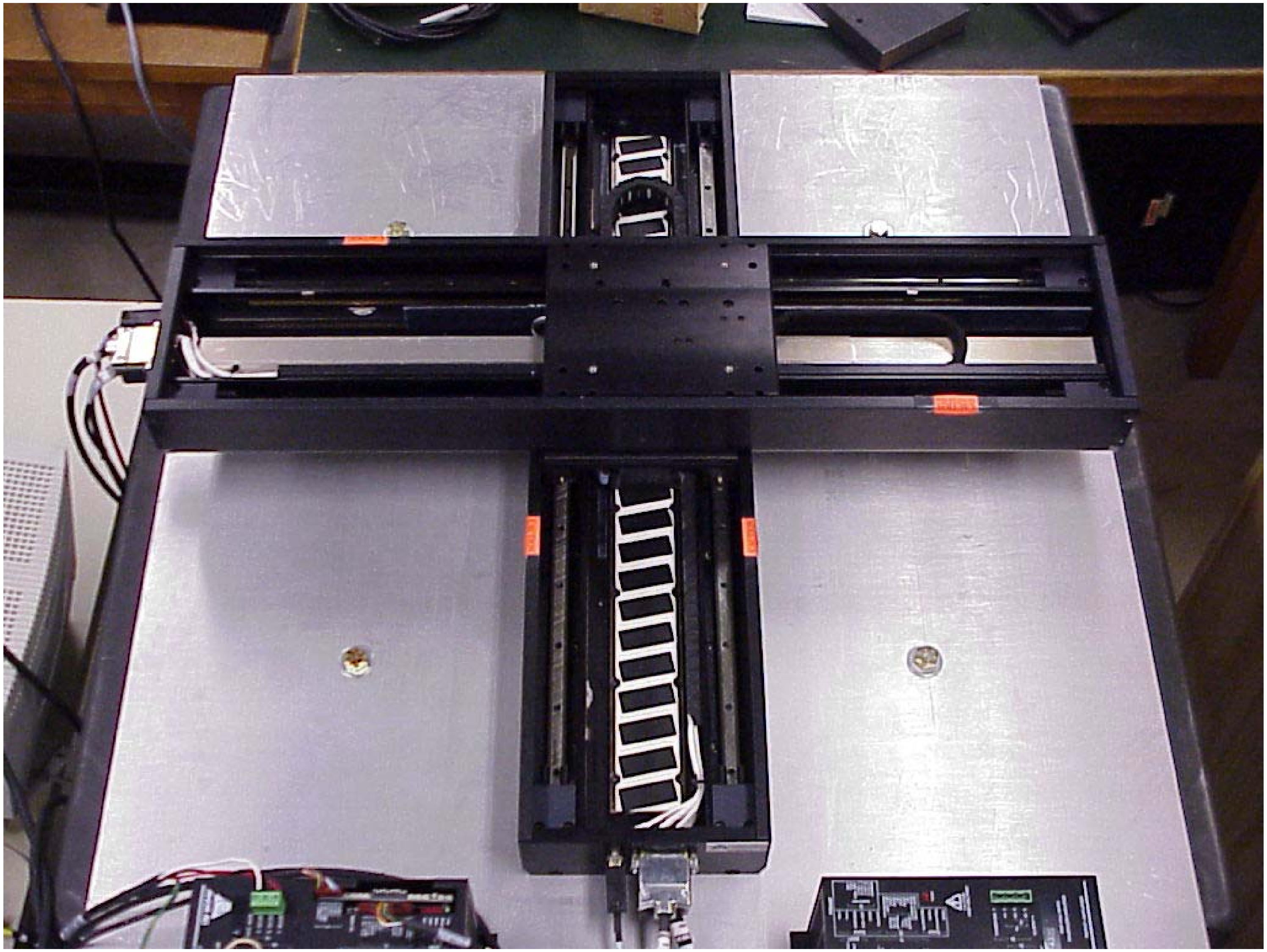
$$\hat{\theta}_i[(j+1)\Delta T] = \begin{cases} \theta_{i\min} & \text{if} \quad \hat{\theta}_{\mathrm{i}} = \hat{\theta}_{\mathrm{imax}} \quad \text{and} \quad \bullet > 0 \\ \theta_{i\max} & \text{if} \quad \hat{\theta}_{\mathrm{i}} = \hat{\theta}_{\mathrm{imin}} \quad \text{and} \quad \bullet < 0 \\ \Theta_i & \text{otherwise} \end{cases}$$

where

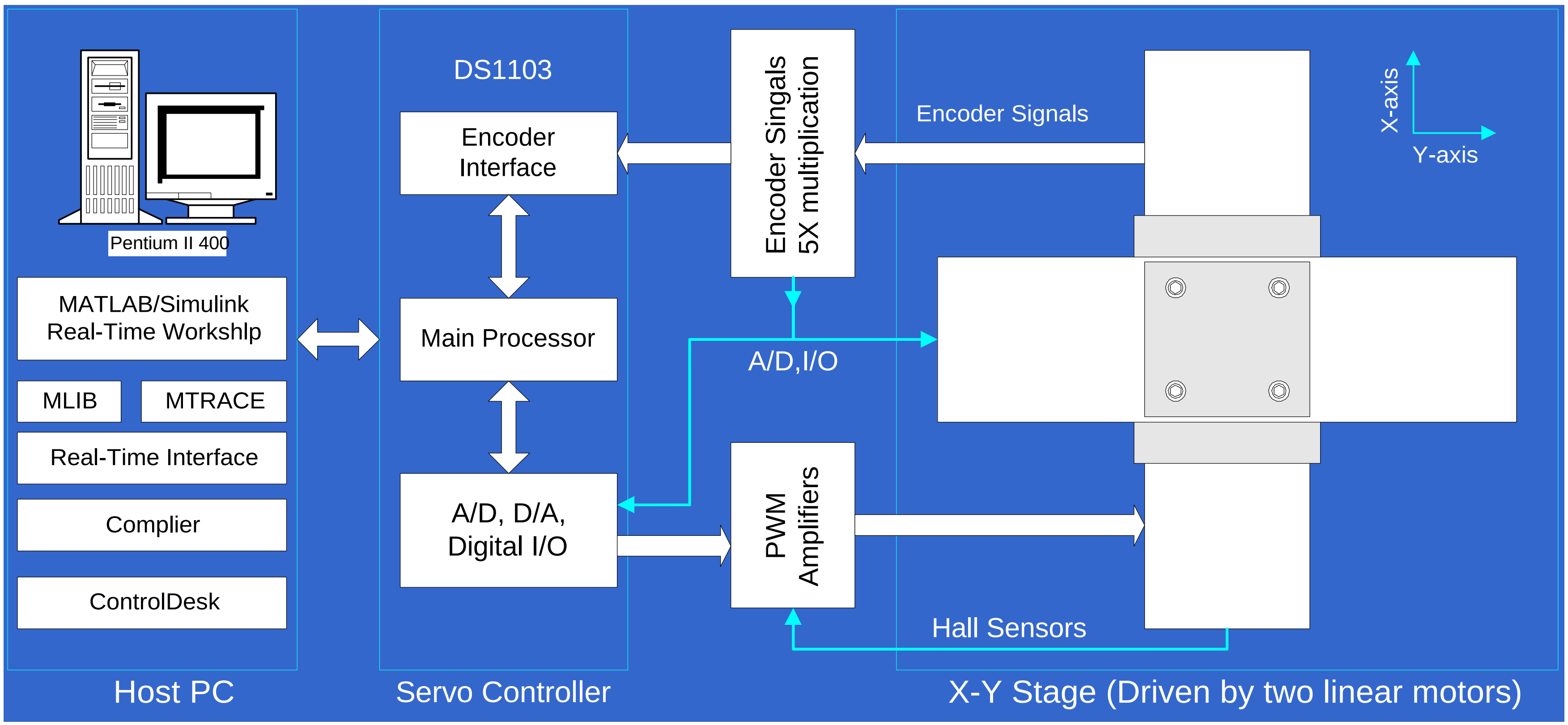
$$\Theta_i = \hat{\theta}_i(j\Delta T) + \gamma_i \int_{j\Delta T}^{(j+1)\Delta T} \varphi_{d,i}(\dot{e} + k_1 e)dt$$

- Velocity-free implementation of adaptation law

$$\Theta_i = \hat{\theta}_i(j\Delta T) + \gamma_i \left(k_1 \int_{j\Delta T}^{(j+1)\Delta T} \varphi_{d,i} e dt + \dot{\varphi}_{d,i} e \Big|_{j\Delta T}^{(j+1)\Delta T} - \int_{j\Delta T}^{(j+1)\Delta T} \dot{\varphi}_{d,i}(t) e dt \right)$$



X-Y Linear Motor Driven Positioning Stage



Experimental Setup

Performance Indexes

- **Transient Performance:** $e_M = \max_t \{|e(t)|\}$
- **Final Tracking Accuracy:** $e_F = \max_{t \in [T_f - 2, T_f]} \{|e(t)|\}$
- **Average Tracking Performance:** $L_2[e] = \sqrt{\frac{1}{T_f} \int_0^{T_f} |e|^2 \, dt}$
- **Average Control Input:** $L_2[u] = \sqrt{\frac{1}{T_f} \int_0^{T_f} |u(t)|^2 \, dt}$
- **Degree of Control Chattering:**

$$c_u = \sum \frac{L_2[\Delta u]}{L_2[u]}, \qquad L_2[\Delta u] = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \left| u(j\Delta T) - u((j-1)\Delta T) \right|^2}$$

Comparative Experimental Results (Y-axis)

- PID with model compensation:

$$u = \hat{\theta}_1(0)\ddot{y}_d(t) + \hat{\theta}_2(0)\dot{y}_d(t) + \hat{\theta}_3(0)S_f(\dot{y}) - K_p e - K_i\int edt - K_d\dot{e}$$

$$S_f(\dot{y}) = \frac{2}{\pi}\arctan(900\dot{y}), \quad K_p = 5.4\times10^3, \quad K_i = 5.4\times10^5, \quad K_d = 18$$

- ARC:

$$k_1 = 400, \quad k_2 = 32, \quad \hat{\theta}(0) = [0.05, 0.25, 0.1, 0]^T$$

$$\Gamma = diag[5,0,2,1000]$$

- DRC:
  - Same controller law with ARC but without parameter adaptation

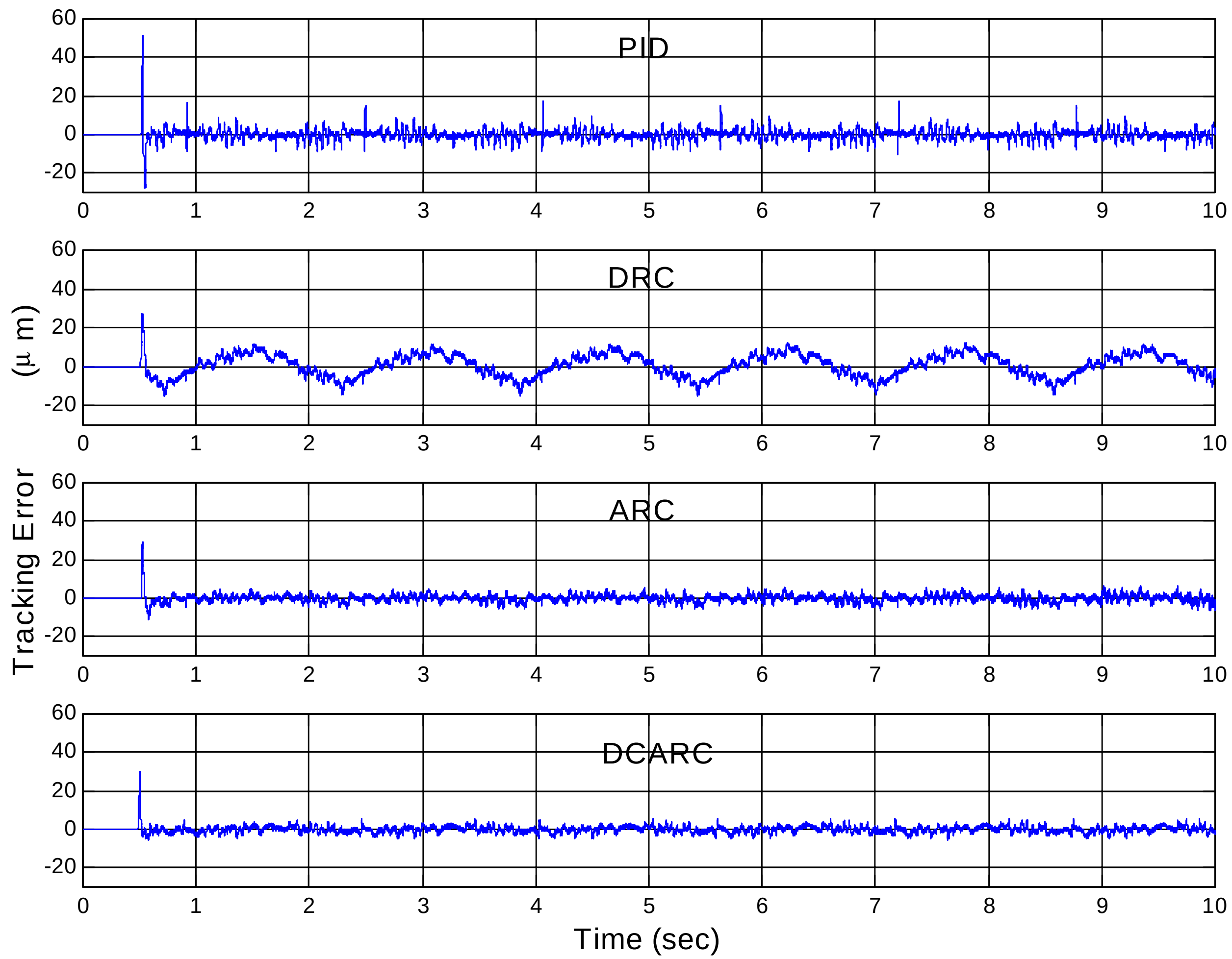
- DCARC:

$$k_1 = 400, \quad k_2 = 32, \quad \hat{\theta}(0) = [0.05, 0.25, 0.1, 0]^T$$

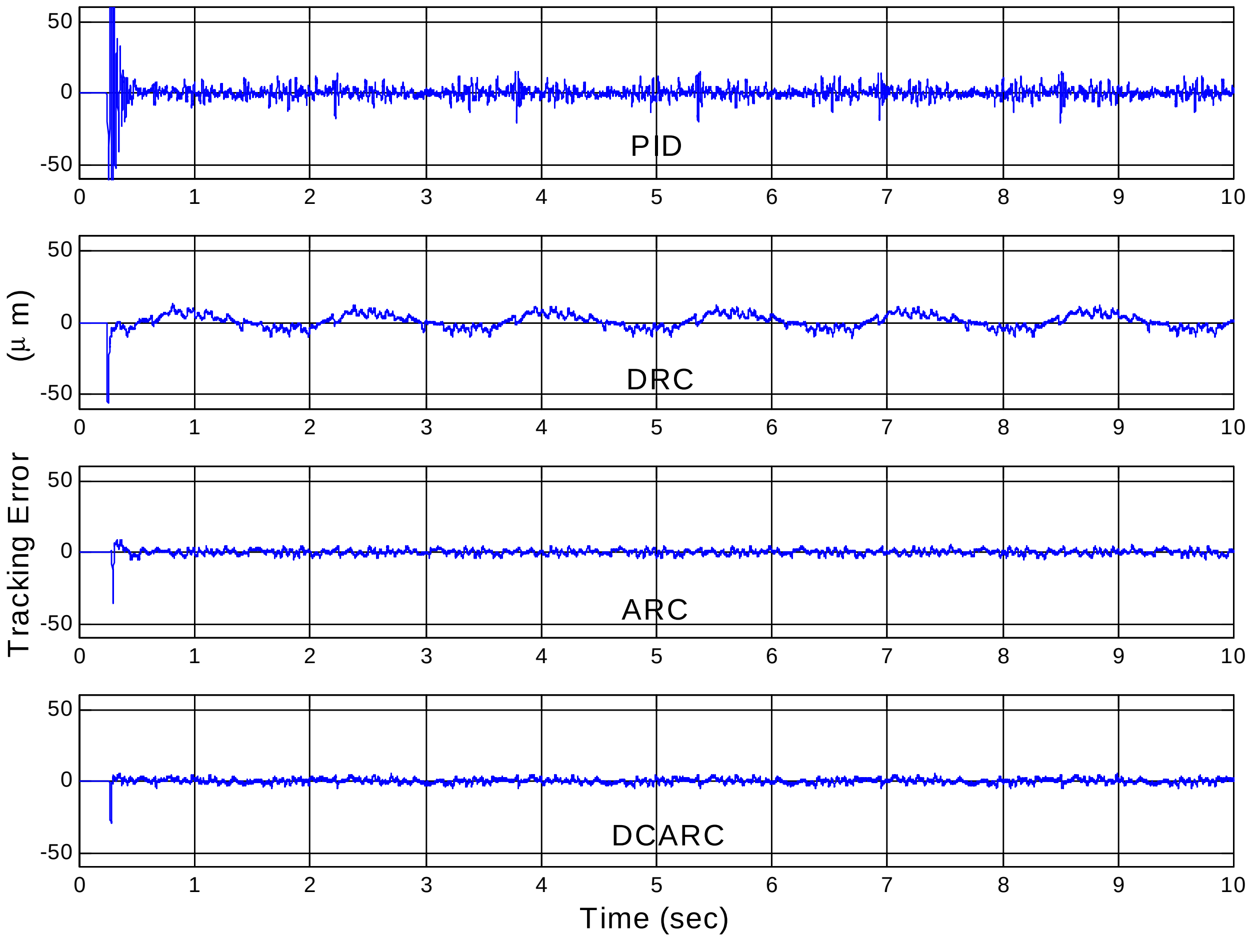
$$\Gamma = diag[25,0,5,1000]$$

Tracking Sinusoidal Trajectory

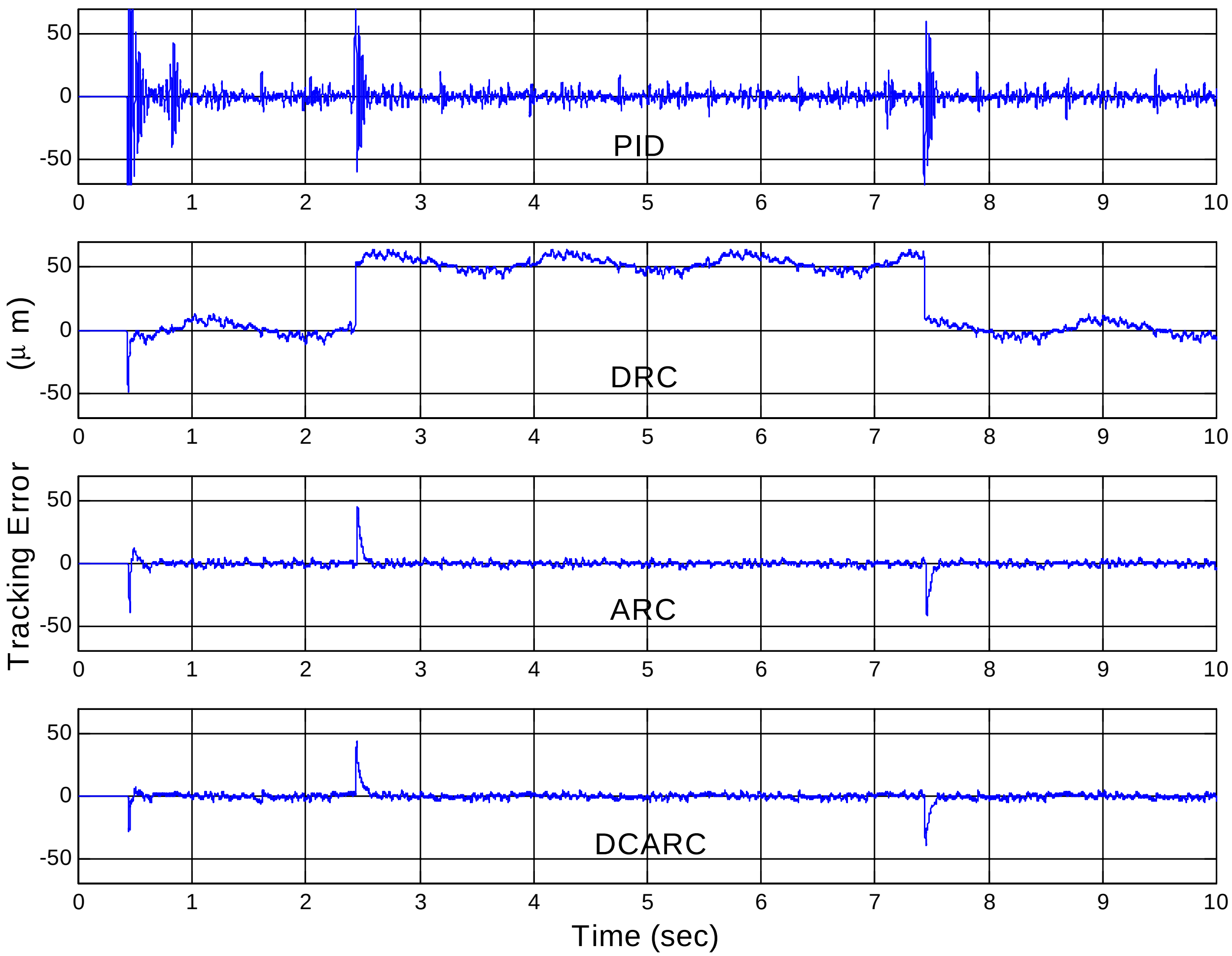
- Set 1: To test the nominal tracking performance, the motors are run without payload
- Set 2: To test the performance robustness of the algorithms to parameter variations, a 20lb payload is mounted on the motor
- Set 3: A large step disturbance (a simulated 0.5V electrical signal) is added at about t=2.5s and removed at t=7.5s to test the performance robustness of each controller to disturbance



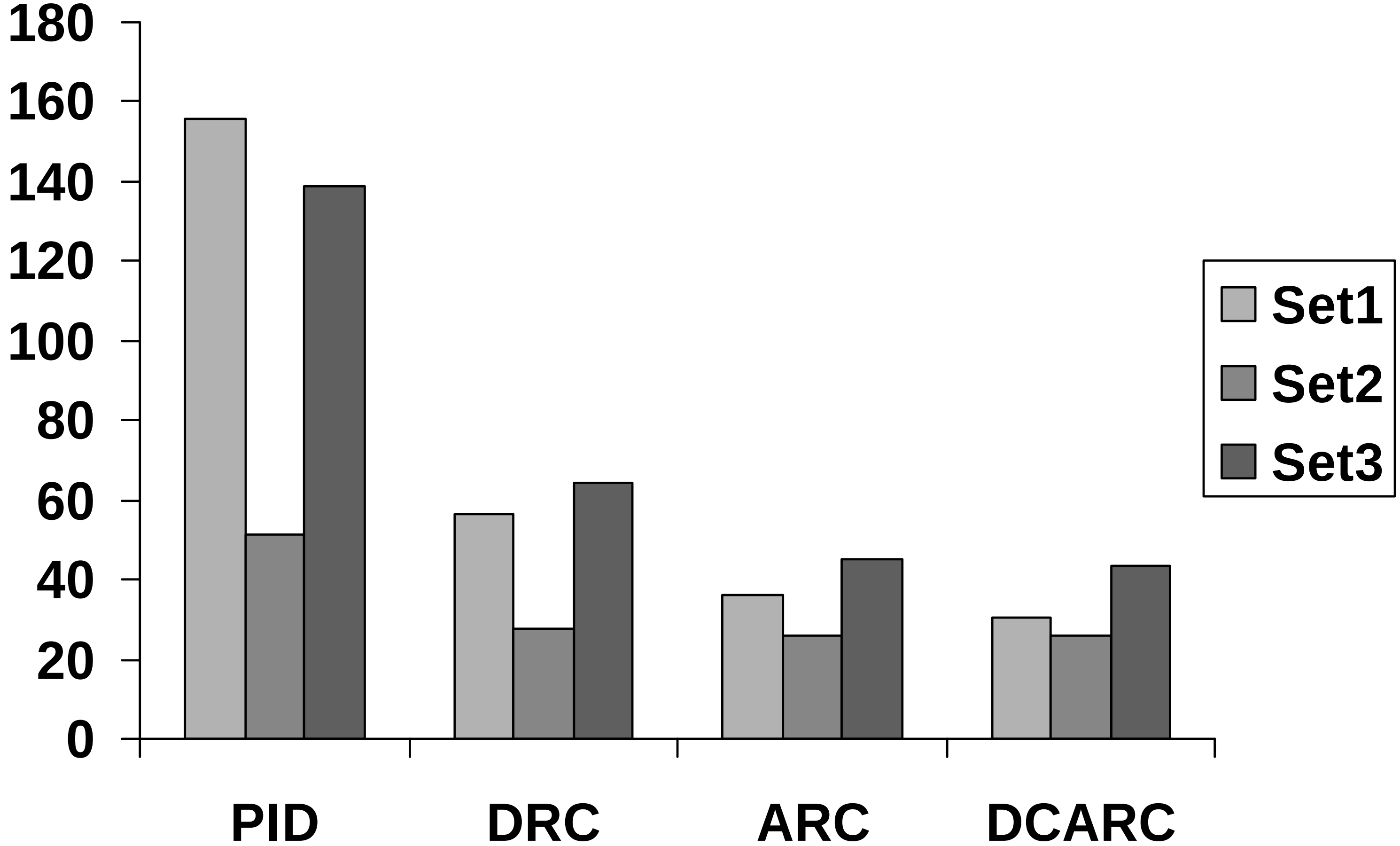
Tracking errors for yr=0.05sin(4t) without load (Y-axis)



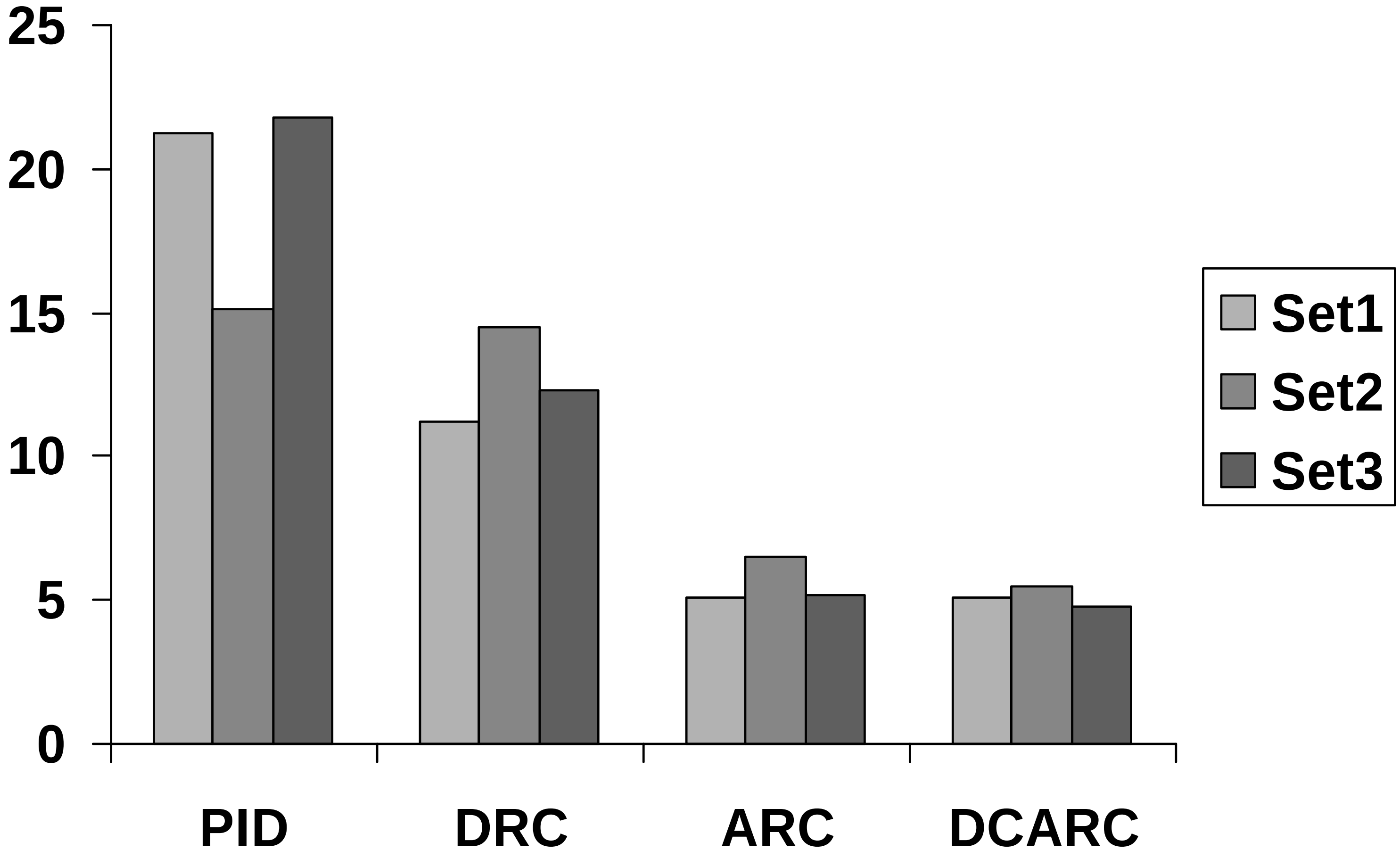
Tracking errors for yr=0.05sin(4t) with 20lb load (Y-axis)



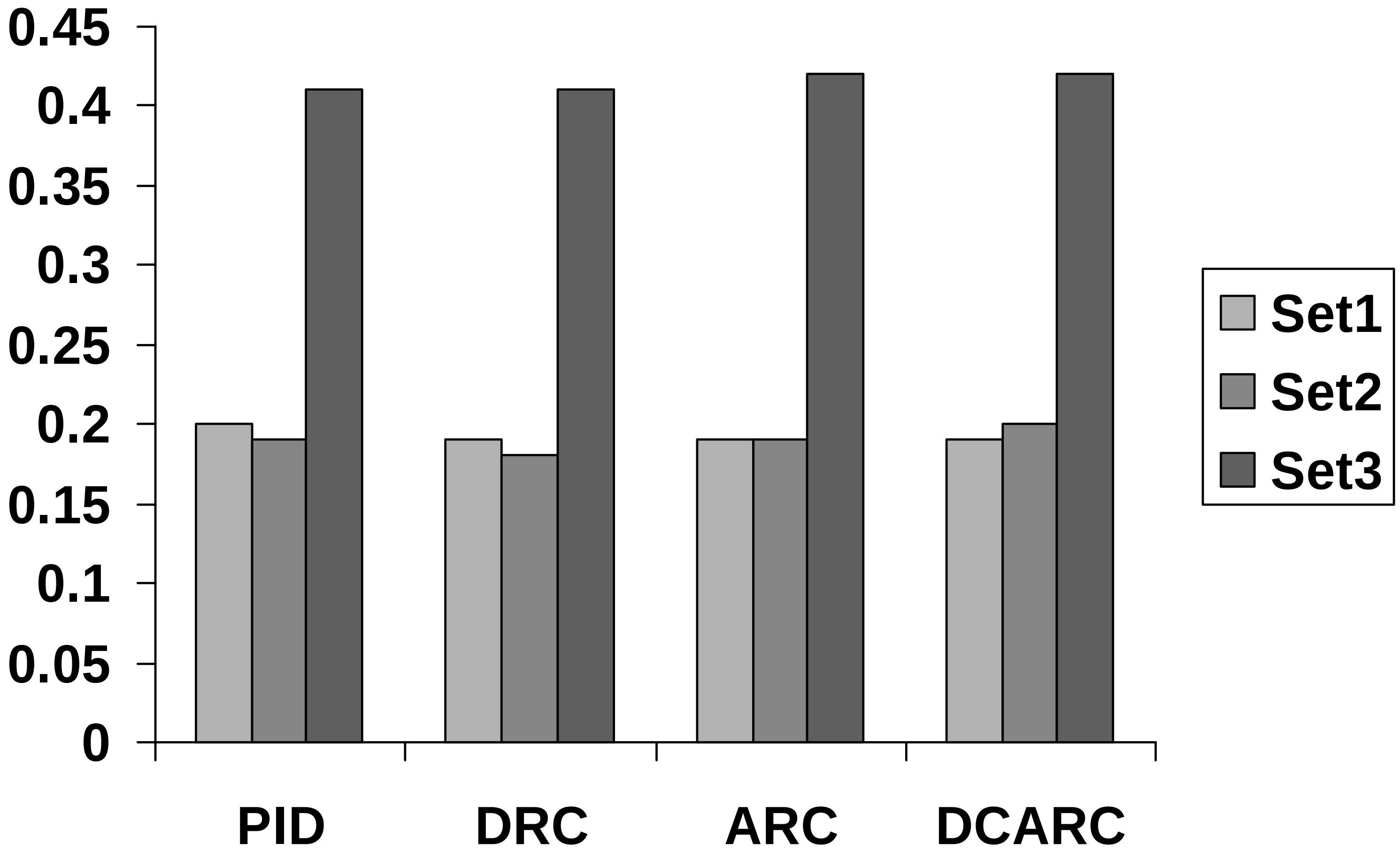
Tracking errors for yr=0.05sin(4t) with disturbance (Y-axis)



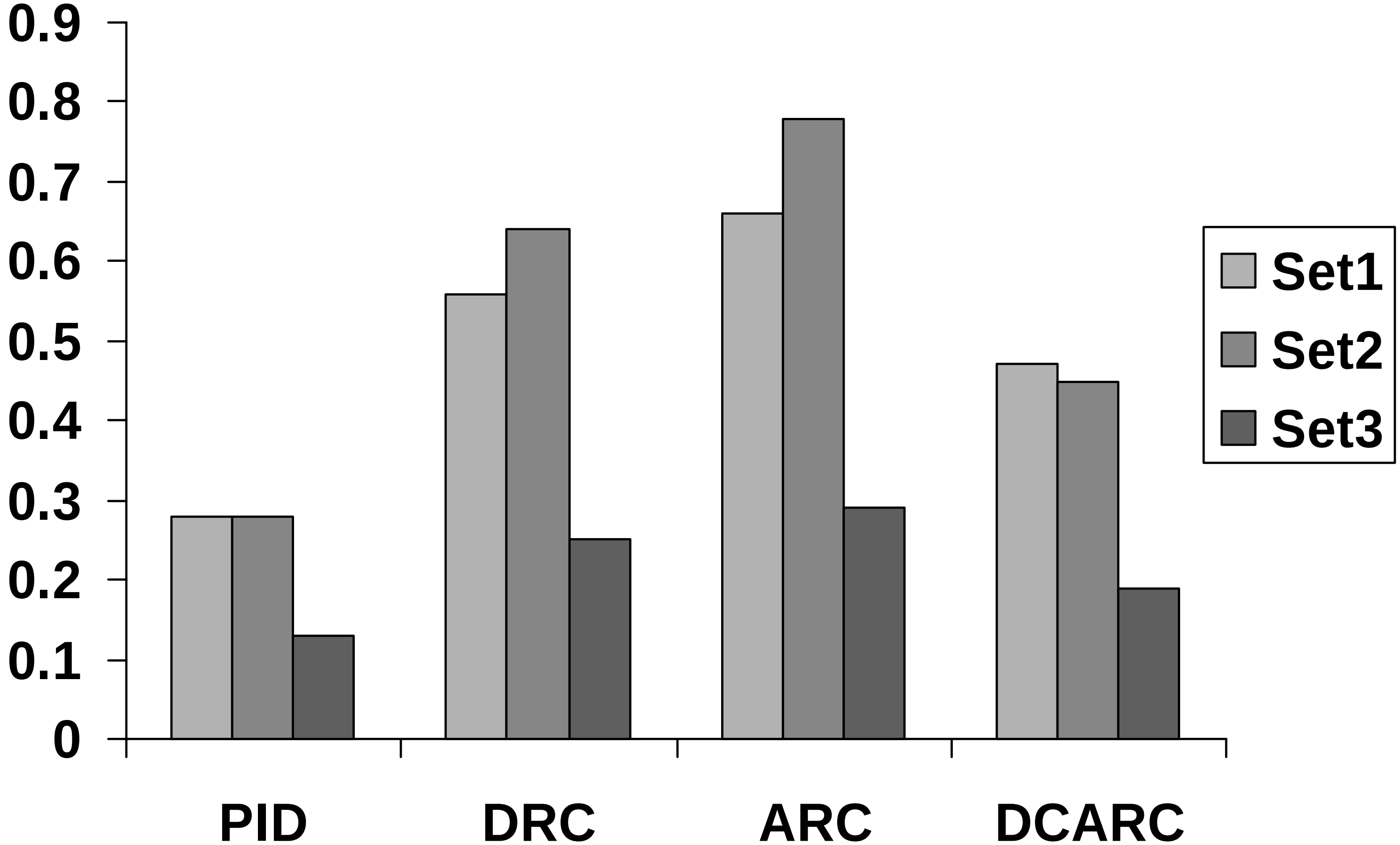
Transient Performance e_M (Y-axis)



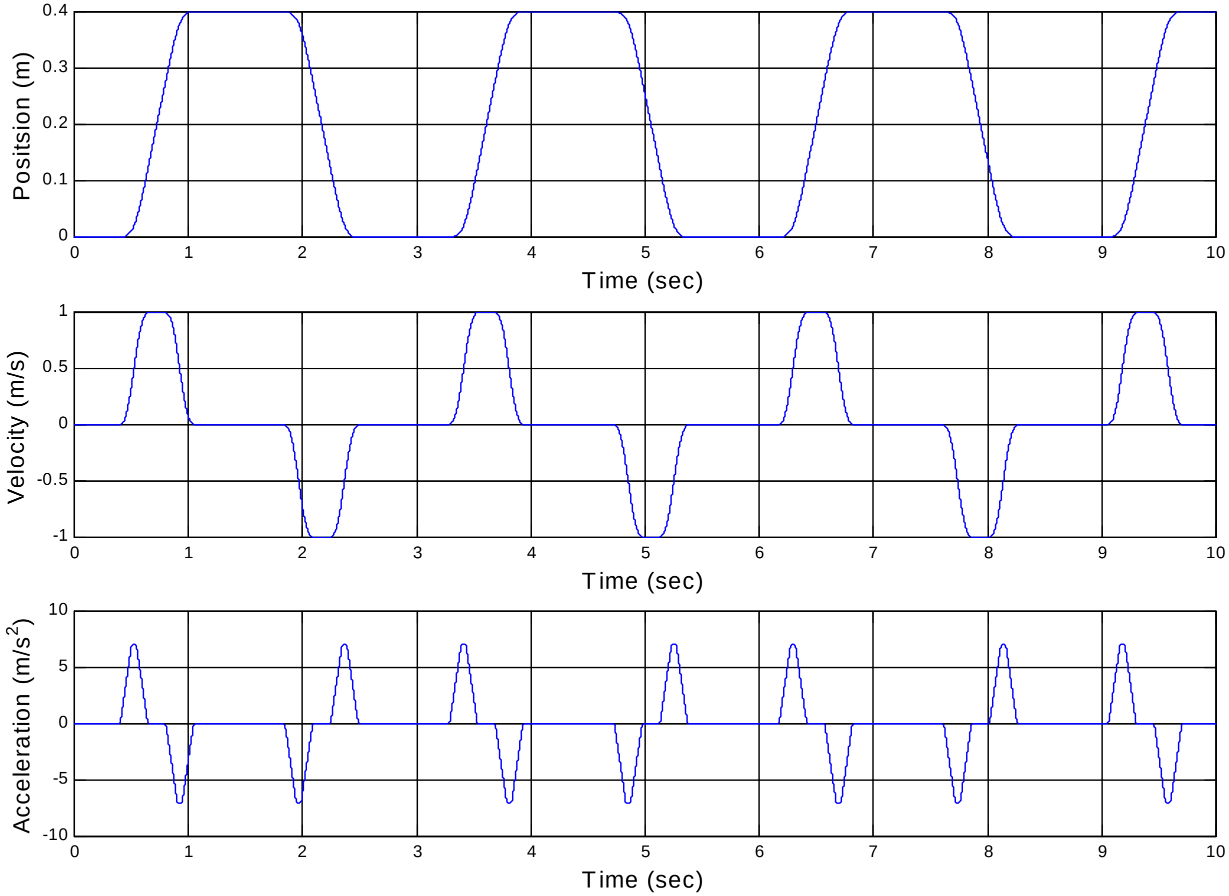
Final Tracking Accuracy e_F (Y-axis)



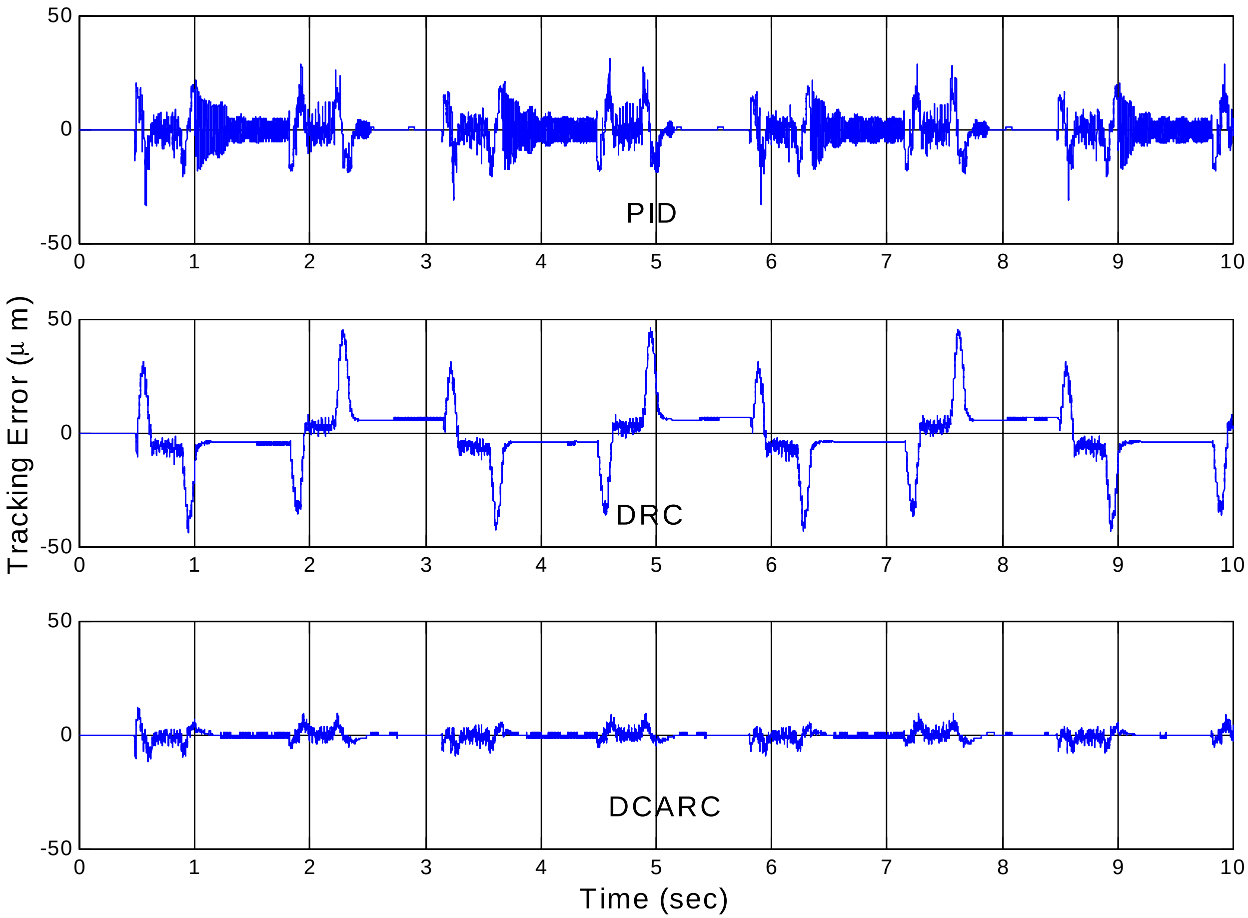
Control Effort L2[u] (Y-axis)



Degree of Control Chattering Cu (Y-axis)



Point-to-Point Motion Trajectory



Tracking errors for point-to-point trajectory (Y-axis)

Conclusion

- **Unique Features of Nonlinear Adaptive Robust Control Strategy**:

  - Nonlinear *physical model* based; easy to incorporate physical intuition into the controller design stage.
  - Address *nonlinearities* associated with linear motor dynamics directly.
  - Address *parameter variations* due to change of load, nominal value of disturbances, …
  - Effectively handle the effect of *hard-to-model terms* (e.g., uncompensated friction)
  - Achieve *high performance* through the use of learning techniques such as on-line parameter estimation.

Conclusion (cont'd)

- **Comparative Experimental Results**
  - Illustrate the above claims
  - The proposed schemes outperform a PID controller with model compensation significantly in terms of output tracking accuracy
  - Tracking errors for high-speed/high-acceleration movements are very small, even during transient period. Final tracking errors are mostly within measurement resolution level of $1\mu m$

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Questions ?

More Information can be downloaded from:

http://widget.ecn.purdue.edu/~byao